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Simultaneously Robust Chaos Synchronization and Identification of Unknown Parameters for Nonlinear Gyro System

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Abstract: This paper concerns robust synchronization and parameter identification for nonlinear gyroscope systems. Gyros are widely utilized in navigational applications where synchronization plays a vital role. A system of nonlinear dynamical equations with some parameters presents a model of gyro systems. The parameters of gyro can vary in time, which can lead to desynchronization of the gyro systems. In this paper, we assume that a gyro system has bounded time-varying unknown parameters and the synchronization problem is considered in two situations. First, the synchronization of two gyroscopes with identical dynamical model and, second, the synchronization of a gyroscope with the Rössler system. The Lyapunov stability theory with control terms is employed to cope with the problem. Also, the identification of time-varying unknown parameters is the side goal of the paper. The proposed scheme synchronizes chaotic nonlinear systems in both situations appropriately. In addition, the slave parameters converge to the nominal values of master parameters despite uncertainty. Simulation results illustrate the superiority of the proposed method.

Keywords: robust synchronization, chaos, parameter identification, gyroscope, Rössler system

1. INTRODUCTION

Numerous studies confirm that complex and chaotic behaviors are observed in physics, mechanics, engineering, etc. [1–4]. The chaotic behavior of dynamical systems is interested in many researches [5,6]. Also, chaos synchronization in applied systems is studied [7]. The intent of chaos synchronization is to synchronize the states of at least two chaotic systems despite the difference in initial conditions, the difference in model parameters, and even the difference in dynamical models. How to reach synchronization under such distinctions is a challenging crux [8]. The identification of unknown system parameters is also the side goal of many studies. Many schemes have been proposed to synchronize two chaotic systems, such as adaptive control [9–11], sliding mode control [12, 13], robust methods [14, 15], intelligent base methods [16–19] and Lyapunov base methods [20–23].

The gyroscopes are widely employed in navigational, aeronautical, and space engineering. Those have complex dynamical motion including periodic, sub-harmonic, quasi-periodic, and chaotic [24]. Gyro synchronization for a couple of chaotic systems with one-way linking is considered in [25] and it is extended in [26] by applying an active control. Fuzzy sliding mode control is employed to synchronize uncertain chaotic nonlinear gyros in [27]. A variable structure control approach using Neural Network with multi-quadratic radial basis

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function is introduced in [28] where gyros with known and unknown system parameters are synchronized. The existence of external disturbance is investigated in [29], where fast non-singular terminal sliding mode surface is employed to derive an adaptive finite-time control in order to synchronize the gyro systems. Synchronization and parameter identification for the symmetric gyroscope system with constant parameters have been done in [30]. It is not robust with respect to parameter's variation and does not converge to the nominal value of parameter.

This paper concerns developing robust synchronization based on the Lyapunov theorem with control terms for nonlinear gyroscope systems with unknown time-varying parameters. In addition, unknown parameter identification is expected. The idea is to find the control input u such that the parameters' uncertainty eliminates in the Lyapunov function. Two situations are considered for the slave system. First, a similar gyro dynamical system, and second, the Rössler system which has another dynamical equation of master gyro. The parameters of slave system or control input converge to the nominal value of master system parameters by a parameter updating law in spite of the uncertainty. It is proved by the Lyapunov stability theorem that the proposed controller can synchronize chaotic systems with time-varying unknown parameters. Simulation results show that method in [30] has a larger margin of error than the proposed robust method here.

Section 2 states the problem. Controller and the parameter identifier are designed in section 3. Simulation results verify achieving synchronization in section 4. Finally, Section 5 concludes the paper.

2. PROBLEM STATEMENTS

Fig. 2.1 shows a gyro which is established on a vibrating base. The dynamical equations of the system are expressed by the Euler's angles θ, ϕ and ψ and a multiple harmonic equation $\sum_{k=1}^{n} A_k \sin \omega_k t$ which represents base vibration. If we define $x_1 = \theta$, $x_2 = \dot{\theta}$ and $x_3 = \dot{\phi}$, the dynamical equations of the system in the state space framework are [30, 31]:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -\frac{(\beta_\phi - \beta_\varphi \cos x_1)(\beta_\varphi - \beta_\phi \cos x_1)}{I^2 \sin^3 x_1} - \frac{C_1(t)}{I} x_2\\ &\quad + \frac{Mgl}{I} \sin x_1 - \frac{Mg}{I} \sum_{k=1}^{n} A_k \sin(\omega_k t) \sin x_1\\ \dot{x}_3 &= -\frac{2\cos x_1}{\sin x_1} x_2 x_3 + \frac{\beta_\varphi x_2}{I \sin x_1}\end{aligned} \tag{2.1}$$

where I, Mg, and l denote the polar moments of gyro inertia, the gravity force, and the distance between origin and the center of gravity. β_ϕ and β_φ are constants of the motion. $C_1(t)$ is a parameter vector which is unknown, time varying, and uncertain. It worth noting that $x_1 = \theta$, which is known as nutation angle, is the angle between the XYZ (fixed) axis and xyz (body) axis. It is shown in Fig.(11-16) of [31] that $\theta(t)$ is limited by two extreme values $0 < \theta_1 < \theta(t) < \theta_2 < \pi$ which correspond to the turning points of the central-force. This fact prevents Eq. (2.1) to become singular. For more details on the derivation of this model we refer to chapter 11 of [31]. However, $C(t)$ has the nominal value C with uncertainty or disturbance $\Delta C(t)$, that is to say $C_1(t) = C + \Delta C(t)$. It is assumed that the uncertainty is bounded, namely:

Assumption 2.1:
(Uncertainty boundedness)

$$\|\Delta C\| \leq k \qquad \forall t \in \mathcal{R}^+ \tag{2.2}$$

where k is a known, positive, and real number.

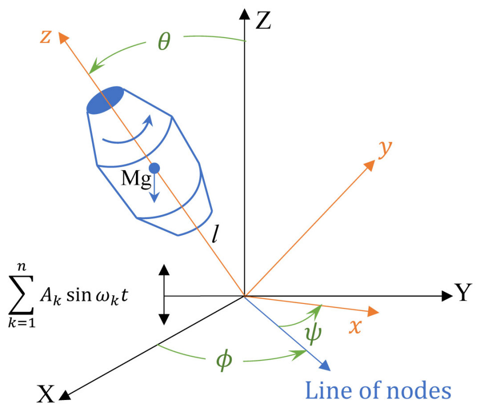

Fig. 2.1. Schematic of a gyroscope on a vibrating base

Eq. (2.1) is the master (drive) system model. The major goal is that the states of a slave (response) system synchronize with the states of the master system. Here, we consider two different dynamical systems as slave system. First, a gyro with state space representation similar to the master system:

$$\begin{aligned} \dot{x}_1^s &= x_2^s + u_1 \\ \dot{x}_2^s &= -\frac{(\beta_\phi - \beta_\varphi \cos x_1^s)(\beta_\varphi - \beta_\phi \cos x_1^s)}{I^2 \sin^3 x_1^s} - \frac{C_2(t)}{I} x_2^s \\ &\quad + \frac{Mgl}{I} \sin x_1^s - \frac{Mg}{I} \sum_{k=1}^{n} A_k \sin(\omega_k t) \sin x_1^s + u_2 \\ \dot{x}_3^s &= -\frac{2\cos x_1^s}{\sin x_1^s} x_2^s x_3^s + \frac{\beta_\varphi x_2^s}{I \sin x_1^s} + u_3, \end{aligned} \tag{2.3}$$

Second, the Rössler system as slave system:

$$\begin{aligned} \dot{x}_1^s &= -x_2^s - x_3^s + u_1 \\ \dot{x}_2^s &= x_1^s + b_1 x_2^s + u_2 \\ \dot{x}_3^s &= a_1 + x_3^s (x_1^s - c_1) + u_3 \end{aligned} \tag{2.4}$$

where a_1, b_1, and c_1 are the Rössler system parameters. Three control inputs u_1, u_2 and u_3 are augmented to the slave dynamical equations to control the synchronization of the systems. $C_2(t)$ is the parameter vector of the slave system. The aim is to not only synchronize the states of two gyros but also $C_2(t)$ converges to C (nominal value of the unknown parameter), simultaneously.

Considering assumption 2.1, any robust controller has to satisfy the following conditions:

1. $\lim_{t\to\infty} \|e(t)\| = 0$
2. $\lim_{t\to\infty} \|\tilde{C}(t)\| = 0$

where $e(t) = x^s(t) - x(t)$ and $\tilde{C}(t) = C_2(t) - C$.

3. DESIGN OF CONTROLLER AND IDENTIFIER

3.1. Gyro system as slave

If gyro dynamical equations (2.3) are considered as a slave system, then the error dynamic can be written in the form:

$$\begin{aligned}
\dot{e}_1 &= e_2 + u_1 \\
\dot{e}_2 &= -\frac{(\beta_\phi - \beta_\varphi \cos x_1^s)(\beta_\varphi - \beta_\phi \cos x_1^s)}{I^2 \sin^3 x_1^s} + \frac{(\beta_\phi - \beta_\varphi \cos x_1)(\beta_\varphi - \beta_\phi \cos x_1)}{I^2 \sin^3 x_1} \\
&\quad -\frac{C_2}{I} x_2^s + \frac{C_1}{I} x_2 + \frac{Mgl}{I}(\sin x_1^s - \sin x_1) \\
&\quad -\frac{Mg}{I} \sum_{k=1}^{n} A_k \sin(\omega_k t)(\sin x_1^s - \sin x_1) + u_2 \\
\dot{e}_3 &= -\frac{2\cos x_1^s}{\sin x_1^s} x_2^s x_3^s + \frac{2\cos x_1}{\sin x_1} x_2 x_3 + \frac{\beta_\varphi x_2^s}{I \sin x_1^s} - \frac{\beta_\varphi x_2}{I \sin x_1} + u_3
\end{aligned} \tag{3.5}$$

where $e_1 = x_1^s - x_1$, $e_2 = x_2^s - x_2$, and $e_3 = x_3^s - x_3$. The following theorem introduces control inputs u_1, u_2, u_3, and parameter identification updating law such that $\lim_{t\to\infty} \|e(t)\| = 0$ and $\lim_{t\to\infty} \|\tilde{C}(t)\| = 0$.

Theorem 3.1:
Consider the master system (2.1) with unknown parameter $C_1(t)$ which satisfies Assumption 2.1 and the slave system (2.3). Then, there exist the control laws

$$\begin{aligned}
u_1 &= -e_2 - e_1 \\
u_2 &= \frac{(\beta_\phi - \beta_\varphi \cos x_1^s)(\beta_\varphi - \beta_\phi \cos x_1^s)}{I^2 \sin^3 x_1^s} - \frac{(\beta_\phi - \beta_\varphi \cos x_1)(\beta_\varphi - \beta_\phi \cos x_1)}{I^2 \sin^3 x_1} \\
&\quad +(\frac{C_2}{I} - 1)e_2 - \frac{Mgl}{I}(\sin x_1^s - \sin x_1) - \frac{k e_2 x_2}{I\|e_2 x_2\|} x_2 \\
&\quad +\frac{Mg}{I} \sum_{k=1}^{n} A_k \sin(\omega_k t)(\sin x_1^s - \sin x_1) \\
u_3 &= \frac{2\cos x_1^s}{\sin x_1^s} x_2^s x_3^s - \frac{2\cos x_1}{\sin x_1} x_2 x_3 - \frac{\beta_\varphi x_2^s}{I \sin x_1^s} + \frac{\beta_\varphi x_2}{I \sin x_1} - e_3
\end{aligned} \tag{3.6}$$

and the parameter updating law

$$\dot{C}_2 = \frac{e_2 x_2}{I} \tag{3.7}$$

which leads to master-slave synchronization. Moreover, the unknown parameter of the slave system converges to the nominal value of the master one.

Proof
Consider the following Lyapunov function:

$$V(e_1, e_2, e_3, \tilde{C}) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + \tilde{C}^2) \tag{3.8}$$

where $\tilde{C}(t) = C_2(t) - C$. The first derivation of the Lyapunov function along the error dynamic Eq.(3.5) is:

$$\begin{aligned}\dot{V} &= e_1(e_2+u_1) + e_2\Big(-\frac{(\beta_\phi - \beta_\varphi \cos x_1^s)(\beta_\varphi - \beta_\phi \cos x_1^s)}{I^2 \sin^3 x_1^s} \\ &\quad + \frac{(\beta_\phi - \beta_\varphi \cos x_1)(\beta_\varphi - \beta_\phi \cos x_1)}{I^2 \sin^3 x_1} - \frac{C_2}{I} x_2^s + \frac{C_1}{I} x_2 \\ &\quad + \frac{Mgl}{I}(\sin x_1^s - \sin x_1) - \frac{Mg}{I}\sum_{k=1}^{n} A_k \sin \omega_k t(\sin x_1^s - \sin x_1) + u_2\Big) \\ &\quad + e_3\Big(-\frac{2\cos x_1^s}{\sin x_1^s} x_2^s x_3^s + \frac{2\cos x_1}{\sin x_1} x_2 x_3 + \frac{\beta_\varphi x_2^s}{I \sin x_1^s} - \frac{\beta_\varphi x_2}{I \sin x_1} + u_3\Big) + \tilde{C}\dot{C}_2 \\ &= -e_1^2 - e_2^2 - e_3^2 + \frac{\Delta C}{I} x_2 e_2 - \frac{k e_2^2 {x_2}^2}{I\|e_2 x_2\|} \end{aligned} \tag{3.9}$$

Taking into account the inequality $\|A+B\| \leq \|A\| + \|B\|$, we obtain the inequality:

$$\dot{V} \leq -\|e\|^2 + \|\frac{\Delta C}{I}\|\|e_2 x_2\| - \|\frac{k}{I}\|\|e_2 x_2\| \tag{3.10}$$

Finally, assumption 2.1, gives:

$$\dot{V} \leq -\|e\|^2 \tag{3.11}$$

So, $\dot{V} < 0$ for $\|e\| \neq 0$. This Lyapunov function and its derivation demonstrate that the error dynamic Eq.(3.5) is asymptotically stable, that is to say, the master-slave synchronization is achieved. It also guarantees the convergence of the slave system parameter $C_2(t)$ to the nominal value of the master system parameter C. □

3.2. *Rössler system as slave*

Now, the Rössler system is considered as a slave. The error dynamic in this situation is:

$$\begin{aligned}\dot{e}_1 &= -x_2^s - x_3^s - x_2 + u_1 \\ \dot{e}_2 &= x_1^s + b_1 x_2^s + \frac{C_1}{I} x_2 - \frac{Mgl}{I}\sin x_1 + \frac{(\beta_\phi - \beta_\varphi \cos x_1)(\beta_\varphi - \beta_\phi \cos x_1)}{I^2 \sin^3 x_1} \\ &\quad + \frac{Mg}{I}\sum_{k=1}^{n} A_k \sin(\omega_k t)(\sin x_1) + u_2 \\ \dot{e}_3 &= a_1 + x_3^s(x_1^s - c_1) + \frac{2\cos x_1}{\sin x_1} x_2 x_3 - \frac{\beta_\varphi x_2}{I \sin x_1} + u_3 \end{aligned} \tag{3.12}$$

The control inputs and parameter identification updating law are given as the following theorem.

Theorem 3.2:
Consider the master system (2.1) with unknown parameter $C_1(t)$ which satisfies Assumption

2.1 and the slave system (2.4). Then, there exist the control laws

$$\begin{aligned} u_1 &= x_2^s + x_3^s + x_2 - e_1 \\ u_2 &= -x_1^s - b_1 x_2^s - \frac{C_2}{I}x_2 - \frac{ke_2x_2}{I\|e_2x_2\|}x_2 + \frac{Mgl}{I}\sin x_1 \\ &\quad -\frac{(\beta_\phi - \beta_\varphi\cos x_1)(\beta_\varphi - \beta_\phi \cos x_1)}{I^2\sin^3 x_1} - \frac{Mg}{I}\sum_{k=1}^{n} A_k \sin(\omega_k t)(\sin x_1) - e_2 \\ u_3 &= -a_1 - x_3^s(x_1^s - c_1) - \frac{2\cos x_1}{\sin x_1}x_2x_3 + \frac{\beta_\varphi x_2}{I\sin x_1} - e_3 \end{aligned} \tag{3.13}$$

and the parameter updating law,

$$\dot{C}_2 = \frac{e_2x_2}{I} \tag{3.14}$$

which leads to master-slave synchronization. Moreover, the unknown parameter of the slave system converges to the nominal value of the master one.

Proof
The proof can be written down in the same way as the proof of Theorem 1 by considering the error dynamic Eq.(3.12). □

So, the two gyro systems synchronize despite initial state and parameter mismatches. The next section shows the simulation results of the proposed method. Furthermore, the outcomes of the proposed method are compared with the results of [30].

4. SIMULATION

Consider the dynamical model parameters $\beta_\phi = 2, \beta_\varphi = 5, I = 1, Mg = 4, l = 0.25, \omega_1 = 1$, and $A_1 = 12.1$. The Rössler system is chaotic where $a_1 = 0.2$, $b_1 = 0.2$, and $c_1 = 5.7$. Also, we consider different initial conditions for the master gyro as $x_1(0) = -0.5$, $x_2(0) = -1.2$, $x_3(0) = 10$, slave gyro as $x_1^s(0) = x_2^s(0) = x_3^s(0) = 0.1$, and slave Rössler system as $x_1^s(0) = -0.1$, $x_2^s(0) = 0.2$, $x_3^s(0) = 0.5$. Assuming the unknown, time varying, and uncertain parameter $C_1(t) = 0.5 + 0.7\sin t$ where the nominal value is $C = 0.5$ and $\Delta C = 0.7\sin t$. Thus, the maximum norm of ΔC can be set as $k = 0.7$. The initial value of slave system parameter is also assumed as $C_2 = 0$. Moreover, the simulation time step is chosen as 0.0001 (sec).

The states of two master and slave gyros as well as the error of their synchronization by the method of [30] are depicted in Fig. 4.2. The mean square errors (MSE) of the states synchronization are 6.617×10^{-4}, 1.151×10^{-2}, and 1.802×10^{-1} for the e_1, e_2, and e_3, respectively. Since the unknown parameter is in the dynamical equation of x_2, the steady state synchronization error is observed in e_2. This can be quantified by neglecting transient time, i.e. considering MSE for data obtained after 10 seconds from the beginning which are 1.721×10^{-12}, 7.749×10^{-03}, and 4.686×10^{-10} for e_1 to e_3, respectively. It can be seen in Fig. 4.2 that e_2 does not converge to zero and it has fluctuations between -0.120 and 0.265. This fact is due to the steady state error in the parameter identification algorithm which is shown in Fig.4.5. The mean value of the identified parameter by the method of [30] after initial transient time is 0.447 which suffers approximately 10.6% steady state error.

Theorems 1 and 2 assert that control inputs (3.6) and (3.13) with the parameter-updating laws (3.7) and (3.14) not only synchronize the master and slave systems but also guarantee the identification of the unknown parameter of the master system correctly. Figs. 4.3 and 4.4 illustrate the states and synchronization error for gyro-gyro and gyro-Rössler systems by the proposed method, respectively. The simulation results show that synchronization

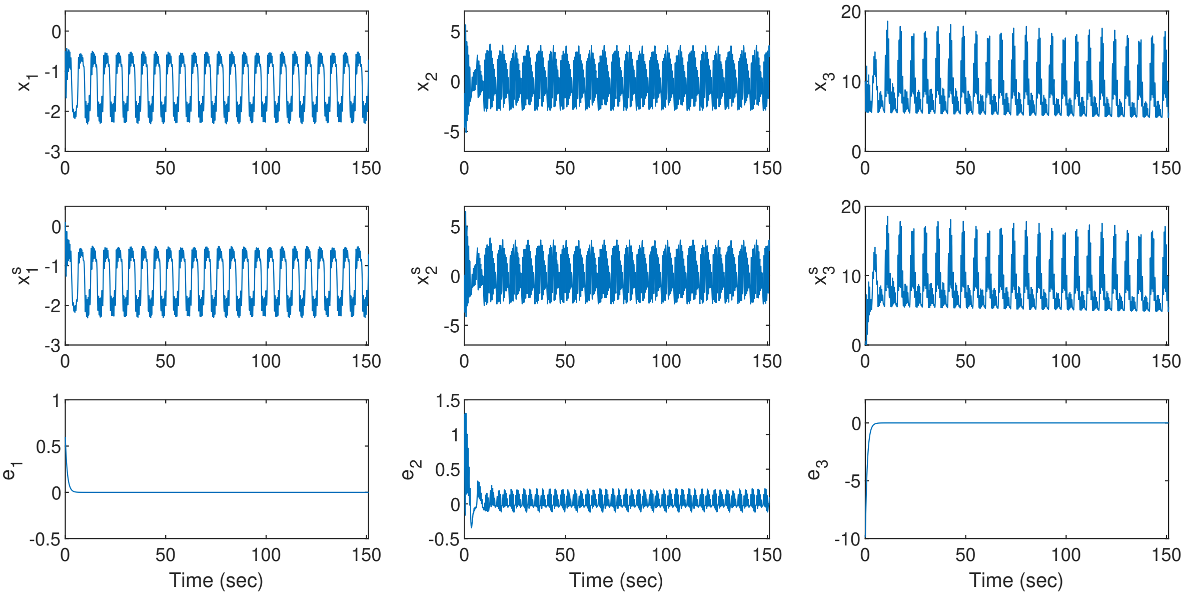

Fig. 4.2. Method of [30]: row 1: Master system states, row 2: Slave gyro system states, row 3: Synchronization error

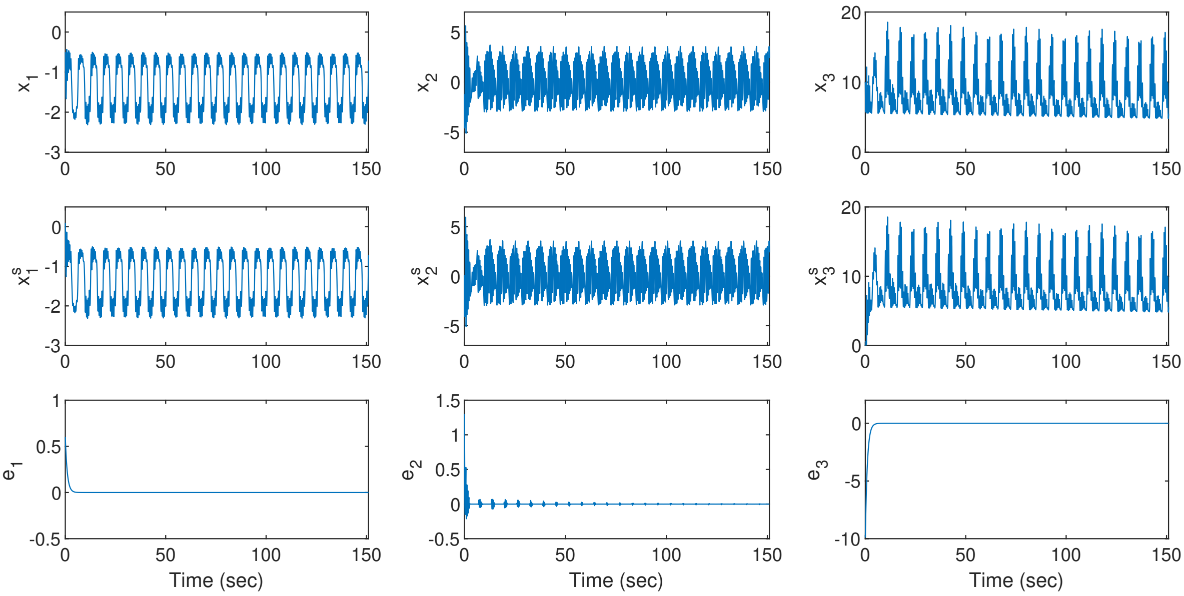

Fig. 4.3. Proposed method (Gyro-Gyro): row 1: Master system states, row 2: Slave gyro system states, row 3: Synchronization error

error asymptotically tends to zero for all states regardless of uncertainty in parameters. In the case of gyro-gyro synchronization, the MSE of the synchronized x_2 reduces to 1.019×10^{-3} which shows a decrease of more than 91%. Omitting initial transient time give MSE equal to 3.45×10^{-5}, which is almost 99.5% more accurate than the previous method. Synchronization MSEs for the Gyro-Rössler situation are 2.941×10^{-4}, 1.202×10^{-3}, and 1.659×10^{-1} related to e_1, e_2, and e_3, respectively. The MSEs decrease to 0.765×10^{-12}, 0.399×10^{-4}, and 4.315×10^{-10} if neglecting the initial transient time. The gyro-Rössler

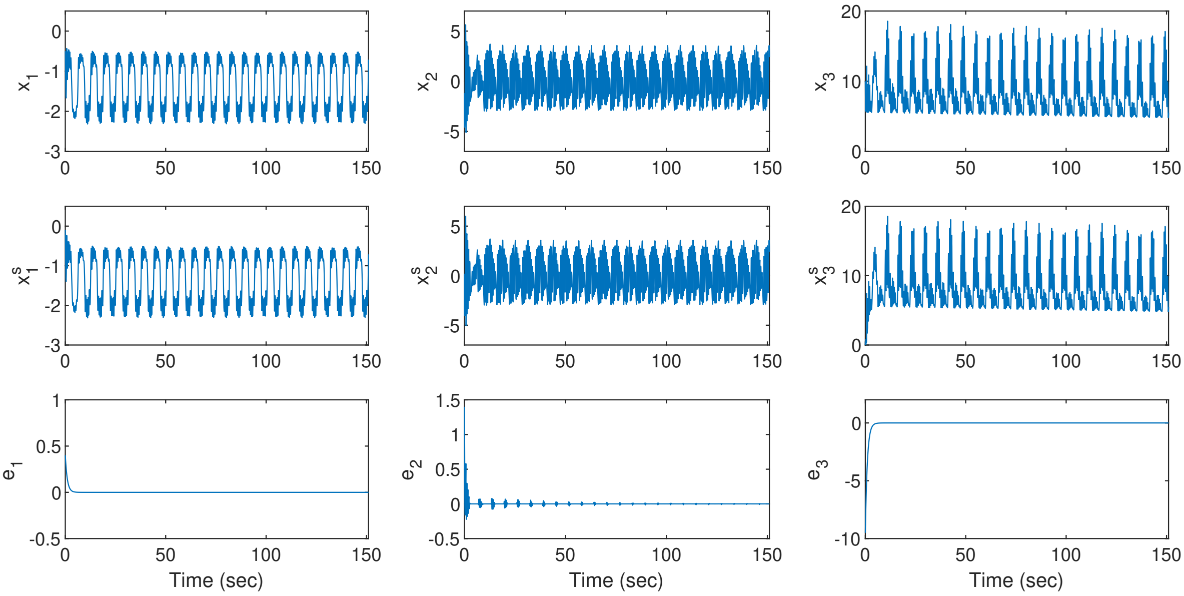

Fig. 4.4. Proposed method (Gyro-Rössler): row 1: Master system states, row 2: Slave gyro system states, row 3: Synchronization error

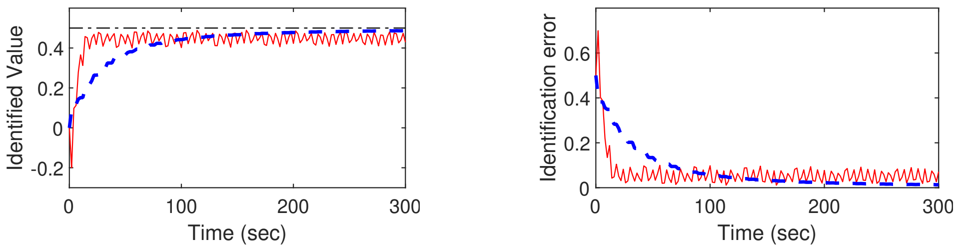

Fig. 4.5. Gyro-gyro synchronization: Left: Parameter identification, Right: Identification error; Dashed-dotted line (black): Nominal value, Dashed line (blue): Proposed method, Solid line (red): Method of [30]

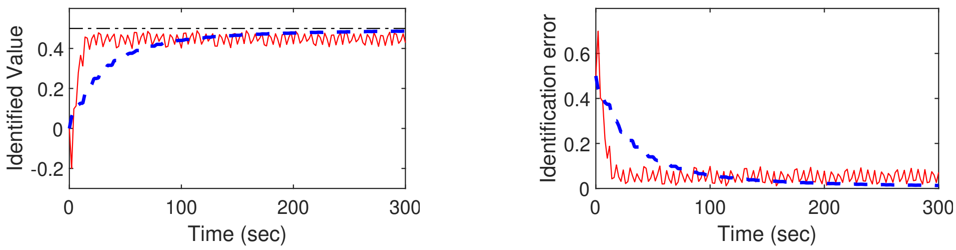

Fig. 4.6. Gyro-Rössler synchronization: Left: Parameter identification, Right: Identification error; Dashed-dotted line (black): Nominal value, Dashed line (blue): Proposed method, Solid line (red): Method of [30]

synchronization also ensures convergence to zero of all errors. The MSEs for all situations are written in table 4.1.

The unknown parameter is also identified perfectly by the developed method. Figs. 4.5 and 4.6 indicate the convergence of the unknown parameter to the nominal value C as well as the error of estimation for the proposed method and that of [30] in the situation of gyro-gyro and gyro-Rössler systems, respectively. It can be seen that the fluctuations in the e_2 decrease as time goes by and the parameter identification error converges to zero in the proposed method while the method of [30] suffers lasting fluctuations as well as steady state error.

Table 4.1. MSE of synchronization

Method		MSE			MSE for time>10 (sec)		
		$e_1(\times 10^{-4})$	$e_2(\times 10^{-3})$	$e_3(\times 10^{-1})$	$e_1(\times 10^{-12})$	$e_2(\times 10^{-4})$	$e_3(\times 10^{-10})$
Method of [30]		6.617	11.51	1.802	1.721	77.49	4.686
Proposed method	Gyro-Gyro	6.617	1.019	1.802	1.721	0.345	4.686
	Gyro-Rössler	2.941	1.202	1.659	0.765	0.399	4.315

5. CONCLUSION

The proposed synchronization of gyro system has been founded based on the elimination of dynamical equations' nonlinearity by the controller. Simultaneously, a parameter identification algorithm estimates the unknown, time-varying, and uncertain parameters. The proposed method not only guarantees convergence of synchronization error to zero but also ensures the convergence of unknown parameter estimation to the nominal value. Considering a typical gyroscope, the synchronization error in order of magnitude 10^{-12}, 10^{-5}, and 10^{-10} for e_1, e_2, and e_3, respectively, are obtained in the situation of gyro-gyro and gyro-Rössler synchronization. Moreover, the unknown parameter is estimated at time 300 second with less than 2.7% of error. However, the proposed algorithm requires the upper bound of the parameter uncertainty to be determined. Another point is that the control inputs are a complex nonlinear function of states that impose high computation load.

Nomenclature

Parameter	Definition
X, Y, Z	Inertia base vectors (Fixed)
x, y, z	Body axis
θ, ϕ, ψ	The Euler's angles
I	The polar moments of gyro inertia
l	The distance between origin and the center of gravity
M	The mass of gyro
g	Gravitational acceleration constant
ω_k	k-th harmony of the base vibration frequency
A_k	Amplitude of base vibration with frequency ω_k
β_ϕ, β_φ	The constants of the motion
a_1, b_1, c_1	The Rössler system parameters
x_i	i-th state of the master system (i=1,2,3)
x_i^s	i-th state of the slave system (i=1,2,3)

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