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Structural Change in Multisector Monopolistic Competition Model

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Abstract: We present a natural generalization of the Dixit-Stiglitz monopolistic competition model (DSM) — we assume that there is a continuum of industries, each of them described as in DSM, and each characterized with its own elasticity of substitution. Although firms in all industries share the same level of productivity and costs, exogenous technological progress leads to non-trivial reallocations of labor and production to industries with lower elasticities of substitution. Thus the model, despite is simplicity and the absence of additional assumptions about industry structure, generates the structural changes described in the economic growth literature.

Keywords: Dixit-Stiglitz model, monopolistic competition, structural change, market reallocations.

1. INTRODUCTION

In *Modern Economic Growth*, Simon Kuznets wrote: "We identify the economic growth of nations as a sustanined increase in per capita or per worker product, most often accompanied by an increase in population and usually by sweeping structutal changes. In modern times these were changes in the industrial structure within which product was turned out and resources employed — away from agriculture toward nonagricultural activities, the process of industrialization..." Existence of these structural changes was considered by Kuznets as one of the main stylized facts of development. Indeed, the loss of relative importance of agricultucal sector in favor of industrial sector and then to services sector is well documented, see for example [3], [5], [10].

A number of economic models were developed to describe these structural changes and their relation to economic growth. The simplest mechanism, which generates structural changes, was proposed in [4] — different economic sectors grow at different rates because they have different rates of technological progress. This idea was developed in [13], [15] and [2] among many others. Our work contributes to another strand of literature based on the idea that structural changes are driven not by differences in production technologies but by demand factors, namely by non-homotheticity of consumer preferences. For example, in [12] authors assume that household consumption consists of agricultural, manufacturing and services goods. A special form of utility function generates reallocations of consumption: after the consumption of certain amount of agricultural good, the household starts to demand other goods, moreover, it consumes service goods only after some level of manufacturing

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consumption is reached. The model in [8] is based on the idea that new goods are introduced in economy as a luxury (a good with high income elasticity), but with time income elasticity decreases and good becomes necessity.

There are also several models incorporating, for example, financial development, demographic transition, urbanization, migration, production organization, human capital, fertility, see [1] for a survey. As we can see, all these models use rather special assumptions about industry structure. The model we present in this paper, on the other hand, is very simple and free of special assumptions, but nevertheless is able to generate the reallocations described by Kuznets.

The key idea of our work is the same as in [8] — the main difference between goods is income elasticity, and structural change is a decline of industries producing goods with low income elasticities and rise of the ones producing goods with high income elasticities. But the model in [8] is highly stylized — there is not much said about the production side of the economy, so it remains unclear what kind of industry structure generates new goods with high income elasticities and why income elasticities of old goods decrease with time. We, on the other hand, describe both consumption and production sides of economy, thus making our model more microfounded.

We present a simple monopolistic competition model, based on the constant elasticity of substitution utility function, proposed in [7]. We assume there is a continuum of industries each of them described as in [7]. The only difference between industries is intra-industry elasticity of substitution. To the best of our knowledge, this description of production side of economy, although rather straigthforward, is new in the literature. Similarly to [8], we assume that "simpler" the commodity, higher the elasticity of intra-industry substitution. This assumption is in line with empirical studies on product differentiation, see for example [11] for the calculations and also [9] for the discussion. As in [8], industries with high elasticities of substitution may be interpreted as agricultural, with middle elasticities — as manufacturing, with low elasticities — as services. Technological progress in our framework is modeled naturally as a decrease of variable costs of a firm (an increase of workers' productivity) at the expense of an increase of fixed costs (which may be interpreted as investments). Although firms in all industries share the same levels of productivity and costs, labor and production flows from less differentiated (higher elasticity of substitution) to more differentiated (lower elasticity of substitution) goods. Thus our model, despite its simplicity and with no additional assumptions on industry structure, generates Kuznets structural changes.

2. MODEL

2.1. Model setup

There is a continuum of industries indexed with $\rho \in (0, 1)$. For every industry, preferences of aggregate consumer are given by a CES function:

$$V(\rho) = \left(\sum_{i=1}^{n(\rho)} c_i \left(\rho\right)^{\rho} \right)^{\frac{1}{\rho}}, \tag{2.1}$$

where $n(\rho)$ is the number of firms working in this industry, $c_i(\rho)$ is the quantity of consumed good produced by i-th firm. Elasticity of substitution between goods in a given industry is $\sigma = 1/(1 - \rho)$. So, if ρ is high then the elasticity of substitution in this industry is high and goods in this industry are weakly differentiated and vice versa.

Preferences of the aggregate consumer over the goods in the continuum of industries are given by the function

$$U = \frac{1}{\nu} \int_0^1 a(\rho) V(\rho)^\nu \, d\rho, \tag{2.2}$$

where $a(\rho) \geq 0$ may be interpreted as consumer's preference for the products of industry ρ. This is also needed for the dimension correctness, because goods in different industries can have different units of measurement.

Firms are assumed to have both fixed and variable costs, so in order to produce $c_i(\rho)$ units of good, i-th firm has to use

$$l_i(\rho) = \alpha \, c_i(\rho) + f \tag{2.3}$$

units of labor. Productivity level $1/\alpha$ and fixed costs f are assumed to be equal across firms. The model becomes highly complicated without this assumption (see, for example [6], who deal with heterogeneous markets). Full stock of labor in the economy is denoted by L, the wage, the same for all workers, is w. So the aggregate consumer has wL units of income.

2.2. Market equilibrium and social welfare

Consider the aggregate consumer's problem of maximizing utility given budget constraint:

$$U = \frac{1}{\nu} \int_0^1 a(\rho) \left(\left(\sum_{i=1}^{n(\rho)} c_i(\rho)^\rho \right)^{\frac{1}{\rho}} \right)^\nu d\rho \to \max,$$

$$\int_0^1 \sum_{i=1}^{n(\rho)} c_i(\rho) \, p_i(\rho) \, d\rho \leq wL.$$

To solve it, we form a Lagrange function in the following form:

$$\mathcal{L} = \frac{1}{\nu} \int_0^1 a(\rho) \left(\left(\sum_{i=1}^{n(\rho)} (c_i(\rho))^\rho \right)^{\frac{1}{\rho}} \right)^\nu d\rho + \xi^{\nu-1} \left(wL - \int_0^1 \sum_{i=1}^{n(\rho)} c_i(\rho) \, p_i(\rho) \, d\rho \right).$$

First of all, we should check if the problem has a solution, for this the integrand must be concave. Applying Sylvester's criterion, we conclude that the function is concave if the two following conditions are valid: $\nu < 1$ and $\nu < \rho$. So we have two possibilities: if $\nu \in (0, 1)$, then the set of possible values of ρ must be restricted to $(\nu, 1)$, or we can assume that $\nu < 0$. We will consider only the second case, because, as we will see later, the first case leads to some very unpleasant degeneracies at the point $\nu = \rho$. From an economic point of view, the first case is problematic because we have to assume a relationship between parameters which does not have an economic meaning. So, from now on, ν is assumed to be negative.

Fixing some $j \in \{1 \ldots n\}$ and equalizing the partial derivative of Lagrange function with respect to $c_j(\rho)$ to zero, we get the optimum level of consumption of $c_j(\rho)$:

$$c_j(\rho) = \left(\frac{p_j(\rho) \, \xi^{\nu-1}}{a(\rho) \, V(\rho)^{\nu-\rho}} \right)^{\frac{1}{\rho-1}}. \tag{2.4}$$

We involute both sides of this equality to the power ρ and find a sum over $j = 1 \ldots n(\rho)$, and after some calculation we get

$$V\left(\rho\right)=\xi\, a\left(\rho\right)^{-\frac{1}{\nu-1}}\, P\left(\rho\right)^{\frac{1}{\nu-1}}\,, \tag{2.5}$$

where

$$P\left(\rho\right)^{\frac{\rho}{\rho-1}}=\sum_{i=1}^{n(\rho)}p_{i}\left(\rho\right)^{\frac{\rho}{\rho-1}} \tag{2.6}$$

is the price index associated with the goods index $V(\rho)$ as in [7] or [14]. Taking (2.5) into account, (2.4) may be rewritten in the following form:

$$c_{j}\left(\rho\right)=p_{j}\left(\rho\right)^{\frac{1}{\rho-1}}\,V\left(\rho\right)P\left(\rho\right)^{-\frac{1}{\rho-1}}\,. \tag{2.7}$$

Now consider the behavior of a firm. First of all, it is obvious that due to the fact that all firms have the same levels of fixed and variable costs, and the symmetry of consumer's preferences over the goods in a fixed industry, all firms in the industry face the same consumer's demand and set the same price. Taking (2.3) into account, the problem of firm in the industry ρ has the form

$$\pi\left(\rho\right)=p\left(\rho\right)c\left(\rho\right)-\left(c\left(\rho\right)\alpha+f\right)w\rightarrow\max\,. \tag{2.8}$$

Substituting (2.7) and finding maximum with respect to $p(\rho)$, we find the price set by firm:

$$p\left(\rho\right)=\frac{\alpha\, w}{\rho}, \tag{2.9}$$

like, again, in [14].

Now we will impose a free entry condition and demand firm profit to be equal to zero. Substituting (2.5), (2.6), (2.9) into (2.8), after some calculations we get the following expression for the number of firms in the industry ρ:

$$n_{M}(\rho)=\left(\frac{(1-\rho)\alpha^{\frac{\nu}{\nu-1}}\xi\, w^{\frac{1}{\nu-1}}}{\rho^{\frac{\nu}{\nu-1}}fa(\rho)^{\frac{1}{\nu-1}}}\right)^{\frac{(\nu-1)\rho}{\nu-\rho}} \tag{2.10}$$

Substituting (2.10) to consumer's budget constraint, we get the equation defining ξ:

$$\int_{0}^{1}a\left(x\right)^{-\frac{x}{\nu-x}}\,w^{\frac{x}{\nu-x}}f^{-\frac{\nu\,(x-1)}{\nu-x}}\alpha^{\frac{\nu\,x}{\nu-x}}\xi^{\frac{(\nu-1)x}{\nu-x}}\left(x^{-\frac{\nu}{\nu-1}}-x^{-\frac{1}{\nu-1}}\right)^{\frac{\nu\,(x-1)}{\nu-x}}\,x^{-\frac{\nu}{\nu-1}}dx=L. \tag{2.11}$$

Unfortunately, ξ can be found from (2.11) only numerically.

Substituting (2.9) into (2.8) and setting firm profit to zero, we find that firm output may be rewritten in the following form:

$$c_{M}\left(\rho\right)=\frac{f\rho}{\alpha\,\left(1-\rho\right)}, \tag{2.12}$$

which is again in line with [7].

Now consider the problem of the benevolent social planner, who maximizes consumer utility with respect to technological limitation.

$$U = \frac{1}{\nu}\int_0^1 a(\rho)\left(\left(\sum_{i=1}^{n(\rho)} c_i(\rho)^{\rho}\right)^{\frac{1}{\rho}}\right)^{\nu} d\rho,$$

$$\alpha\int_0^1 \sum_{i=1}^{n(\rho)} c_i(\rho)\,d\rho + f\int_0^1 n(\rho)\,d\rho \le L.$$

Form the Lagrange function in the following form:

$$\mathcal{L} = \frac{1}{\nu}\int_0^1 a(\rho)\left(\left(\sum_{i=1}^{n(\rho)} c_i(\rho)^{\rho}\right)^{\frac{1}{\rho}}\right)^{\nu} d\rho +$$

$$\lambda\left(L - \alpha\int_0^1 \sum_{i=1}^{n(\rho)} c_i(\rho)\,d\rho + f\int_0^1 n(\rho)\,d\rho\right).$$

Differentiating with respect to $c(\rho)$ and $n(\rho)$, after some calculations we get

$$n_W(\rho) = \left(\frac{(1-\rho)\,\alpha^{\frac{\nu}{\nu-1}}\lambda}{\rho f a(\rho)^{\frac{1}{\nu-1}}}\right)^{\frac{(\nu-1)\rho}{\nu-\rho}}, \tag{2.13}$$

$$c_W(\rho) = \frac{f\rho}{\alpha\,(1-\rho)}.$$

Substituting (2.12) into the planner's technological constraint, we get the formula defining λ:

$$\int_0^1 \alpha^{\frac{\nu x}{\nu-x}} a(x)^{-\frac{x}{\nu-x}} \lambda^{\frac{(\nu-1)x}{\nu-x}} f^{-\frac{\nu(x-1)}{\nu-x}} x^{-\frac{(\nu-1)x}{\nu-x}} (1-x)^{\frac{\nu(x-1)}{\nu-x}}\,dx = L. \tag{2.14}$$

Similarly to (2.11), it can be solved only numerically.

As we can see, outputs in market equilibrium and in the social welfare problem are the same, but the numbers of firms are different. This proves the following

Proposition 1. For any $\nu \in (-\infty, 0)$ and any parameters of the economy L, f, α, w market equilibrium is inefficient.

Thus, in the market equilibrium consumer cannot optimally distribute her expenses across industries, but can do it within an industry. Recall that in one-sector Dixit-Stiglitz model the equilibrium is efficient, but in the presence of second market of homogenous product (as it was proposed in the original paper), it is not. This result is disappointing, but rather expectable — market efficiency is rare thing in the presence of monopolists. It is also worth noting that this result cannot be considered as a trivial corollary of general theorems about the efficiency in monopolistic competition models proven, for example, in [6] and [16], because utility function (2.1)–(2.2) does not belong to the class of variable elasticity of substitution utility functions analyzed in these papers.

Figure 1 shows the distribution of the number of firms in the market equilibrium and in social welfare state for arbitrarily chosen parameters of the economy $\alpha = 0.01, f = 0.5, L = 30, a(\rho) = 1, w = 1$ and consumer's preferences characterized with $\nu = -1$. Figure 2 shows the distribution of firms in the same economy and for consumer's preferences characterized with $\nu = -5$.

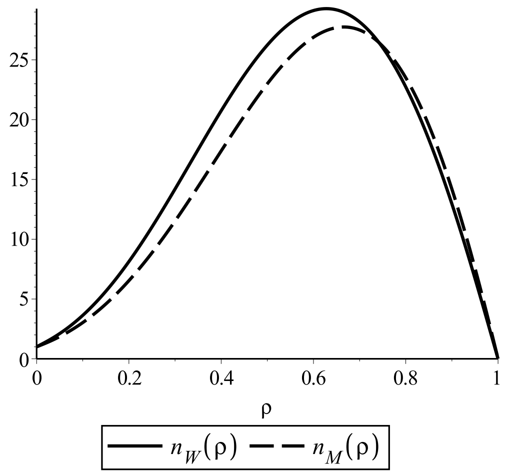

Fig. 2.1. $\nu = -1$.

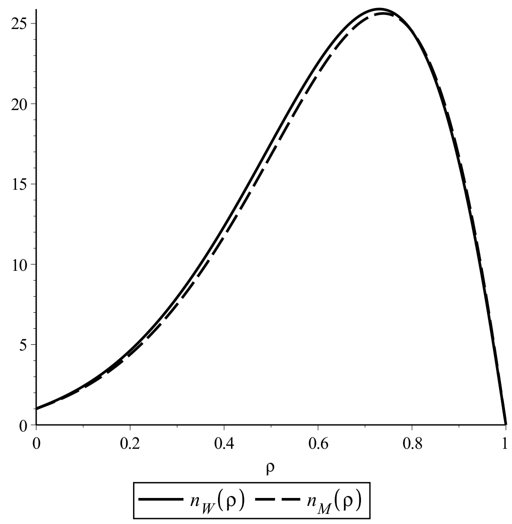

Fig. 2.2. $\nu = -5$.

As we see, smaller ν, closer the market equilibrium to the social welfare state. Note that as ν tends to $-\infty$, utility function (2.2) converges to Leontieff-like minimum function:

$$U_{-\infty} = \min_{\rho\in(0,1)} V(\rho). \tag{2.15}$$

This observation leads us to

Proposition 2. For consumer's preferences defined by (2.15), market equilibrium is efficient.

Proof. As ν tends to $-\infty$ in (2.10) and (2.11), we get

$$n_M^*(\rho) = \left(\frac{f\rho}{\xi\,(1-\rho)\,\alpha}\right)^{-\rho}, n_W^*(\rho) = \left(\frac{f\rho}{\lambda\,(1-\rho)\,\alpha}\right)^{-\rho},$$
$$\int_0^1 \xi^x f^{-x+1}\alpha^x x^{-x}(-x+1)^{x-1}\,dx = L,$$
$$\int_0^1 \lambda^x f^{-x+1}\alpha^x x^{-x}(-x+1)^{x-1}\,dx = L.$$

Obviously, $\lambda = \xi$ and $n_M^*(\rho) = n_W^*(\rho)$. ■

2.3. Effects of technological progress and population growth

Consider the effects of technological progress in this model. We assume that in our framework technological progress means an increase of workers' productivity $1/\alpha$. This, however, doesn't come without cost — we assume that fixed costs f also increase. Thus, progress is due to the increase in capital expenditures. Figures 3–6 show the effect of technological progress in economy with parameters $\alpha = 0.01, f = 0.5, L = 30, w = 1$ and consumer's preferences defined by parameters $a(\rho) \equiv 1, \nu = -\infty$, on the distribution of the number of firms, output and labor. Technological progress is modeled by decreasing α 3 times and increasing f 3 times. The solid line is before progress, dashed — after.

Conclusions are summed up in the following

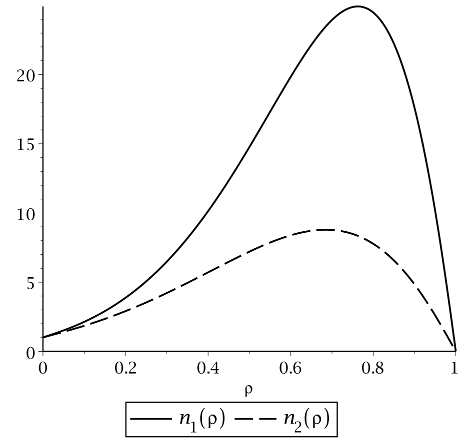

Fig. 2.3. Number of firms.

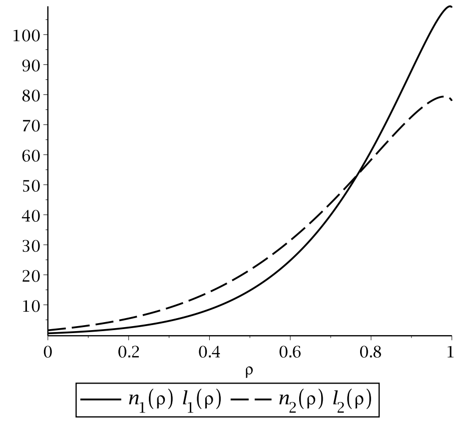

Fig. 2.4. Number of workers.

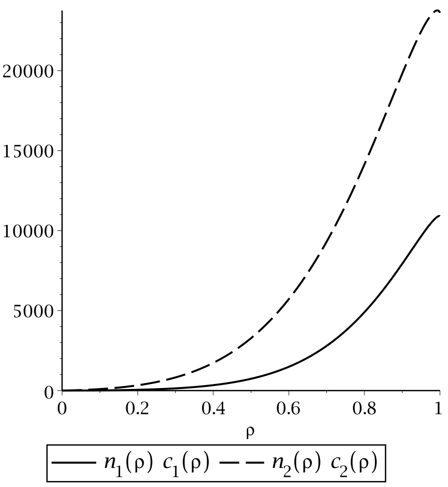

Fig. 2.5. Output.

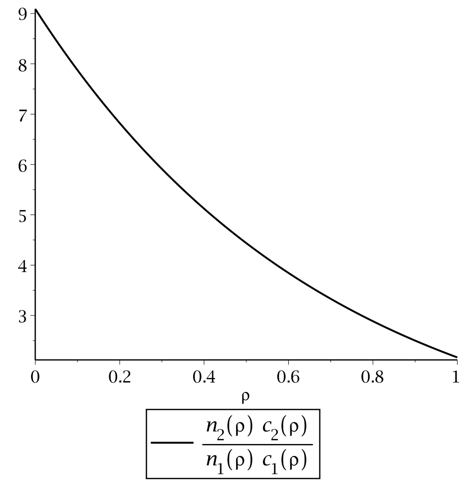

Fig. 2.6. Ratio of outputs.

Proposition 3. With decrease of variable costs and increase of fixed costs: $\alpha' = \frac{1}{k}\alpha, f' = k^{\mu} f, k > 1, \mu > 0$, for any parameters of economy and consumer's preferences of the type (2.1)-(2.2) with $a(\rho) = 1$ for any ρ,

1. for $\mu \leq 1$ output increases in all industries $\rho \in (0, 1)$,
2. number of workers increases in industries with $\rho \in (0, \rho^*)$ and decreases for industries with $\rho \in (\rho^*, 1)$ for some $\rho^* \in (0, 1)$.
3. if $\mu + \nu \leq 0$, consumer's utility increases.

Proof. 1. Obviously, with the change of costs, ξ will change as well: $\xi' = \theta\xi$, where θ can depend on μ. We need to prove that $c(\rho)\, n(\rho) < c'(\rho)\, n'(\rho)$. Substituting the expressions from (2.11) and (2.12), we get

$$\frac{\rho\, f'}{\alpha'\,(1-\rho)\, w}\left(\frac{(1-\rho)\alpha^{\frac{\nu}{\nu-1}}\xi\; w^{\frac{1}{\nu-1}}}{\rho^{\frac{\nu}{\nu-1}} f a(\rho)^{\frac{1}{\nu-1}}}\right)^{\frac{(\nu-1)\rho}{\nu-\rho}} = k^{-\frac{\nu(\mu+1)(\rho-1)+\rho}{\nu-\rho}}\theta^{\frac{(\nu-1)\rho}{\nu-\rho}} \times \tag{2.16}$$
$$\times\, c(\rho)\, n(\rho) > c(\rho)\, n(\rho)\,.$$

Note that $-\frac{\nu(\mu+1)(\rho-1)+\rho}{\nu-\rho} > 0$ and $\frac{(\nu-1)\rho}{\nu-\rho} > 0$ for all $\nu < 0$ and $\rho \in (0,1)$. So if we prove that $\theta > 1$, inequality (2.16) will be proven. In order to prove that $\theta > 1$, consider budget constraint (2.11) of economy after technological progress, which may be rewritten in the following form:

$$\int_0^1 k^{\frac{-\nu(\mu x-\mu+x)}{\nu-x}} \theta^{\frac{(\nu-1)\rho}{\nu-\rho}} (1-x)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx = \frac{L}{f}, \tag{2.17}$$

where A is a combination of parameters of the model. Budget constraint for economy before technological progress is then

$$\int_0^1 (1-x)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx = \frac{L}{f}. \tag{2.18}$$

Note that if we prove that

$$\int_0^1 k^{\frac{-\nu(\mu x-\mu+x)}{\nu-x}} (1-x)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx < \frac{L}{f}, \tag{2.19}$$

that will mean exactly that for (2.17) to be true, θ must be greater than 1. Due to (2.18), inequality (2.19) may be rewritten in the following form:

$$\int_0^1 \left(1 - k^{-\frac{\nu(\mu x-\mu+x)}{\nu-x}}\right) (1-x)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx > 0. \tag{2.20}$$

Denote the integrand in (2.20) as $g(x)$. Figure 6 shows the graph of $g(x)$. This function is positive for $\rho > \frac{\mu}{1+\mu}$ and negative otherwise.

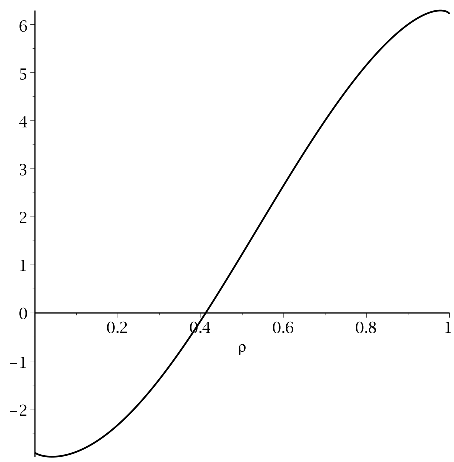

Fig. 2.7. $g(x)$ for $\mu = 0.7$.

We will prove that if $\mu \leq 1$, than

$$g(x) > -g(1-x) \text{ for all } x \in \left(\frac{\mu}{1+\mu}, 2\frac{\mu}{1+\mu}\right), \tag{2.21}$$

which proves (2.20). Note that for $\mu > 1$ it doesn't have to be true. Consider the function $h(x) = -\frac{g(x)}{g(1-x)}$. Analysis of this function shows that $\lim_{x\to\frac{\mu}{1+\mu}} h(x) = 1$ and it is monotonically increasing. So $h(x) > 1$ for $\rho \in (0.5, 1)$ and hence (2.21) holds, hence (2.20) holds, hence $\theta > 1$, which concludes the proof.

2. We will prove that $l'(1)n'(1) < l(1)n(1), l'(0)n'(0) < l(0)n(0)$ and $l \cdot n$ is a monotonic function of ρ. $\lim_{\rho\to 1} l(\rho)n(\rho) = \alpha^{\frac{\nu}{\nu-1}}\xi\, w^{\frac{1}{\nu-1}}$, so $\frac{l'(\rho)n'(\rho)}{l(\rho)n(\rho)} = k^{-\frac{\nu}{\nu-1}}\theta$. We need to prove that

$$\theta < k^{\frac{\nu}{\nu-1}}. \tag{2.22}$$

Assume otherwise: $\theta \geq k^{(\nu/(\nu-1))}$ and substitute it to the budget constraint in the form

$$L = \int_0^1 k^{-\frac{\nu(\mu x-\mu+x)}{\nu-x}} \theta^{\frac{(\nu-1)\rho}{\nu-\rho}} (1-x)^{\frac{\nu\,(x-1)}{\nu-x}} x^{-\frac{\nu\, x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx \geq$$
$$\geq \int_0^1 k^{\frac{\nu\mu(1-x)}{\nu-x}} (1-x)^{\frac{\nu\,(x-1)}{\nu-x}} x^{-\frac{\nu\, x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx >$$
$$\int_0^1 (1-x)^{\frac{\nu\,(x-1)}{\nu-x}} x^{-\frac{\nu\, x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx = L$$

This contradiction proves (2.21) and the fact that $l'(1)n'(1) < l(1)n(1)$.

$\lim_{\rho\to 0} l(\rho) = f$, so obviously $l'(0) > l(0)$. Finally, analysis of the derivative of l shows that is indeed monotonically increasing.

3. Consumer's utility may be written in the following form:

$$U' = \frac{1}{\nu}\int_0^1 \left(c'\left(\rho\right) n'\left(\rho\right)^{\frac{1}{\rho}}\right)^{\nu} d\rho = \frac{1}{\nu}\int_0^1 \left(k^{-\frac{\mu\rho-\mu+\rho}{\nu-\rho}} \theta^{\frac{\nu-1}{\nu-\rho}} c\left(\rho\right) n\left(\rho\right)^{\frac{1}{\rho}}\right)^{\nu} d\rho >$$
$$> \frac{1}{\nu}\int_0^1 k^{\frac{\nu(\mu\rho-\mu-\nu+\rho)}{\nu-\rho}} \left(c\left(\rho\right) n\left(\rho\right)^{\frac{1}{\rho}}\right)^{\nu} d\rho > \frac{1}{\nu}\int_0^1 \left(c\left(\rho\right) n\left(\rho\right)^{\frac{1}{\rho}}\right)^{\nu} d\rho = U.$$

Here we used (21) in the first inequality and the assumption $\nu \leq -1$ in the second one $(\mu\rho - \mu - \nu + \rho > -\mu - \nu > 0)$. ■

There are few things regarding these results we should point out. First, the decrease of the number of firms in all industries, which can be seen in Figure 3, is expectable result of increasing fixed costs and may be interpreted as creation of big corporations. The second part of proposition 3 is the most important to us: it means that because of technological progress, labor flows from less differentiated to more differentiated industries. This reallocation is exactly of the type described in the citation in the beginning of the article and the main feature of this model.

Figure 6 indicates another good property of output: the rate of output growth is higher in more differentiated industries, which is in line with what we observe in the modern economy. We do not include a proof because it can be done trivially by analyzing the derivative of the ratio. Note that another important indicator, output per worker, does not show non-trivial dynamics in our framework: it can be easily calculated that $c(\rho)/l(\rho) = \rho/\alpha$, so with technological progress this ratio increases in all industries equally.

Now consider the effect of population growth. Figures 7, 8, 9 show the effects of population growth of 150% in the same economy as in the previous figures, but everything is now per capita.

Proposition 4. With the population growth, for any parameters of economy and consumer's preferences of the type (2.1)-(2.2) with $a(\rho) = 1$ for any $\rho \in (0, 1)$,

1. output per capita decreases in industries with $\rho \in (0, \rho_1)$ and increases in industries with $\rho \in (\rho_1, 1)$ for some $\rho_1 \in (0, 1)$.

2. labor per capita decreases in industries with $\rho \in (0, \rho_2)$ and increases in industries with $\rho \in (\rho_2, 1)$ for some $\rho_2 \in (0, 1)$.

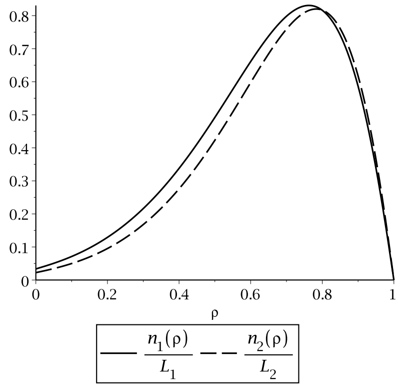

Fig. 2.8. Number of firms per capita.

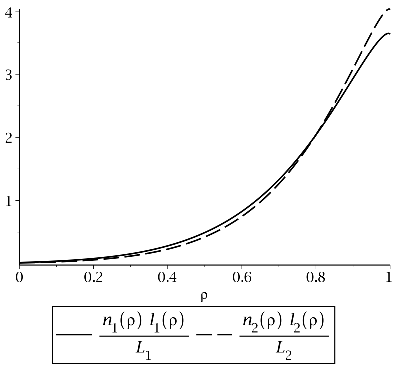

Fig. 2.9. Output per capita.

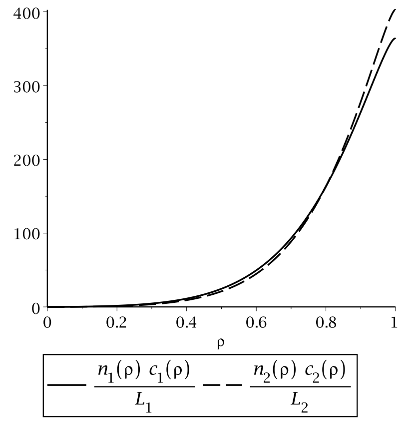

Fig. 2.10. Number of workers in industry per capita.

Proof. 1. Denote $L' = kL, k > 1, \xi' = \delta\xi$. First, we shall prove that $\delta > 1$. Assume otherwise, $\delta \leq 1$, then

$$L' = kL = \int_0^1 \delta^{\frac{(\nu-1)x}{\nu-x}} (1-x)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx \leq$$
$$\leq \int_0^1 (1-x)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx = L,$$

which contradicts to the assumption $k > 1$. Next, we prove that $\delta > k$. To do so, consider budget constraint in the form (2.17):

$$L' = kL = \int_0^1 \delta^{\frac{(\nu-1)x}{\nu-x}} (1-x)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx <$$
$$< \delta \int_0^1 (1-x)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx = \delta L.$$

$\lim_{\rho\to 0}\frac{c(\rho)n(\rho)}{L}=0$, so we have to analyze derivatives: $\lim_{\rho\to 0}\left(\frac{c(\rho)n(\rho)}{L}\right)'=\frac{f}{L}$ and $\frac{f}{L'}<\frac{f}{L}$, hence $\frac{c(\rho)n'(\rho)}{L'(x)}<\frac{c(\rho)n(\rho)}{L(x)}$ for small ρ's.

$\lim_{\rho\to 1}\frac{c(\rho)n(\rho)}{L}=\frac{\alpha^{\frac{1}{\nu-1}}\xi w^{\frac{1}{\nu-1}}}{L}$ and $\frac{xi'}{L'}=\frac{\delta\xi}{kL}>\frac{\xi}{L}$, hence $\frac{c(\rho)n'(\rho)}{L'(x)}>\frac{c(\rho)n(\rho)}{L(x)}$ for big ρ's. Analysis of derivative of the function $\frac{c(\rho)n(\rho)}{L(x)}$ shows that it is monotonically increasing.

2. Similarly to 1: $\lim_{\rho\to 0}\frac{c(\rho)n(\rho)}{L}=\frac{f}{L}$, $\lim_{\rho\to 1}\frac{c(\rho)n(\rho)}{L}=\frac{\alpha^{\frac{\nu}{\nu-1}}\xi w^{\frac{1}{\nu-1}}}{L}$. ■

As we can see, and as it might be expected, population growth leads the economy to, in a sense, opposite direction compared to economic growth, consumer now cares more about "simple" homogenous goods. This behavior may be interpreted in the following way: as the population increases, it becomes harder and harder to "feed" them, so people start to care less about luxury (differentiated goods) and more about simple material goods.

3. CONCLUSION

We developed a simple and natural model, which generalizes the Dixit-Stiglitz monopolistic competition model. It provides a natural way to describe technological progress, which leads to non-trivial labor and product reallocations. These reallocations may be interpreted as transfers from "simple" homogeneous to "more complicated" differentiated goods (from agricultural to manufacturing goods and then to services). Under some assumptions, it leads to an increase in consumer utility. So, despite its simplicity, our model is able to reproduce essential features of modern economies, described in economic growth literature.

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