

Costs Function Optimization For Prevention Of Firm's Industrial Risks With Regard To Reinvestment Of Profit

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Abstract: We consider the problem of the firm's cost function optimization to prevent the industrial risks (i.e., the voluntary risk costs), by the criterion of the total costs with regard to a reinvestment of the firm's profit. In the paper the voluntary risk costs function of the firm in the perfect competition by the total costs criterion is optimized. The rational strategy of the firm's risk management consists in a gradual increase of the costs to prevent the risk situations with the production volume growth. The voluntary risk costs function is determined for the power function of the production costs and the exponential decreasing function of the industrial damage for the firm in the perfect competition.

Keywords: industrial risk, industrial damage, costs, optimization, risk management.

1. INTRODUCTION

The industrial risks are treated as the risks, arising in the commercial activity of the industrial firms. These risks were comprehensively considered in the economic theory. The problems of the industrial risks management cover various levels of the risks: first, the risks, connected with firm's entry into the world market (i.e., risk of globalization) [13], second, the risks of commercial activity's impact on the regional economic systems [20], [22], [26], and the risks, caused by the non-performance of some technological operations. In the context of the further analysis, the risks are considered in the firm's level. In this aspect, the traditional reactive environmental management and the active enterprise's risk management (ERM) [30] was compared as the behaviors of the firm's management in the conditions of an uncertainty. The effects of the equipment maintenance frequency [6] and the organizational environment in industrial damage were analyzed: the risk analysis was conducted in the framework of the Unified Modeling Language (UML) models [5], the Analysis Method of Dysfunctional Systems (MADS), and then the risk management is simulated to ensure the safety of personnel from the possible industrial hazards. The effectiveness of the ERM system was investigated for nine firms from various industries [2]. The risks of the goods and resources supply breaks chain were investigated as part of the technological process within the firms [29], and in the external supply chain [19]. Researches of the decision-making models under conditions of uncertainty [21] proved an applicability of the system approach to the complex challenges modeling of the industrial equipment maintenance imperfection on the basis of the UML model and the Method Organized for a Systemic Analysis of Risk (MOSAR) [12]. Additionally, authors investigated the regional level risks [26]. For example, Shelkov A. B. investigated the problems of the regional security [16], [24], [15] by means of the scenario approach. The risks in the firms were analyzed taking into account a human factor [1] by means of the business games [7], the mechanisms of penalties [9], and on the basis of the innovations [3], [11].

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In general, it was shown [30], [4] that the production activity in any economy sector is connected with the uncertainty and the risk. In contrast to the aforementioned authors, we consider the problem of the firm's total costs minimization. The solution of the problem enables to define the voluntary risk costs function. Additionally, we determine the production volume, when the firm's profit is maximum for derived voluntary risk costs function with regard to a reinvestment of the firm's profit.

Further the industrial risk is treated [28], [25], [17] as a certain technogenic factor of the probabilistic nature, resulting in a loss of the firm's assets. Consequently, the industrial risk leads to the additional firm's expenses, but does not lead to the revenue decrease. The risk measure is the mathematical expectation of the damage from risk factors (e.g., accident, incident, equipment stopping, production failure, etc.), measured in the monetary units of the firm's costs; the degree of risk represents the probability of this event.

The risk costs are divided into two components [8]:

- 1) the unexpected expenses (e.g., the penalties for an exceeding the extent of environmental damage over existing norms);
- 2) the planned expenses, caused by a necessity of the risk situation prevention or a decrease in the risk emergence probability.

We divide the risk damage in the internal damage and the external damage in relation to the firm. The internal damage includes assets loss and, additionally, the expenses for a mitigation of the consequences and recovery works. The size of the possible internal damage is proportional to the firm's output, which growth increases the production capacity of the firm (i.e., production facilities, equipment, personnel). Consequently, the potentially internal damage is affected an assets increases. Therefore, the reduction of the possible internal damage can be reached by the additional expenses on the actions for the risk decrease, which we call further the voluntary risk costs (VRC). The structure of the VRC includes the costs of the following types: the training personnel, the advanced equipment installation, the sewage treatment plants installation, the control systems and alarms, etc. The firm's external damage (called further the obligatory risk costs) we accept equal to a sum of the penalties, imposed on the firm for an excess of the environmental damage level over the established norms. Thus, the total costs of the industrial firm, among with the production costs, additionally include the VRC, the obligatory risk costs, and the internal damage. Further, the model of the firm without regard to the obligatory risk costs is considered.

The choice of the VRC size is predetermined by the following contradictory factors. On the one hand, a reduction of these costs leads to the firm's total costs reduction that contributes to the profit's growth and further expansion of the production; however, at the same time, an increasing in the internal damage and the obligatory risk costs, on the contrary, increases the total costs. On the other hand, an increase in the VRC leads to a decrease in the internal damage and the obligatory risk costs, however, the total costs of the firm increase. Additionally, a reinvestment of the firm's profit causes contradictory influence on the potential costs, revenue and assets in the future periods: the using of the current profit for the risk prevention measures leads to the further production decreasing, and, to an increasing in the further VRC and the obligatory risk costs. Consequently, a reinvestment of the firm's profit influences on the firm's VRC function in the current period, because the optimal VRC function is derived, taking into account the both tendencies.

Thus, the relevant problem of the industrial risks management is the choice of firm's VRC function, which is optimized by criterion of the total costs. Additionally, the problem of the firm under the VRC optimal function consists in optimization of the output by criterion of the profit. The problems represent the model of the firm's management taking into account the industrial risks. This problems is the subject of the paper.

2. METHODOLOGICAL FRAMEWORK

To search for the optimal VRC function, the firm's total costs are decomposed into the production costs, the internal damage costs, and the VRC. In contrast to the traditional cost accounting methodology [10], according to which the last two components are included in the production costs, we consider the risk costs separately.

The problem of the determining a nonnegative, real, limited from above function VRC is considered, taking into account the minimization of the firm's total costs function:

$$f^*(Q) = \arg \min_{f(\bullet) \in A_f} C_\Sigma(Q, f(\bullet)), \quad (2.1)$$

$$A_f = \{f(\bullet) \in R^+ : f(\bullet) \leq f^{\max}, f^{\max} \in (0, C_\Sigma^{\max})\},$$

where Q is the output, $C(Q)$ is the production costs function, $f(Q)$ is the VRC function, f^{\max} is the VRC maximum possible amount. The symbol «*» indicates the optimal value.

The firm's total costs function $C_\Sigma(Q, f)$ is considered in the two types. First, the total costs in the current period ($t=0$), indicated by $C_{0\Sigma}(\bullet)$, is considered as a function of the following type:

$$C_{0\Sigma}(Q, f) = C_0(Q) + f_0(Q) + X_0(Q, f), C_{0\Sigma}(\bullet) \leq C_\Sigma^{\max}, \quad (2.2)$$

where $X(Q, f)$ is the internal damage function, C^{\max} is the maximum possible total costs of the period on the basis of the firm's manufacturing capacity. Second, the cumulative costs of the future periods is considered as a sum of the costs $C_\Sigma^\tau(\bullet)$ up to the period τ , which is discounted to the current period:

$$C_{\tau\Sigma}(Q, f) = \sum_{t=1}^{\tau} \frac{C_{t\Sigma}}{(1+r)^t}, \quad (2.3)$$

$$C_{t\Sigma}(Q, f) = C_t(Q) + f_t(Q) + X_t(Q, f), C_{t\Sigma}(\bullet) \leq C_\Sigma^{\max},$$

where r is the discount rate, t is the index of the period.

We formulate the firm's main objective as the problem of the output choice under the following conditions: first, the output is limited from above, second, it maximizes the profit, and, third, the VRC function is optimal according to (2.1). Thus, the problem is as follows:

$$Q_\tau^* = \arg \max_{Q_\tau \in A_Q} \Pi_\tau(Q_\tau, f_\tau^*(Q)), \quad (2.4)$$

$$A_Q = \{Q_\tau \in R^+ : Q_\tau \leq Q^{\max}, Q^{\max} > 0\},$$

$$\Pi_\tau(Q, f) = R_\tau - C_{\tau\Sigma}(Q, f^*), \quad (2.5)$$

$$R_\tau(Q) = pQ_\tau. \quad (2.6)$$

where Π_τ , R_τ are the profit and the revenue in the period τ , respectively, p is the product price, Q^{\max} is the maximum possible manufacturing output taking into account the production capacity.

The profit function (2.5) doesn't include the increment from prevention of industrial risk. Because, according to the accepted in the article concept, the risk costs reduce in the internal damage from the industrial risk, but it does not affect the revenue growth.

We introduce the following assumptions that determine the limits of the model (2.1), (2.3) applicability on the basis of the generally accepted provisions of the economic theory.

1. The hypothesis of the perfect competition in the goods market [28]: for the firm the product price is an exogenous variable, that is, the firm does not affect the market price:

$$p'_Q(Q) = 0.$$

2. The hypothesis of the return from production increased scale, which corresponds to a relatively large firm [29]:

$$C''_Q(Q) > 0.$$

3. The hypothesis of the control parameters influence on the internal damage [10]: an increase in the volume of the production assets leads to an increasing in the damage; the internal damage is reduced with an increase in the VRC, the internal damage is limited from above due to the nature of technology and limited by the production volume:

$$X'_Q(Q, f) > 0, X'_f(Q, f) < 0, X(Q, f) \in [0, X^{\max}], X^{\max} > 0,$$

where X^{\max} is the maximum possible internal damage.

4. The hypothesis of the reinvested profit influence [18]: the profit in the current period is invested only in the fixed capital, that is, the reinvested profits affects the intensity (flow) of the capital, but does not affect the flow of other resources in the future periods. The costs are covered by the revenue of the corresponding period. Therefore, the production function of the firm in the t -th period is:

$$Q_t = [K_t(f, X)]^\alpha L_t^\beta,$$

where $K_t(\bullet)$, L_t are the capital flow and non-capital resources flow in the t -th period, α , β are the elasticity coefficients of resources. Here the Cobb-Duglas's function (i.e., the constant elasticity production function) is considered, but the further results are not proved for the general type of the constant elasticity function.

The production costs function and the damage function, satisfying the hypotheses 2, 3, have the forms [10], [32]

$$C_t(Q) = BQ_t^\gamma, \gamma \in (1, \gamma^{\max}], \gamma^{\max} \in (1, 2], B > 0, \quad (2.7)$$

$$X_t(Q, f) = \chi_t(Q)e^{-\xi f}, \xi \in (0, \xi^{\max}], \xi^{\max} \in (0, 1], \chi_t(Q)'_Q \geq 0. \quad (2.8)$$

The power-type production cost function (2.7) [10], [32] is introduced for a long-term period, because it includes only variable costs. The damage function (2.8) expresses the exponential distribution of damage [14], which corresponds to man-made accidents. Therefore, the solution of the problems (2.1), (2.3) with the functions (2.7), (2.8) is not general, but it corresponds to the typical firm's characteristics.

Under consideration the VRC and the industrial damage, in the t -th period, the capital flow is equal to the initial capital K_0 , which is reduced by the amount of the not reinvested profit; the reduction in the profit in accordance with (2.1), (2.5) is equal to the amount of the VRC and the industrial damage:

$$K_t = K_0 + \sum_{i=1}^{t-1} \frac{\pi_i}{(1+r)^i} - \sum_{i=1}^{t-1} \frac{f_i + X_i}{(1+r)^i}, \quad (2.9)$$

where π_i is the potential profit of the company in the i -th period, which is calculated according to the model (2.5) without taking into account the risk costs, that is, when $f_i(Q_i) = 0, X_i(Q_i, f_i) = 0$.

In contrast to the classical model [10] of the firm's capital growth, the model of the capital flow (2.9) describes the process of capital growth as the annual returns, and the process of the capital reduction due to the influence of the damage and the VRC, which enables to determine the risk costs.

Taking into account hypothesis 4, the production function in the t -th period is:

$$Q_t = \left(K_0 + \sum_{i=1}^{t-1} \frac{\pi_i}{(1+r)^i} - \sum_{i=1}^{t-1} \frac{f_i + X_i}{(1+r)^i} \right)^\alpha L_t^\beta. \quad (2.10)$$

Here the financial capital is considered, that is, the capital is equal to the share capital and the retained earnings.

Let's consider the optimal control problem: to search for the pair $\langle f^*(\cdot), Q^* \rangle$, which is optimized by the criteria (2.1), (2.3) on the respective admissible sets for the costs function and the damage function (2.6), (2.7), and taking into account the restrictions (2.10).

3. RESULTS

In the first stage, we define the VRC function $f^*(\cdot)$ under condition (2.1) in the following propositions.

Proposition 1. For the continuously differentiable functions $C(\cdot), f(\cdot)$, and the function $X(\cdot)$ of the type (2.8) the function

$$f^*(Q) = \frac{1}{\xi} \ln[\xi \chi(Q)] \quad (3.1)$$

is the solution of the problem (2.1) $\forall Q \in A_Q$ at $\xi \chi(Q) \geq 1$.

Proof. Let's solve the problem (2.1) by the Lagrange multipliers method. This method is applicable as the criterion function $C_{\Sigma}^0(Q, f)$ is continuously differentiable, the function $X(Q, f) - \chi(Q) e^{-\xi f} = 0$ is continuously differentiable with the partial derivatives, which are not equal to zero simultaneously (the equation $X(Q, f) - \chi(Q) e^{-\xi f} = 0$ defines a convex curve).

We write the Lagrange function:

$$L(Q, f, \lambda) = C(Q) + f + X(Q, f) + \lambda(X(Q, f) - \chi(Q) e^{-\xi f}). \quad (3.2)$$

We search for the partial derivatives, which are equal to zero:

$$\begin{cases} L'_X = 1 + \lambda = 0, \\ L'_f = 1 + \lambda \chi(Q) \xi e^{-\xi f} = 0, \\ L'_\lambda = X - \chi(Q) e^{-\xi f} = 0. \end{cases}$$

We solve the resulting system:

$$\begin{cases} \lambda = -1, \\ \chi(Q) \xi = e^{\xi f}, \\ X = \chi(Q) e^{-\xi f}, \end{cases}$$

and we derive the VRC function, subject to the total costs minimizing:

$$f^*(Q) = \frac{1}{\xi} \ln(\xi \chi(Q)).$$

We check for the Lagrange function (3.2) the fulfillment of the minimum sufficient conditions at $f=f^*$. For this purpose we define a sign of $d^2L|_{f=f^*}$:

$$\begin{aligned} d^2L &= L_{ff} df^2 + 2L_{fQ} df dQ + L_{QQ} dQ^2 = \chi(Q) \xi^2 e^{-\xi f} df^2. \\ d^2L|_{f=f^*} &= \xi df^2 > 0. \end{aligned}$$

This condition are fulfilled $\forall Q \in A_Q$. Then function $f^*(Q)$ is the solution of the problem (2.1) for $\min_{f \in A_f} C_{\Sigma}(Q, f)$ in the case of $\xi \chi(Q) \geq 1$. ■

Proposition 2. For the continuously differentiable functions $C_t(\cdot), f_t(\cdot), X_t(\cdot)$ and Q_t form (2.10) the function

$$f_t^*(Q) = \frac{1}{\xi} \ln[\xi \chi_t(Q)] \quad (3.3)$$

is the solution of the problem (2.1), if the costs functions are of the type (2.2) and (2.3) $\forall Q \in A_Q$ at $\xi \chi(Q) \geq 1$ and $\chi \chi'' + 2\chi \chi' - \chi'^2 > 0$.

Proof. Let's solve the problem (2.1), (2.2), (2.3) by Lagrange multipliers method. This method is applicable as the criterion function $C_X(Q, f)$ is continuously differentiable, the function (2.10) is continuously differentiable with partial derivatives that are not equal to zero simultaneously.

We write the Lagrange function:

$$L(Q, f, \lambda) = C_{\tau \Sigma}(Q, f) + \lambda(Q - K_t^\alpha L_t^\beta), \quad (3.4)$$

where $Q = (Q_1, Q_2, \dots, Q_\tau)$, $f = (f_1, f_2, \dots, f_\tau)$.

We transform the function (3.4):

$$L(Q, f, \lambda) = \sum_{t=1}^{\tau} \frac{BQ_t^\gamma + f_t + \chi_t(Q) e^{-\xi f_t}}{(1+r)^t} + \lambda(Q_t - K_t^\alpha L_t^\beta). \quad (3.4')$$

We search for the partial derivatives of the function (3.4), which are equal to zero:

$$\begin{cases} L'_{Q_t} = \sum_{t=1}^{\tau} \frac{B\gamma Q_t^{\gamma-1} + \chi_t(Q) \xi e^{-\xi f_t}}{(1+r)^t} + \lambda = 0, \\ L'_{f_t} = \sum_{t=1}^{\tau} \frac{1 - \chi_t(Q) \xi e^{-\xi f_t}}{(1+r)^t} = 0, \\ L'_\lambda = Q_t - K_t^\alpha L_t^\beta = 0. \end{cases}$$

We solve the resulting system, and transform the second equation of the system:

$$\sum_{t=1}^{\tau} \frac{1}{(1+r)^t} = \sum_{t=1}^{\tau} \frac{\chi_t(Q) \xi e^{-\xi f_t}}{(1+r)^t}.$$

We can write the last equality as:

$$\frac{1}{(1+r)^t} = \frac{\chi_t(Q) \xi e^{-\xi f_t}}{(1+r)^t} \quad \forall t = \overline{1, \tau}$$

or

$$\chi_t(Q) \xi e^{-\xi f_t} = 1 \quad \forall t = \overline{1, \tau}.$$

Here we derive the VRC function, subject to the total costs minimizing in t -th period:

$$f_t^* = f^*(Q_t) = \frac{1}{\xi} \ln(\xi \chi_t(Q)).$$

We will check for the Lagrange function (3.4') fulfillment of the minimum sufficient conditions at $f=f^*$. For this purpose we define a sign of $d^2L|_{f=f^*}$.

$$\begin{aligned} d^2L &= (L''_{QQ} dQ^2 + L''_{ff} df^2 + 2L''_{fQ} df dQ) |_{f=f^*} = \\ &= \sum_{t=1}^{\tau} \frac{B\gamma(\gamma-1)Q_t^{\gamma-2} + \chi_t'' e^{-\xi f_t}}{(1+r)^t} dQ^2 + 2 \sum_{t=1}^{\tau} \frac{1 - \xi \chi_t' e^{-\xi f_t}}{(1+r)^t} dQ df + \sum_{t=1}^{\tau} \frac{\xi^2 \chi_t e^{-\xi f_t}}{(1+r)^t} df^2. \end{aligned}$$

$$d^2L|_{f=f^*} = \sum_{t=1}^{\tau} \frac{1}{(1+r)^t} \left(B\gamma(\gamma-1)Q_t^{\gamma-2} + \frac{\chi_t'' + 2\chi_t'}{\xi \chi_t} - \frac{\chi_t'^2}{\xi \chi_t^2} \right) dQ^2.$$

$$d^2L|_{f=f^*} > 0 \text{ if } \chi_t \chi_t'' + 2\chi_t \chi_t' - \chi_t'^2 > 0.$$

This condition are fulfilled $\forall Q_t \in A_Q$. Then function $f_t^*(Q)$ is the solution of the problem

(2.1) for $\min_{f \in A_f} C_{\tau \Sigma}(Q, f)$ in the case of $\xi \chi_t(Q) \geq 1$. ■

The logarithmic function of the firm's output in the t -th period (3.1) minimizes the total costs under the exponentially decreasing function of the industrial damage. This function enables to determine the level of the VRC, which minimizes the total cost, depending on the production volume and the parameter ξ , characterizing the VRC efficiency.

In the next stage, we determine the production volume, when the firm's profit is maximum taking into account the function (3.1).

Proposition 3. For the continuously differentiable functions $C(\cdot), f(\cdot), \chi(\cdot)$ the equation

$$p = B\gamma Q_\tau^{\gamma-1} + \frac{\chi'}{\xi\chi} \quad (3.5)$$

is the solution of the problem (2.4), (2.5), (2.6), if

$$\chi'^2 - \chi\chi'' < 0. \quad (3.6)$$

Thus, when Q_0^* satisfies (3.5) and (3.6), the profit function (2.5) has the maximum value.

Proof. Let's determine the production volume Q_τ^* in the current period, in which the criterion function of the profit reaches its maximum value. For this purpose, we search for the partial derivative of function (2.5) on Q_τ , and we equate it to zero taking into account that $f_t = f_t^*$ according to (3.1):

$$\Pi'_{\tau Q} = R'_{\tau Q} - C'_{\tau\Sigma} Q = 0.$$

$$\Pi'_{\tau Q} = p - B\gamma Q_\tau^{\gamma-1} - \frac{\chi'}{\xi\chi} = 0.$$

$$p = B\gamma Q_\tau^{\gamma-1} + \frac{\chi'}{\xi\chi}.$$

We check the fulfillment of the maximum sufficient condition of the function $\Pi(Q, f)$ subject to (3.1):

$$\Pi''_{\tau QQ} = -B\gamma(\gamma-1)Q_\tau^{\gamma-2} - \frac{\chi''\xi\chi - \xi\chi'^2}{\xi^2\chi^2} = -B\gamma(\gamma-1)Q_\tau^{\gamma-2} - \frac{\chi''}{\xi\chi} + \frac{\chi'^2}{\xi\chi^2}.$$

$$\Pi''_{\tau QQ} < 0 \text{ at } \forall Q_\tau \in A_Q, \text{ if } \chi'^2 - \chi\chi'' < 0. \blacksquare$$

We consider two variants of the function $\chi(Q)$: the power function and the exponential function. If $\chi(Q) = Q^n$, then from (3.1) the follows function may be written:

$$f^*(Q) = \frac{1}{\xi} \ln(\xi Q^n) = \frac{\ln \xi}{\xi} + \frac{n}{\xi} \ln Q. \quad (3.1')$$

If $\chi(Q) = e^Q$, then from (3.1) the follows function may be written:

$$f^*(Q) = \frac{1}{\xi} \ln(\xi e^Q) = \frac{\ln \xi}{\xi} + \frac{Q}{\xi}. \quad (3.1'')$$

The results of the simulation of these functions are presented in tables 1, 2. The calculations are performed in the program Maple for the various parameters, whose values are indicated for each variant of the model.

For the function $\chi(Q) = Q^3$, under all values of the parameters γ and ξ , satisfying (2.7) and (2.8), respectively, the problem (2.1) has no solution for sufficiently small values of the price p and the parameter B (model 1). That is, with low market price and low coefficient of the production costs function, the firm produces such low volume of the product that does not have revenue, sufficient for making the VRC. With the price growth, the revenue increases, and, as a result of the solution (3.4), we receive two possible values of Q^* , from which only $Q^* < 1$ satisfies (3.5) (model 2). With higher values of the parameter B , the production costs increase, therefore, the problem (2.1) has no solution (models 3 to 5) with higher levels of the internal damage and the VRC. That is, the production costs are so high that the firm's revenue does not cover the expenses for the control of the risks.

For the function $\chi(Q) = e^Q$, under small values of the price p (model 6), the problem (2.1) has no solution for the reason, described above for model 1.

The problem (2.1) has the solution, when the price increases that speaks about the increased revenue, which provides the VRC (model 7). An increasing in the parameters γ and B leads to an increase in the production costs that does not leave to the firm the revenue for making the VRC, therefore, the problem (2.1) has no solution under (2.8), (2.9). Model 10 illustrates the ratio of the parameters, when the price is high enough to bring the revenue that is able to cover the expenses for the control of the risks.

Let's consider the simulation of optimal mechanisms (3.1), (3.5) with example of the real industrial company (Samara bearing plant) at various values of p, B, γ, ζ , presented in table 1.

Table 1. Results of simulation for $\chi(Q)=Q^3$.

| Model number | p | B | ζ | γ | Q^* |
|--------------|------|--------------------|---------|----------|-------------------------------------|
| 1 | 1 | 1 | (0; 1] | (1; 2] | $Q^* \notin R$ |
| 2 | 100 | 1 | 0,7 | 1,2 | $Q^*_1=0,08 \quad Q^*_2=4018775720$ |
| 3 | 100 | [80; $+\infty$) | 0,7 | 1,2 | $Q^* \notin R$ |
| 4 | 1000 | 1000 | 0,7 | 1,2 | $Q^*_1=0,018 \quad Q^*_2=0,3557$ |
| 5 | 1000 | (1257; $+\infty$) | 0,7 | 1,2 | $Q^* \notin R$ |

Table 2. Results of simulation for $\chi(Q)=e^Q$.

| Model number | p | B | ζ | γ | Q^* |
|--------------|--------|-----------------|---------|----------|---------------------|
| 6 | (0; 2) | 1 | (0; 1] | 2 | $Q^* < 0$ |
| 7 | 10 | 1 | 1 | 1,3 | 8 |
| 8 | 10 | 1 | 1 | (1,8; 2] | |
| 9 | 10 | [6; $+\infty$) | 0,7 | 1,2 | |
| 10 | 100 | 3 | 0,7 | 1,2 | $14,306 \cdot 10^6$ |

The graphic illustration of these examples is presented in the figure 1.

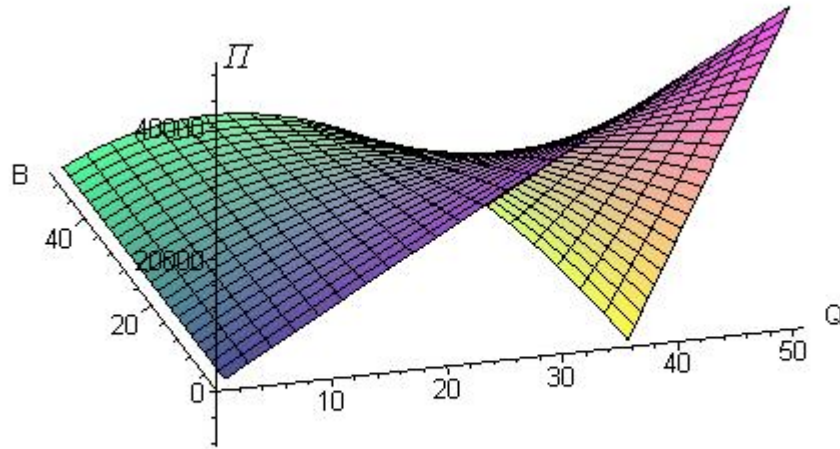


Fig. 1. Function of profit at $p = 1000, \zeta = 0,7, \gamma = 1,9$.

Let's consider modeling with the change of parameters Q and B .

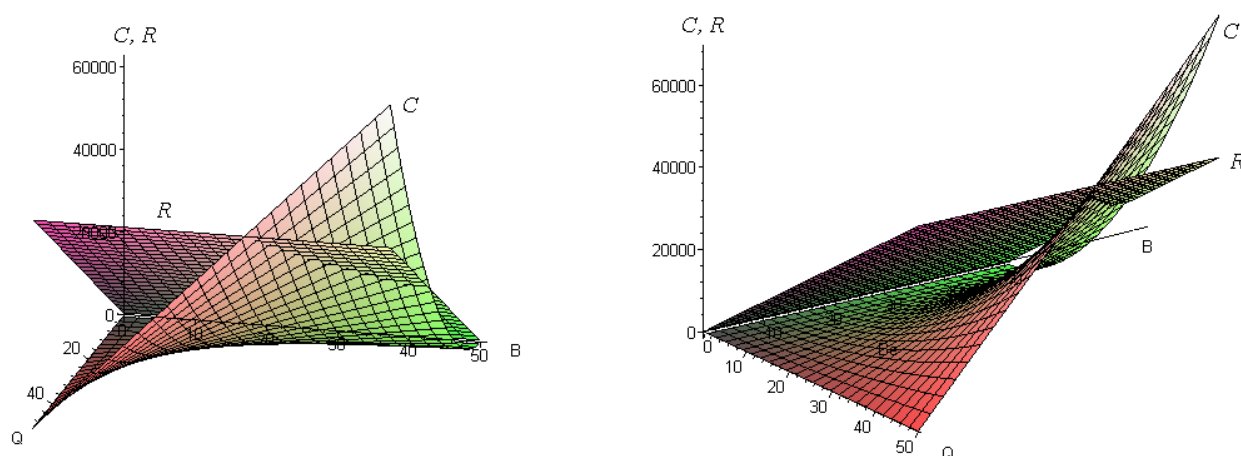


Fig. 2. Results of modeling at $p=1000$, $B=80$, $\zeta=0,7$, $\gamma=1,9$ (different view).

The results of the graphic simulation show that at high values of parameter B revenue is not enough for a covering of the expenses. It means that with increase in the costs, the production volume has to increase. In this case the revenue exceeds the expenses.

4. DISCUSSIONS

The problem of the firm's risk management including the industrial damage from unexpected production situations is examined. The probable firm's expenses in these situations are classified into the voluntary costs and the obligatory costs. It enables to identify the optimization problem of searching for the VRC as a separate component of the overall objectives for the costs management, which, in this case, is the component of the profit maximization problem.

The VRC function optimum is determined by the total costs criterion for the power function and the exponential function of the production costs and for the decreasing function of the industrial damage. The optimal VRC function represents the logarithmic dependence on the monotonic function of the output volume. Therefore, with the output growth, the rational strategy of firm's risk management consists in smooth increase in the costs for prevention of risk situations.

The influence of the costs function and the industrial damage function parameters on an existence of the solution of the profit maximization problem is analyzed. Under the VRC optimal function, low firm's goods price leads to low revenue, which together with high values of the cost function parameters does not enable to implement the VRC. Under sufficiently high price, the existence of firm's output is proved, which enable to make the VRC, with the profit maximizing, that is, the revenue is sufficient to conduct the risk reduction activities. Thus, the boundaries of the market prices and parameters of the cost function are a set, in which the firm's risk management is critical for the firm's existence.

5. CONCLUSION

The results of the research enable to determine the VRC depending on the price and the production function parameters. The expediency of the VRC also depends on these parameters. Thus, the firm can simulate the different development options and determine the VRC depending on the output volume, the product prices, the production functions parameters and the internal damage function. As a result of modeling, the high value of production costs together with low market price do not provide to make the VRC. With the price increase, the firm's revenue increases, and, together with the large output volume, enable to search for an optimal solution of the problems of profit maximizing and expenses minimization.

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