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The Optimal Control of the Vessel's Automatic Dynamic Positioning System Under Deviation

Vadim Kramar [1], Vasilii Alchakov[2], Alexander Osadchenko[3*]

[1)] *Sevastopol State University, Sevastopol, Russia*

E-mail: kramarv@mail.ru

[2)] *Sevastopol State University, Sevastopol, Russia*

E-mail: alchakov@sevsu.ru

[3)] *Sevastopol State University, Sevastopol, Russia*

E-mail: aeosadchenko@rambler.ru

Abstract*:* An optimal algorithm for linear control of the unmanned vessel dynamic positioning system with provision for vessel characteristics, equipment, and the process has been developed. A block diagram for the automatic dynamic positioning system has been suggested which allows presenting a necessary set of devices for its technical implementation.

Keywords*:* automatic dynamic positioning system, deviation, control system, vessel, the optimal control

1. INTRODUCTION

Automatic dynamic positioning systems or position-keeping systems are the main part of unmanned vessels that are developed in modern practice. Vessel movement while station keeping is due not only to the type or nature of the external influence (wind, current, sea disturbance, the propulsive force of the steering) but also the type of the performed action since it can change dynamic properties of the vessel [1–3, 10]. When the vessel is under the action of current, wind, sea disturbance – manual control, if high accuracy of keeping the station is required, becomes almost impossible. There is consequently a need for automatic dynamic positioning system development.

The main tasks of dynamic positioning system (stabilization of horizontal shifts of the ship with the required accuracy; vessel control by the heading angle so that its current value φ align with its optimal value φ* with the required accuracy, which ensures minimal energy consumption while steering etc.) [4], inaccuracy in the calculations of perturbation strengths from wind forcing and current predetermines system design. Principles for system designing, its structure, performance, properties are mainly defined by the control object – the vessel, its mathematical model, and control problems.

Existing methods of dynamic positioning systems design are based on the application of MIMO PID controllers [6, 9], neural networks [12], and fuzzy logic controllers [11], as well as approaches that rely on the application of adaptive [5] and hybrid [7] controllers. Ways of angle speed control and vessel heading under "strong" maneuvers using the method of final conditions are suggested in the article. Many control methods are based on the principle of optimal control and Kalman filtering theory [1].

Questions of designing a block diagram of the automatic dynamic positioning system and optimal linear control of unmanned vessels under deviation are considered in this article.

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* Corresponding author: aeosadchenko@rambler.ru

2. A BLOCK DIAGRAM OF AUTOMATIC POSITION-KEEPING SYSTEM

Position-keeping is possible with the use of two active control means (ACM), each of them can have two control responses $P_{i,}, \alpha_i,\ i = 1,2$, where P_i – force intensity value, α_i – ACM turn angles.

The vessel control must be performed to the coordinates x, y and the heading angle φ. This indicates the need for introducing intermediate control responses, which are vessel axis-direction forces F_{xy}, F_{yy} and the yawing moment M_{zy} that must be compensated by the ACM, and the ones in the system must be converted into real control response $P_1, \alpha_1, P_2, \alpha_2$.

Thus, a device that can perform a one-to-one transformation of these controls is needed. Coordination of both digital and analog elements must be performed by digital-to-analog (DAC) and analog-to-digital (ADC) converters. The system must also contain a deviation control unit (DCU) and a vessel horizontal shifts measurement system along with a coordinates calculator.

A general block diagram of the automatic vessel position-keeping system is shown in Fig. 1.

The object of control (O) is the vessel. Variables ξ, θ, ψ characterize motions of the vessel and are disturbers for the sonar sensors system (SS). External disturbance influence on the vessel in the form of wind, current, sea disturbance is characterized by the variables ν – direction of current relative to a fore-and-aft axis of the vessel, β – wind direction, υ_b, υ_T – wind speed and current speed respectively, f – uncontrolled disturbance, which characterizes the sea forces and moments. Control actions on the vessel are thrust forces of the ACM P_1, P_2 and their steering angles α_1, α_2.

Sonar sensors system is intended for measuring vessel horizontal shifts, presented in the form of variables $\tau_1, \tau_2, \tau_3, \tau_4$ which are parameters for calculation of the coordinates x, y by coordinates calculator (CC). Variables ξ, θ, ψ that define the motion of the vessel are used as additional variables for calculations of the coordinates x, y. They can be measured and are disturbances.

The deviation control unit generates intermediate control actions $F^0_{xy}, F^0_{yy}, M^0_{zy}$ in the function of deviations of horizontal shifts x, y and deviations of the heading angle from the optimal value φ^*. The choice of these control is a separate task.

Thrust block optimizer unit (OT) performs an optimal conversion of intermediate controls F_{xy}, F_{yy}, M_{zy}, presented in digital form, into controls $P_{10}, \alpha_{10}, P_{20}, \alpha_{20}$ that are also presented in digital form.

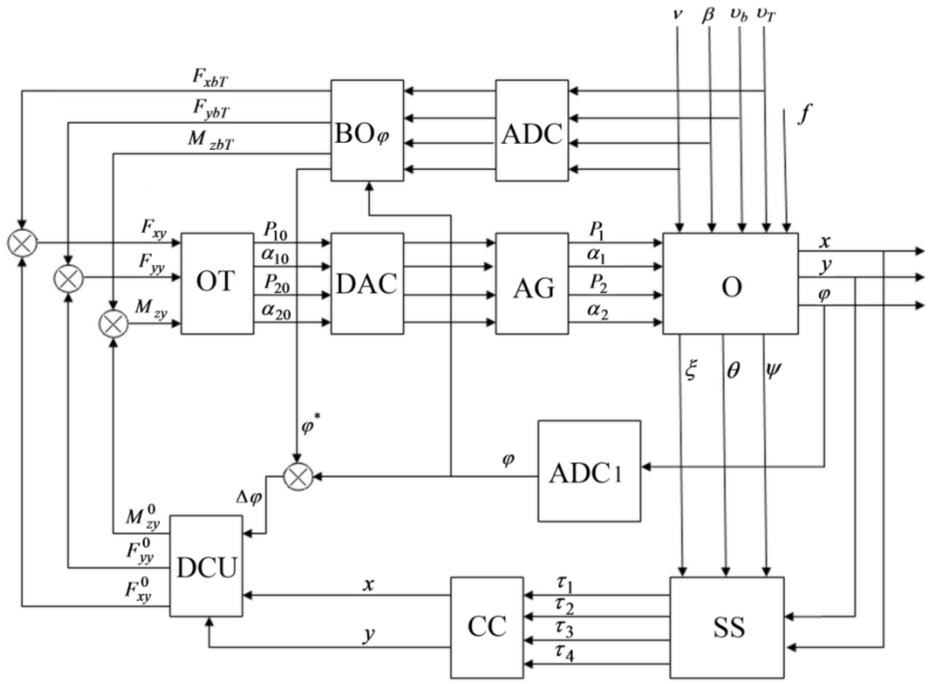

Fig. 2.1. The general architecture of the system of the automatic hold of an unmanned vessel

Automatic gear (AG) tracks the forces of thrusts P_{10}, P_{20} and their turn angles α_{10}, α_{20}, presented in analog form, into real thrusts and turn angles ACM $P_1, \alpha_1, P_2, \alpha_2$. The connection between OT and AG is accomplished by DAC.

Optimization unit by heading angle from measured magnitude $\nu, \beta, \upsilon_b, \upsilon_T, \varphi$ calculates the optimal value of heading angle φ^* to the direction of wind effect and current. It determines the value of force projection F_{xbT}, F_{ybT} and moment M_{zbT} from the wind and current.

A block diagram of the automatic position-keeping system assumes the following parameters to be measurable: wind angle and current angle to the center plane of the vessel, wind speed and current speed, horizontal shifts, vessel relative azimuth. To design an automatic position-keeping system, it is necessary to know many design and hydrodynamic parameters.

3. SYSTEM MODEL AND PROBLEM FORMULATION

The mathematical model of the vessel can be presented in different forms. The spatial motion of the vessel considering all the coordinates correlations and vessel motion is described by a complex nonlinear nonstationary system of differential equations. The use of such a model for solving vessel motion optimization problems in the modes of position-keeping currently presents insuperable theoretical difficulties. The position-keeping regime is characterized by many favorable features: 1) the coordinates of vessel position and their velocity change in proximity to zero values; 2) all types of vessel motions must be insignificant; 3) the vessel heading angle in a time of vessel control process can be considered constant, equal to some required value. Neglecting the influence of the oscillatory motion and performing linearization of ship motion control, neglecting also the influence of the equipment on the dynamics of vessel motion, neglecting the influence of hydrodynamic coefficient λ_{26} [4, 10], we get a simplified vessel motion model in the horizontal plane in the aligned with the vessel coordinate system in the form of:

$$\left.\begin{aligned}(m+\lambda_{11})\ddot{x}&=F_x\\(m+\lambda_{22})\ddot{y}&=F_y\\(J_z+\lambda_{66})\ddot{\varphi}&=M_z\end{aligned}\right\},\qquad (1)$$

where m – vessel mass, J_z – body centroidal moment of inertia as regard to the z-axis, $\lambda_{11},\lambda_{22},\lambda_{66}$ – hydrodynamical coefficient.

It is also possible to present the vessel motion model in the fixed coordinate system. In addition to the system (1) let us introduce constraint equations

$$\left.\begin{aligned}\dot{x}&=\dot{\xi}\cos\varphi+\dot{\eta}\sin\varphi\\\dot{y}&=-\dot{\xi}\sin\varphi+\dot{\eta}\cos\varphi\end{aligned}\right\},\qquad (2)$$

where ξ,η – coordinates of a fixed coordinate system connected with the vessel center of gravity.

Let us find second derivatives of coordinates:

$$\left.\begin{aligned}\ddot{x}&=\ddot{\xi}\cos\varphi+\ddot{\eta}\sin\varphi-\dot{\xi}\dot{\varphi}\sin\varphi+\dot{\eta}\dot{\varphi}\cos\varphi\\\ddot{y}&=-\ddot{\xi}\sin\varphi+\ddot{\eta}\cos\varphi-\dot{\xi}\dot{\varphi}\cos\varphi-\dot{\eta}\dot{\varphi}\sin\varphi\end{aligned}\right\}.\qquad (3)$$

Substituted (2), (3) to (1), we get:

$$\left.\begin{aligned}&(m+\lambda_{11})\cos\varphi^{*}\ddot{\xi}+(m+\lambda_{11})\sin\varphi^{*}\ddot{\eta}=F_x\\&(m+\lambda_{22})\cos\varphi^{*}\ddot{\eta}-(m+\lambda_{22})\sin\varphi^{*}\ddot{\xi}=F_y\\&(J_z+\lambda_{66})\ddot{\varphi}=M_z\end{aligned}\right\},\qquad (4)$$

where:

$\cos(\varphi^{*}+\Delta\varphi)\approx\cos\varphi^{*}-\Delta\varphi\sin\varphi^{*}$,

$\sin(\varphi^{*}+\Delta\varphi)\approx\sin\varphi^{*}+\Delta\varphi\cos\varphi^{*}$,

and variables $\xi,\ \eta,\ \varphi$ – the deviations from the set values $\xi_0=\eta_0=0,\ \varphi=\varphi^{*}$.

System (4) can be represented as:

$$\left.\begin{aligned}\ddot{\xi}&=\frac{c_2}{c_1c_2+c_1'c_2'}F_x-\frac{c_1'c_2}{c_2'(c_1c_2+c_1'c_2')}F_y\\\ddot{\eta}&=\frac{c_1}{c_1c_2+c_1'c_2'}F_y+\frac{1}{c_1'c_2'+c_1c_2}F_x\\\ddot{\varphi}&=\frac{1}{c_3}M_z\end{aligned}\right\},\qquad (5)$$

where $(m+\lambda_{11})\cos\varphi^{*}=c_1$, $(m+\lambda_{11})\sin\varphi^{*}=c_1'$, $(m+\lambda_{22})\cos\varphi^{*}=c_2$, $(m+\lambda_{22})\sin\varphi^{*}=c_2'$, $J_z+\lambda_{66}=c_3$.

Model (1) in the connection with the vessel frame of the axis, reflects only the inertial properties of the object. Coordinate motions are independent. This vessel motion model reflects the main properties of the object in the position-keeping mode under the effect of external disturbance forces, presented in the form of force projection and the moment as regards the z-axis. Apart from the disturbance forces, the control force will affect the ACM.

If we present these forces in the form of projection of forces and moment, then a simplified vessel motion model in the horizontal plane will have the form:

$$\left.\begin{aligned}(m+\lambda_{11})\ddot{x} &= F_{xy}+F_x\\ (m+\lambda_{22})\ddot{y} &= F_{yy}+F_y\\ (J_z+\lambda_{66})\ddot{\varphi} &= M_{zy}+M_z\end{aligned}\right\}. \tag{6}$$

Forces and moment projection values, created by ACM, depend on the number of ACM, their placement on the vessel, thrusts values created by them, and their turn angles as regards the center plane of the vessel.

For two ACM:

$$\left.\begin{aligned}F_{xy} &= P_1\cos\alpha_1+P_2\cos\alpha_2\\ F_{yy} &= P_1\sin\alpha_1+P_2\sin\alpha_2\\ M_{zy} &= y_1P_1\cos\alpha_1+y_2P_2\cos\alpha_2+x_1P_1\sin\alpha_1+x_2P_2\sin\alpha_2\end{aligned}\right\}, \tag{7}$$

where P_1, P_2 – ACM thrusts values; x_1, x_2, y_1, y_2 – coordinates of ACM placement on the vessel; α_1, α_2 – ACM turn angles. Forces projection F_x, F_y and moment M_z represent the algebraic sum of projections of forces and moments from wind, current, and sea disturbance.

4. SYNTHESIS OF THE OPTIMAL LINEAR CONTROL

For the synthesis problem, we shall accept an object model in the form (6), assuming that: $m+\lambda_{11}=m_x$, $m+\lambda_{22}=m_y$, $J_z+\lambda_{66}=m_z$, $F_{xy}=F^0_{xy}$, $F_{yy}=F^0_{yy}$, $M_{zy}=M^0_{zy}$. It means that we shall design a control contour, which works on the principle of removal of random deflections of variables from the required values.

A mathematical model of the control object can be given by:

$$\left.\begin{aligned}m_x\ddot{x} &= F^0_{xy}+F_x\\ m_y\ddot{y} &= F^0_{yy}+F_y\\ m_z\ddot{\varphi} &= M^0_{zy}+M_z\end{aligned}\right\}. \tag{8}$$

We shall accept limitations on vessel horizontal shifts:

$$x^2+y^2\le r^2, \tag{9}$$

where r – circle radius, which determines legitimate vessel horizontal shifts.

For an object (8) with limitations (9) let us consider a problem of synthesis of the optimal linear control under random stationary perturbations. As optimal control under random perturbations criterion, when the full-time of the control is big enough, we shall accept the root-mean-square criterion, which depends on the control coordinates

$$J=\lim_{T\to\infty}\frac{1}{T}\int_0^T (m_1^2x^2+m_2^2y^2+m_3^2\varphi^2+m_4^2F^{0^2}_{xy}+m_5^2F^{0^2}_{yy}+m_6^2M^{0^2}_{zy})dt, \tag{10}$$

where $m_i\,(i=1,2,\dots,6)$ – weight coefficients, which determine the contribution of each sum and in the quality criterion value; T – the full time of the control.

Let us convert inequality (9) into equality constraint by extending the number of variables. For this purpose, let us introduce an additional real variable $v_1\in(-\infty,\infty)$, nonlinear conversion e^{v_1} , and write over (9) in the form of equivalent constraint

$$e^{v_1} - r^2 + x^2 + y^2 = 0\,. \tag{11}$$

To minimize the performance functional (10) under constraints (8), (11) let us make use of the Lagrange multiplier method and the necessary conditions in the form of the Euler-Poisson equation. Lagrange function will be written in the form of

$$\begin{aligned} L = {} & m_1^2 x^2 + m_2^2 y^2 + m_3^2 \varphi^2 + m_4^2 F_{xy}^{0^2} + m_5^2 F_{yy}^{0^2} + m_6^2 M_{zy}^{0^2} + \lambda_1 (m_x \ddot{x} - \\ & - F_{xy}^0 - F_x) + \lambda_2 (m_y \ddot{y} - F_{yy}^0 - F_y) + \lambda_3 (m_z \ddot{\varphi} - M_{zy}^0 - M_z) + \\ & + \lambda_4 (e^{v_1} - r^2 + x^2 + y^2), \end{aligned} \tag{12}$$

where $\lambda_j \ (j = 1,2,3,4)$ – Lagrange indefinite multipliers, some functions of time.

Necessary conditions for the minimum criterion (10) in the form of the Euler-Poisson equation will be given by

$$\left.\begin{aligned} & 1)\, 2m_1^2 x + 2\lambda_4 x + m_x \ddot{\lambda}_1 = 0 \\ & 2)\, 2m_2^2 y + 2\lambda_4 y + m_y \ddot{\lambda}_2 = 0 \\ & 3)\, 2m_3^2 \varphi + m_z \ddot{\lambda}_3 = 0 \\ & 4)\, m_x \ddot{x} - F_{xy}^0 - F_x = 0 \\ & 5)\, m_y \ddot{y} - F_{yy}^0 - F_y = 0 \\ & 6)\, m_z \ddot{\varphi} - M_{zy}^0 - M_z = 0 \\ & 7)\, 2m_4^2 F_{xy}^0 - \lambda_1 = 0 \\ & 8)\, 2m_5^2 F_{yy}^0 - \lambda_2 = 0 \\ & 9)\, 2m_6^2 M_{zy}^0 - \lambda_3 = 0 \\ & 10)\, \lambda_4 (r^2 - x^2 - y^2) = 0 \end{aligned}\right\}. \tag{13}$$

It should be noted that the last equation in (13) appeared due to constraints (12) under differentiation of the last term in (13) by λ_4 and v_1 combination of the obtained correlations into one equation. This equation is satisfied under any x, y if $\lambda_4 = 0$ or under any finite λ_4, if $r^2 - x^2 + y^2 = 0$. The last condition corresponds with the presence of the vessel in the boundary of the admissible deviation domain. From a physical standpoint, it cannot be realized for a long time, as the vessel under the reduced output of the ACM can only cross this boundary. Thus, there is no need to switch over controls and $\lambda_4 = 0$ must be set. It means, that constraints (8) and equivalent to them (11) can be disregarded under the synthesis of optimal controls. It is enough to have optimal controls when the vessel faces admissible boundaries. The synthesis of optimal linear controls satisfies these conditions. At that, the set of equations (13) falls into three independent subsystems: 1) equations number (1), (4), (7); 2) equations number (2), (5), (8); 3) equations number (3), (6), (9), when $\lambda_4 = 0$. Each subsystem gives equations for extremals which correspond to the coordinates and controls

$$(m_x^2 m_4^2 p^4 + m_1^2) x = m_4^2 m_x p^2 F_x\,, \tag{14}$$

$$(m_x^2 m_4^2 p^4 + m_1^2) F_{xy}^0 = -m_1^2 F_x\,, \tag{15}$$

$$(m_y^2 m_3^2 p^4 + m_2^2) y = m_5^2 m_y p^2 F_y\,, \tag{16}$$

$$(m_y^2 m_5^2 p^4 + m_2^2) F_{yy}^0 = -m_2^2 F_y\,, \tag{17}$$

$$(m_z^2 m_5^2 p^4 + m_3^2)\varphi = m_6^2 m_z p^2 M_z, \tag{18}$$

$$(m_z^2 m_6^2 p^4 + m_3^2)M_{zy}^0 = -m_3^2 M_z, \tag{19}$$

where $p = \dfrac{d}{dt}$ – operator of differentiation. Controls (15), (17), (19) cause unstable coordinates motion. For control synthesis, which ensures stable extremals motions, let us make use of the results [8]. For perturbances, set by correlation functions of the form $k(\tau) = e^{-\alpha_i \tau}$, $(i = 1,2,3)$, we get control laws in the form of

$$F_{xy}^0 = [m_x p^2 - \frac{Y_2(\alpha_1)}{m_4^2 m_x \alpha_1^2} Y_1(p)]x, \tag{20}$$

$$F_{yy}^0 = [m_y p^2 - \frac{Y_4(\alpha_2)}{m_5^2 m_y \alpha_2^2} Y_3(p)]y, \tag{21}$$

$$M_{zy}^0 = [m_z p^2 - \frac{Y_6(\alpha_3)}{m^2 m_z \alpha_3^2} Y_5(p)]\varphi, \tag{22}$$

where

$$Y_1(p) = p^2 + \sqrt{\frac{2m_1}{m_4 m_x}} p + \frac{m_1}{m_4 m_x};\ Y_2(\alpha_1) = \alpha_1^2 - \sqrt{\frac{2m_1}{m_4 m_x}} \alpha_1 + \frac{m_1}{m_4 m_x};$$

$$Y_3(p) = p^2 + \sqrt{\frac{2m_2}{m_5 m_y}} p + \frac{m_2}{m_5 m_y};\ Y_4(\alpha_2) = \alpha_2^2 - \sqrt{\frac{2m_2}{m_5 m_y}} \alpha_2 + \frac{m_2}{m_5 m_y};$$

$$Y_5(p) = p^2 + \sqrt{\frac{2m_3}{m_6 m_z}} p + \frac{m_3}{m_6 m_z};\ Y_6(\alpha_3) = \alpha_3^2 - \sqrt{\frac{2m_3}{m_6 m_z}} \alpha_3 + \frac{m_3}{m_6 m_z}.$$

For perturbances, set by correlations functions of the form of

$$k(\tau) = D_i e^{-\alpha_i \tau}(\cos\beta_i \tau + \frac{\alpha_i}{\beta_i}\sin\beta_i \tau),\ i = 1,2,3$$

we will have

$$F_{xy}^0 = [m_x p^2 - \frac{Y_1(p)}{m_4^2(a_1 + b_1 p)}]x, \tag{23}$$

where $a_1 = c_1 + \dfrac{\alpha_1}{\beta_1} c_2$; $b_1 = \dfrac{c_2}{\beta_1}$; coefficients c_1, c_2 are determined from the equation

$$\frac{m_x(\alpha_1 - j\beta_1)^2}{Y_2(\alpha_1 - j\beta_1)} = c_1 + jc_2, \qquad F_{yy}^0 = [m_y p^2 - \frac{Y_3(p)}{m_5^2(a_2 + b_2 p)}]y, \tag{24}$$

where a_2, b_2 are determined from the expressions

$$a_2 = c_3 + \frac{\alpha_2}{\beta_2} c_4;\ b_2 = \frac{c_4}{\beta_2};\ \frac{m_y(\alpha_2 - j\beta_2)^2}{Y_4(\alpha_2 - j\beta_2)} = c_3 + jc_4,$$

$$M_{zy}^0 = [m_z p^2 - \frac{Y_5(p)}{m_6^2(a_3 + b_3 p)}]\varphi, \tag{25}$$

where a_3, b_3 are determined from the expressions

$$a_3 = c_5 + \frac{\alpha_3}{\beta_3} c_6 \; ; \; b_3 = \frac{c_6}{\beta_3} \; ; \; \frac{m_z(\alpha_3 - j\beta_3)^2}{Y_4(\alpha_3 - j\beta_3)} = c_5 + jc_6 \, .$$

For a complete definition of optimal control laws (20) – (25) it is necessary to find weight coefficients $m_i \; (i = 1, 2, \ldots, 6)$.

Deviation control block which realizes optimal linear control laws (20), (21), (22) or (23), (24), (25), is comparatively complex, as it requires second differential coefficient coordinates measuring. From system engineering, for inertial objects control experience, it is known that control of such objects is quite efficient when the deviation signal and its first-order derivative are used. Hence, control can be set in proportional-differential form.

The optimal stop values will be determined by the relations [4]:

$$\left.\begin{aligned} P_1 &== \frac{\left|M_z - x_2F_y\right|\sqrt{F_x^2(x_1 - x_2)^2 + \left(\left|M_z - x_2F_y\right| + \left|x_1F_y - M_z\right|\right)^2}}{\left(\left|M_z - x_2F_y\right| + \left|x_1F_y - M_z\right|\right)\left|x_1 - x_2\right|} \\ P_2 &= \frac{\left|x_1F_y - M_z\right|\sqrt{F_x^2(x_1 - x_2)^2 + \left(\left|M_z - x_2F_y\right| + \left|x_1F_y - M_z\right|\right)^2}}{\left(\left|M_z - x_2F_y\right| + \left|x_1F_y - M_z\right|\right)\left|x_1 - x_2\right|} \end{aligned}\right\} . \qquad (26)$$

The rotation angles are determined by the relations [4]:

$$\left.\begin{aligned} \alpha_1 &= arctg \frac{2(M_z - x_2F_y)}{F_x(x_1 - x_2)} \\ \alpha_2 &= arctg \frac{2(x_1F_y - M_z)}{F_x(x_1 - x_2)} \end{aligned}\right\} ; \qquad (27)$$

where $x_i, \; i = 1{,}2$ are the coordinates of the stops.

Let us cite the results of the study of the dynamics of the automatic position-keeping system. The model under study performed the calculation of optimal values of controls, presented in the form of projections of forces and moment on vessel coordinate axes with the subsequent calculation of the stop values P_1, P_2 (see Fig. 4.1) and their rotation angles α_1, α_2 (see Fig. 4.2).

The studies were conducted for the vessel of the length of $L = 104m$ and mass of $m = 734{,}7T$, which has the following characteristics: $\lambda_{11} = 22\,TM^{-1}s^2$; $\lambda_{22} = 410\,TM^{-1}s^2$; $\lambda_{66} = 2{,}2 \cdot 10^5\,TM^{-1}s^2$; $J_z = 48{,}9 \cdot 10^4\,TM \cdot s^2$.

Fig. 4.1, 4.2 shows the results of the calculations of thrust change of the vessel ACM, provided that the following conditions for vessel deviation in the set time interval $t \le 600$ s are met: $|x| \le 0{,}15$m, $|y| \le 0{,}05$m, $|\varphi| \le 0{,}2$ rad.

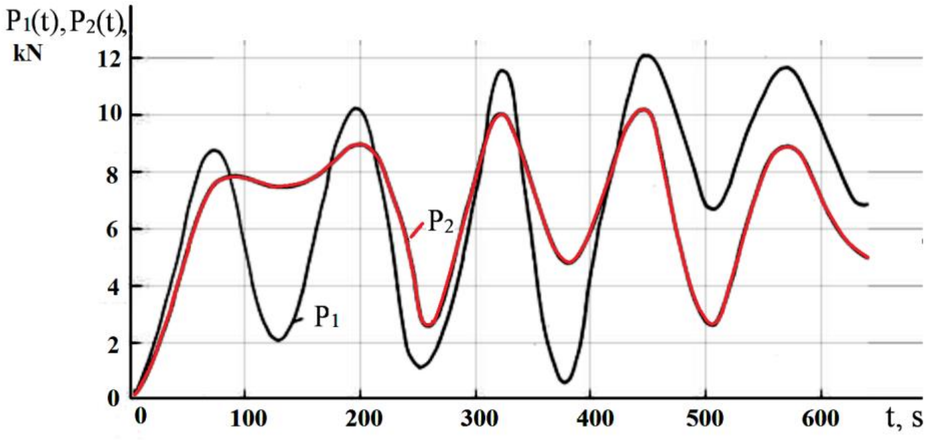

Fig 4.1. Thrusts change $P_1(t)$, $P_2(t)$

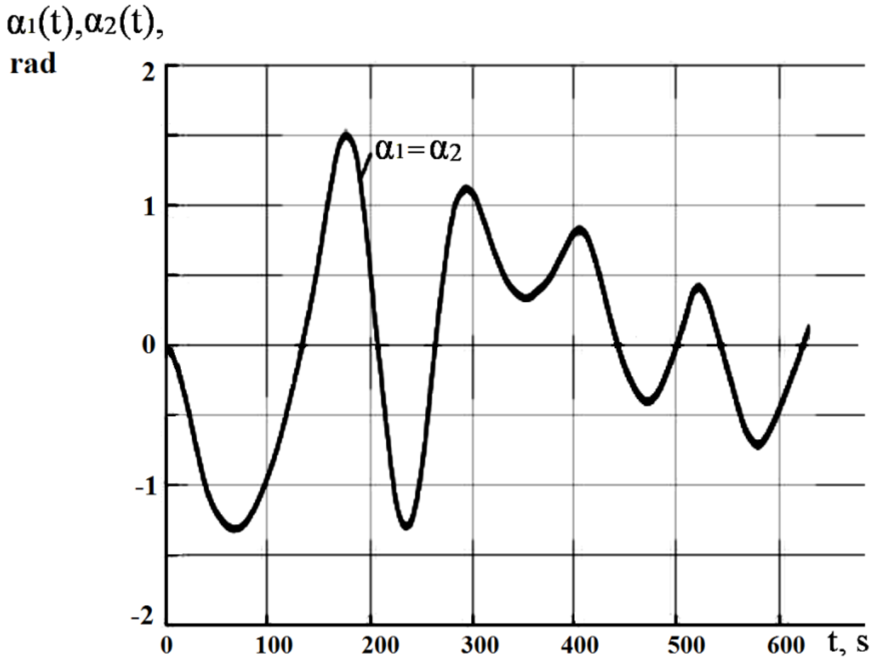

Fig 4.2. Angles change $\alpha_1(t)$, $\alpha_2(t)$

5. CONCLUSIONS

A block diagram of the automatic vessel position-keeping system and the development of optimal linear control laws of unmanned coastal vessels under deviation was suggested.

A designed block diagram and algorithms of its implementation allow presenting a necessary complex of devices for its technical implementation.

An automatic vessel position-keeping system represents a digital-to-analog complex. A digital part of the system performs control laws generation, problems solution of optimization of variables conversion, makes calculations of vessel horizontal shifts based on the measurement system data.

An analog part represents an automated control actions task drive and the control object. An optimal linear control law suggested in the study allows for effective position-keeping.

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