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Multistate System Performance Analysis Incorporating Human Error, Mathematical modeling and Reliability Approach

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Abstract: This paper proposes a novel framework for analyzing the performance of a Sugar mill (Wahid Sugar Mill, situated in Punjab, India) with human errors, mathematical modeling, and reliability approach. The proposed framework considers the complex interactions between the different components of the Sugar mill and the potential impact of human error on the same. The performance of the considered system is affected by the different components failures and unplanned outages. Considering these facts, a mathematical model is developed for the sugar mill to calculate different reliability measures such as availability, reliability, and MTTF. The mathematical modeling aspect of the framework utilizes Markov chains to model the stochastic behavior of the sugar mill. The authors also perform sensitivity analysis to identify the impact of different components' failure on the performance of the same. The results of the paper demonstrate the potential of the proposed framework in providing valuable insights into the performance of the Sugar Mill under different scenarios, including the impact of human error.

Keywords: Reliability measures; Unplanned outages; Sensitivity analysis; Sugar mill.

1. INTRODUCTION

In this age of science and technology, heavy and automated machines are being used to enhance production in industries, but at the same pace, it increases the complexity of the associated industry. So, to maintain the desired production of a system, the maintenance team takes good care of different components of the same. It can be done by obtaining the various performance measures of the system as well as by keeping the proper track of the maintenance of the components. Hence, keeping all these things into consideration, in the present study, we consider a sugar mill in Punjab, India, a complex system comprising many components like an unloader, conveyor, cutter, crusher, bagasse carrying machine, and boiler. Failure of any of these components may lead the whole system to a degraded or failed state. These failures may be mechanical, electrical or due to human operators. Human operators sometimes give wrong commands to the machines, which may cause the failure of costly components or machines. Sometimes, humans do not pay proper attention to their work because they feel tired or bored inside the mill. Sometimes failures also occur due to power outages, corrosion, manufacturing defects and wear out, natural calamities like earthquakes or tornadoes, etc. These system failures can't be avoided (expect natural calamities) but can be mitigated with proper repair and maintenance or using the redundancy in the system. Nandhini and Padmavathy [10] analyzed the year-wise production of sugarcane and described the reason for the production change in the sugar cane industry from 2000-2010. Zhao and Li [20] presented the effects of climate change on sugar cane production and described the strategies to mitigate the effects of climate change. Performance analysis of the sugar mill taking various parameters into consideration was presented by [12]. Sharma and Vishwakarma [16] utilized the Genetic

Algorithm technique to analyze the performance of the feeding system in the sugar industry. Zaidi [19] examined the transient and steady-state behavior of the feeding system within the sugar industry. Dahiya et al. [4] employed a fuzzy reliability approach to develop a mathematical model for the A-pan crystallization system in the sugar industry. Navyata et al. [11] evaluated the reliability, availability, and MTTF of a dual-channel logic communication system using the Boolean function technique. Tewari and Kumar [17] presented the availability analysis of the milling system in the rice milling plant. Bansal et al. [2] applied the Boolean function technique for the evaluation of reliability parameters for a milk powder plant manufacturing plant. Li [9] discussed the system's comparison between active and standby redundancy. Some industrial systems take rest after working for some specific amount of time, such as an industrial system that works under a cost-free warranty and rest policy were presented by Kumar and Kumar [8]. Reliability and sensitivity analysis of a thermal power plant were presented [13]. k-out-of-n: The F/G system has gained popularity in the industrial system and is used for improving the reliability of the system. Ram and Kumar [14] presented a study on the performance analysis of an industrial system using a *2*-out-of-*3*: *F* configuration considering human error. In a separate work, Ram and Manglik [15] investigated a system with parallel redundancy with human error, partial failure, and catastrophic failure and evaluated the various reliability parameters.

In the literature, it has been observed that repair facilities may not always be available with the system. A new approach that repairmen can take multiple vacations when there is no product for repair was presented by [18]. Kalaiarasi et al. [6] analyzed a system consisting of four components with a human error rate with the help of Markov modeling. Haggag [5] presented the profit analysis and availability of 3-out-of-4 systems under preventive maintenance. Chatterjee and Nath [3] presented a case study on an Indian railway passenger reservation system using a smart computing application. Aly et al. [1] presented the RAM analysis of a *3*-out-of-*4* system and identified the most critical components that affect the system's performance.

Markov modeling is a highly effective tool for analyzing different ofsystems' performance, as it focuses on the potential states a system can assume throughout its functioning. In the initial state, referred to as the perfect state, all system components are in optimal working condition. As time passes, components of the system start to degrade, and the performance of the system reduces significantly. If proper maintenance is done at the right time it saves the system from major failures. If this component maintenance is not done at the right time, then the system is bound to fail, which may cost a lot to the organization. There can be many possible failure states in the Markov model. But after the failure of the component system is repaired and failed components are either repaired or replaced to bring the system back to the good working state. Kumar and Kumar [7] used Markov modeling to analyze the performance of Automatic ticket vending machines for the same performance analysis.

In the above studies, it has been observed that elite researchers investigated many industrial systems through different techniques for finding their various system measures. Also, specific authors were investigating some of the components of sugar mill. But no one has ever tried to investigate a sugar mill as a whole for performance analysis by taking human error into consideration, and also sensitivity analysis of the sugar mill plant regarding its components failure/repair has never been performed. Hence, in the present study, authors have investigated a sugar mill by taking its various important components along with a human operator. The next section briefly describes the sugar mill's components, which have been taken into consideration for the reliability analysis of the system.

1. SYSTEM DESCRIPTION

The description of the components of the sugar mill is as follows

Component-A: Unloader is represented by component A. Basically; it is used to unload the cane from the means of transport. In the present study, two unloaders in parallel configuration have been taken into consideration. If one of them fails, then the sugar mill goes into the degraded state.

Component-B: The conveyor is represented by component B. Once the cane is unloaded, then it is kept on the conveyor for further process. Failure of the conveyer results in the failure of the whole system.

Component-C: Cutter is represented as component C. Basically, it used to cut the cane into specific size of pieces.

Component-D: Component D represents the crushing system, which serves the purpose of cane crushing and juice extraction.

Component-E: Bagasse carrying system is represented by component E. After the juice extraction from canes, Bagasse is used as a fuel in the sugar mill. It is used in the heat-generating system of the mill. The bagasse carrying machine is used to carry the bagasse to the heat-generating system.

Component-F: The boiler is represented by component F. It is used to generate heat in the various stages of production in the sugar mill. Optimizing the boiler's performance can significantly improve the reliability of the sugar mill. In this study, two boilers are taken to enhance the overall production of the mill. In the event of a single boiler failure, the sugar mill operates in a degraded state, leading to reduced production. However, complete failure occurs only when both boilers cease to function.

The interconnection of these components (flow diagram) in the sugar mill is represented in the following Fig. 2.1.

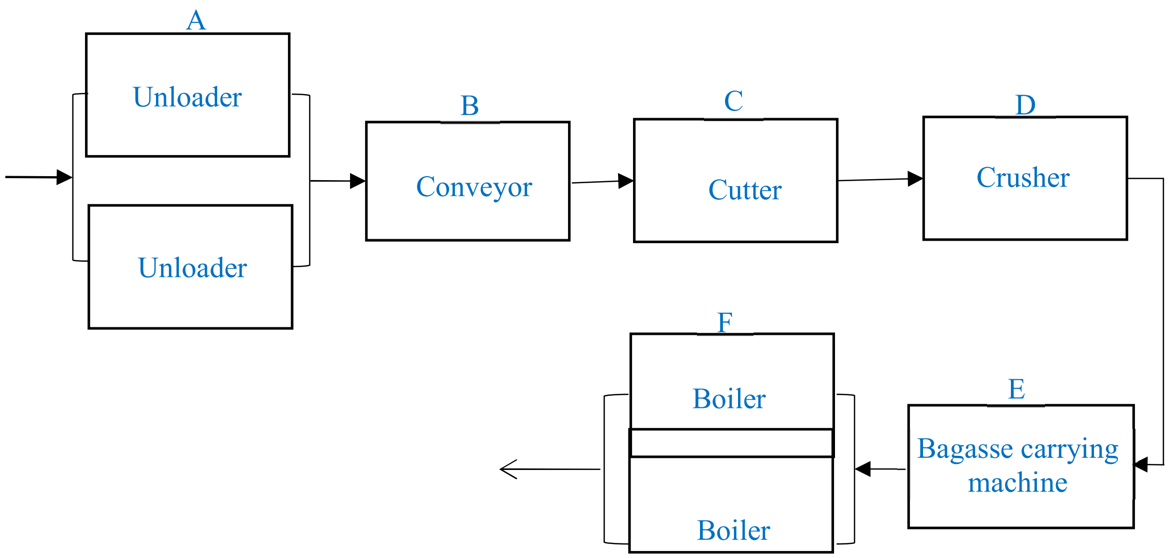


Fig. 2.1: Configuration of the System

2. ASSUMPTIONS

The following assumptions are associated with this model.

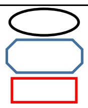
- Initially, the system is in good condition and all the components are working with full efficiency.
- System components can be in working, partially failed, or in a failed state.
- A repair facility is always available.
- Failure and repair rates have been taken as constant and follow exponential distribution.
- Human operators always available to operate the system.

- The raw material is always available for production.

3. NOMENCLATURE

The description of the various nomenclature and states, which followed throughout the manuscript, is given in the following Table 4.1.

Table 4.1. Nomenclature

 t	Signifies that the system is in a satisfactory condition. Indicates that the system is in a compromised state. Shows that the system has experienced a failure. Time frame.
s	Laplace Transformation variable.
$P_i(t)$	Likelihood of the system being in a state $S_i$at instant t $(i = 0,1,2,3,\ldots,27)$.
$\overline{P}_i(t)$	Laplace transform of $P_i(t)$.
α_i	Failure rate of i^{th} component of the system.
α_{HE}	Human error failure rate.
β_i	Repair rate of i^{th} component of the system.
β_{HE}	Human error repair rate.
S_0	Good state: All components of system are operating properly and in optimal working condition.
S_1	Degraded state: The state in which the first unloader experiences a failure.
S_2	Failed state: State in which the second unloader fails after the failure of the first unloader.
S_3	Failed state: State in which the conveyer of the system fails.
S_4	Failed state: State in which the cutter of the system fails.
S_5	Failed state: State in which the crusher of the system fails.
S_6	Failed state: State in which the bagasse carrying machine of the system fails.
S_7	Failed state: A state in which the system fails due to human error.
S_8	Degraded state: State in which the first boiler of the system fails.
S_9	Failed state: State in which the conveyer of the system fails after the failure of the first boiler.
S_{10}	Failed state: State in which the cutter of the system fails after the failure of the first boiler.
S_{11}	Failed state: State in which the crusher of the system fails after the failure of the first boiler.
S_{12}	Failed state: State in which the bagasse carrying machine of the system fails after the failure of the first boiler.
S_{13}	Failed state: State in which the second boiler fails after the failure of the first boiler.
S_{14}	Failed state: A state in which the system fails due to human error after the failure of the first boiler.
S_{15}	Degraded state: State in which the first unloader and first boiler fail.
S_{16}	Failed state: State in which the second unloader fails after the failure of the first unloader and first boiler.
S_{17}	Failed state: The state in which the conveyer fails after the failure of the first unloader and first boiler.
S_{18}	Failed state: State in which the cutter fails after the failure of the first unloader and first boiler.
S_{19}	Failed state: State in which the crusher fails after the failure of the first unloader and first boiler.
S_{20}	Failed state: The state in which the bagasse carrying machine fails after the failure of the first unloader and first boiler.
S_{21}	Failed state: State in which second boiler fails after the the first unloader and first boiler fails.
S_{22}	Failed state: State in which the system fails due to human error after the failure of the first unloader and first boiler.
S_{23}	Failed state: State in which the conveyer fails after the failure of the first unloader.
S_{24}	Failed state: State in which the cutter fails after the failure of the first unloader.
S_{25}	Failed state: State in which the crusher fails after the failure of the first unloader.
S_{26}	Failed state: The state in which the bagasse carrying machine fails after the failure of the first unloader.
S_{27}	Failed state: State in which the system fails due to human error after the failure of the first unloader.

4. MATHEMATICAL MODELING OF THE SUGAR MILL PLANT

Based on the system analysis, the authors would have developed a mathematical model to represent the behavior of the sugar mill system. This model would take into account the various parameters and variables that influence the reliability of the system, such as failure rates, repair times, maintenance policies, and external factors like human error. Critically analyzing the probability of various failure/repair of the components of the sugar mill during its production, different possible states and their interconnection are identified and represented in the following state transition diagram (Fig. 4.2). All the possible states $S_i: i = 0,1,2,\dots,27$ are shown in the diagram below. For a better understanding of these states Table 4.1 is given previously. The Chapman-Kolmogorov differential equations are developed from the state transition diagram in the interval $(t, t + \Delta t)$ as follows.

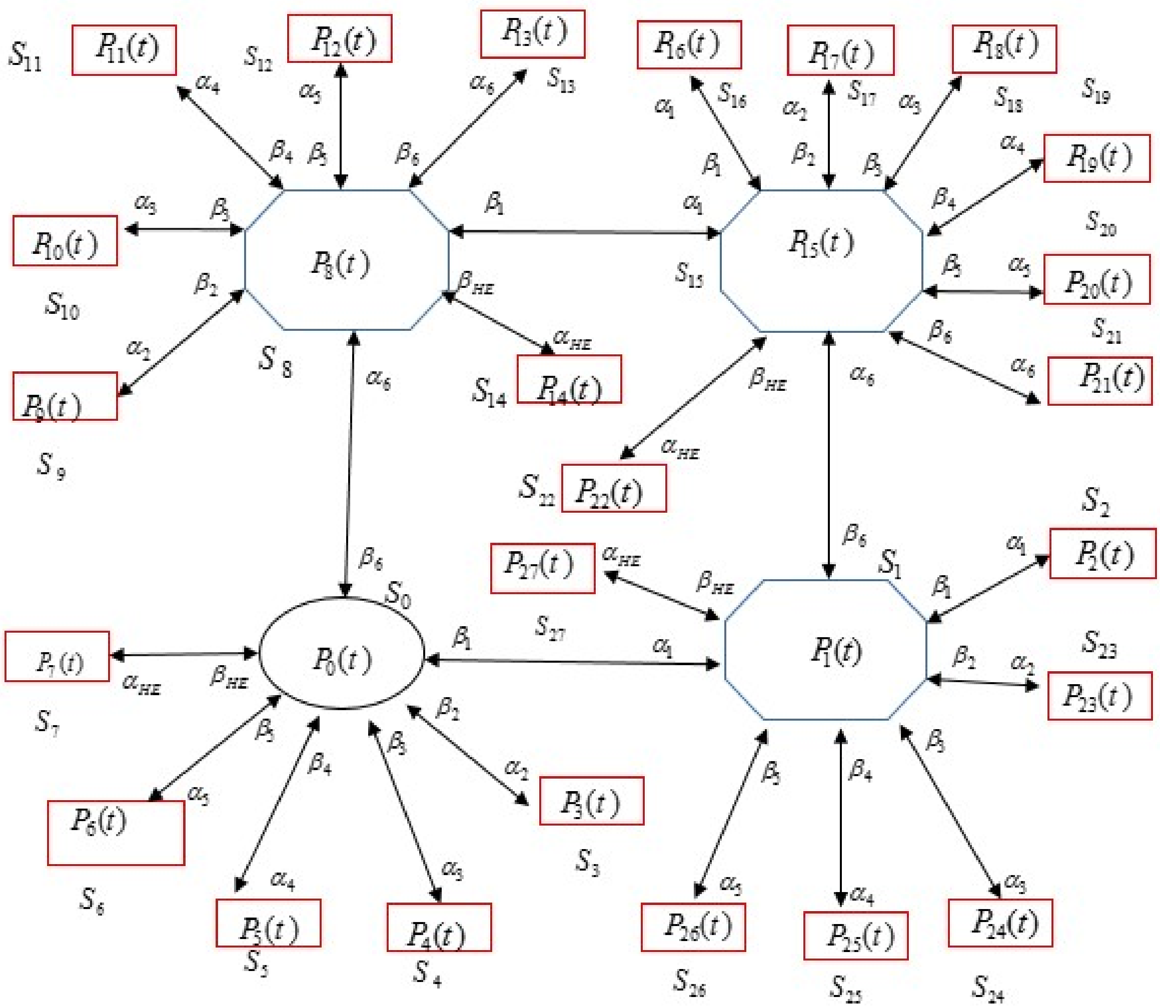


Fig. 4.2: State Transition diagram

5. FORMULATION AND SOLUTION OF THE MODEL

The state transition diagram for the considered process (Fig. 4.2) shows the transition between the i^{th} state to the j^{th} state in a small time interval Δt, which can be represented by the following set of Chapman- Kolmogorov differential equations, with the aid of Markov birth-death process, as follows.

$$\left[\frac{d}{dt}+\alpha_{HE}+\sum_{i=1}^{6}\alpha_i\right]P_0(t)=\beta_1P_1(t)+\sum_{i=2}^{5}\beta_iP_{i+1}(t)+\beta_6P_8(t)+\beta_{HE}P_7(t) \qquad (5.1)$$

$$\left[\frac{d}{dt}+\beta_1+\alpha_{HE}+\sum_{i=1}^{6}\alpha_i\right]P_0(t)= \qquad (5.2)$$

$$\alpha_1P_0(t)+\beta_1P_2(t)+\beta_6P_{15}(t)+\beta_{HE}P_{27}(t)+\sum_{i=2}^{5}\beta_iP_{2(i+1)}(t)$$

$$\left[\frac{d}{dt}+\beta_1\right]P_2(t)=\alpha_1P_1(t); \qquad (5.3)$$

$$\left[\frac{d}{dt}+\beta_i\right]P_{i+1}(t)=\alpha_iP_0(t), i=2,3,4,5,6,7; \qquad (5.4)$$

$$\left[\frac{d}{dt}+\beta_{HE}\right]P_7(t)=\alpha_{HE}P_0(t); \qquad (5.5)$$

$$\left[\frac{d}{dt}+\alpha_{HE}+\beta_6+\sum_{i=1}^{6}\alpha_i\right]P_8(t)=\beta_1P_{15}(t)+\beta_2P_9(t)+ \qquad (5.6)$$

$$\beta_3P_{10}(t)+\beta_4P_{11}(t)+\beta_5P_{12}(t)+\beta_6P_{13}(t)+\beta_{HE}P_{14}(t)+\alpha_6P_0(t);$$

$$\left[\frac{d}{dt}+\beta_i\right]P_{i+7}(t)=\alpha_iP_8(t), i=2,3,4,5,6; \qquad (5.7)$$

$$\left[\frac{d}{dt}+\beta_{HE}\right]P_{14}(t)=\alpha_{HE}P_8(t); \qquad (5.8)$$

$$\left[\frac{d}{dt}+\alpha_{HE}+\beta_1+\beta_6+\sum_{i=1}^{6}\alpha_i\right]P_{15}(t)= \qquad (5.9)$$

$$\beta_{HE}P_{22}(t)+\alpha_1P_8(t)+\alpha_6P_1(t)+\sum_{i=1}^{6}\beta_iP_{i+15}(t);$$

$$\left[\frac{d}{dt}+\beta_i\right]P_{i+15}(t)=\alpha_iP_{15}(t), i=1,2,3,4,5,6; \qquad (5.10)$$

$$\left[\frac{d}{dt}+\beta_{HE}\right]P_{22}(t)=\alpha_{HE}P_{15}(t); \qquad (5.11)$$

$$\left[\frac{d}{dt}+\beta_i\right]P_{i+21}(t)=\alpha_iP_1(t), i=2,3,4,5; \qquad (5.12)$$

$$\left[\frac{d}{dt}+\beta_{HE}\right]P_{27}(t)=\alpha_{HE}P_1(t). \qquad (5.13)$$

Initial condition

$$P_i(t)=\begin{cases}1, t=0 \text{ and } i=0;\\ 0, \text{otherwise.}\end{cases} \qquad (5.14)$$

On taking the Laplace transformation in equation (5.1)–(5.14), we get

$$\left[s+\alpha_{HE}+\sum_{i=1}^{6}\alpha_i\right]\bar{P}_0(s)=1+\beta_1\bar{P}_1(s)+\sum_{i=2}^{5}\beta_i\bar{P}_{i+1}(s)+\beta_6\bar{P}_8(s)+\beta_{HE}\bar{P}_7(s); \qquad (5.15)$$

$$\left[s+\beta_1+\alpha_{HE}+\sum_{i=1}^{6}\alpha_i\right]\overline{P}_0(s)=$$
$$\alpha_1\overline{P}_0(s)+\beta_1\overline{P}_2(s)+\beta_6\overline{P}_{15}(s)+\beta_{HE}\overline{P}_{27}(s)+\sum_{i=2}^{5}\beta_i\overline{P}_{2(i+1)}(s)\,; \tag{5.16}$$

$$[s+\beta_1]\overline{P}_2(s)=\alpha_1\overline{P}_1(s); \tag{5.17}$$

$$[s+\beta_i]\overline{P}_{i+1}(s)=\alpha_i\overline{P}_0(s), i=2,3,4,5,6,7; \tag{5.18}$$

$$[s+\beta_{HE}]\overline{P}_7(s)=\alpha_{HE}\overline{P}_0(s); \tag{5.19}$$

$$\left[s+\alpha_{HE}+\beta_6+\sum_{i=1}^{6}\alpha_i\right]\overline{P}_8(s)=\beta_1\overline{P}_{15}(s)+\beta_2\overline{P}_9(s)+\beta_3\overline{P}_{10}(s)+$$
$$\beta_4\overline{P}_{11}(s)+\beta_5\overline{P}_{12}(s)+\beta_6\overline{P}_{13}(s)+\beta_{HE}\overline{P}_{14}(s)+\alpha_6\overline{P}_0(s); \tag{5.20}$$

$$[s+\beta_i]\overline{P}_{i+7}(s)=\alpha_i\overline{P}_8(s), i=2,3,4,5,6; \tag{5.21}$$

$$[s+\beta_{HE}]\overline{P}_{14}(s)=\alpha_{HE}\overline{P}_8(s); \tag{5.22}$$

$$\left[s+\alpha_{HE}+\beta_1+\beta_6+\sum_{i=1}^{6}\alpha_i\right]\overline{P}_{15}(s)=$$
$$\beta_{HE}\overline{P}_{22}(s)+\alpha_1\overline{P}_8(s)+\alpha_6\overline{P}_1(s)+\sum_{i=1}^{6}\beta_i\overline{P}_{i+15}(s)\,; \tag{5.23}$$

$$[s+\beta_i]\overline{P}_{i+15}(s)=\alpha_i\overline{P}_{15}(s), i=1,2,3,4,5,6; \tag{5.24}$$
$$[s+\beta_{HE}]\overline{P}_{22}(s)=\alpha_{HE}\overline{P}_{15}(s); \tag{5.25}$$
$$[s+\beta_i]\overline{P}_{i+21}(s)=\alpha_i\overline{P}_1(s), i=2,3,4,5; \tag{5.26}$$
$$[s+\beta_{HE}]\overline{P}_{27}(s)=\alpha_{HE}\overline{P}_1(s). \tag{5.27}$$

Initial condition

$$\overline{P}_i(s)=\begin{cases}1, s=0 \text{ and } i=0;\\ 0, \text{otherwise.}\end{cases} \tag{5.28}$$

In order to find the various performance indicators of the considered system, the authors solve the above set of equations and find the various state probabilities $\overline{P}_i(s); i=0,1,\ldots,27$ for the sugar mill. Equation (5.29 and equation (5.30) give the up state (working) and down states (failed state) probability of the sugar mill.

$$\overline{P}_{up}(s)=\overline{P}_0(s)+\overline{P}_1(s)+\overline{P}_8(s)+\overline{P}_{15}(s), \tag{5.29}$$

$$\overline{P}_{down}(s)=\sum_{i=0}^{27}\overline{P}_i(s)-\overline{P}_{up}(s). \tag{5.30}$$

The following state probabilities are obtained when solving the above set of equations using initial and boundary conditions.

$$\overline{P}_0(s)=\frac{1}{H_1}, \tag{5.31}$$

$$\overline{P}_1(s)=\left[\frac{\alpha_1}{H_2}+\frac{\beta_6\alpha_6\alpha_1}{H_2H_3H_4}\right]\overline{P}_0(s), \tag{5.32}$$

$$\overline{P}_8(s)=\left[\frac{\beta_1\beta_6\alpha_1{\alpha_6}^2}{H_2H_3^2H_4^2}+\frac{\alpha_6}{H_3}+\frac{\beta_1\alpha_1\alpha_6}{H_2H_3H_4}\right]\overline{P}_0(s), \tag{5.33}$$

$$\overline{P}_{15}(s)=\left[\frac{\alpha_6^2\beta_6\alpha_1}{H_2H_3H_4^2}+\frac{\beta_1\beta_6\alpha_1^2\alpha_6^2}{H_2H_3^2H_4^3}+\frac{\alpha_1\alpha_6}{H_3H_4}+\frac{\alpha_1\alpha_6}{H_2H_4}+\frac{\alpha_1^2\beta_1\alpha_6}{H_2H_3H_4^2}\right]\overline{P}_0(s), \tag{5.34}$$

where

$$H_1 = \left[s + \alpha_{HE} + \sum_{i=1}^{6} \alpha_i - \frac{\beta_1 \alpha_1}{H_2} - \frac{2\beta_1 \beta_6 \alpha_1 \alpha_6}{H_2 H_3 H_4} - \sum_{i=2}^{5} \frac{\beta_i \alpha_i}{s + \beta_i} - \frac{\beta_{HE} \alpha_{HE}}{s + \beta_{HE}} - \frac{\beta_6 \alpha_6}{H_3} - \frac{\beta_1 \beta_6^2 \alpha_6^2 \alpha_1}{H_2 H_3^2 H_4^2} \right]; \tag{5.35}$$

$$H_2 = \left[s + \beta_1 + \alpha_{HE} + \sum_{i=1}^{6} \alpha_i - \sum_{i=1}^{5} \frac{\beta_i \alpha_i}{s + \beta_i} - \frac{\beta_{HE} \alpha_{HE}}{s + \beta_{HE1}} - \frac{\beta_6 \alpha_6}{H_4} - \frac{\beta_1 \beta_6 \alpha_1 \alpha_6}{H_3 H_4^2} \right], \tag{5.36}$$

$$H_3 = \left[s + \beta_6 + \alpha_{HE} + \sum_{i=1}^{6} \alpha_i - \frac{\beta_1 \alpha_1}{H_{4_1}} - \sum_{i=2}^{6} \frac{\beta_i \alpha_i}{s + \beta_i} - \frac{\beta_{HE} \alpha_{HE}}{s + \beta_{HE}} \right], \tag{5.37}$$

$$H_4 = \left[s + \beta_1 + \beta_6 + \alpha_{HE} + \sum_{i=1}^{6} \alpha_i - \sum_{i=1}^{6} \frac{\beta_i \alpha_i}{s + \beta_i} - \frac{\beta_{HE} \alpha_{HE}}{s + \beta_{HE}} \right]. \tag{5.38}$$

6. NUMERICAL CALCULATION AND EVALUATION OF DIFFERENT RELIABILITY MEASURES

6.1. Availability

System availability is a performance metric used to assess the operational effectiveness of a system while accounting for appropriate maintenance practices. To calculate the time-dependent availability of the system, substitute the numerical values of different failure and repair parameters as $\alpha_2 = 0.01$, $\alpha_3 = 0.02$, $\alpha_4 = 0.03$, $\alpha_5 = 0.015$, $\alpha_6 = 0.028$, $\alpha_{HE} = 0.04$, $\beta_1 = 1$, $\beta_2 = 1$, $\beta_3 = 1$, $\beta_4 = 1$, $\beta_5 = 1$, $\beta_6 = 1$, $\beta_{HE} = 1$ in equation (5.29) and apply the Inverse Laplace Transform. The resulting equation (6.1) provides the time-dependent availability of the sugar mill.

$$A(t) = \begin{bmatrix} -0.00096837955 e^{-1.476376154t} - \\ 0.0001949522158 e^{-1.460089466t} + \\ 0.1073957516 e^{-1.117248193t} + \\ 0.0001833977962 e^{-0.8655224585} - \\ 0.09037726155 e^{-2.327216180t} - \\ 0.002608553632 e^{-1.525425094t} - \\ 0.1570307560 e^{-2.387741855t} - \\ 0.01092951713 e^{-2.404259749t} + \\ 0.002032653966 e^{-0.6969340258t} + \\ 0.002724385526 e^{-0.6908052982t} + \\ 0.0004439964442 e^{-0.8633203241} - \\ 0.0009123318364 e^{-0.8540851581} - \\ 0.0009921283752 e^{-0.6822587943t} + \\ 0.8938297613 e^{.0.0002827503194t} \end{bmatrix}. \tag{6.1}$$

By fitting the time unit *t* in equation (6.1), the time-dependent availability of the sugar mill can be observed. Table 6.1 and the corresponding Fig. 6.1 display the availability values for the sugar mill.

Table 6.1. Behaviors of Availability of the sugar mill with time

Time unit (t)	Availability $A(t)$
0	1.00000
1	0.90691
2	0.90425
3	0.89853
4	0.89626

5	0.89560
6	0.89553
7	0.89567
8	0.89588
9	0.89611
10	0.89636

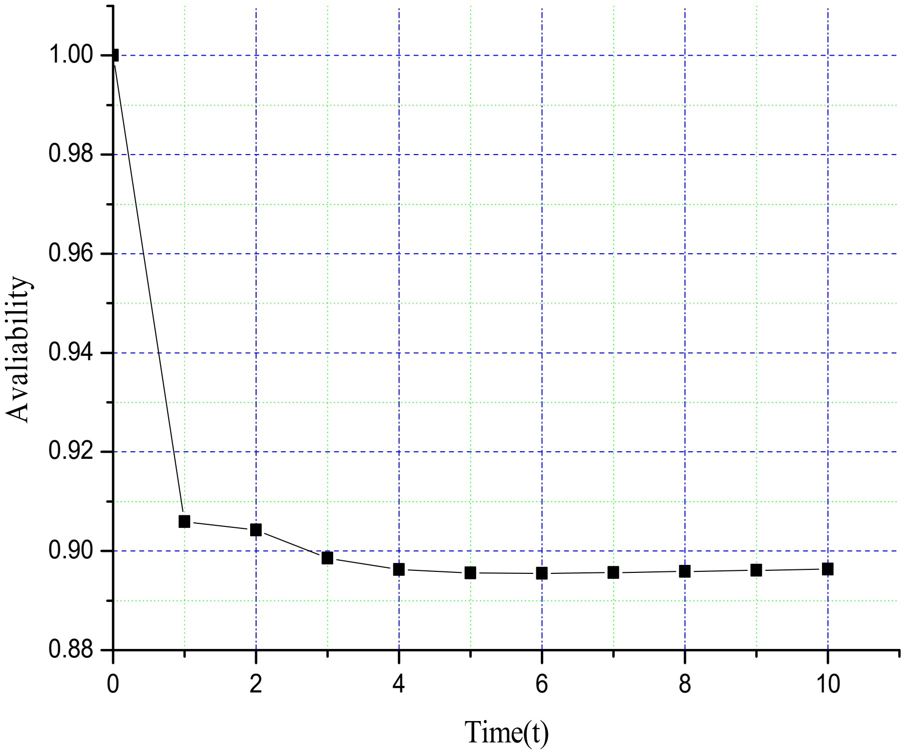


Fig. 6.1. Behaviour of Availability of the sugar mill with time

6.2. Reliability

Reliability refers to the probability of a system successfully carrying out its intended task within specified operating conditions for a given duration. To determine the reliability of the sugar mill, the various failure rates as $\alpha_1 = 0.05$, $\alpha_2 = 0.01$, $\alpha_3 = 0.02$, $\alpha_4 = 0.03$, $\alpha_5 = 0.015$, $\alpha_6 = 0.028$, $\alpha_{HE} = 0.04$ while the repair rate is set to zero in equation (5.29). The resulting reliability of the sugar mill is obtained as given in equation (6.2).

$$R(t) = [0.00020000(5000 + 390t + 7t^2)e^{-0.19300000t}] \tag{6.2}$$

The behavior of time-dependent reliability of sugar mill can be obtained by varying time unit *t* in (6.2). Table 6.2 and corresponding Fig. 6.2 represent the reliability of the sugar mill.

Table 6.2. Behavior of Reliability of the system with time unit

Time unit (*t*)	Reliability R(*t*)
0	1.00000
1	0.88994
2	0.78962
3	0..69866
4	0.61661
5	0.54290
6	0.47695
7	0.41815
8	0.36589
9	0.31959
10	0.27868

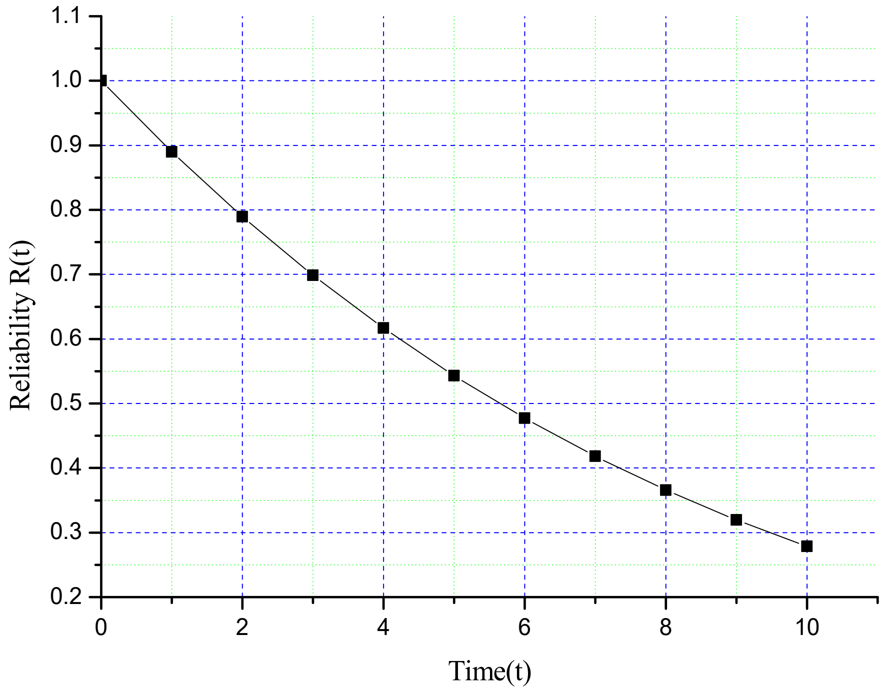


Fig. 6.2. Behavior of Reliability of the System with Time Unit

6.3. Mean Time to Failure (MTTF)

Mathematically, the MTTF of a system is calculated as given in equation (6.3).

$$MTTF = \int_0^t R(t)dt = \lim_{s\to 0} \overline{R}(s) \tag{6.3}$$

Now, using equation (6.2) in (6.3), authors obtained the MTTF of the considered system as:

$$MTTF = \left[\begin{array}{c} \dfrac{1}{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_{HE}} + \\ \dfrac{\alpha_1+\alpha_6}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_{HE})^2} + \\ \dfrac{2\alpha_1\alpha_6}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_{HE})^3} \end{array} \right]. \tag{6.4}$$

Varying failure rates one by one from 0.01 to 0.09 with an interval of 0.01 and fixing other failure rates, Table 6.3 and Fig. 6.3 are obtained for MTTF of the considered system.

Table 6.3. MTTF of the Sugar mill with various failure rates

Variations in Failure rates	MTTF with respect to failure rates						
	α_1	α_2	α_3	α_4	α_5	α_6	α_{HE}
0.01	8.31561	7.66484	8.25048	8.92729	7.94742	7.86005	9.71726
0.02	8.20020	7.15360	7.66484	8.25048	7.40081	7.76656	8.92729
0.03	8.04273	6.70381	7.15360	7.66484	6.92172	7.63667	8.25048
0.04	7.86050	6.30529	6.70381	7.15360	6.49869	7.48393	7.66484
0.05	7.66484	5.94995	6.30529	6.70381	6.12266	7.31754	7.15360
0.06	7.46321	5.63130	5.94995	6.30529	5.78639	7.14403	6.70381
0.07	7.26054	5.34404	5.63130	5.94995	5.48403	6.96762	6.30529
0.08	7.06005	5.08387	5.34404	5.63130	5.21081	6.79138	5.94995
0.09	6.86383	4.84719	5.08387	5.34404	4.96279	6.61736	5.63130

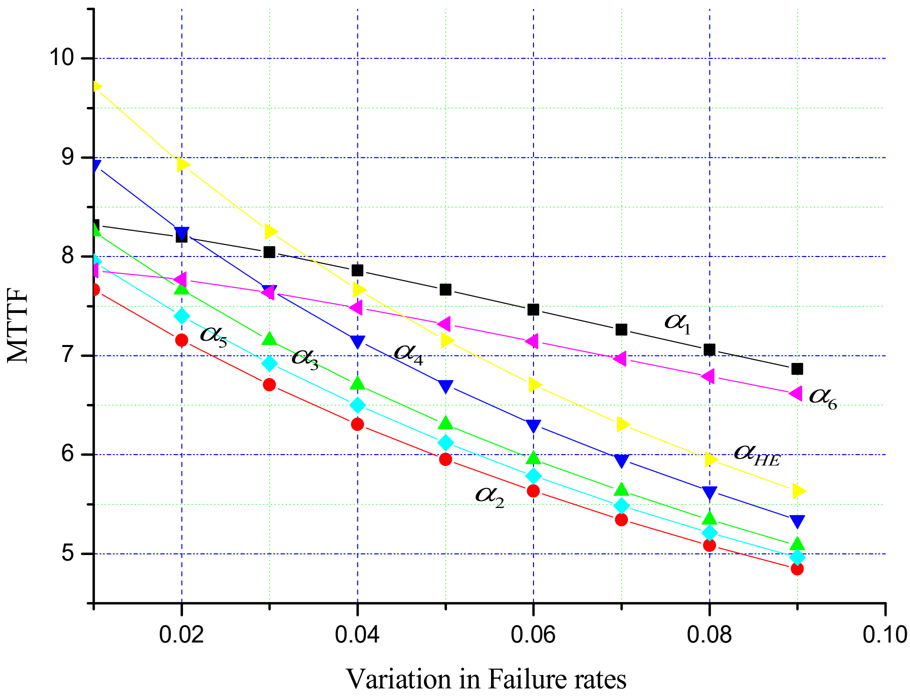


Fig. 6.3. MTTF w.r.t. failure rates

6.4. Sensitivity Analysis of MTTF

The objective of the sensitivity analysis is to determine the input variables which affect the system performance most. Here authors perform the sensitivity analysis on the MTTF of the sugar mill. Table 6.4 shows the change in the meantime to failure MTTF of the system resulting from changes in parametersα_1,α_2,α_3,α_4,α_5,α_6,α_{HE}. The same is depicted in Fig. 6.4.

Table 6.4: Sensitivity of the MTTF

Variation in failure rates	Sensitivity with respect to MTTF						
	$\frac{\partial(MTTF)}{\partial\alpha_1}$	$\frac{\partial(MTTF)}{\partial\alpha_2}$	$\frac{\partial(MTTF)}{\partial\alpha_3}$	$\frac{\partial(MTTF)}{\partial\alpha_4}$	$\frac{\partial(MTTF)}{\partial\alpha_5}$	$\frac{\partial(MTTF)}{\partial\alpha_6}$	$\frac{\partial(MTTF)}{\partial\alpha_{HE}}$
0.01	-8.64993	-54.60011	-62.80532	-72.91918	-58.49511	-6.93044	-85.55885
0.02	-13.99608	-47.86119	-54.60011	-62.80532	-51.06985	-11.43978	-72.91918
0.03	-17.21471	-42.26548	-47.86119	-54.60011	-44.93705	-14.31633	-62.80532
0.04	-19.04569	-37.57298	-42.26548	-47.86119	-39.81881	-16.08062	-54.60011
0.05	-19.96413	-33.60267	-37.57298	-42.26548	-35.50705	-17.08201	-47.86119
0.06	-20.28051	-30.21609	-33.60267	-37.57298	-31.84373	-17.55829	-42.26548
0.07	-20.20068	-27.30604	-30.21609	-33.60267	-28.70721	-17.67328	-37.57298
0.08	-19.86272	-24.78852	-27.30604	-30.21609	-26.00272	-17.54093	-33.60267
0.09	-19.36012	-22.59706	-24.78852	-27.30604	-23.65563	-17.24117	-30.21609

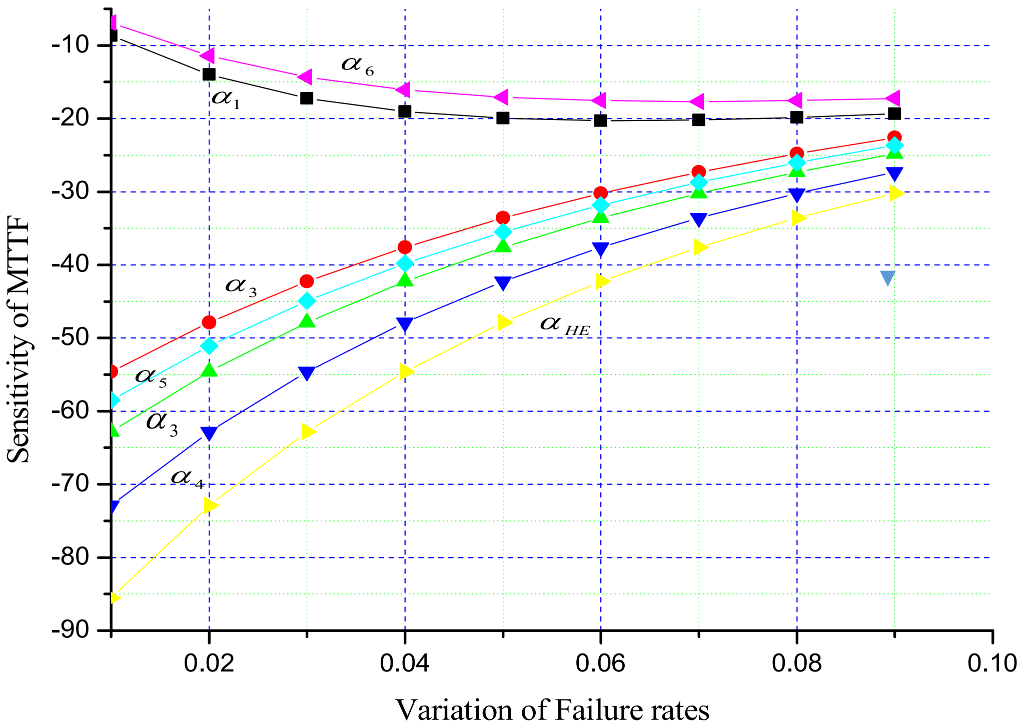


Fig. 6.4. Sensitivity of the MTTF

6.5. Estimated Profit from the Sugar Mill

The estimated profit of the sugar mill in the interval $[0, t)$ is calculated by using

$$E_P(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \tag{6.5}$$

Equation (6.5) will provide the expected profit from the sugar mill for the various revenue and service cost. Using equation (5.29) in equation (6.5), authors obtain the profit function for the considered sugar mill as follows.

$$E_P(t) = K_1 \left\{ \begin{array}{c} 0.0006559165477e^{-1.476376154t} + \\ 0.0001335207330e^{-1.460089466t} - \\ 0.09612524081e^{-1.117248193} - \\ 0.0002118925909e^{-0.8655224585t} + \\ 0.03883492317e^{-2.327216180t} + \\ 0.001710050295e^{-1.525425094t} + \\ 0.06576538233e^{-2.387741855t} + \\ 0.004545896979e^{-2.40425974t} - \\ 0.002916565831e^{-0.6969340258} - \\ 0.003943782037e^{-0.6908052982t} - \\ 0.005142893452e^{-0.8633203241} + \\ 0.001068197741e^{-0.8540851581} + \\ 0.001454181879e^{-0.6822587943t} + \\ 3161.198061e^{0.0002827503194} - \\ 3161.208517 \end{array} \right\} - K_2 t. \tag{6.6}$$

Now vary the service cost $K_2 = 0.1, 0.2, 0.3, 0.4, 0.5$, fix revenue K_1 as one, and vary time unit t in (6.6). Table 6.5 and corresponding Fig. 6.5 are obtained as follows.

Table 6.5. Expected profit of the system

Time unit (t)	Expected Profit from the system				
	$K_2 = 0.1$	$K_2 = 0.2$	$K_2 = 0.3$	$K_2 = 0.4$	$K_2 = 0.5$
0	0	0	0	0	0
1	0.76029	0.66029	0.56029	0.46029	0.36029
2	1.56722	1.36722	1.16722	0.96722	0.76722
3	2.36828	2.06828	1.76828	1.46828	1.16828
4	3.16547	2.76547	2.36547	1.96547	1.56547
5	3.96133	3.46133	2.96133	2.46133	1.96133
6	4.75687	4.15687	3.55687	2.95687	2.35687
7	5.55246	4.85246	4.15246	3.45246	2.75246
8	6.34824	5.54824	4.74824	3.94824	3.14824
9	7.14424	6.24424	5.34424	4.44424	3.54424
10	7.94048	6.94048	5.94048	4.94048	3.94048

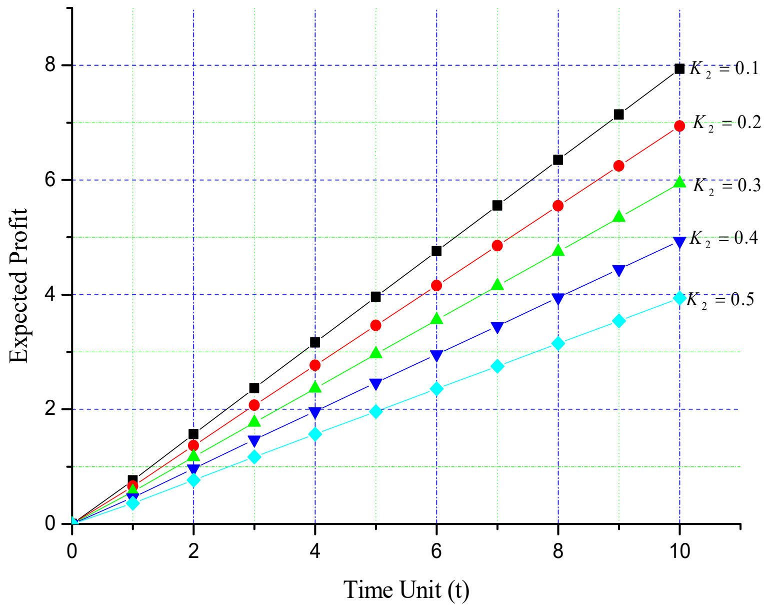


Fig. 6.5. Expected Profit of the sugar mill vs. time unit t

4. RESULT DISCUSSION

In this paper, the authors have developed a mathematical model based on the working of a sugar mill to determine the various reliability measures of the sugar mill. For this, the six components of the plant have been taken into consideration. After analyzing the system mathematically, the following results are obtained.

The behavior of sugar mill availability is shown in Fig. 6.1. It is observed that system availability decreases very slowly as time passes. Also, the availability of sugar mill at ten units of time is 0.89636. Fig. 6.2 reflects the reliability of the sugar mill concerning time unit. It is found that the reliability of the sugar mill at ten units of time is 0.27868. This means that with the passage of time unit system’s unit reliability is also decreasing. This may be due to the system component's aging, corrosion, stress, etc. Fig. 6.3 shows the nature of MTTF of the sugar mill concerning variation in failure rates. It reflects that the MTTF of the sugar mill with respect to the failure rate of the unloader is the highest. So, the performance of the sugar mill is less effected by the failure of the unloader. Despite increasing the failure rate of the unloader MTTF is higher as compared to other components of the system. The graph of sugar mill sensitivity for its MTTF is shown in Fig. 6.4, it is observed that the system’s MTTF is highly sensitive with respect to human error. As the rate of human error increases, it adversely affects the system MTTF. Fig. 6.5 shows the behavior of expected profit from the sugar mill. It shows that the expected profit of the sugar mill decreases as the service cost increases. Hence, to optimize the profit function, the management needs to give more attention to the maintenance policy.

CONCLUSION

In this paper, we utilized the concept of mathematical modeling and the Markov model to obtain the various reliability measures of the "Wahid Sugar Mill" situated in Punjab, India. From the above discussion presented in section 7, the authors conclude that the mill needs to pay more attention to human error along with other failures. Failures can be mitigated by employing skilled labor and imparting them training from time to time. It asserts that this research is beneficial for the management of the same for improving productivity.

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