The Vibration Response Analysis on a 6-UPS PKM Based on FEM

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Abstract

In this paper, a finite element full-scale entity model for 6-UPS PKM and its accessories were established based on substructure method and were further confirmed through a modal experiment and a finite element modal analysis. The displacements on response curves of the nodes along the virtual x, y and z axes of PKM were obtained through a random vibration response analysis on PKM performed with a finite element method. Based on these resultant curves, the law of vibration response on the PKM was investigated, and the resultant parameters clearly demonstrated the vibration response characteristic of PKM and its response scope magnitude, which can provide an important theoretical base for the optimal structure design of PKM.

Keywords PKM, Random Vibration Response, Stewart platform, FEM, Dynamics

1 Introduction

The BKX–I parallel kinematic machine (PKM)[1] is a typical using of Stewart platform in machine. It is a 6 -UPS PKM, shown as Fig.1.

Fig.1 BKX-I PKM

The PKM's rigid body dynamics model can be built by nearly all classical mechanics methods, such as Newton-Euler method, Lagrange method, virtual displacement principal, Kane equation[2-3], etc. So, these methods based on dynamic can be chosen proper and simply according to different type machines which are extensively used in control system. But, the telescopic shafts in PKM shows the flexible characters that the rigid parallel kinematic structures don't have during the high speed working. The shaft's elastic deformation, dynamic stress and elastic vibration can cause the whole machine's impact, noise and fatigue[4]. The PKM's dynamic system is actual a multi-elastic system, having the characters of mechanism-structure couple, time-varying, nonlinearity, etc. So, during the period of the PKM's design and optimization, the finite element method is usually used in the modal analysis and the response analysis to reflect the machines' dynamic characters and the convenience of solution accurately[5].

The machine structure's random vibration response analysis is one of spectral analysis in the dynamic analysis. It is mainly used in the probability statistics of structure to random vibration response analysis. It is also a relationship curve of power spectral destiny and frequency which reflects the intensity of load and the frequency in time changing.

2 The Machine's Finite Element Solid Modeling Method

The BKX-I PKM's geometric structure is a complex spatial structure. Directly establish its finite element model by using the software of ANSYS would be complex. In this paper, the geometry solid model including 14 substructures was established by 3D software PRO/E. The substructures included the machine frame, motional platform, 6 telescopic shafts and 6 crosses of universal joint which the universal joint's other accessories belonged to the machine frame and telescopic shaft. The main idea for making substructures was that it could make every moving structure as the substructure while the machine was working. The motional platform is a substructure because it is the PKM's actuator and also an individual moving part. The machine's frame is an entirety and holds stationary relative the earth when the machine is working. The universal joint's upper part was fixed the frame firmly by all kinds of connection, so they could be ranged as a substructure. Inside the telescopic shaft assembly, although it's lower slide bar and upper swing bar had the tendency of motion relatively, both of them and the universal joint's lower part which connected with them were supposed as a substructure in order to simplify the structure because their connections and power transmission were obtained by the ball screw. The universal joint's cross as the main member of connecting the machine frame and telescopic shaft is always an individual active structure in any time, so it also was a substructure.

After establishing the geometry solid model, it could be transferred to finite element model by the interface between the PRO/E and ANSYS. During the transferring, the relationship between the original PRO/E substructures would disappear. The couple configuration of every joint face needs to be made again in ANSYS to simulate spherical hinge, universal joint and other structures. The couple configuration was that it equalized the entire contact node's displacement DOF which between the two contact face, and each other's rotational DOF need not to be constrained to make the contact node have the tendency of rotation relatively. After coupling, it could use the SOLID45 unit to meshing every solid model freely. Then the PKM's finite element solid model including 170 thousand solid units was established, shown as Fig.2.

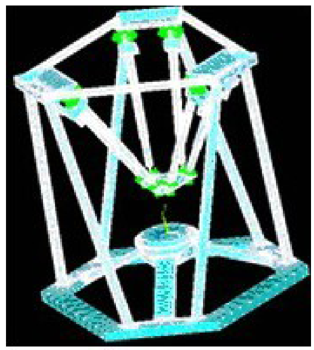

Fig.2 BKX-I PKM FEM model

3 The Confirmation and the Modal Analysis on the Finite Element Model of PKM

The modal experiment and finite element theoretic modal analysis on the machine were carried out respectively. Then, the comparison of results was made to confirm the validity of finite element model.

During the experiment modal analysis the method of single point exciting and multi-point receiving[6] was used. In the experiment, the machine was fixed on the foundation firmly first, which its position and posture were adjusted to as the same as finite element model's. Then the excitation with the hammer were made, while the 4 acceleration sensors picked up and recorded the system's single exciting signal and multi-response signals at the same time. The experiment statistic's acquisition and disposal were made by the software of DASP. After a series of signals disposing like analog digital conversion and faster Fourier transformation, the system's transfer function, amplitude-frequency were obtained which can reflect the functional relationship of machine's dynamic characteristics and then the machine's natural frequency could be got by normalizing the modal parameter.

The boundary condition of finite element theoretic modal analysis was that the machine basement's DOF of displacement was zero. Then the Block-lanczos method was used during the finite element modal analysis. After that, all of the machine's 10 order natural frequency, vibration shape and its animation were obtained in ANSYS.

The comparable result between the machine's natural frequency got from the finite element modal analysis and the machine's natural frequency got from the experiment was shown in the Table 1. From the table we could get that the proper finite element model could be build with the error less than 10% after many times of simplifying the structures and choosing proper finite element unit.

Table 1 The results comparison between FEM's analysis and experimental modal analysis

Modal Order	FEM Calculation	Actually Value	Error
1	25.836	23.357	9.6%
2	49.436	45.472	8.0%
3	67.988	63.434	6.7%
4	76.475	72.365	5.4%

4 The Theoretic Basis of Response Analysis on PKM

When n order natural frequencies of PKM are $\omega_1, \omega_2, \ldots \omega_n$ and the corresponding vibration shapes are $\mathbf{X}_1, \mathbf{X}_2, \ldots \mathbf{X}_n$, which natural frequencies are arranged according to sort ascending and the ω_1 and $\mathbf{X}_1$ are the PKM's basic frequency and basic vibration shape, the relationship of them can be expressed as follows:

$$[K]\mathbf{X}_i = {\omega_i}^2[M]\mathbf{X}_i \qquad (i = 1, 2, \ldots n) \tag{1}$$

Because the integer stiffness matrix $[K]$ and the integer mass matrix [M] are real symmetric positive definite matrix which the vibration shape vectors $\mathbf{X}_1, \mathbf{X}_2, \ldots \mathbf{X}_n$ are a group of basis of n-dimensional space vector, the node displacement vector $\{\phi\,(t)\}$ can be expressed as follows:

$$\{\phi\,(t)\} = q_1\mathbf{X}_1 + q_2\mathbf{X}_2 + \ldots + q_n\mathbf{X}_n = [X]\{\mathbf{q}\} \tag{2}$$

Here, $q_1, q_2, \ldots q_n$ is the coordinate of $\{\phi\,(t)\}$ in the coordinate system which the vibration shape vectors are basis and they are the time function. So, $\{q\} = \{q_1 \;\; q_2 \;\; \ldots \;\; q_n\}^T \cdot [X]$ is the vibration shape matrix of system which [x] $= [\mathbf{x}_1 \;\; \mathbf{x}_2 \;\; \ldots \;\; \mathbf{x}_n]$.

The dynamic equation of machine in the entire coordinate system can be obtained as follows:

$$[M]\{\ddot{\phi}\} + [C]\{\dot{\phi}\} + [K]\{\phi\} = \{F\} \tag{3}$$

The Eq.(2) is substituted into Eq.(3) and the equation can be obtained as follows:

$$[\mathbf{M}][X]\{\ddot{\mathbf{q}}\} + [\mathbf{C}][X]\{\dot{\mathbf{q}}\} + [\mathbf{K}][X]\{\mathbf{q}\} = \{F(t)\} \tag{4}$$

The equation is left multiplicity by $[X]^T$ and the equation can be obtained as follows after collate:

$$\overline{\mathbf{M}}\ddot{\mathbf{q}} + \overline{\mathbf{C}}\dot{\mathbf{q}} + \overline{\mathbf{K}}\mathbf{q} = \overline{\mathbf{F}}(\mathbf{t}) \tag{5}$$

Here,

$\overline{\mathbf{M}}$–Main mass matrix of system.

$$\overline{\mathbf{M}} = [X]^T[\mathbf{M}][X] = diag(\overline{m}_1, \overline{m}_2, \ldots, \overline{m}_n)$$

$\overline{\mathbf{K}}$–Main stiffness matrix of system.

$$\overline{\mathbf{K}} = [X]^T[\mathbf{K}][X] = diag(\overline{k}_1, \overline{k}_2, \ldots, \overline{k}_n)$$

$$\text{Here, } {\omega_i}^2 = \overline{k}_i / \overline{m}_i$$

$\overline{\mathbf{C}}$–Main damp matrix of system

$$\overline{\mathbf{C}} = [X]^T[\mathbf{C}][X] = diag(\overline{c}_1, \overline{c}_2, \ldots, \overline{c}_n) \text{ and}$$

$\overline{c}_i = 2\xi_i\omega_i\overline{m}_i$. Here, the ξ_i is the damp ratio corresponding to the i order vibration shape.

$\overline{F}\,(t)$ –Main active load vector.

$$\overline{F}\,(t) = [X]^T\{F\,(t)\}.$$

Expand the Eq.(5) and every equation is divided by $\bar{m}_i$, the equation can be obtained as follows after collate:

$$\ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + {\omega_i}^2 q_i = \overline{F}_i\,(t)\,/\overline{m}_i \qquad (i = 1, 2, \ldots, n) \tag{6}$$

This is a typical single freedom vibration equation belong to linear constant coefficient quadratic differential equation. The chief response of structure system in modal space can be got after solving the equation.

$$\begin{aligned} q_i = {} & e^{\xi_i\omega_i t}\left(q_{i0}\cos\omega_{di}t + \frac{\dot{q}_{i0} + \xi_i\omega_i q_{i0}}{\omega_{di}}\sin\omega_{di}t\right) \\ & + \frac{1}{\omega_{di}}\int_0^t \frac{F_i\,(\tau)}{\overline{m}_i} e^{-\xi_i\omega_i(t-\tau)}\sin\omega_{di}\,(t-\tau)\,d\tau \end{aligned} \tag{7}$$

$$\text{Here,}\omega_{di} = \omega_i\sqrt{1 - {\xi_i}^2}$$

$$\{q\}_0 = [X]^T[M]\{\phi_0\}$$

$$\{\dot{q}\}_0 = [X]^T[M]\{\dot{\phi}_0\}$$

Last, when q_i are replaced by Eq.(7) in Eq.(2), the relationship equation between the displacement response in the original physics coordinate system and the excitation could be obtained.

5 The Random Vibration Response Analysis Based on FEM

The random vibration analysis based on the FEM was solved on the base of modal analysis after extending the modal. In ANSYS, the nodes' DOF of machine base plane were fixed. After extending the modal, the random displacement vibration loads of 1×10^{-5} m amplitude were imposed on servo motors which located on the 6 telescopic shafts and the main axis motors which located on the center of motional platform along the machine's virtual x and y axes. According to the machine structure damper rate which got from the modal experiment, it could definite the damper rate as 2%. Finally, random vibration analysis was implemented to the PKM's general structure to get the curve of the machine structure node's displacement response when the random vibration frequency was between zero and 200Hz.

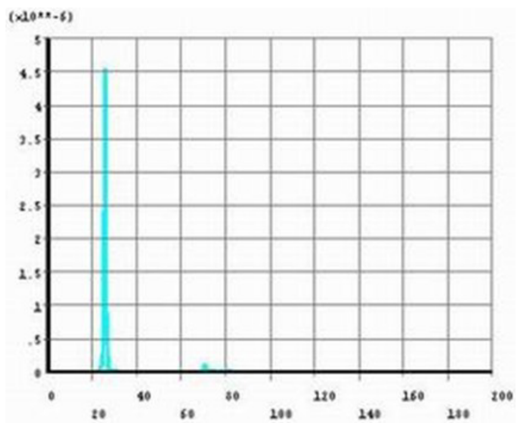

Fig.3 The random vibration response curve of motional platform centre node along the x axis

The displacement response curve of the node centre on the motional platform along the virtual x axis was shown as Fig3, on the y axis curve shown as Fig 4, on the z axis curve shown as Fig5. For saving the words, other nodes' curves were not shown. In Fig3, Fig4 and Fig5, the horizontal axis was frequency which its unit was Hz, the longitudinal axis was displacement which its unit was meter.

6 The Analysis and Conclusions of Calculation Results

Some conclusions were got from the PKM's random vibration response curves:

First, all of the virtual x, y, z axes had a great response around the 26Hz of the machine's first order natural frequency. It was the resonance response which didn't happen in the second and third order natural frequency. At the same time,

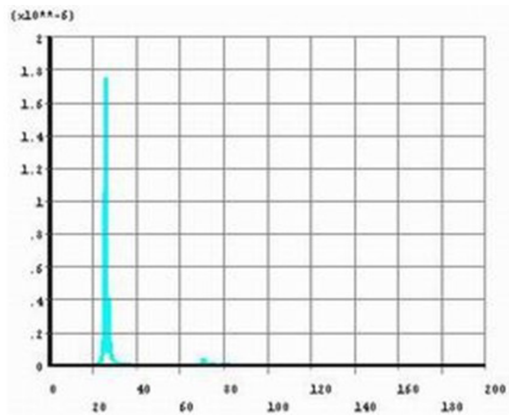

Fig.4 The random vibration response curve of motional platform centre node along the y axis

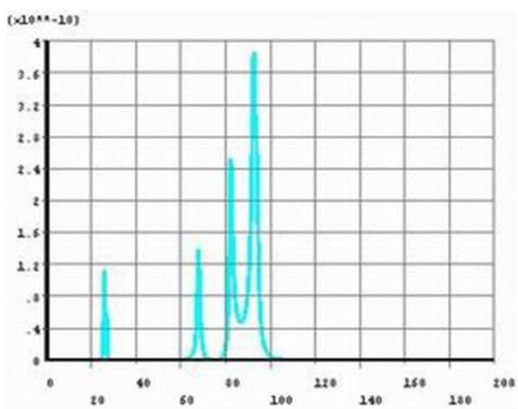

Fig.5 The random vibration response curve of motional platform centre node along the z axis

a smaller resonance happened in the fourth order natural frequency. From the modal shape of machine and its animation, the result could be made that the PKM's first order modal shape was the machine's whole modal shape, the second and third order modal shape were machine's local modal shape. The vibration response excitation which loaded on the centre of motional platform didn't excite the machine's second and third order response. The result of machine's vibration analysis and the result of machine's modal analysis were closely fit each other.

Second, the motional platform centre's maximum resonance amplitude was 4.5×10^{-6} m which was half of the loaded maximum resonance amplitude. Comparing to the vibration excitation amplitude, the vibration response value was less than 2 orders of magnitude when the PKM didn't resonate, so its influence could be neglected during the actually manufacturing progress.

Third, the motional platform centre resonated in some low-order natural frequency along the virtual z axis, but compared with the biggest random vibration amplitude, its values were less than 4 orders of magnitude in vibration magnitude. So it could be nearly neglected.

Fourth, the result, which the random response along the x and y axis' was greater than that along the z axis', was just like the rigidity which along the machine x and y axis' was far lower than that along the z axis'. The result of machine's vibration analysis and the result of machine's rigidity analysis[6] were closely fit each other.

Acknowledgements

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