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Design of Robust Stabilizing PI/PID Controller for time delay Interval Process Plants Using Particle Swarm Optimization

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Abstract: Generally, most of the real plants operate in a wide range of unknown operating conditions, but bounded parameter uncertainties in the control system. These uncertainties in the control system cause degradation of system performance and destabilization. In general, it is not easy to design a controller for interval time –delay process plant, because of interval dead time Therefore, robust control of these uncertainties is a vital to operate the plant under stabilized condition. With a view to conquering the uncertainty, in this paper a new stability conditions are developed for determining the stability of interval process plants based on Rouche's Theorem and then a robust PI/PID controller is designed for the interval process plant with and without time delay based on these newly developed stability conditions for stability of interval polynomial by using Particle Swarm Optimization algorithm. A set of inequalities for a closed loop characteristic polynomial of an interval process plant in terms of controller parameters are derived from these newly developed stability conditions. These inequalities are solved to obtain controller parameters with the help of PSO algorithm. The PI/PID controller designed in this proposed method stabilizes the given interval plant with and without time delay at all operating conditions. The proposed method has the advantage of having less computational complexity and easy to implement on a digital computer. The viability of the proposed methodology is illustrated through numerical examples of its successful implementation. The efficacy of the proposed methodology is also evaluated against the available approaches presented in the literature and the results were successfully implemented.

Keywords: Kharitonov's theorem, parametric uncertainty, robust controller, Interval polynomial, particle swarm optimization.

1. INTRODUCTION

Generally, many of the real plants operate in a wide range of unknown operating conditions bounded under parametric uncertainties called interval plants, in control systems. The large uncertainty present in the control system causes degradation of system performance and destabilization. Therefore, robust control of these uncertainties is vital to operate the plant under stabilized condition. This necessitates a robust controller design which could stabilize the plant for all the operating conditions. Hence designing a robust controller for the parametric uncertain plants having unknown, but bounded parameter uncertainties has become the problem of research nowadays. With a view to minimizing the stated

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uncertainties, many solutions are proposed in the literature for the simulation, design and tuning of controllers [1-3]. Recently, affordable results have been reported on computation of all stabilizing P, PI and PID controllers which are mentioned here. The problem in [4] of stabilizing a linear time-invariant plant using a fixed order compensator was considered by using the Hermite-Biehler Theorem. A feasible Robust PID controllers have been developed in [5] using the minimax search, a co-evolutionary algorithm based on Particle Swarm Optimization. The problem of designing robust and optimal PID controllers for a given linear time-invariant plant was proposed in[6-7]. Design of a robust PID controller for a first-order lag with pure delay (FOLPD) model in [8] using PSO enabled automated quantitative feedback theory (QFT) and compared with manual graphical techniques. A design method was proposed for calculating the optimum values PID controller for interval plants using the PSO algorithm. Most of the practical systems operate based on approximate polynomial models; the parameters of these models would lie within an interval but not have specific values and are unknown. Therefore, the stability analysis of polynomials subjected to parameter uncertainty has received considerable attention after the celebrated theorem of Kharitonov [10], which assesses robust stability under the condition that four specially constructed extreme polynomials, called Kharitonov polynomials are Hurwitz. Robust stability of interval polynomial is also discussed here by many researchers. Among these discussions, some important methods have been presented here from the literature. A robust controller has been designed [11] for interval plants based on Kharitonov'theorem and the results Nie of [12] for fixed polynomials. A systematic optimization approach was proposed [13] to design a robust controller using the well-known Kharitonov and Hermite-Biehler stability theorems for single-input/single-output process systems in the presence of unknown but bounded parameter uncertainties. The problem of robust stabilization [15] of a linear time-invariant system was considered subject to variations of a real parameter vector used to design a robust controller. The design of a robust course controller was proposed in [16] for a cargo ship interacting with an uncertain environment using PSO enabled automated Quantitative Feedback Theory. A fractional-order proportional-integral controller was proposed and designed in [17] for a class of nonlinear integer-order systems to guarantee the desired control performance and the robustness of the designed controllers to the loop gain variations. A robust controller was designed in [18-19] for interval plants based on the result of Kharitonov'theorem. With a view to reducing the test of Hurwitz stability of the entire family, several investigations have been presented in the literature. Among these, a few imperative investigations are discussed here, including; an algorithm has been presented in the design of a robust PI and PID controller [20]. This method is based on approximating the fuzzy coefficients by the nearest interval system and then a robust controller is designed using the necessary and sufficient conditions for stability of the interval systems. The Inverse Bilinear Transformation (IBT) is proposed in [21] to design a robust controller using the necessary and sufficient conditions for a discrete-time interval plants. They designed a robust controller using the necessary and sufficient conditions for a chemical process plant with delay subjected to unknown, but bounded parameter uncertainties referred to an interval process plant with interval time delay. Srinivasa Rao *et.al* [22] presented a new algorithm for the design of the robust PI controller for a process control interval plant using Routhe's theorem and Kharitonov's theorem. A robust PI controller design approach was discussed in [23] by finding the controllers using pole placement method for active suspension system with parametric uncertainty. A pure gain compensator $c(s) = K$ stabilizes the entire interval plant family such that a distinguished set of eight of the extreme plants are stable [24]. The first order controller is made by the experimental setup in [25] which was developed by Ghosh. They prove that to robustly stabilize the extreme plants which are obtained by taking all possible combinations of extreme values of the plant numerator has degree m and the plant denominator is monotonic with degree n, the number of extreme plants can be high as Next $=2^{m+n+1}$ in [26]. An explicit equation of control parameters defining the stability boundary in parametric space was derived based on the plant model in time domain and by using the extraordinary feature results from the Kronecker sum operation [27]. The stabilizing values of the parameters of a PI controller were computed based on plotting the stability boundary locus method in [28]. A complete survey of these extreme points is given in [29]. The necessary and sufficient conditions in [18] and [30] for interval polynomials are proposed using the results of [12] for fixed polynomials.

In process industries due to the presence of transportation lag, recycle loops, and dead time corresponding to composition analysis, time delays frequently occur. The mathematical model of uncertain processes has described by the interval time-delay model in the presence of time delay. Unfortunately, as compared with the successful development of controller design for the rational interval model, much less effort has been devoted to the processes described by an interval time-delay model; this is because the process delay is a source of instability and can render many established techniques inadequate. A new approach [32] was proposed to determine the entire set of stabilizing PI/PID parameters for time delay process with bounded uncertainties using the combination of the generalized Kharitonov theorem and the Hermit Biehler theorem. The design of a Smith predictor for the operation of processes under the variation of process gain, time constant and dead time based on the concept of an inferential control framework was presented in [32]. By considering the interval time-delay process, an alternative Smith predictor design in [33] was proposed for the purpose of ensuring robust performance. The use of the structured singular value to design robust controllers was presented in [34] for interval time-delay processes. The designers of the robust stabilizing controller and construction of pre-filter with interval time delay have been considered in [30] to guarantee both robust stability and performance.

Particle swarm optimization (PSO), first introduced by Kennedy and Eberhart [35] is one of the modern heuristic algorithms. It was developed through simulation of a simplified social system, and has been found to be robust in solving continuous nonlinear optimization problems [9] and [36-37]. The PSO technique can generate a high-quality solution within shorter calculation time and stable convergence characteristic than other stochastic methods.

In this note, a PI/PID controller is designed for an interval process plant with and without time delay based on the newly developed necessary and sufficient stability conditions. These conditions are used to derive a set of inequalities in terms of controller parameters. The inequality constraints from the characteristic polynomial are solved consequently to obtain the controller parameters with the help of PSO algorithm. The efficacy of the proposed method is demonstrated by implementing with typical numerical examples available in the literature. In comparison with the method available in the literature [5], [8], [18], [30] and [31] the proposed method in this paper is simple and involves less computational complexity. The paper is organized as follows: Section 2 describes development of stability conditions for robust stability of interval polynomial. Section 3 gives the design of robust stabilizing PI/PID controller with and without time delay process plant. Section 4 proposes PSO algorithm to find the controller parameters. In Section 5, the proposed method is applied to design a robust PI/PID controller for an interval process plant.

2. DEVELOPMENT OF STABILITY CONDITIONS FOR INTERVAL POLYNOMIAL

According to Anderson *et.al* [38] the necessary and sufficient condition for robust stability of interval polynomials of order $n \leq 3$ is positive lower bounds on the coefficients of an interval polynomial.

Therefore, consider an interval polynomial of order $n=1$

$$P(s)=\sum_{i=0}^{1} p_i s^i \text{, where } p_i \in [a_i, b_i].$$

$$P(s)= p_1 s + p_0 = [a_1, b_1]s + [a_0, b_0].$$

Therefore, as per Anderson [38], the robust stability condition is

$a_1 > 0$ and $a_0 > 0$ i.e. $a_i > 0$ for $i=0,1$.

Similarly for order $n=2$

$$P(s)=\sum_{i=0}^{2} p_i s^i = p_2 s^2 + p_1 s + p_0 = [a_2, b_2]s^2 + [a_1, b_1]s + [a_0, b_0].$$

Therefore, the robust stability condition is

$a_2 > 0$, $a_1 > 0$ and $a_0 > 0$ i.e. $a_i > 0$ for $i=0,1,2$.

Lemma 2.1

Consider a real Hurwitz polynomial Q(s) of the form

$$Q(s)=q_n s^n + q_{n-1}s^{n-1} + \dots + q_i s^i + \dots + q_1 s + q_0 \tag{2.1}$$

$i=0,1,2,\dots\dots,n$

Where q_i is real and positive, $q_0 > 0$.

If any complex number Z such that $R_e > 0, |f(z)| > |f(-z)|$, moreover, $|f(z)|_{z\, on\, C} > |f(-z)|_{z\, on\, C}$, where C is a Closed contour, then, according to Routhe's theorem [39] the following two polynomials can be formulated.

$$Q_0 = \frac{1}{2}[Q(s)+Q(-s)]\Big|_{s^2=x} \tag{2.2}$$

$$Q_1 = \frac{1}{2s}[Q(s)-Q(-s)]\Big|_{s^2=x} \tag{2.3}$$

Theorem 2.1: For stability of $Q(s)$ the two polynomials Q_0 and Q_1 formed by the alternate coefficients of a Hurwitz polynomial in accordance with equations (2.2) and (2.3) must have negative real zeros. The proof of this is given in [39].

2.1 Necessary conditions for stability of an interval polynomials

Consider an interval polynomial of order n > 3 of the form

$$P(s)= p_n s^n + p_{n-1}s^{n-1} + \dots + p_i s^i + \dots + p_1 s + p_0,$$

Where $p_i \in [a_i, b_i]$ for $i=0,1,2,3,\dots\dots,n$.

The necessary conditions for an interval polynomial to be stable is given as

$$b_i \ge a_i > 0 \text{ for } i=0,1,2,3,\dots\dots,n \tag{2.4}$$

2.2 Sufficient conditions for stability of interval polynomial for n > 3

2.2.1. *For Fourth-order Interval Polynomial (n= 4)*

Consider the fourth-order interval polynomial as

$$P(s)= p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0 \tag{2.5}$$

Where $p_0 \in [a_0, b_0]$, $p_1 \in [a_1, b_1]$, $p_2 \in [a_2, b_2]$, $p_3 \in [a_3, b_3]$ and $p_4 \in [a_4, b_4]$

Using Lemma 2.1, The P(s) can be represented into two polynomials P_0 and P_1 as given below.

$$P_0 = \frac{1}{2}[P(s)+P(-s)]\Big|_{s^2=x} = [a_4, b_4]x^2 + [a_2, b_2]x + [a_0, b_0] \tag{2.6}$$

$$P_1 = \frac{1}{2s}[P(s) - P(-s)]\Big|_{s^2=x} = [a_3, b_3]x + [a_1, b_1] \tag{2.7}$$

According to the Theorem 2.1, for robust stability of interval polynomial P(s) the above polynomials P_0 and P_1 must have negative real zeros i.e.

$$[a_2, b_2]^2 > 4[a_0, b_0][a_4, b_4] \tag{2.8}$$

$$\frac{-[a_1, b_1]}{[a_3, b_3]} < 0. \tag{2.9}$$

Apply interval arithmetic to the above equations (2.8) and (2.9), the stability conditions for the interval polynomial are

$$a_2^2 > 4b_0b_4 \tag{2.10}$$

$$b_2^2 > 4a_0a_4 \tag{2.11}$$

$$\frac{-a_1}{b_3} < 0 \tag{2.12}$$

$$\frac{-b_1}{a_3} < 0 \tag{2.13}$$

From the above four equations, the sufficient conditions for the robust stability of fourth order interval polynomial P(s) are

$$a_2^2 > 4b_0b_4 \tag{2.14}$$

$$\frac{-a_1}{b_3} < 0 \tag{2.15}$$

2.2.2. *For Fifth-order Interval Polynomial (n = 5)*

Consider the fifth-order interval polynomial as

$$P(s) = p_5s^5 + p_4s^4 + p_3s^3 + p_2s^2 + p_1s + p_0 \tag{2.16}$$

Where

$p_0 \in [a_0, b_0]$, $p_1 \in [a_1, b_1]$, $p_2 \in [a_2, b_2]$, $p_3 \in [a_3, b_3]$, $p_4 \in [a_4, b_4]$ and $p_5 \in [a_5, b_5]$

Using Lemma 2.1, The P(s) can be represented into two polynomials P_0 and P_1 as given below.

$$P_0 = \frac{1}{2}[P(s) + P(-s)]\Big|_{s^2=x} = [a_4, b_4]x^2 + [a_2, b_2]x + [a_0, b_0] \tag{2.17}$$

$$P_1 = \frac{1}{2s}[P(s) - P(-s)]\Big|_{s^2=x} = [a_5, b_5]x^2 + [a_3, b_3]x + [a_1, b_1] \tag{2.18}$$

According to the Theorem 2.1, for robust stability of interval polynomial P(s) the above polynomials P_0 and P_1 must have negative real zeros i.e.

$$[a_2, b_2]^2 > 4[a_0, b_0][a_4, b_4] \quad (2.19)$$

$$[a_3, b_3]^2 > 4[a_1, b_1][a_5, b_5] \quad (2.20)$$

Apply interval arithmetic to the above equations (2.19) and (2.20), the stability conditions for the interval polynomial are

$$a_2^2 > 4b_0b_4 \quad (2.21)$$

$$b_2^2 > 4a_0a_4 \quad (2.22)$$

$$a_3^2 > 4b_1b_5 \quad (2.23)$$

$$b_2^2 > 4a_1a_5 \quad (2.24)$$

From the above four equations, the sufficient conditions for the robust stability of fifth order interval polynomial P(s) are

$$a_2^2 > 4b_0b_4 \quad (2.25)$$

$$a_3^2 > 4b_1b_5 \quad (2.26)$$

In a similar manner, the robust stability conditions for interval polynomial of degree $n > 4$ can be determined. The robust stability conditions for higher-order interval polynomials are represented in a tabular form in Table 2.1.

. **Table2.1. Robust** Stability conditions for various higher order interval polynomials

Order of the polynomial	Robust stability conditions	
	Necessary Conditions	Sufficient conditions
$n = 3$	$a_i > 0$ Where i = 0,1,2,3.	$a_1^2 > 3b_0b_2$
$n = 4$	$a_i > 0$ Where i = 0, 1,2,3,4.	$a_2^2 > 4b_0b_4$ and $\frac{-a_1}{b_3} < 0$.
$n = 5$	$a_i > 0$ Where i = 0, 1,..,4, 5.	$a_2^2 > 4b_0b_4$ and $a_3^2 > 4b_1b_5$.
$n = 6$	$a_i > 0$ Where i = 0, 1,...5, 6.	$a_2^2 > 3b_0b_4$ and $a_3^2 > 4b_1b_5$.
$n = 7$	$a_i > 0$ Where i = 0, 1,.... 6, 7.	$a_2^2 > 3b_0b_4$ and $a_3^2 > 3b_1b_5$.
$n = 8$	$a_i > 0$ Where i = 0, 1,2,..,7,8.	$a_3^2 > 3b_1b_5$, $b_4^2 > 4a_0a_8$ and $\frac{-a_2}{b_6} < 0$.
$n = 9$	$a_i > 0$ Where i = 0, 1,2,..,8,9..	$b_4^2 > 4a_0a_8$ $b_5^2 > 4a_1a_9$, $\frac{-a_2}{b_6} < 0$ and $\frac{-a_3}{b_7} < 0$
		$b_4^2 > 4a_0a_8$, $b_5^2 > 4a_1a_9$,

$n=10$	$a_i > 0$ Where i = 0, 1,2,..,10.	$b_6^2 > 4a_2a_{10}$ and $\frac{-a_3}{b_7} < 0$

Using these developed stability conditions, the stability of interval polynomials can be determined easily without formulating the four Kharitonov's polynomials, unlike Kharitonov's theorem. A robust PI/PID controller (which can stabilize the given plant under large uncertainty) can be designed easily using these stability conditions. The design procedure is given in following section.

3. DESIGN OF ROBUST STABILIZING PI/PID CONTROLLER

3.1 Interval plant without time delay

Consider a plant with parametric uncertainty without time delay represented by its transfer function as

$$G(s,c,d) = \frac{N(s,c)}{D(s,d)} = \frac{c_0 + c_1 s + ... + c_{m-1}s^{m-1} + c_m s^m}{d_0 + d_1 s + ... + d_{n-1}s^{n-1} + d_n s^n} \quad (3.1)$$

Where

$$c \in C = [c_i^-, c_i^+] \text{ for } i = 0,1,2,....,m$$

$$d \in D = [d_i^-, d_i^+] \text{ for } i = 0,1,2,....,n$$

and the bounds c_i^-, c_i^+, d_i^- and d_i^+ are specified a priori and $n \le m$.

Let the stabilizing PI/PID controller transfer function of the form given below

$$C_{PI}(s) = K_P + \frac{K_I}{s} = \frac{N_c(s)}{D_c(s)} \quad \text{For PI controller} \quad (3.2)$$

$$C_{PID}(s) = K_P + \frac{K_I}{s} + K_D s = \frac{N_c(s)}{D_c(s)} \quad \text{For PID controller} \quad (3.3)$$

Where K_P = Proportional gain, K_I =Integral gain and K_D = Derivative gain

3.2. Interval plant with time delay

Consider a plant with parametric uncertainty with time delay represented by its transfer function as

$$\overline{G}(s,\overline{c},\overline{d}) = \frac{\overline{N}(s,\overline{c})}{\overline{D}(s,\overline{d})} e^{-t_d s} \quad (3.4)$$

Where the numerator and denominator polynomials are of the form

$$\overline{N}(s,\overline{c}) = \overline{c}_0 + \overline{c}_1 s + ... + \overline{c}_{m-1}s^{m-1} + \overline{c}_m s^m$$

$$\overline{D}(s,\overline{d}) = \overline{d}_0 + \overline{d}_1 s + ... + \overline{d}_{n-1}s^{n-1} + \overline{d}_n s^n$$

With the parameters being specified by their lower and upper bounds as follows:

$$\overline{c} \in \overline{C} = [\overline{c}_i^-, \overline{c}_i^+] \text{ for } i = 0,1,2,....,m.$$

$$\overline{d} \in \overline{D} = [\overline{d}_i^-, \overline{d}_i^+] \text{ for } i = 0,1,2,....,n.$$

$$t_d^- \le t_d \le t_d^+. \quad t_d \ge 0$$

It is not easy work to design the stabilizing controller for interval time-delay process plants, because of interval dead time. In order to extend the technique, the first- order rational function of the approximation of the interval delay plant is obtained using the procedure given in Corollary 3.1 to solve the design of the robust stabilizing controller problem of an interval time-delay plant.

3.2.1. An interval approximate time delay model

The first- order rational function, $\prod(s,t_d)$ of the approximation of the interval delay part is given by

$$e^{-t_d s} \cong \prod(s,t_d).\ \text{for} \qquad t_d^- \le t_d \le t_d^+ \tag{3.5}$$

The simplex form for $\prod(s,t_d)$ is given by the following corollary which is expressed in[13].

Corollary 3.1 : The interval function $e^{-t_d s}$, where $t_d^- \le t_d \le t_d^+$ can be approximated by a first-order interval rational function $\prod(s,\delta)$, which is given by

$$\prod(s,t_d) \equiv \frac{1-\left[\frac{t_d^-}{2},t_d^+ + \frac{t_d^-}{2}\right]s}{1+\left[\frac{t_d^-}{2},t_d^+ + \frac{t_d^-}{2}\right]s} \tag{3.6}$$

with the following properties:

$$\left|\prod(j\omega,t_d)\right|_{max} \equiv \left|\frac{1-\left[\frac{t_d^-}{2}+t_d^+\right]j\omega}{1+\left[\frac{t_d^-}{2}\right]j\omega}\right|_{max} > 1 \tag{3.7}$$

$$\left|\prod(j\omega,t_d)\right|_{min} \equiv \left|\frac{1-\left[\frac{t_d^-}{2}\right]j\omega}{1+\left[\frac{t_d^+}{2}+t_d^+\right]j\omega}\right|_{min} < 1 \tag{3.8}$$

$$\angle\prod(j\omega,t_d)\big|_{max} \ge \angle e^{-t_d j\omega}. \tag{3.9}$$

$$\angle\prod(j\omega,t_d)\big|_{min} < \angle e^{-t_d j\omega}. \tag{3.10}$$

Where $\omega < \omega*$ and $\omega*$ is the limited frequency that the inequalities in Equations (3.7) to (3.10) hold. The arbitrarily extended approximation of a desired degree is the first-order approximation, since it covers the properties of phase frequency of original function for a wider range of frequencies, whereas the approximate original function can be in the other forms of approximations for control in the loss of some system information within a very limited frequency range.

The rational interval function, then the approximate system model is given by

$$G(s,c,d) = \frac{\overline{N}(s,\bar{c})}{\overline{D}(s,\bar{d})}\prod(s,t_d). \tag{3.11}$$

After simplification the (3.11) will become

$$G(s,c,d)=\frac{N(s,c)}{D(s,d)} \tag{3.12}$$

Where

$$N(s,c)=c_0+c_1s+...+c_{m-1}s^{m-1}+c_ms^m$$

$$D(s,d)=d_0+d_1s+...+d_{n-1}s^{n-1}+d_ns^n$$

$$c\in C=[c_i^-,c_i^+] \text{ for } i=0,1,2,....,m.$$

$$d\in D=[d_i^-,d_i^+] \text{ for } i=0,1,2,....,n.$$

$t_d^-\le t_d\le t_d^+$. and the bounds $c_i^-,c_i^+,d_i^-,d_i^+,t_{-d}$ *and* $\bar{t}_d$ are specified a priori and $n\le m$.

Now the system with robust stabilizing controller for Parametric Uncertainty is as shown in Fig.3.1.

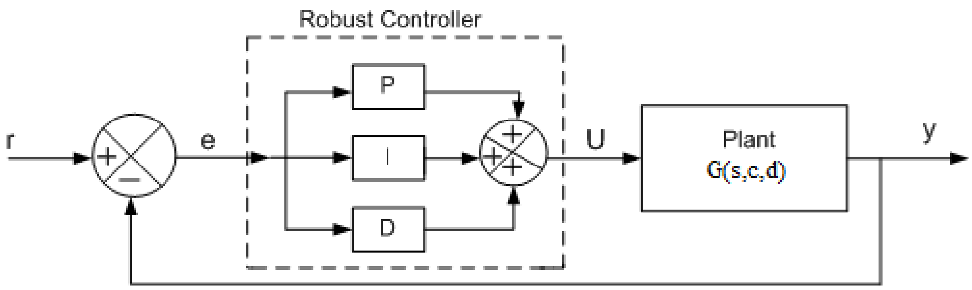

Fig.3.1. Block diagram of interval Plant with a Robust Controller

Let the stabilizing PI/PID controller transfer function of the form given below

$$C_{PI}(s)=K_c(1+\frac{1}{\tau_is})=\frac{N_c(s)}{D_c(s)} \quad \text{For PI controller} \tag{3.13}$$

$$C_{PID}(s)=K_c(1+\frac{1}{\tau_is}+\tau_d)=\frac{N_c(s)}{D_c(s)} \quad \text{For PID controller} \tag{3.14}$$

Where Kc = Proportional gain, τ_i =Integral gain and τ_d = Derivative gain.
Then the closed loop transfer function with a PI / PID controller can be defined as

$$T(s)=\frac{C_{PI}(s)G(s,c,d)}{1+C_{PI}(s)G(s,c,d)}=\frac{N_c(s)N(s,c)}{N_c(s)N(s,c)+D_c(s)D(s,d)} \quad \text{For PI controller} \tag{3.15}$$

$$T(s)=\frac{C_{PID}(s)\,G(s,c,d)}{1+C_{PID}(s)G(s,c,d)}=\frac{N_c(s)N(s,c)}{N_c(s)N(s,c)+D_c(s)D(s,d)} \quad \text{For PID controller} \tag{3.16}$$

The Characteristic equation of this system with a PI / PID controller is given as

$$1+C_{PI}(s)G(s,c,d)=N_c(s)N(s,c)+D_c(s)D(s,d) \quad \text{for PI controller} \tag{3.17}$$

$$1+C_{PID}(s)G(s,c,d)=N_c(s)N(s,c)+D_c(s)D(s,d) \text{for PID controller} \tag{3.18}$$

Where $N(s,c)$ and $D(s,d)$ are the numerator and denominator polynomials of the plant considered respectively, and $N_c(s)$ and $D_c(s)$ are the numerator and denominator polynomials of PI/PID controller transfer function respectively. This PI/PID controller robustly stabilizes the interval plants family, if for all $c\in C$ and $d\in D$, then the characteristic polynomial of a closed loop transfer function given in equations (3.17) and (3.18) has all zeros have negative real values. Now apply the necessary and sufficient conditions of robust stability conditions given in Table.1 to the closed-loop polynomial $N_c(s)N(s,c)+D_c(s)D(s,d)$ which leads to a set of inequalities in terms of controller parameters. Then these inequalities can be solved by using PSO with MATLAB Optimization [40] programming so as to minimize the objective function to obtain controller parameters. Then after obtaining the Controller parameters, form four sets of Kharitonov's polynomials to check the stability and the closed-loop step response to verify the results. The PSO algorithm for the proposed method is given in section 4.

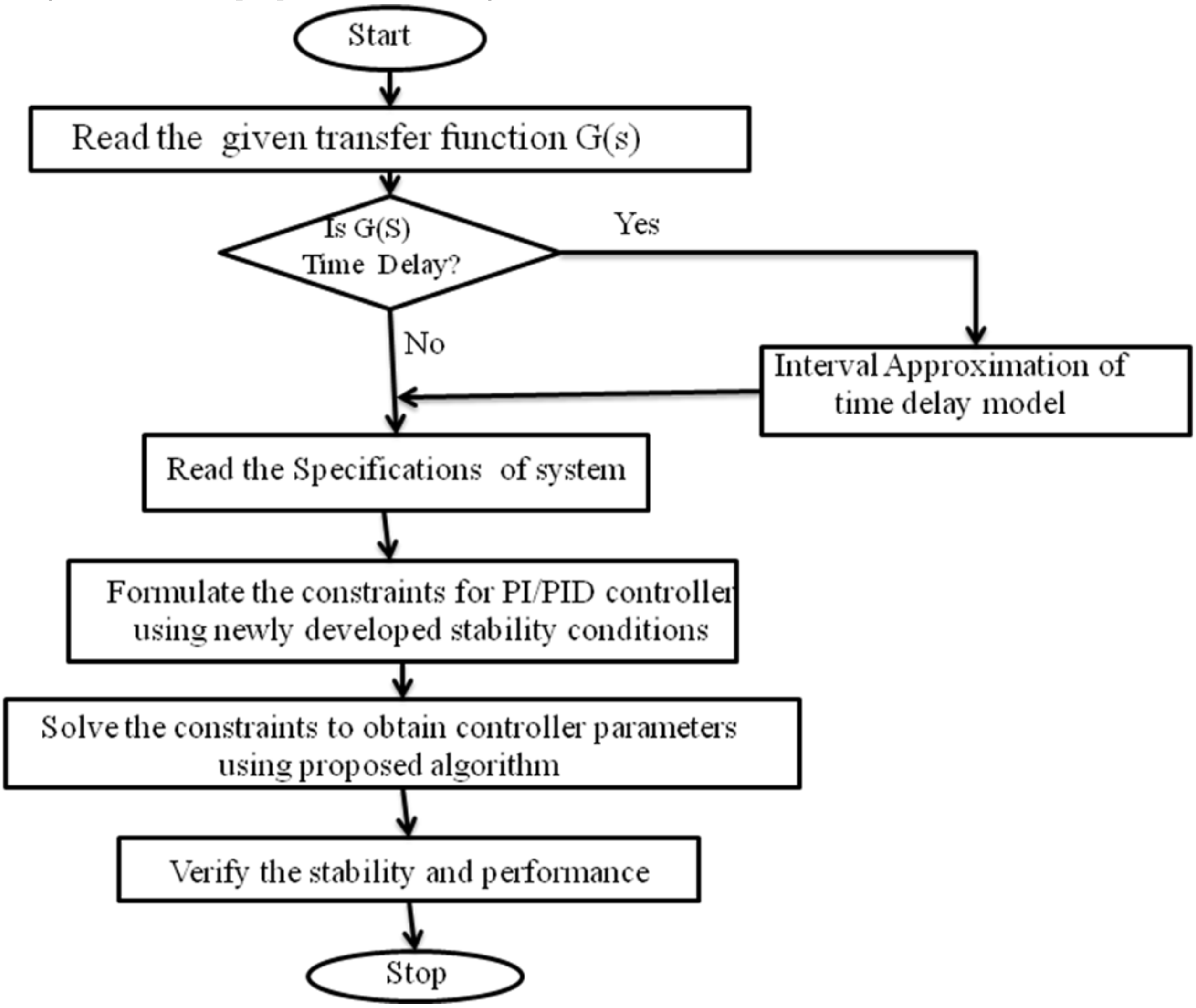

Fig.3.2. Flowchart for the Proposed Algorithm.

4. APPLICATION OF PARTICLE SWARM OPTIMIZATION ALGORITHM

Kennedy and Eberhart [35] first introduced the PSO method. It is one of the optimization algorithms and a kind of evolutionary computation algorithm. The method has been found to be robust in solving problems featuring nonlinearity and no differentiability, multiple optima, and high dimensionality through adaptation, which is derived from the social-psychological theory. PSO is inspired by social and cooperative behaviour displayed by various species to fill their needs in the search space. The algorithm is guided by personal experience (P best), the overall experience (G best) and the present movement of the particles to decide their positions in the next space. Further the experiences are accelerated by two factors C1 and C2, and two random numbers generated between [0, 1]. Whereas the present movement is multiplied by an inertia factor w varying between [w_{min},w_{max}].

Application of PSO algorithm for determining the controller parameters is as follows:

Step 1: Initialization:

PSO parameters are chosen as

Population size (P) = 100

Number of Iterations (n) =1000

Cognitive coefficients C1 =2 and C2 =2

Inertia weight $W = W_{max} - iter\ x(W_{max} - W_{min})/n$

Where *Wmax* =0.9, *Wmin* =0.4

Step 2: Initial search space limits of control variables in PI/PID Controller are selected as

For PI controller:

$0 < K_P < 10$ and $0 < K_I < 10$.(Without time delay)

$0 < K_c < 1$ and $0 < \tau_i < 15$ (With time delay)

For PID controller:

$0 < K_P < 10, 0 < K_I < 10$ and $0 < K_D < 10$ (Without time delay)

$0 < K_c < 1, 0 < \tau_i < 15$ and $0 < \tau_d < 10$ (With time delay)

Step 3: Initial search space populations of Xi are generated from specified intervals using the given below equation

$$X_i^0 = X_{i,min} + rand(\ ).(X_{i,max} - X_{i,min}), \tag{4.1}$$

Where i =1, 2......, N, and rand () represents a uniformly distributed random number within the range of [0,1]

Step 4: Initialize the iteration index n=1

During the initialization, parameters of a PI / PID controller are randomly generated within allowable limits.

Step 5: Evaluate the fitness function

To get effective performance, in this paper, the fitness function J is defined as Minimize J

For PI controller: The fitness function J is chosen as Integral Square Error (ISE)

$$J = \int_0^\infty e^2(t)dt \quad \text{(Without time delay)} \tag{4.2}$$

Where $e(t) = 1 - output$

$$J = \left\| \frac{K_c - K_c^0}{K_c^0} \right\|^2 + \left\| \frac{\tau_i - \tau_i^0}{\tau_i^0} \right\|^2 \quad \text{(With time delay)} \tag{4.3}$$

For PID controller: The fitness function J is chosen as Integral Square Error (ISE)

$$J = \int_0^\infty e^2(t)dt \quad \text{(Without time delay)} \tag{4.4}$$

Where $e(t) = 1 - output$

$$J = \left\| \frac{K_c - K_c^0}{K_c^0} \right\|^2 + \left\| \frac{\tau_i - \tau_i^0}{\tau_i^0} \right\|^2 + \left\| \frac{\tau_d - \tau_d^0}{\tau_d^0} \right\|^2 \quad \text{(With time delay)} \tag{4.5}$$

J is determined when the controller parameters are subjected to

$K_{P\,min} \le K_P \le K_{P\,max}$; $K_{Im\,in} \le K_I \le K_{Im\,ax}$; and $K_{D\,min} \le K_D \le K_{D\,max}$;

$K_{c\min} \le K_P \le K_{c\max}$; $\tau_{i\min} \le \tau_i \le \tau_{i\max}$; and $\tau_{d\,min} \le \tau_d \le \tau_{d\,max}$;

Step 6: Update velocity. For each particle, the velocity can be updated by

$$V_i^{n+1} = w \times V_i^n + C_1 \times rand(\ ) \times (Pbest_i^n - X_i^n) + C_2 \times rand(\ ) \times (Gbest_i^n - X_i^n) \quad (4.6)$$

Step 7: Update position. Each particle changes its position by adding the updated velocity to the previous position and it is represented as.

$$X(i,j) = X_i^0 + V_i^{n+1} \quad (4.7)$$

Step 8: Repeat steps 5 to 7 until maximum generations are completed PSO algorithm is run for each particle to evaluate fitness function several times and better results are saved and applied to the proposed PI/PID controller.

5. DESIGN OF ROBUST STABILIZING CONTROLLER

In this section, a design procedure for a robust PI/PID controller of a plant with parametric uncertainty is illustrated.

Example 1

Consider a wing aircraft [18] whose transfer function with parametric uncertainty is given by

$$G(s,c,d) = \frac{[54,74]s + [90,166]}{s^4 + [2.8,4.6]s^3 + [50.4,80.8]s^2 + [30.1,33.9]s + [-0.1,0.1]} \quad (5.1)$$

As the necessary conditions $b_i \geq a_i > 0$ (for i=0, 1, 2, 3, 4) are not satisfied for the above characteristic polynomial, Hence the given interval plant is unstable. Thereby, it is required to design a robust controller, which stabilizes the given plant.

Design of PI controller:

The transfer function of the PI controller is given by

$$C_{PI}(s) = K_P + \frac{K_I}{s} = \frac{N_c(s)}{D_c(s)}$$

Then the closed loop transfer function with a PI controller becomes

$$T(s) = \frac{[54K_P, 74K_P]s^2 + [90K_P + 54K_I, 166K_P + 74K_I]s + [90K_I, 166K_I]}{[1,1]s^5 + [2.8, 4.6]s^4 + [50.4, 80.8]s^3 + [54K_P + 30.1, 74K_P + 33.9]s^2 + [90K_P + 54K_I - 0.1, 166K_P + 74K_I + 0.1]s + [90K_I, 166K_I]} \quad (5.2)$$

From the above equation, the characteristic equation of the closed loop interval system with PI controller can be taken as

$$[1,1]s^5 + [2.8, 4.6]s^4 + [50.4, 80.8]s^3 + [54K_P + 30.1, 74K_P + 33.9]s^2 + [90K_P + 54K_I - 0.1, 166K_P + 74K_I + 0.1]s + [90K_I, 166K_I] = 0 \quad (5.3)$$

The step response PI controller (K_P = 1.3172 and K_I = 1.9378) using Ziegler-Nichols settings [41] are shown in Figure 5.1. From this Figure 5.1, it has been observed that the designed PI controller from Ziegler-Nichols tuning method cannot stabilize the given interval plant at all operating conditions. Hence it is necessary to redesign the PI controller to stabilize given interval plant. By applying the necessary and sufficient conditions from Table 1 to the above 5th order polynomial (5.3), the following set of inequality constraints are obtained. In order to make this set of constraints into the feasible closed set, a small positive number 'ε' is introduced into the constraints. Hence the optimization problem can be stated as to find K_P and K_I such that the objective function $J = \int_0^\infty e^2(t)dt$ is minimized, subjected to the following constraints.

Inequality constraints for proposed method:

Necessary conditions:

$-90K_I + \varepsilon < 0$

$-90K_P - 54K_I + 0.1 + \varepsilon < 0$

$-54K_P - 30.1 + \varepsilon < 0$

Sufficient conditions:

$-2916K_P^2 - 3250.8K_P - 906.01 + 3054.4K_I + \varepsilon < 0$

$-2540.16 + 664K_P + 296K_I + 0.4 + \varepsilon < 0$

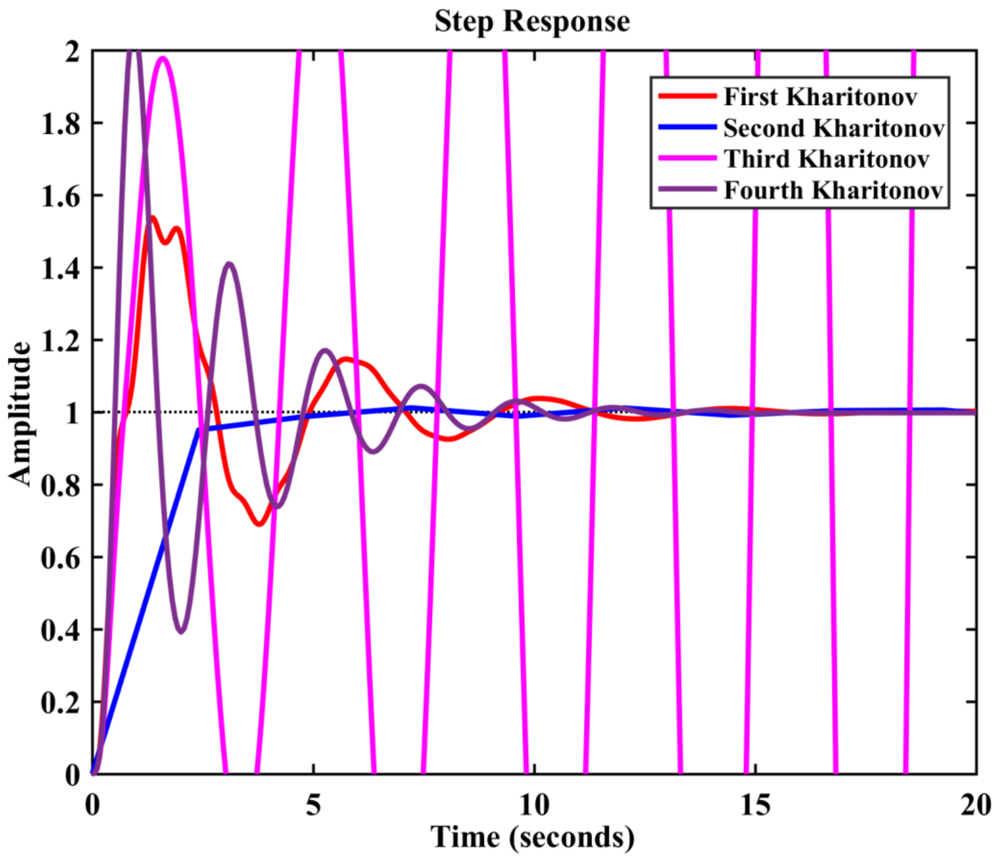

Fig.5.1. Closed loop step response with PI controller for all extreme Plants using the Ziegler-Nichols

The linear programming problem consists of two decision variables and five constraints. The controller parameters K_P and K_I are restricted to small values by choosing the objective function J properly. The purpose of using a small positive number ε is to formulate a feasible set closed. In this work, the PSO algorithm proposed in section 4 is used to minimize objective function. It attempts to explain the problems of minimization, subjected to linear as well as nonlinear constraints. By applying the proposed algorithm, then the values of controller parameters are obtained as K_P = 0.5766 and K_I = 0.01. The closed loop step response of the system with a PI controller for both proposed methods (K_P = 0.5766 and K_I = 0.001) and the method given in [18] (K_P = 0.5 and K_I = 0.1) are shown in Figures 5.2 and 5.3 for ε = 1 respectively. The nominal PI controller parameters $K_c^0 = 0.6$ and $\tau_i^0 = 11.655$ and nominal PID controller parameters $K_c^0 = 0.68$ $\tau_i^0 = 6.993$ and $\tau_d^0 = 1.74825$ are designed by the Ziegler-Nichols settings. The time domain specifications of Figures 5.2 and 5.3 are shown in Table 5.1 and which describes the efficacy of the proposed method for the design of the robust PI controller. The step response comparison of four extreme plants with a PI controller obtained by the proposed method and the method given in [18] is shown in Figure

5.4.

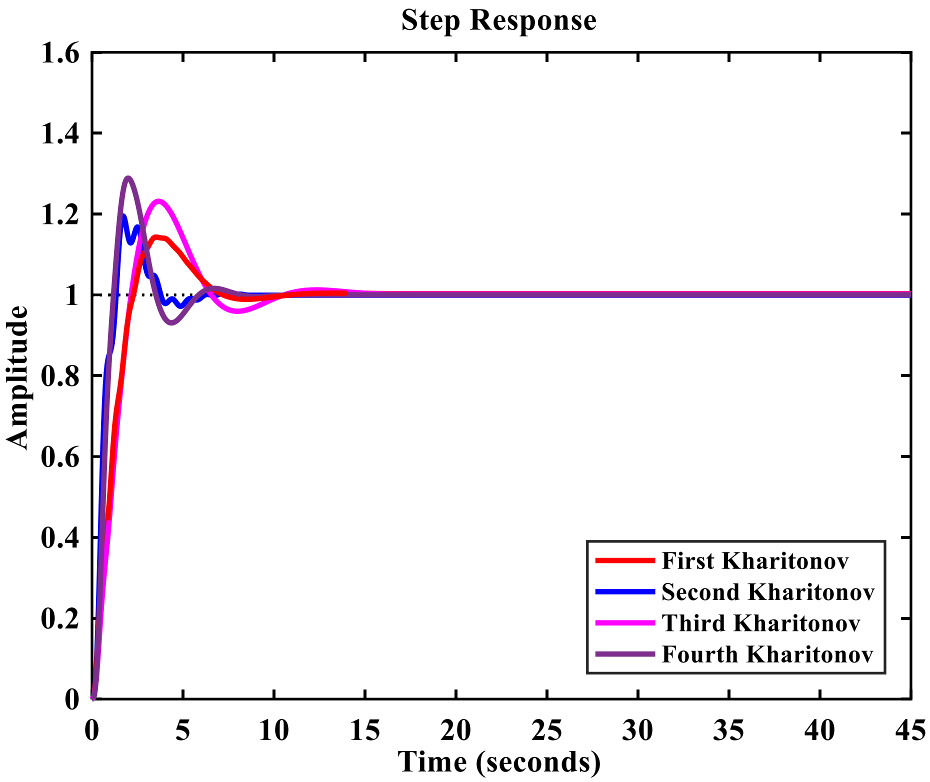

Fig. 5.2. Closed loop step response to the PI controller for all extreme plants using the proposed method**.**

In order determine the robustness of proposed method, the given interval plant G(s,c,d) from equation (5.1) can be written as

$$G(s)=\frac{64s+128}{s^4+3.7s^3+64s^2+32s} \tag{5.4}$$

Here G(S) is obtained by averaging the upper and lower bound of s coefficients of G(s,c,d). The closed transfer function of wing aircraft with PI controller is given by

$$T^1(s)=\frac{64K_Ps^2+(128K_P+64K_I)s+128K_I}{s^5+3.7s^4+65.6s^3+(64K_P+32)s^2+(128K_P+64K_I)s+128K_I} \tag{5.5}$$

The closed loop step response of the system with PI controller (K_P = 0.5766 and K_I = 0.001) from proposed methods is shown in Figure 5.5. The PI controller designed from proposed method can stabilize the given plant if any changes occur in the uncertain parameters within the bounds

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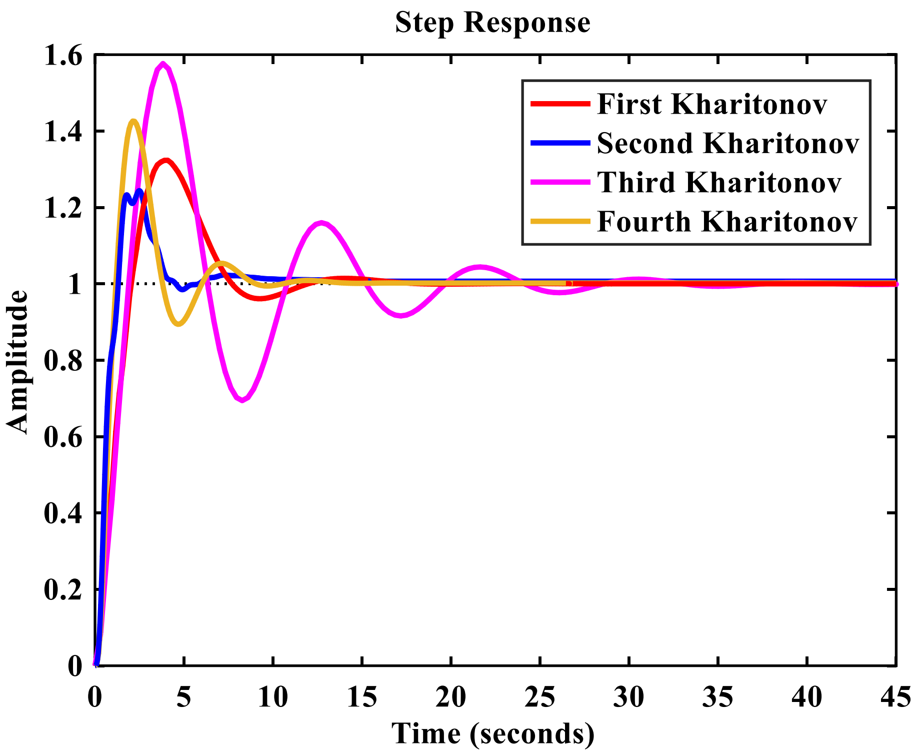

Fig.5.3. Closed loop step response to the PI controller for all extreme plants using the method given in [18].

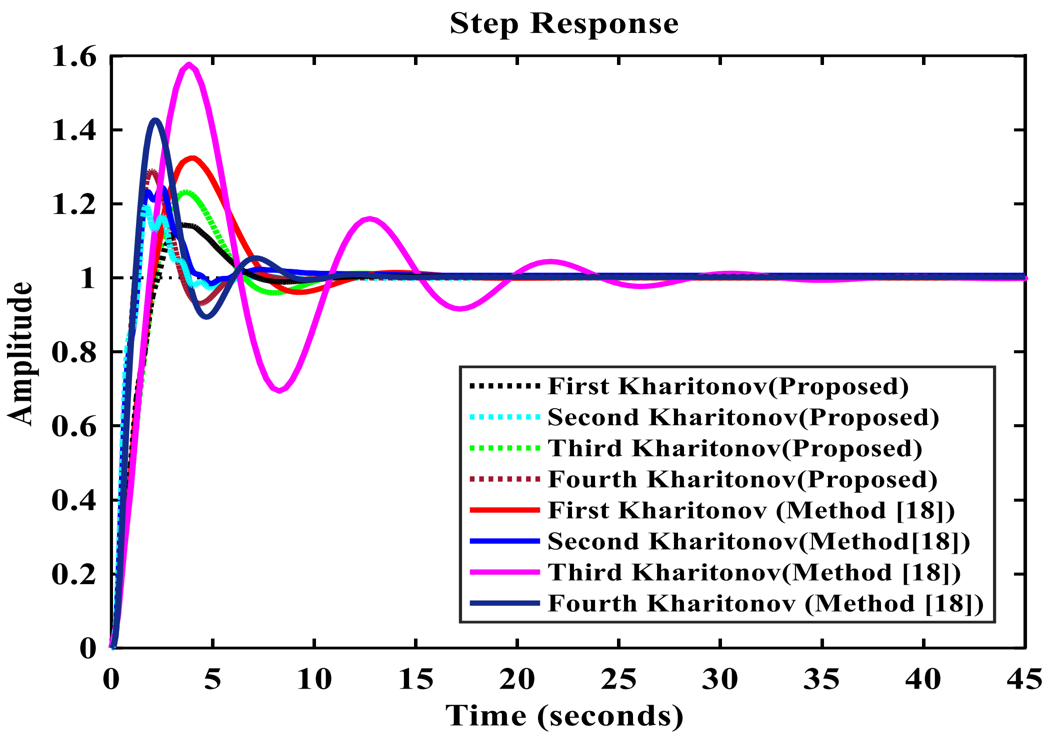

Fig.5.4. Closed loop step response to the PI controller for all extreme plants for proposed method and the method given in[18].

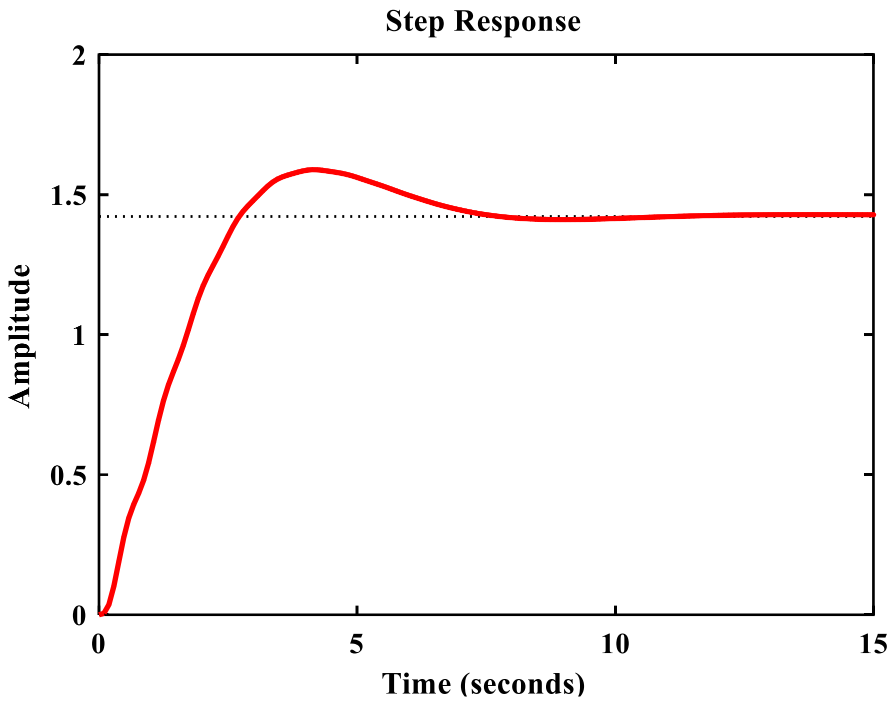

Fig.5.5. Closed loop step response to the PI controller for changes in the uncertain parameter to determine the robustness for proposed method

Table 5.1. Time domain specifications for proposed method and Existing method in [18]

Name of the Kharitonov Polynomial	Proposed method					Existing method in [18]				
	% peak overshoot MP	Peak Time t_p(sec)	Rise time t_r(sec)	Settling time t_s(sec)	ISE 10^{-4}	% peak overshoot MP	Peak Time t_p(sec)	Rise time t_r(sec)	Settling time t_s(sec)	ISE 10^{-3}
First	17.492	3.514	1.501	7.0534	4.22	32.4147	4.008	1.508	10.808	1.101
Second	20.826	1.707	0.878	5.0156	0.093	24.3994	2.488	0.880	8.2771	0.158
Third	29.702	3.683	1.349	14.229	1.53	57.6729	3.820	1.378	26.828	16.00
Fourth	31.419	1.988	0.718	7.7175	0.631	42.659	2.166	0.741	8.326	0.022

It has been observed from Figures 5.2 and 5.3 that the designed PI controller, which uses the proposed stability conditions, robustly stabilizes the plant very quickly when compared to the method given in [18]. From Table 5.1, the designed PI controller stabilizes the plant with lesser time domain parameters than the existing method. It has been observed from figure 5.5, that the designed PI controller from the proposed method is used to determine the robustness in any changes in the uncertain parameters of the given plant. In our proposed method, the controller parameters are obtained based on the minimization of the objective function (ISE). This Integral Square Error obtained from our proposed method is less compared to other methods available in the literature. This shows the efficacy of the proposed method in terms of time domain specifications and the ISE. The proposed method involves five sets of equations for NLP to solve. Whereas the method in [18] requires eight set of equations. Thus, the proposed method requires less computational complexity than the method given in [18]. It is also observed that the computation time required for solving the

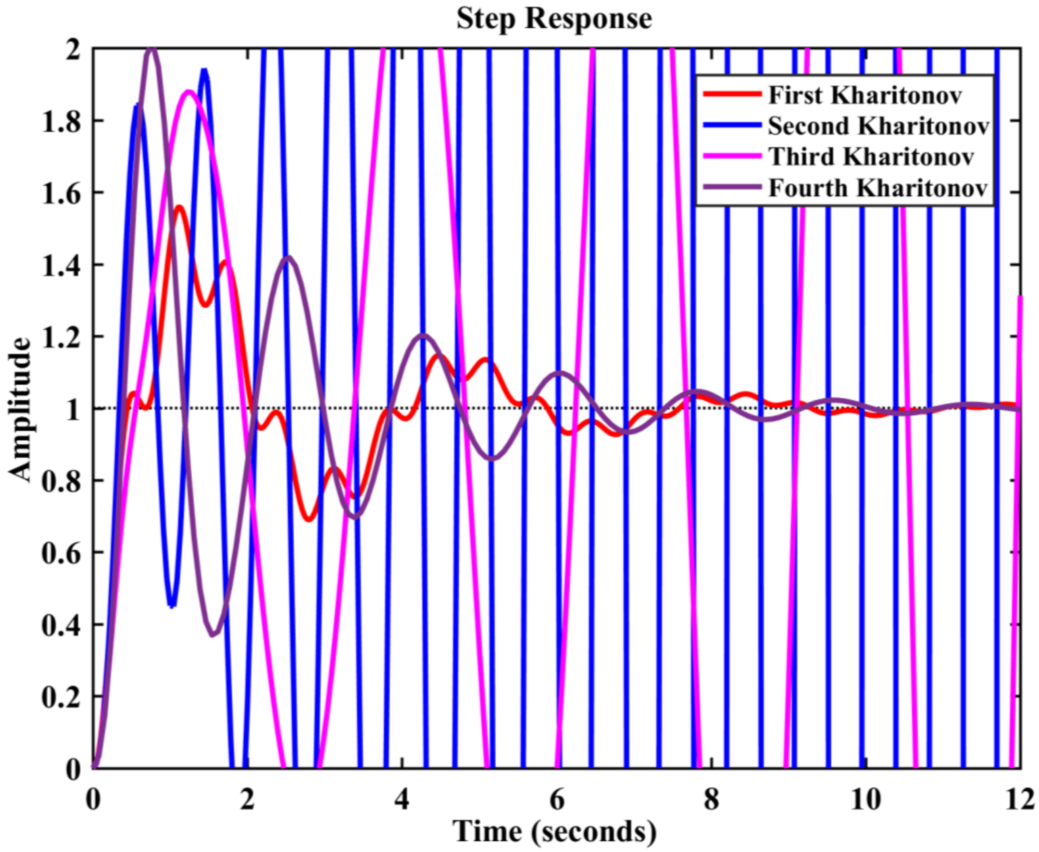

Fig.5.6. Closed loop step response with PID controller for all extreme Plants using the Ziegler-Nichols

NLP problem with minimum number of equations using proposed stability conditions and PSO algorithm is much less than the method given in [18]. Thus, the developed PI controller using necessary and sufficient conditions of interval polynomial robustly stabilizes the Wing aircraft. These stability conditions can be implemented easily for determining the stability of higher order interval plants.

Design of PID controller

The transfer function of the PID controller is given by

$$C_{PID}(s) = K_P + \frac{K_I}{s} + K_D s = \frac{N_c(s)}{D_c(s)}$$

Then the closed loop transfer function with PID controller becomes

$$T(s) = \frac{[54K_D, 74K_D]s^3 + [90K_D + 54K_P, 166K_D + 74K_P]s^2 + [90K_P + 54K_I, 166K_P + 74K_I]s + [90K_I, 166K_I]}{[1,1]s^5 + [2.8, 4.6]s^4 + [54K_D + 50.4, 74K_D + 80.8]s^3 + [90K_D + 54K_P + 30.1, 166K_D + 74K_P + 33.9]s^2 + [90K_P + 54K_I - 0.1, 166K_P + 74K_I + 0.1]s + [90K_I, 166K_I]} \quad (5.6)$$

From the above equation, the characteristic equation of the closed loop interval system with PID controller can be taken as

$$[1,1]s^5 + [2.8, 4.6]s^4 + [54K_D + 50.4, 74K_D + 80.8]s^3 + [90K_D + 54K_P + 30.1, 166K_D + 74K_P + 33.9]s^2 + [90K_P + 54K_I - 0.1, 166K_P + 74K_I + 0.1]s + [90K_I, 166K_I] = 0 \quad (5.7)$$

The step response PID controller (K_P = 1.3172, K_I = 1.9378 and K_D=0.1791) using

Ziegler-Nichols settings [41] are shown in Figure 5.6. From this Figure 5.6, it has been observed that the designed PI controller from Ziegler-Nichols tuning method cannot stabilize the given interval plant at all operating conditions. Hence it is necessary to redesign the PID controller to stabilize given interval plant. By applying the necessary and sufficient conditions for the above 5th order polynomial (5.7), six sets of inequality constraints are obtained. The controller parameters K_P, K_I and K_D are obtained by solving these inequality constraints using PSO such that the objective function $J = \int_0^\infty e^2(t)dt$ is minimized. Then the controller parameters $K_P = 0.5933, K_I = 0.001$ and $K_D = 0.252$ are obtained. The closed loop step response of system with PID controller using the proposed method for all extreme plants is shown in Figure 5.7 for ε = 1 respectively. The time domain specifications of Figure 5.7 are tabulated in Table 5.2 which describes the efficacy of the proposed method.

From the equation (5.4) the closed transfer function of wing aircraft with PID controller is given by

$$T^1(s) = \frac{64K_D s^3 + (128K_D + 64K_P)s^2 + (128K_P + 64K_I)s + 128K_I}{[1, s^5 + 3.7s^4 + (64K_D + 65.6)s^3 + (128K_D + 64K_P + 32)s^2 + (128K_P + 64K_I)s + 128K_I]} \quad (5.8)$$

The closed loop step response of the system with PID controller ($K_P = 0.5933, K_I = 0.001$ and $K_D = 0.252$) from proposed methods is shown in Figure 5.8.The PID controller designed from proposed method can stabilize can stabilize the given plant if any changes occur in the uncertain parameters within the bounds.

Table 5.2. Time domain specifications for proposed method

Name of the Kharitonov Polynomial	% peak overshoot MP	Peak time t_p(sec)	Rise time t_r(sec)	Settling time t_s(sec)	ISE $*10^{-6}$
First	4.934	4.179	1.4975	6.3771	10.381
Second	15.79	2.096	0.9614	5.0017	0.1072
Third	12.41	4.015	1.8559	6.9367	56.76
Fourth	15.26	2.189	0.9886	5.4216	0.4.348

It has been observed from the simulation results of Figure 5.7 that the designed PID controller robustly stabilizes the plant very quickly. From Table 5.2, it is evident that the designed PID controller stabilizes the plant with lesser time domain parameters. The proposed method involves six sets of equations for NLP to solve. Thus, the proposed method requires less computational complexity than the methods in the literature It has been observed from figure 5.8, that the designed PID controller from the proposed method is used to determine the robustness in any changes in the uncertain parameter of the given plant This shows the efficacy of the proposed method in terms of time domain specifications. It is also observed in the computation time required for solving the NLP problem using stability conditions and PSO algorithm is much less. Thus, the developed PID controller using necessary and sufficient conditions of interval polynomial robustly stabilizes the Wing aircraft.

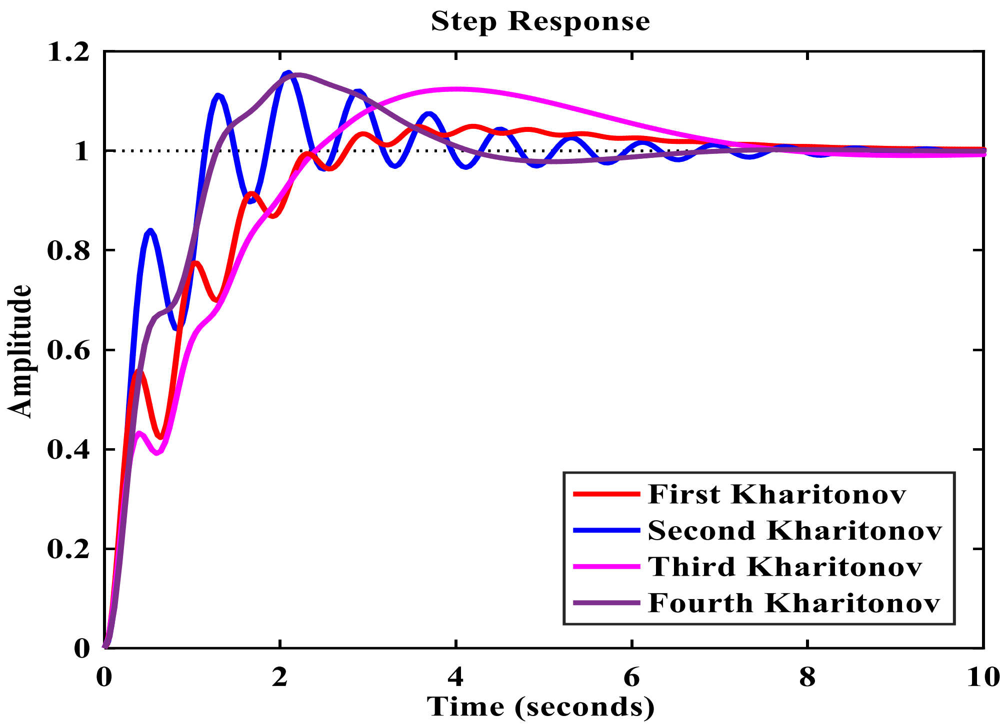

Fig.5.7. Closed loop step response with PID controller for all extreme Plants using the proposed method.

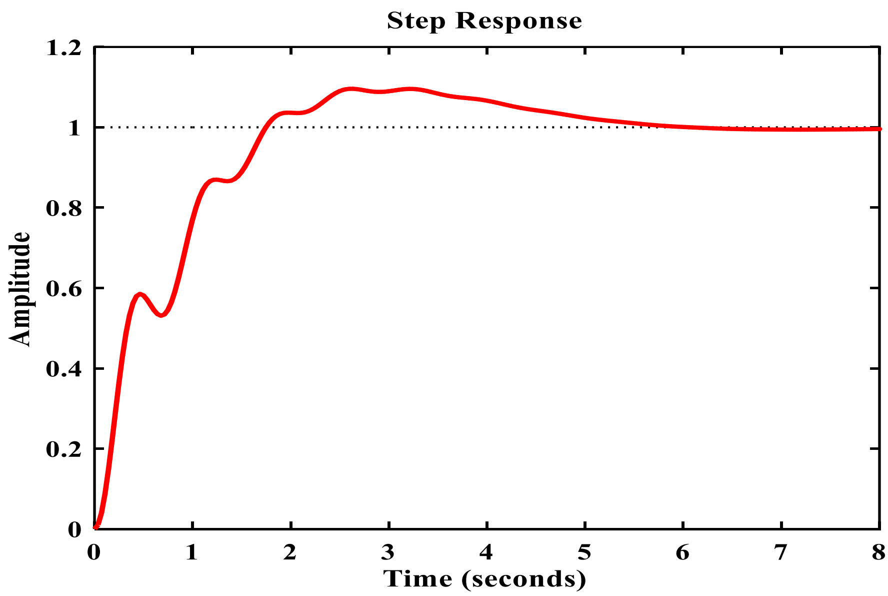

Fig.5.8. Closed loop step response to the PID controller for changes in the uncertain parameter to determine the robustness of the proposed method

Example 2

Consider the dynamics of a vast range of chemical processes which is described by the interval First-Order Plus Dead Time (FOPDT) model [30] is given by

$$\overline{G}(s) = \frac{K}{\tau s + 1} e^{-t_d s} \tag{5.9}$$

Where $K \in [3,9]$, $\tau \in [10,18]$, $t_d \in [1,6]$

In this, the nominal value variations in the process gain K, time constant τ and dead time TD are taken as ±50%, ±28.57%, and ±71.43%. The nominal design execution is continued based on the nominal values of $K^0 = 6, \tau = 14, t_d = 3.5$. The nominal PI controller parameters $K_c^0 = 0.6 \text{ and } \tau_i^0 = 11.655$ and nominal PID controller parameters

$K_c^0 = 0.68$ $\tau_i^0 = 6.993$ and $\tau_d^0 = 1.74825$ are designed by the Ziegler-Nichols settings [41]. The step response PI/PID controller using Ziegler-Nichols is shown in Figure 9 and 10. From this Figure 9 and 10,it has been observed that the designed PI/PID controller from Ziegler-Nichols cannot stabilize the given interval plant at all operating conditions. Hence it is necessary to redesign the PI/PID controller to stabilize given interval time delay plant. From the Corollary 3.1, $e^{-[1\ 6]s}$ can be written as $\frac{1-[0.5\ \ 6.5]s}{1+[0.5\ \ 6.5]s}$. Then the following delay-free model for original interval FOPDT is given as

$$G(s,c,d) = \frac{-[1.5, 58.5]s + [3, 9]}{[5, 117]s^2 + [10.5, 24.5]s + 1} \tag{5.10}$$

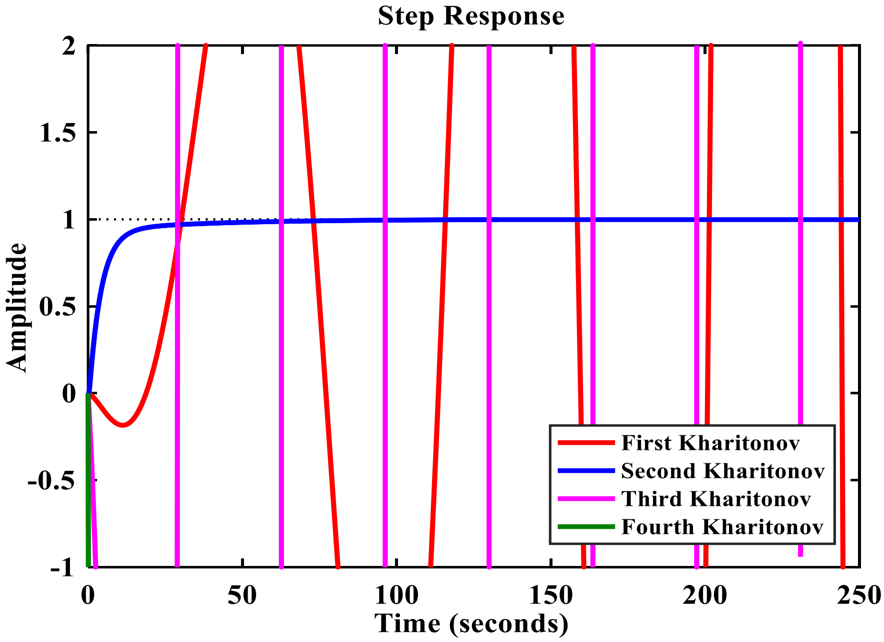

Fig. 5.9. : Closed loop step response to the PI controller for all extreme plants using Ziegler-Nichlos method with time delay.

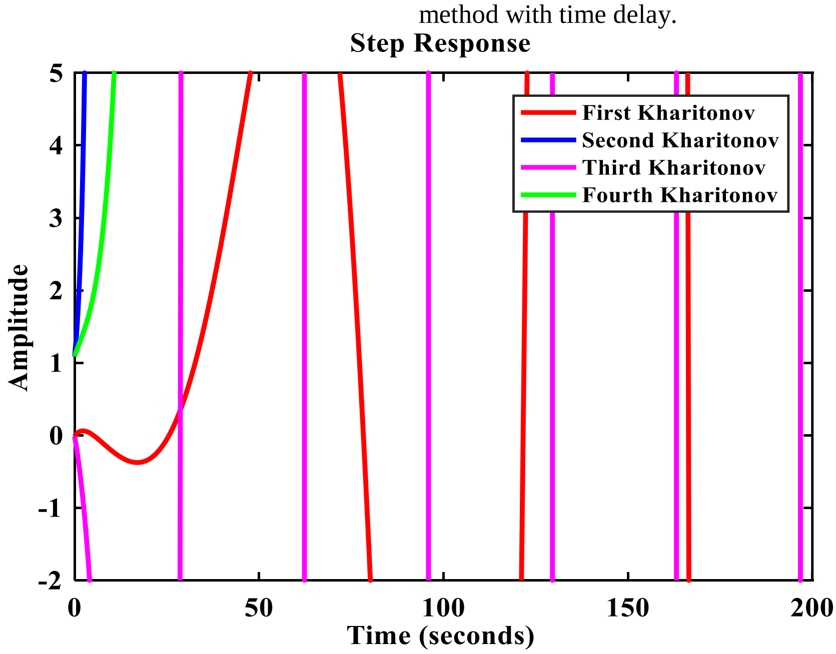

Fig. 5.10: Closed loop step response to the PID controller for all extreme plants using Ziegler-Nichlos method with time delay.

Design of PI controller:

Consider the PI controller of the form

$$C_{PI}(s) = K_c(1+\frac{1}{\tau_i s}) = \frac{N_c(s)}{D_c(s)}$$

Then the closed loop transfer function with a PI controller becomes

$$T(s) = \frac{-[1.5K_c\tau_i, 58.5K_c\tau_i]s^2 + [3K_c\tau_i - 58.5K_c, 9K_c\tau_i - 1.5K_c]s + [3K_c, 9K_c]}{[5\tau_i, 117\tau_i]s^3 + [10.5\tau_i - 58.5K_c\tau_i, 24.5\tau_i - 1.5K_c\tau_i]s^2 + [\tau_i + 3K_c\tau_i - 58.5K_c, \tau_i + 9K_c\tau_i - 1.5K_c]s + [3K_c, 9K_c]} \tag{5.11}$$

From the above equation, the characteristic equation of the closed loop interval system with PI controller can be taken as

$$\begin{aligned}&[5\tau_i, 117\tau_i]s^3 + [10.5\tau_i - 58.5K_c\tau_i, 24.5\tau_i - 1.5K_c\tau_i]s^2 + \\ &[\tau_i + 3K_c\tau_i - 58.5K_c, \tau_i + 9K_c\tau_i - 1.5K_c]s + [3K_c, 9K_c] = 0\end{aligned} \tag{5.12}$$

By applying the necessary and sufficient conditions for the above 3rd order polynomial (5.12), the following set of inequality constraints is obtained. In order to make this set of constraints into the feasible closed set, a small positive number 'ε' is introduced into the constraints. Hence the optimization problem can be stated as to find K_c and τ_i such that the objective function $J = \left\|\frac{K_c - K_c^0}{K_c^0}\right\|^2 + \left\|\frac{\tau_i - \tau_i^0}{\tau_i^0}\right\|^2$ is minimized, subjected to the following constraints.

Necessary conditions:

$$-3K_c + \varepsilon < 0$$

$$-3K_c\tau_i + 58.5K_c - \tau_i + \varepsilon < 0$$

$$-10.5\tau_i + 58.5K_c\tau_i + \varepsilon < 0$$

$$-5\tau_i + \varepsilon < 0$$

Sufficient condition:

$$-(\tau_i + 3K_c\tau_i - 58.5K_c)^2 + 27K_c(24\tau_i - 1.5K_c\tau_i) + \varepsilon < 0$$

The linear programming problem consists of two decision variables and five constraints. The controller parameters Kc and τ_i are restricted to small values by choosing the objective function J purposely. In this work, the PSO algorithm proposed in section 4 is used to minimize the objective function J. By applying the proposed algorithm, then the values of controller parameters are obtained. As shown in Table 5.3, the values of the controller parameters K_c and τ_i increase as 'ε' are increased which shows the sensitivity of the controller parameters with respect to the NLP parameter 'ε'. The closed loop step response of the system with the PI controller by the proposed method (Kc = 4.0432 and τ_i = 0.00926) and the method given in [30] (Kc = 0.0684 and τ_i = 12.5102) are shown in Figures 5.11 and 5 .12 for ε = 0.05 respectively. The time domain specifications of Figures 5.11 and 5.12 are shown in Table 5.4 which describes the efficacy of the proposed method.

In order determine the robustness of proposed method, the given interval plant G(s,c,d) from equation (5.10) can be written as

$$G(s) = \frac{-30s + 6}{61s^2 + 17.5s + 1} \tag{5.13}$$

Here G(s) is obtained by averaging the upper and lower bound of s coefficients of G(s,c,d). The closed transfer function of wing aircraft with PI controller is given by

$$T(s)=\frac{-30K_c\tau_i s^2+(6K_c\tau_i-30K_c)s+6K_c}{61\tau_i s^3+(17.5\tau_i-30K_c\tau_i)s^2+(\tau_i+6K_c\tau_i-30K_c)s+6K_c} \tag{5.14}$$

The closed loop step response of the system with PI controller (Kc = 4.0432 and τ_i = 0.00926) from proposed method is shown in Figure 5.13. The PI controller designed from proposed method can stabilize the given plant if any changes occur in the uncertain parameters within the bounds .

Table 5.3. Variation of K_c and τ_i for different values of ε

Controller set	ε	Proposed method		Existing method	
		K_c	τ_i	K_c	τ_i
1	0.05	0.00926	4.0432	0.0684	12.5102
2	1	0.01389	8.0934	0.0683	12.5110
3	1.5	0.01803	10.4032	0.0683	12.5116
4	2	0.02059	11.9023	0.0683	12.5121

Table 5.4. Time domain specifications for proposed method

Name of the Kharitonov Polynomial	Rise time t_r (sec)	Settling time t_s (sec)
First	222.2482	422.5395
Second	294.5659	526.1731
Third	43.3195	139.2935
Fourth	92.4901	166.2970

It has been observed from the Figure 5.11 and 5.12 that the designed PI controller, which uses the proposed stability conditions robustly stabilizes the plant when compared to the method given in [30]. From Figure 5.12, it is observed that the closed loop interval time delay system with a controller is unstable by the method in [30]. Whereas closed loop interval time delay system with proposed method is stable. It has been observed from figure 5.13, that the designed PI controller from the proposed method is used to determine the robustness in any changes in the uncertain parameters of the given plant. This shows that proposed method robustly stabilizes the interval time delay process plant. The proposed method has five inequality constraints which are less compared with the method (which has seven inequality constraints) in [30]. Hence the proposed method is simple and requires less computation as it uses a lesser number of inequality constraints than the method given in [30]. It is also observed that the computation time required for solving the NLP problem using proposed stability conditions and PSO algorithm is 3.872603 seconds, which is much less than the method given in [30]. Thus, the developed PI controller using necessary and sufficient conditions of interval polynomial robustly stabilizes the interval process time delay system. These stability conditions can be implemented easily for determining the stability of higher order interval plants.

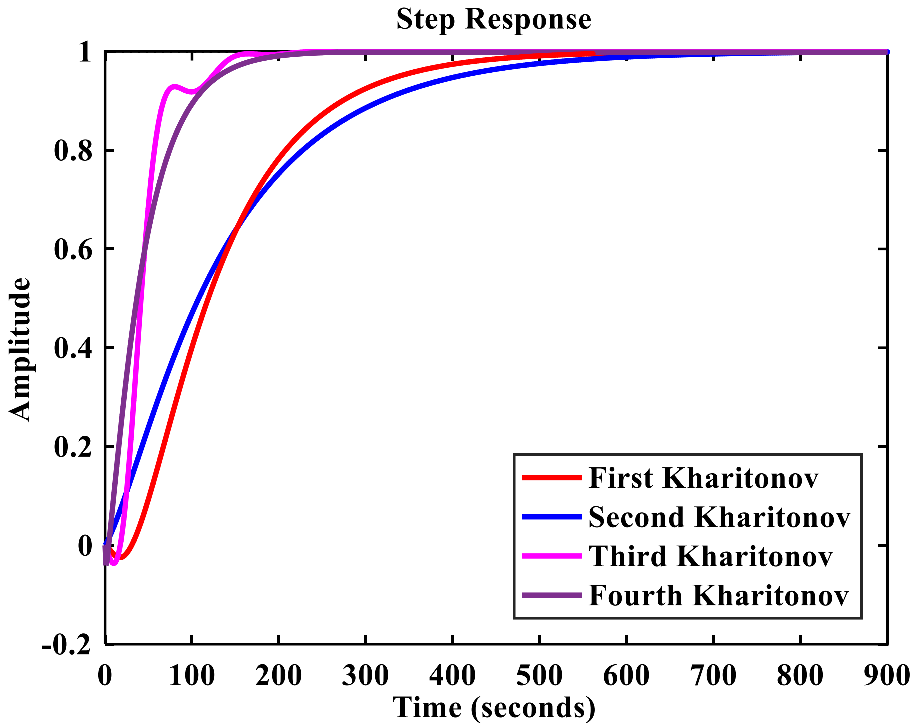

Fig.5.11. Closed loop step response with a PI controller for all extreme plants using proposed method with time delay.

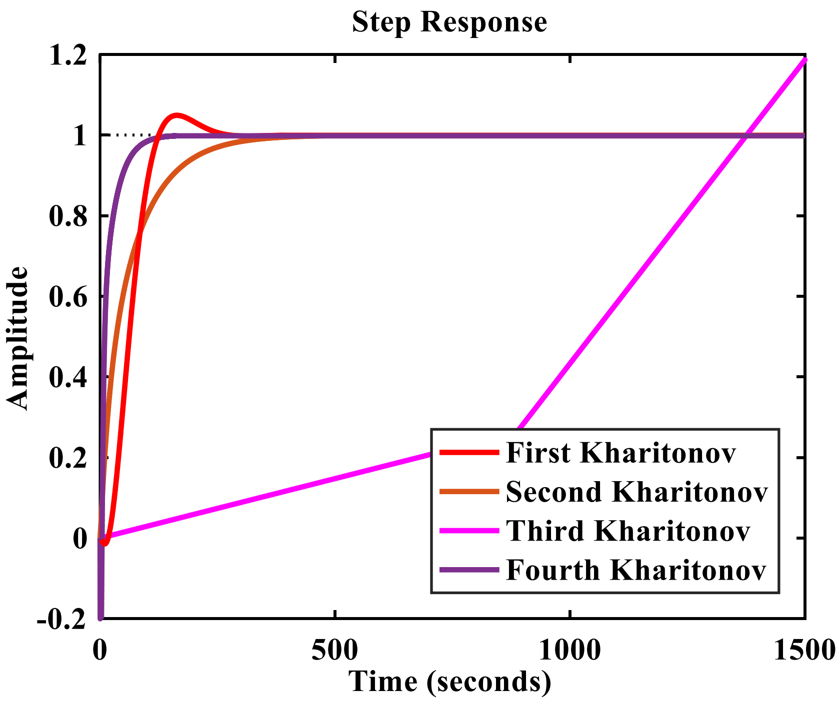

Fig.5.12. Closed loop step response to the PI controller for all extreme plants using the existing method [30].

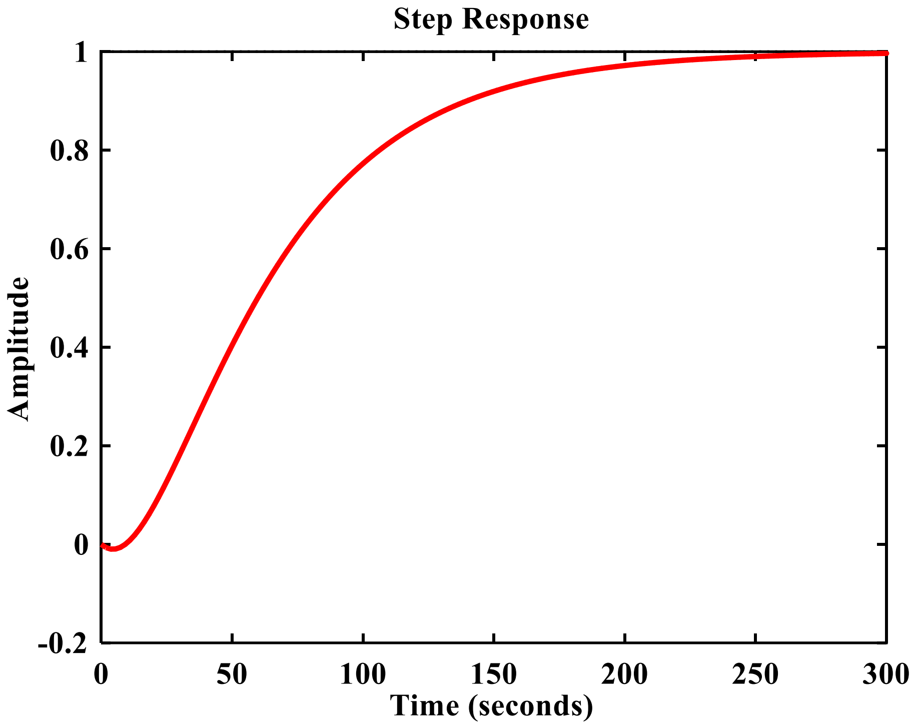

Fig.5.13. Closed loop step response to the PI controller for changes in the uncertain parameter to determine the robustness for proposed method

Design of PID controller:

Consider the PID controller of the form

$$C_{PID}(s) = K_c\left(1+\frac{1}{\tau_i s}+\tau_d\right) = \frac{N_c(s)}{D_c(s)}$$

Then the closed loop transfer function with PID controller becomes

$$T(s) = \frac{-[1.5K_c\tau_i\tau_d\ ,58.5K_c\tau_i\tau_d\ ]s^3+[3K_c\tau_i\tau_d\ -58.5K_c\tau_i\ ,9K_c\tau_i\tau_d\ -1.5K_c\tau_i]s^2 \quad [3K_c\tau_i-58.5K_c\ ,9K_c\tau_i-1.5K_c]s+[3K_c\ ,\ 9K_c]}{[5\tau_i-58.5K_c\tau_i\tau_d,\ 117\tau_i-1.5K_c\tau_i\tau_d]s^3+[10.5\tau_i+3K_c\tau_i\tau_d-58.5K_c\tau_i\ , \ 24.5\tau_i+9K_c\tau_i\tau_d-1.5K_c\tau_i]s^2+[\tau_i+3K_c\tau_i-58.5K_c\ ,\tau_i+9K_c\tau_i-1.5K_c]s+[3K_c,9K_c]} \tag{5.15}$$

From the above equation, the characteristic equation of the closed loop interval system with PID controller can be taken as

$$[5\tau_i-58.5K_c\tau_i\tau_d,\ 117\tau_i-1.5K_c\tau_i\tau_d\ ]s^3+[\ 10.5\tau_i+3K_c\tau_i\tau_d-58.5K_c\tau_i\ ,24.5\tau_i+9K_c\tau_i\tau_d-1.5K_c\tau_i\ ]s^2 + [\ \tau_i+3K_c\tau_i-58.5K_c\ ,\tau_i+9K_c\tau_i-1.5K_c\ ]s+[\ 3K_c, 9K_c\ ]=0 \tag{5.16}$$

By applying the necessary and sufficient conditions of equations for the above 3rd order polynomial (5.16), set of inequality constraints are obtained. Hence the optimization problem can be stated as to find K_c, τ_i and τ_d such that the objective function $J=\left\|\frac{K_c-K_c^0}{K_c^0}\right\|^2+\left\|\frac{\tau_i-\tau_i^0}{\tau_i^0}\right\|^2+\left\|\frac{\tau_d-\tau_d^0}{\tau_d^0}\right\|^2$ is minimized. By using the proposed method given in section 4 to the above NLP problem, the controller parameters K_c, τ_i and τ_d are obtained. As shown in Table 5.5, the values of the controller parameters K_c, τ_i and τ_d increase as 'ε' are increased which shows the sensitivity of the controller parameters with respect to the NLP parameter 'ε'. They are given by $K_c=0.00914, \tau_i=3.1520$ and $\tau_d=$ 1.2040. The closed loop step response of system with PID controller for proposed method is shown in Figure 5.14 for ε = 1. The time domain specifications of Figure 5.14 are shown in Table 5.6 which describes the efficacy of the proposed method in terms of the time domain specifications.

Table 5.5. Variation of Kc, τ_i and τ_d for different values of ε

Controller Set	ε	K_c	τ_i	τ_d
1	0.05	0.00914	3.1520	1.2040
2	1	0.00925	6.8461	1.7481
2	1.5	0.00930	7.1864	1.8998
4	2	0.00977	7.4979	2.1075

Table 5.6. Time domain specifications for proposed method

Name of the Kharitonov Polynomial	Rise time t_r (sec)	Settling time t_s(sec)
First	152.3465	283.6998
Second	230.6303	381.7915
Third	29.0473	131.9949
Fourth	72.5310	119.3718

From equation (5.10) the closed loop transfer function of wing aircraft with PID controller is given by

$$T(s)=\frac{-30K_c\tau_i\tau_d\, s^3+(6K_c\tau_i\tau_d-30K_c\tau_i)s^2+(6K_c\tau_i-30K_c)s+6K_c}{(61\tau_i-30K_c\tau_i\tau_d)s^3+(17.5\tau_i+9K_c\tau_i\tau_d-30K_c\tau_i)s^2+[\tau_i+6K_c\tau_i-30K_c)s+6K_c} \quad (5.17)$$

The closed loop step response of the system with PID controller ($K_c=0.00914, \tau_i=3.1520$ and τ_d = 1.2040.) from proposed methods is shown in Figure 5.15. The PID controller designed from proposed method can stabilize the given plant if any changes occur in the uncertain parameters within the bounds.

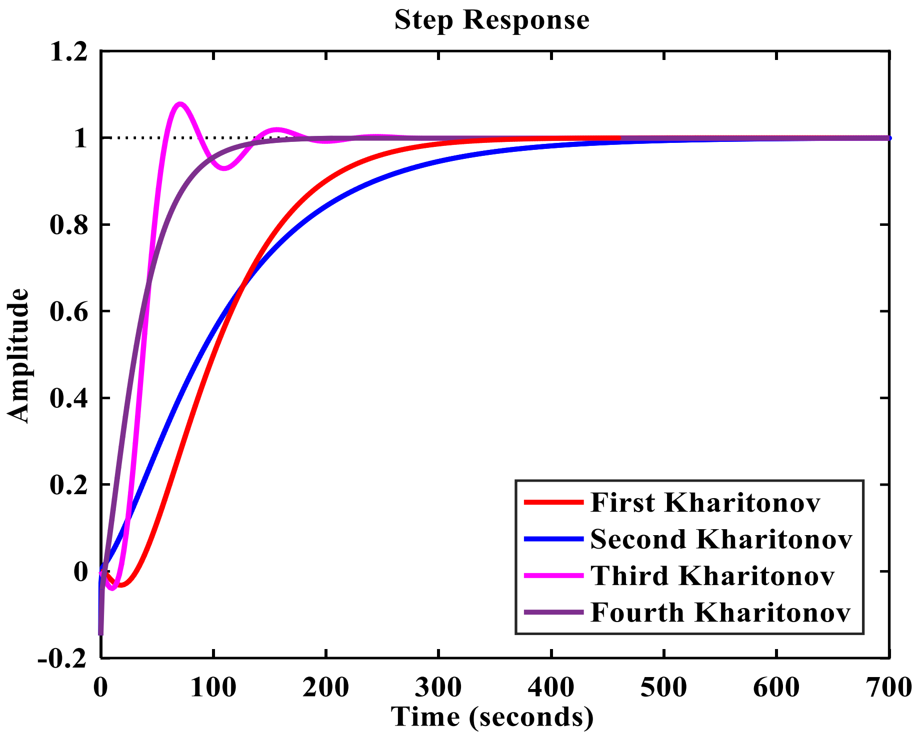

Fig.5.14. Closed loop step response with PID controller for all extreme plants using the proposed method with time delay.

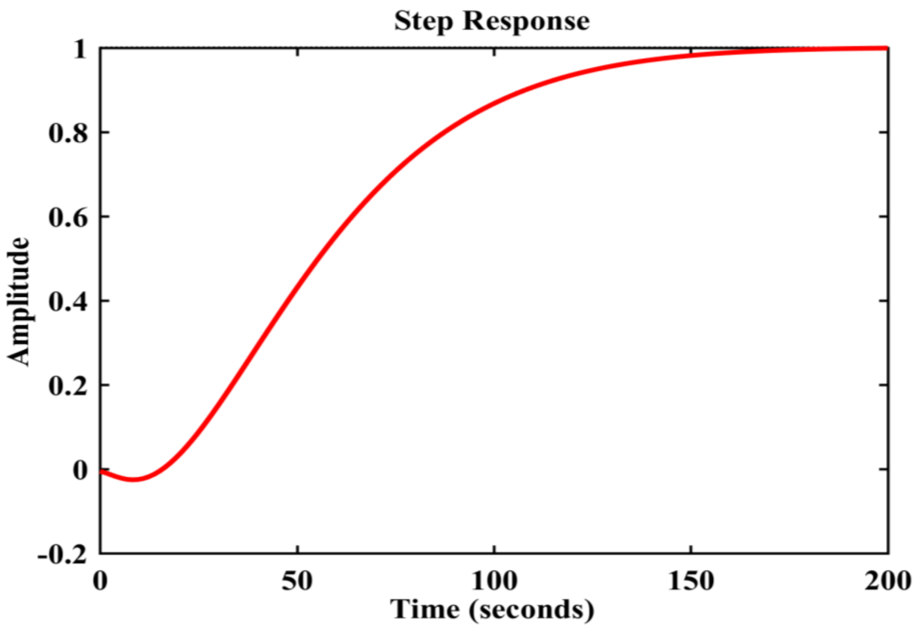

Fig.5.15. Closed loop step response to the PID controller for changes in the uncertain parameter to determine the robustness with time delay for the proposed method

It has been observed from the simulation results of Figure 5.14 that the designed PID controller robustly stabilizes the plant very quickly. From Table 5.6, it is evident that the designed PID controller stabilizes the plant with lesser time domain parameters. This shows that proposed method robustly stabilizes the interval time delay process plant. It has been observed from figure 5.15, that the designed PI controller from the proposed method is used to determine the robustness in any changes in the uncertain parameters of the given plant. The proposed method has five inequality constraints which are less compared with the method (which has seven inequality constraints) in [30]. Hence the proposed method is simple and requires less computation as it uses a lesser number of inequality constraints than the method given in [30]. This shows the efficacy of the proposed method in terms of time domain specifications. It is also observed that the computation time (4.63 seconds) required for solving the NLP problem using the proposed stability conditions and PSO algorithm is much less. Thus, the developed PID controller using necessary and sufficient conditions of interval polynomial robustly stabilizes the interval process time delay system.

6. CONCLUSIONS

A robust stabilizing PI/PID controller is designed for interval process plant with and without time delay based on the newly developed necessary and sufficient conditions using PSO algorithm. A set of inequality constraints in terms of controller parameters are derived from interval polynomial based on the new necessary and sufficient conditions. Consequently, these inequalities are solved using PSO to obtain controller parameters. The proposed PI/PID controller procedure is also applied and demonstrated through typical numerical examples. It is observed that the designed PI/PID controller robustly stabilizes the plant with and without time delay with lesser time domain parameters than the existing methods. The PI/PID controller designed from proposed method can stabilize the given plant if any changes occur in the uncertain parameters within the bounds The simulation results are evidence for its robustness in stabilizing the interval process time delay plant using a PI/PID controller. Also, the proposed method is simple and involves less computational complexity in comparison with the methods available in the literature.

REFERENCES

1. Astrom, K. J., Hagglund, T., Hang, C. C. & Ho, W.K. (1993). Automatic tuning and adaptation for PID controllers- a survey, *Control Engineering Practice*, 1(4), 699-714.

2. Astrom, K. J. & Hagglund, T. (1995). PID Controllers: Theory, Design, and Tuning, ISA: The Instrumentation, *Systems, and Automation Society*, Research Triangle Park, NC.

3. Ho, W. K., Hang, C. C. & Cao, L. S. (1995). Tuning of PID controllers based on gain and phase margin specifications, *Automatica*, 31(3), 497-502.

4. Ho, M. T., Datta, A. & Bhattacharyya, S. P. (1996). A new approach to feedback stabilization, *Proc. of the 35th IEEE conf. on Decision and Control*, 4(7), 4643-4648.

5. Yongling Zheng, Longhua, Ma, Liyan Zhang & Jixin Qian. (2003). Robust PID controller design using the particle swarm optimizer, *Intelligent Control*,IEEE International Symposium, paper-12, 974-979.

6. Ho, M. T., Datta, A. & Bhattacharyya, S. P. (1997). A linear programming characterization of all stabilizing PID controllers, *Proc. of American Control Conf,* 6(23,). 3922-3928.

7. Ho, M. T., Datta, A.& Bhattacharyya, S. P. (1997). A new approach to feedback design Part II: PI and PID controllers, *Dept. of Electrical Eng.*, Texas A&M Univ., College Station, TX, Tech. Report TAMUECE 97- 001-B.

8. Satpati, Biplab, Chiranjib Koley & Subhashis Datta.(2014). Robust PID controller design using particle swarm optimization-enabled automated quantitative feedback theory approach for a first-order lag system with minimal dead time, *Systems Science and Control Engineering*, 2 (1), 502-511.

9. Kalyani, T. & Siva Kumar, M. (2013). Computational of optimal stabilizing PID controller for interval plants using PSO, *International Journal Engineering Research and Technology*,2(10), 2993-2998.

10. Barmish, B. R. (1989). A generalization of Kharitonov's four polynomial concept for robust stability problems with linearly Dependent coefficient perturbations, *IEEE Trans. Automatic Control*, 34(2),157- 165.

11. Deore, P.J. & Patre, B.M. (2005). Design of robust compensator for jet engine: An interval analysis approach, *proc. of the IEEE conf. on control*, Toronto, Ont., Canada,376-381.

12. Nie, Y.Y. (1976). A new class of criterion for the stability of the polynomial, *Act.Mechnica sinica*, 15(1),110-116.

13. Chen, C.T. & Wang, M.D. (1997). Robust controller design for interval process system', *computer and chemical engineering*, .21(7), 739-750.

14. Chen, CT. Fan, CK. & Peng, ST. (2004). Robust control system design for interval time delay process systems. *Chin Inst Chem Engrs*, 35(2), 213–223.

15. Bhattachrya,S.P.(1987). Robust stabilization against structured perturbations', L*ect. Notes in Control and Information Sciences,* Springer Verlag, Berlin, 99, 78-99.

16. Satpati, B., & Sadhu, S. (2008). Course changing control for a cargo mariner class ship using quantitative feedback theory. IE(I), *Marine Divisional Journal*, 88, 1–8.

17. Patil, M. D., Nataraj, P. S. V., & Vyawahare, V. A. (2012). Automated design of the fractional PI QFT controller using interval constraint satisfaction technique (ICST). *Nonlinear Dynamics*, 69(3), 1405–1422.

18. Patre, B.M. & Deore, P.J. (2003). Robust Stabilization of Interval Plants', *European Control Conf. ECC-03*, University of Cambridge (UK), pp.1660-1663.

19. Patre, B.M. & Deore, P.J. (2007). Robust stability and performance for interval process plants, *ISA transactions*, 46(3),343-349.

20. Siva Kumar, M., Ramalinga Raju,M., Srinivasa Rao, D. & Bala Bhargavi, T. (2015). Design of Robust PI and PID Controller for DC Motor Fuzzy Parametric Approach, *Global Journal of Mathematical Sciences (GJMS), Special Issue for Recent Advances in Mathematical Sciences and Applications-13*, 2(2),77–83.

21. Siva Kumar, M. & Bala Bhargavi, T. (2013). Design of Robust PI/PID Controller Discrete-Time Parametric Uncertain System', *International Journal of Scientific Research*,.2(10),5-9.

22. Srinivasa Rao,D., Siva Kumar,M. & Ramalinga Raju,M.(2018). New algorithm for the design of robust PI controller for plants with parametric uncertainty, *Trans. of the Institute of Measurement and Control*, 40(5), 1481-1489.

23. Rahul Mittal & Manisha Bhandari. (2015). Design of robust PI controller for active suspension system with uncertain parameters, *IEEE conf. on signal processing, computing and control at Waknaghat* ISBN: 978-1-4799-8435-0, 333-337.

24. Ghosh B. K. (1985). Some new results in the simultaneous stabilization of a family of single input, single output Systems, *Systems and Control Letters*, 6(1), 39-45.

25. Hollot, C.V. & Yang, F. (1990). Robust stabilization of interval plants using lead or lag compensators, *Systems and Control Letters*, 14(1), 9-12.

26. Barmish, B. R., Hollot, C. V., Kraus, F. J. & Tempo, R. (1992). Extreme point results for robust stabilization of interval plants with first order compensators, *IEEE Trans. on Automatic Control,* 37(6),707- 714.

27. Jian'an Fang, Da Zheng, & Zhengyun Ren. (2009). Computation of stabilizing PI and PID controllers by using Kronecker summation method, *Energy Conversion and Management*, 50(7),1821–1827.

28. Nusret Tan, Ibrahim Kaya, Celaleddin Yeroglu & Derek P. Atherton. (2006). Computation of stabilizing PI and PID controllers using the stability boundary locus, *Energy Conversion and Management*, 47(18–19),3045–3058.

29. Barmish, B. R. & Kang, H. I. (1993). A survey of extreme point results for robustness of control systems', *Automatica*, 29(1),13-35.

30. Patre, B.M. & Deore, P.J. (2011). Robust stability and performance of interval process plant with interval time delay', *Transactions of the Institute of measurements and Control*, 34 (5), 627-634.

31. Farkh, Rihem, Kaouther Laabidi & Mekki Ksouri. Robust PI/PID controller for interval first order systems with time delay, *International Journal of Modelling, Identification and Control*13.1-2 (2011): 67-77.

32. Brosilow CB. (1979. The structure and design of smith predictors from viewpoint of inferential control, *In: Joint Automatic Control Conference*, 16, 288.

33. Laughlin DL, Rivera DE & Morari M. (1987). Smith predictor design for robust performance, *International Journal of Control*, 46(2), 477-504.

34. Wang, ZQ. & Skogestad, S. (1993). Robust control of time delay systems using the Smith predictor, *International Journal on control 57*: 1405.

35. Kennedy, J. & Eberhart, R.C.(1995) 'Particle Swarm Optimization', *Proc. IEEE International Conf. on Neural Networks IV*, Piscataway, NJ, 1942-1948

36. Gaing, Z-L. (2004) 'A particle swarm optimization for optimum design of PID Controller in AVR system', *IEEE Trans. on energy conversion*,19 (2), 384-391.

37. Angeline P.J. (1998). Using selection to improve particle swarm optimization, *Proc.IEEE Int. Conf. Evol. Comput.*, Anchorage, A K, 84–89.

38. Anderson, BDO., Jury, EI. & Mansour, M. (1987). On Robust Hurwitz polynomials', *IEEE Trans. on Automatic control*, 32 (10), 909-913.

39. Guillemin, E.A. (1962). *The Mathematics of Circuit Analysis*, 7th Ed. Calcutta, India: Oxford and IBH Publishing, 327-330.

40. *Optimization Toolbox*: Math works Inc, User guide 2010.

41. Ziegler, J.B. and Nichols, N.B. (1942). Optimum settings for automatic controllers, *Trans. ASME*, 64,759-768.

