

# Determining the Vertical Force When Steering

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**Abstract:** When the vehicles move at high velocity and quickly steer, the vehicles can be rollover. The first sign of this phenomenon is that two wheels on the same side are completely separated from the road surface. The typical value for this sign is the vertical force  $F_z$  at each wheel. If this value gradually decreases to zero, the wheel runs the risk of separating from the road surface that may lead to rollover. Therefore, the value of the vertical force  $F_z$  in different conditions needs to be determined to detect the imminent limit of the rollover phenomenon. This research established the spatial dynamic model of the vehicle to determine the vertical force  $F_z$  based on the simulation method. Besides, the equation describing the relationship between vertical force  $F_z$ , velocity  $v$ , and acceleration steering  $\varepsilon$  corresponding to the value of steering angle  $\delta$  is also established by the calculation process. From this equation, the value of the vertical force  $F_z$  can be simply calculated with relatively high accuracy based on determining conditions. The results of this research are the basis for determining and establishing the vehicle's rollover limit.

**Keywords:** dynamic vehicle model, vertical force, function, steering acceleration, rollover

## 1. INTRODUCTION

### 1.1. The Instability Problem of the Vehicle

Automobiles are a popular vehicle in everyday life. However, the number of vehicle accidents is also very large. When the vehicles move at high velocity and quickly steer, the vehicles often encounter the situation: side slip or rollover.

The side slip phenomenon usually occurs when the vehicles go on slippery roads, wheels are not able to contact the road surface. When the side slip occurs, the driver could not control the trajectory and direction of the motion of the vehicles. The sideslip problem is usually less dangerous than the rollover problem.

The rollover phenomenon occurs when two wheels on the same side are completely separated from the road surface. If there is only one wheel separated from the road surface, the vehicle will be in an unstable state and at risk of rollover [15, 17]. The main cause of this phenomenon is due to the lateral acceleration  $a_y$  appeared suddenly when quickly steering, especially in the case of fishhook steering [11, 18]. The parameter that warns the risk of rollover is the vertical force at wheels  $F_z$ . If this value is sufficiently close to zero, the wheel runs the risk of separating from the road surface. Therefore, if it is possible to determine the vertical force value  $F_z$ , the timing of the rollover of the vehicle can be predicted.

Some solutions have been proposed to improve this situation such as the use of the active stabilizer bar, the electronic power steering, the active suspension, the electronic stability

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program, [etc.] [1, 16, 19]. The above systems use parameters of input signals such as the lateral acceleration  $a_y$ , the roll angle  $\varphi$ , the vertical force at wheels  $F_z$  [5, 20].

### 1.2. Literature Review

The topic of "Rollover Vehicle" has been researched by many authors. Several authors have introduced a rollover index  $R$  [6, 8, 9, 10, 14]. This index is determined based on the difference in the vertical force of the wheels. If  $|R| < 1$ , the car operates stably. In the case of  $|R| = 1$ , both wheels on one side are separated from the road surface, the vehicle rollovers completely. However, this index only really makes sense in case both wheels of the same side are separated from the road surface. If only one of the wheels is lifted off the road surface, the rollover index  $R$  cannot be determined.

Besides, the value of lateral acceleration  $a_y$  is often used to determine the rollover threshold of the vehicles [3, 7, 13]. The value of the lateral acceleration  $a_y$  can be calculated and simulated through link equations or experiments. However, this value is only really meaningful in case of ignoring the influence of the dimensions of the vehicles. Some other studies use the roll angle of the vehicle  $\varphi$  to determine the vehicle's rollover limited [2, 4, 12]. The value of the roll angle of the vehicles  $\varphi$  is calculated by the lateral acceleration  $a_y$ , which includes the influence of the dimensions.

To accurately determine the limits of the vehicle instability, it is necessary to find the time when the wheels are separated from the road surface ( $F_z = 0$ ). This research focuses on identifying the limits at which the wheels are separated from the road surface. At the same time, the research also established the equation showing the relationship between vertical force  $F_z$ , steering angle  $\delta$ , and steering acceleration  $\varepsilon$ . Therefore, it is easy to determine the vertical force  $F_z$  at different times and conditions.

## 2. METHODOLOGY

### 2.1. Nomenclature

$\theta$ :	Pitch angle, rad
$\varphi$ :	Roll angle, rad
$\delta$ :	Steering angle, rad
$\psi$ :	Yaw angle, rad
$a_y$ :	Lateral acceleration, $m/s^2$
$b$ :	Half of the base width, m
$C_{ij}$ :	Damping coefficient, Ns/m
$F_{Cij}$ :	Force of the damper, N
$F_{Kij}$ :	Force of the spring, N
$F_{KTij}$ :	Force of the tire, N
$F_{Xij}$ :	Longitudinal force, N
$F_{Yij}$ :	Lateral force, N
$F_{Zij}$ :	Vertical force, N
$h$ :	Distance from center to roll axis, m
$I_z$ :	Moment of inertia of the z-axis, $kgm^2$
$I_x$ :	Moment of inertia of the x-axis, $kgm^2$
$K_{ij}$ :	Stiffness of the spring, N/m
$K_{Tij}$ :	Stiffness of the tire, N/m
$l_1$ :	Distance from the center to the front axle, m
$l_2$ :	Distance from the center to the rear axle, m
$m$ :	Sprung mass, kg
$m_{ij}$ :	Unsprung mass, kg

- $u_{ij}$ : Bump on the road, m
- $v_x$ : Longitudinal velocity, m/s
- $v_y$ : Lateral velocity, m/s
- $z$ : Displacement of the sprung mass, m
- $z_{ij}$ : Displacement of the unsprung mass, m

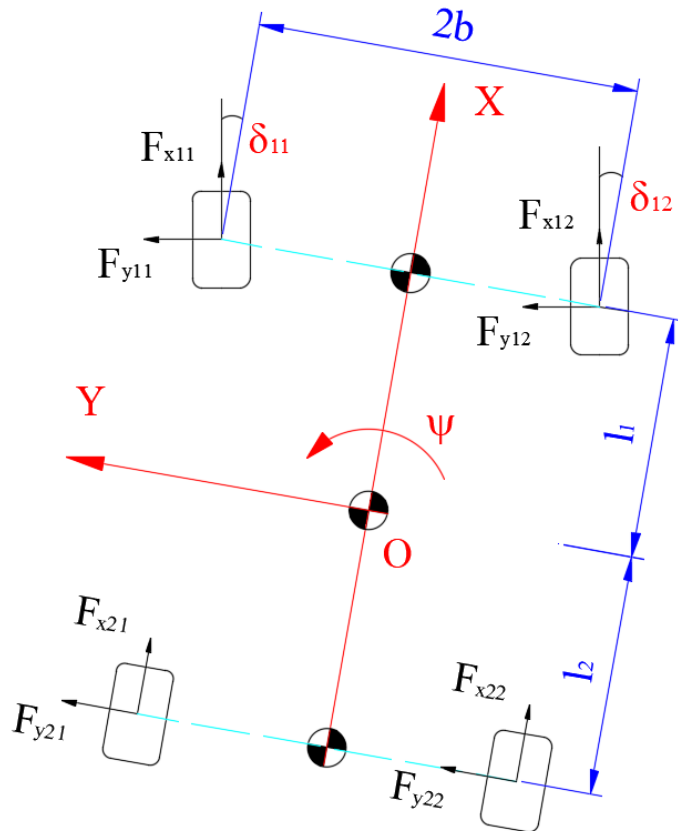
**2.2. Double-track Dynamic Vehicle Model**

The motion of the vehicle is set based on the model of 10 degrees of freedom. The double-track model is set as below (Fig. 2.1).

Assuming that the vehicle is moving at a constant velocity, the steering angle is small. Ignore the influence of other factors. The motion determination matrix is described as follows [2]:

$$(\dot{v}_y \ddot{\psi}) = (F_{y1} \ F_{y2}) \begin{pmatrix} \frac{1}{m + \sum_{i,j=1}^2 m_{ij}} & \frac{l_1}{I_z} \\ \frac{1}{m + \sum_{i,j=1}^2 m_{ij}} & -\frac{l_2}{I_z} \end{pmatrix} - v_x (\dot{\psi} \ 0) \tag{2.1}$$

Where:  $F_{y1} = F_{y11} + F_{y12}$        $F_{y2} = F_{y21} + F_{y22}$



**Fig. 2.1.** The double-track model

**2.3. Spatial Dynamic Model 7 DOF**

To determine the oscillation of the vehicle, it is necessary to establish the spatial dynamic model with 7 degrees of freedom as Fig. 2.2.

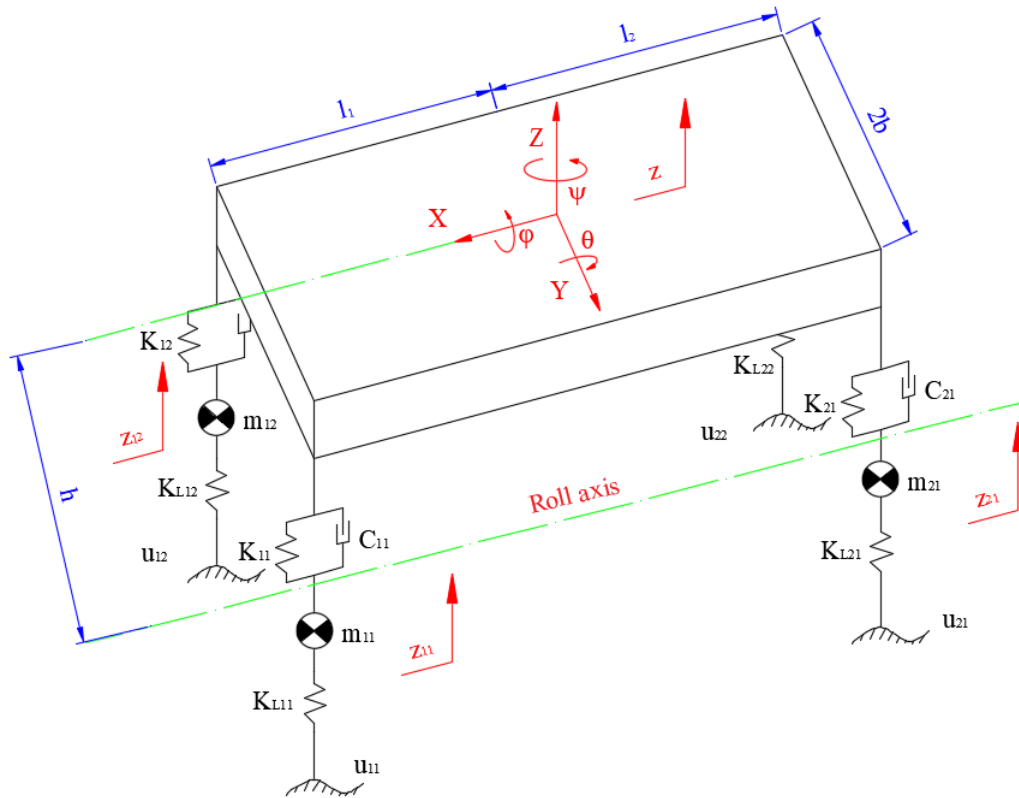


Fig. 2.2. Dynamic vehicle model 7 DOF

The matrix of vehicle oscillation is given as follows [2]:

$$(\ddot{z} \quad \ddot{\phi}) = (F_1 \quad F_2) \begin{pmatrix} \frac{1}{m} & \frac{b}{I_x + mh^2} \\ \frac{1}{m} & -\frac{b}{I_x + mh^2} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{(a_y \cos^2 \phi + g \sin \phi) mh}{I_x + mh^2} \end{pmatrix} \quad (2.2)$$

Where:  $F_1 = F_{C11} + F_{K11} + F_{C21} + F_{K21}$        $F_2 = F_{C12} + F_{K12} + F_{C22} + F_{K22}$

To be able to identify the above matrix, use the corresponding link equations.

The vertical displacement of the unsprung mass:

$$m_{ij} \ddot{z}_{ij} = F_{KTij} - F_{Cij} - F_{Kij} \quad (2.3)$$

The elastic force of the suspension system:

$$F_{Kij} = K_{ij}(z_{ij} - z \pm b\phi) \quad (2.4)$$

The damping force of the suspension system:

$$F_{Cij} = C_{ij}(\dot{z}_{ij} - \dot{z} \pm b\dot{\phi}) \quad (2.5)$$

The elastic force of the tire:

$$F_{KTij} = K_{Tij}(u_{ij} - z_{ij}) \quad (2.6)$$

The vertical force  $F_z$  is calculated by using the equation below:

$$F_{Zij} = F_{KTij} - F_{Cij} - F_{Kij} \quad (2.7)$$

When the vertical force  $F_z$  becomes close to zero, the wheel tends to separate from the road surface, the vehicle is in an unstable state.

### 3. RESULT AND DISCUSSION

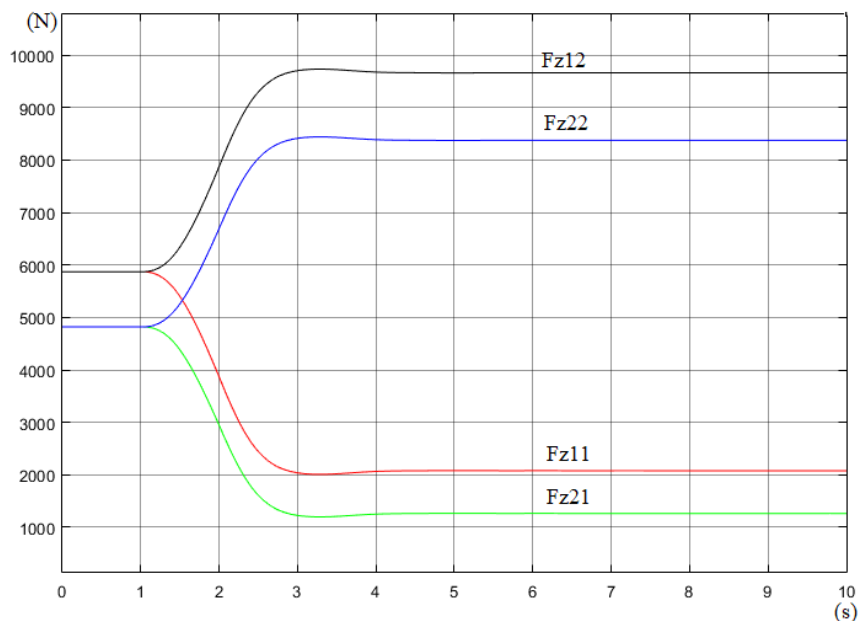
#### 3.1. Simulation Conditions

The established model is used for most of the common vehicles. To conduct simulation and review, the specifications of the reference vehicles are given in Table 3.1.

**Table 3.1.** Vehicle specifications

Symbol	Description	Value	Unit
m	Sprung mass	2000	kg
m <sub>ij</sub>	Unsprung mass	45	kg
l <sub>1</sub>	Distance from center of gravity to front axle	1250	mm
l <sub>2</sub>	Distance from center of gravity to rear axle	1550	mm
b	Half of the base width	750	mm
I <sub>x</sub>	Moment of inertia of the x-axis	800	kgm <sup>2</sup>
I <sub>z</sub>	Moment of inertia of the z-axis	2400	kgm <sup>2</sup>
h	Distance from the center to roll axis	650	mm

At the velocity  $v = 70$  km/h, the steering angle  $\delta = 0-5^0$  (linear increase in 1 second), the graph in Fig. 3.1 shows the value of the vertical force  $F_z$  at the wheels at the same time.



**Fig. 3.1.** The value of the vertical force  $F_z$

From Fig. 3.1 it can be seen that the value of the  $F_{z21}$  is the smallest, this wheel tends to separate from the road surface first. Therefore, simulation is concentrated at this position to determine the time of separating the wheels from the road surface.

The research will simulate the oscillation of the vehicle with two cases:

Case 1: Steering angle  $\delta = 0-4^0$  (linear increase), corresponding to the different values of steering acceleration and velocity of the vehicle.

Case 2: Steering angle  $\delta = 0-5^0$  (linear increase), corresponding to the different values of steering acceleration and velocity of the vehicle.

### 3.2. Results

Using the parameters of the reference vehicle and the two proposed cases, the graph in Fig. 3.2 shows the relationship between steering acceleration  $\varepsilon$ , velocity  $v$ , and vertical force  $F_{Z21}$  of the wheel at position (21).

The graph in Fig. 3.2 indicates that:

+ At the same value of the velocity, if the steering acceleration increases, the vertical force at the wheel will decrease. In case the velocity value is small, this decrease is not much. Otherwise, if the velocity value is large, this attenuation will be significant.

+ At the same value of steering acceleration, if the velocity increases, the vertical force will drop sharply. This decrease is almost linear (in case the value of steering acceleration is large).

+ At the same value of velocity and steering acceleration, if the steering angle is larger, the value of vertical force will be smaller.

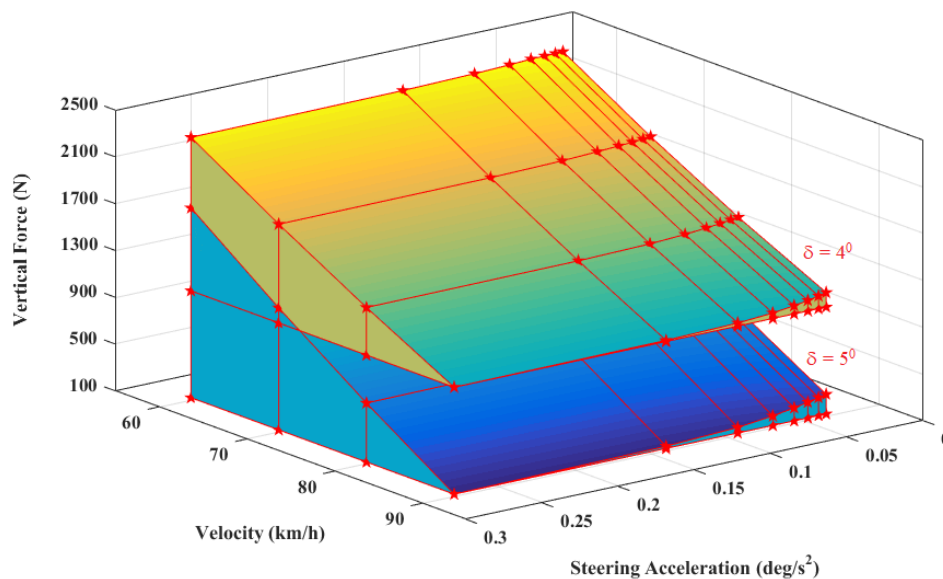
The relationship between the parameters in Fig. 3.2 has the form of irregular planes. Therefore, it is possible to set up many plane equations to determine the vertical force value  $F_z$  when the velocity  $v$  and steering acceleration  $\varepsilon$  are known.

The equation for determining the value of the vertical force  $F_z$  is as follows:

$$F_z = A_1 + A_2\varepsilon + A_3v \quad (3.1)$$

Where:  $A_1$ ,  $A_2$ , and  $A_3$  are the coefficients of the equation for each specific determination interval.

Based on the results obtained from the simulation process, the above coefficients are given in Table 3.2. The coefficients  $A_1$ ,  $A_2$ , and  $A_3$  correspond to different values of steering angle and steering acceleration.



**Fig. 3.2.** The relationship between steering acceleration, velocity, and vertical force

**Table 3.2.** The coefficients of the equation

Steering angle (deg)	Steering acceleration (deg/s <sup>2</sup> )	The coefficients of the equation		
		A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>
4 <sup>0</sup>	$\varepsilon \geq 0.14$	4989.07	-71.94	-43.73
4 <sup>0</sup>	$0.07 \leq \varepsilon < 0.14$	5064.91	-913.04	-43.57
4 <sup>0</sup>	$\varepsilon < 0.07$	5099.01	-2571.43	-42.80
5 <sup>0</sup>	$\varepsilon \geq 0.14$	5023.14	-136.69	-54.07
5 <sup>0</sup>	$0.07 \leq \varepsilon < 0.14$	5127.43	-1434.78	-53.63
5 <sup>0</sup>	$\varepsilon < 0.07$	4704.57	-2714.29	-48.03

The vertical force value  $F_z$  can easily be determined through the coefficients of the equation.

Table 3.3 shows the difference between the results of calculating vertical force  $F_z$  by two different methods (simulation and equation).

**Table 3.3.** The value of the vertical force  $F_z$

Velocity (km/h)	Steering acceleration (deg/s <sup>2</sup> )	Steering angle $\delta = 0-4^0$			Steering angle $\delta = 0-5^0$		
		Fz simulation (N)	Fz equation (N)	Tolerance (%)	Fz simulation (N)	Fz equation (N)	Tolerance (%)
60	0.279	2345	2345	0.0	1741	1741	0.0
60	0.140	2354	2355	0.0	1757	1760	0.2
60	0.093	2366	2366	0.0	1776	1776	0.0
60	0.070	2377	2387	0.4	1784	1810	1.4
60	0.056	2387	2387	0.0	1787	1771	0.9
60	0.047	2388	2411	1.0	1791	1797	0.3
60	0.040	2389	2428	1.6	1795	1815	1.1
60	0.035	2391	2441	2.0	1796	1828	1.8
70	0.279	1877	1908	1.6	1165	1200	2.9
70	0.140	1884	1918	1.8	1179	1219	3.3
70	0.093	1896	1930	1.8	1199	1240	3.3
70	0.070	1909	1951	2.2	1216	1273	4.5
70	0.056	1919	1959	2.0	1224	1191	2.8
70	0.047	1926	1983	2.9	1231	1216	1.2
70	0.040	1931	2000	3.5	1237	1234	0.2
70	0.035	1939	2013	3.7	1241	1248	0.6
80	0.279	1440	1471	2.1	635	659	3.6
80	0.140	1448	1481	2.2	649	678	4.3
80	0.093	1461	1494	2.2	674	704	4.3
80	0.070	1477	1516	2.6	705	737	4.3
80	0.056	1493	1531	2.5	722	711	1.5
80	0.047	1506	1555	3.2	736	736	0.0
80	0.040	1516	1572	3.6	750	754	0.5
80	0.035	1525	1585	3.8	760	767	0.9
90	0.279	1033	1033	0.0	119	119	0.0
90	0.140	1043	1043	0.0	138	138	0.0
90	0.093	1059	1059	0.0	167	167	0.0
90	0.070	1080	1080	0.0	200	201	0.5
90	0.056	1103	1103	0.0	230	230	0.0
90	0.047	1123	1127	0.4	253	256	1.2
90	0.040	1142	1144	0.2	273	274	0.4
90	0.035	1157	1157	0.0	287	287	0.0

The data in the Table show that there is a difference between the two methods, but the tolerance is quite small. In the case of steering angle  $\delta = 0-4^{\circ}$ , the biggest tolerance is only 3.8%. In the case of steering angle  $\delta = 0-5^{\circ}$ , this value does not exceed 4.5%.

In general, this difference is not much and this equation can be used in many different cases.

#### 4. CONCLUSION

The stability and safety of the vehicle are expressed through the value of vertical force at the wheel  $F_z$ . The velocity, steering angle, and steering acceleration have a great influence on this problem.

When the steering angle increases or the steering acceleration increases or both factors increase, the value of the vertical force  $F_z$  will decrease significantly. If the value of  $F_z$  gets close to zero, the wheels tend to separate from the road surface. At this time, the vehicle is in an unstable situation and very dangerous.

To determine the value of the vertical force  $F_z$  at different velocities, steering angle, and steering acceleration, the established dynamic vehicle model can be used. However, the setup and simulation process is complicated and inconvenient. This research gave the function of determining the value of the vertical force  $F_z$  at specific conditions. Therefore, it is easy to identify  $F_z$  without using the dynamic vehicle model to simulate.

The function is not perfectly accurate, there is a difference in the vertical force value  $F_z$  when determined by this function and simulation process. However, this difference is quite small (less than or equal to 3.8% when the steering angle is from  $0^{\circ}$  to  $4^{\circ}$  and less than or equal to 4.5% when the steering angle is from  $0^{\circ}$  to  $5^{\circ}$ ). The results of the research can be used in many different survey cases.

This research has only focused on the theoretical calculation process and establish corresponding equations, it is necessary to experiment to accurately evaluate the results. The results of this research will be the basis for the development of other research in the future.

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