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Advanced Solow model as a tool for coordination of interests of spatial economic systems' development (on the materials of the South Russian macro-region)

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Abstract: The paper presents the results of the initial stage of the study of the dynamic socio-ecological-economic model of the synergetic development of individual entities within the macro-region, aimed at coordinating the common and private interests of each region. The maximization of specific consumption is proposed as a target parameter. In relation to private interests - for the region, in relation to common interests - for the macro-region. The model provides the possibility of using the available resources of each region both in the tasks of its own development, and for the general development goals of other regions of the macro-region. The model is studied on the materials of the subjects of the Southern Federal District. The study came to several debatable conclusions. In particular, calculations show that in the current conditions it is not profitable for any region to develop its own production sector, the optimal strategy for each of them is only to increase consumption in the hope of production activity in neighboring regions (a free-rider problem). In view of the selection of this rational strategy by all regions at the same time, general degradation of the production sphere and stagnation of the regional economy are forecasted. At the same time, it was revealed that a further reduction in production becomes unprofitable for almost all lagging regions (republics of the Southern Federal District), while inaction remains profitable for leading regions (regions of the Southern Federal District). The results are debatable and require further discussion.

Keywords: modeling of the coordination of interests, management of regional development, interregional interaction, production activity of the region, macro-region of the Southern Federal District.

1. INTRODUCTION

A methodology for the socio-ecological and economic development of the region modeling based on the theory of optimal control was proposed in the works by Gurman et al. [15]. Analytical methods were used along with simulation modeling in the tasks of regional development research.

In the monograph [7], the well-known Solow macroeconomic model [14] is modified taking into account the spatial aspect and environmental pollution. A detailed review of models and decision support systems in the field of sustainable development management is presented in [16], models of economic growth are discussed in [5].

An important class of dynamic models of territories' economic development is formed by the Computable General Equilibrium (CGE) models [6,8,18]. These models have a solid microeconomic justification and ensure that the sectoral structure of the economy is fully taken into account, as well as the impact of changes in some economic sectors on others. The main disadvantage of CGE models is that they are economic-mathematical and do not adequately

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describe environmental and social aspects, as well as effects associated with dynamics and uncertainty; their identification is also difficult.

In the works [8, 9, 11] the authors of this paper introduced an original integrated approach to the modeling of the processes of accumulation and productive use of tangible and intangible assets – resources for territorial sustainable development as well as to the modeling of coordination processes of social and private interests for resource allocation in hierarchical control systems. An implementation of this approach allowed to establish the system compatibility conditions for different control problems setups; to justify a control strategy design methodology for balancing the social and economic interests of national, regional (local) and global economic agents in the reproduction and utilization of welfare resources for the sustainable development of a regional system; to propose system coordination mechanisms and study their properties; to initiate application of the coordination models of social and private interests to real regional management problems, more specifically, to the design of administrative and economic coordination mechanisms (engines) for the interests of territorial subjects [2, 4, 8, 9, 10, 12, 17].

The multidimensional experience of applying various modifications of the Solow model to solve economic problems demonstrates both efficiency and certain difficulties associated with the reflection of conditions and parametric characteristics adequate to the ongoing transformations. The “weakness” of the toolkit for reconciling the interests of spatial economic systems of various hierarchy levels is among the problems of the analytical support of the process of modern economic growth [1].

The proposed paper contains a detailed description of the approach to the process of coordinating the interests of these systems’ modeling on the basis of development, identification and software implementation of the advanced Solow model. The Solow model is supplemented by the optimization (game-theoretic) block as well as it is modified taking into account the spatial factor.

The list of key assumptions of the model includes the endogenous nature of the model; three-level structuring of the national spatial and economic system “subjects - federal districts - Russian Federation”; lack of parameters characterizing significant external financial flows (federal subsidies); insufficiently complete parametric characteristics of the comparative advantages of regional systems. Despite these assumptions, testing the proposed model on a representative information base of the Southern Federal District showed its applicability in the management system of modern spatial economic systems’ sustainable development. The improvement of the model by leveling of simplifying assumptions will make it possible to use it as a tool for designing a strategy for reconciling the interests of spatial economic systems in the process of implementation of various types of territorial development projects.

Thus, the results of the initial stage of research on the application of models for reconciling public and private interests in managing regional development, where agents are not separate municipalities of one subject, but different regions, will be presented below. At this stage, the adequacy of the model in the constraints of a self-contained system is investigated and the highest level of the management hierarchy (federal center) is not considered. As an assumption, we study the optimal behavior of the regions in the macro-region in order to increase both their own specific consumption (living standards) and increase the specific consumption of the entire region. To achieve this goal, regions can spend their funds both for their development and the development of neighboring regions.

The further structure of the article is as follows. Section 2 describes a mathematical model of the interaction of regions within the macro-region. Section 3 substantiates in detail the process of model parameters for the regions of the Southern Federal District identifying. Section 4 presents the procedure for finding a Nash equilibrium for the regions of the Southern Federal District in the region's investment in production. Section 5 summarizes the findings of the study.

2. MATHEMATICAL MODEL

Regions in a macro-region are considered. The system that makes up the macro-region is considered to be closed. Interaction with the external world for the system occurs due to the fact that the regions can spend their money on external goals for the macro-region. But this is reflected on the regional level only because the region theoretically does not manage all its means. Feedback, or receiving funds by the macro-region from the outside, does not occur.
The model of a regional social-ecological-economic system has the form:

$$Y_i(t) = A_i(t)K_i^{\alpha_i}(t)(R_iL_i)^{1-\alpha_i}(t); \quad (2.1)$$

$$I_i(t) = s_i(t)Y_i(t); \quad (2.2)$$

$$C_i(t) = [1 - s_i(t)]Y_i(t); \quad (2.3)$$

$$R_i(t+1) = (1+\eta_i)R_i(t); \quad (2.4)$$

$$K_i(t+1) = (1-\mu_i)K_i(t) + \sum_{j=1}^{n} k_{ji}(t)I_j(t)\,; \quad (2.5)$$

$$L_i(t+1) = (1+\ b_i - m_i)L_i(t); \quad (2.6)$$

$$P_i^a(t) = [1 - c_i^a v_i^a(t)I_i(t)]\big[B_{K_i}^a K_i(t) + B_{L_i}^a L_i(t)\big]; \quad (2.7)$$

$$P_i^w(t) = [1 - c_i^w v_i^w(t)I_i(t)]\big[B_{K_i}^w K_i(t) + B_{L_i}^w L_i(t)\big]; \quad (2.8)$$

$$K_i(0) = K_i^0; L_i(0) = L_i^0; R_i(0) = R_i^0; \quad (2.9)$$

$$\sum_{j=0}^{n} k_{ij}(t) + v_i^a(t) + v_i^w(t) = 1; 0 \le s_i(t) \le 1; v_i^a(t) \ge 0; \quad (2.10)$$

$$v_i^w(t) \ge 0; k_{ij} \ge 0; i = 1, \dots, n; j = 0, \dots, n;\ t = 0,1,2 \dots, T. \quad (2.11)$$

Index i denotes a region within a macro-region. The model has discrete time t = 0, 1, 2, ... with a step of one year. The model also includes other variables and parameters as follows.
$Y_i(t)$ as the final output of agent i in year t (in financial terms);
$K_i(t)$ as the agent's basic production assets (capital) in year t;
$L_i(t)$ as the agent's labor resources in year t;
$R_i(t)$ as the efficiency of the agent's labor resources in year t;
$A_i(t)$ as the influence function of the agent's innovative activity on the final output in year t;
α_i as the parameter of the agent's Cobb-Douglas production function;
$I_i(t)$ as the agent's production investments in year t;
$C_i(t)$ as the agent's nonproduction consumption in year t;
$s_i(t)$ as the share of the agent's production investments in its final output in year t;
η_i as the efficiency growth parameter of the agent's labor resources;
μ_i as the depreciation factor of the agent's basic assets;
$k_{ij}(t)$ as the share of the investments of agent i in the activity of agent j (the cooperation coefficient of these agents); here index j=0 describes an external agent for the whole system (for example, the federal state);
b_i, m_i as the reproduction and retirement coefficients of the agent's labor resources, respectively;
$P_i^a(t), P_i^w(t)$ as the agent's pollutant emissions into air and water in year t, respectively;
$v_i^a(t), v_i^w(t)$as the agent's allocations to prevent air and water pollution in year t, respectively;
c_i^a, c_i^was the efficiency coefficients of environmental allocations;
$B_{K_i}^a, B_{K_i}^w$as the specific rates of industrial pollution into air and water, respectively;
$B_{L_i}^a, B_{L_i}^w$ as the specific rates of human pollution (labor resources) into air and water, respectively;
K_i^0, R_i^0, L_i^0as given initial values of the model variables.
Therefore, the agent's state vector is:

$$X_i(t) = (Y_i(t), I_i(t), C_i(t), K_i(t), L_i(t), R_i(t), P_i^a(t), P_i^w(t)), \tag{2.12}$$

the control vector is

$$U_i(t) = (s_i(t), v_i^a(t), v_i^w(t), \{k_{ij}(t)\}_{j=0}^n), \tag{2.13}$$

and the parameter vector is

$$Z_i = (\alpha_i, \eta_i, b_i, m_i, \mu_i, c_i^a, c_i^w, B_{K_i}^a, B_{K_i}^w, B_{L_i}^a, B_{L_i}^w). \tag{2.14}$$

The following quantities were taken as model parameters:
$Y_i(t)$ − gross regional product (GRP);
$K_i(t)$ − fixed assets (FA) in the economy;
$L_i(t)$ − employed population (EP);
$R_i(t)$ − gross output (ratio of manufactured products to labor costs GRP / EP);
α_i − fixed assets elasticity;
η_i − increase (dynamics) of production;
μ_i − wear of FA, i.e. share of the value transferred to the manufactured product (GRP);
$b_i,\ m_i$ −increase and decrease in the employed population, respectively;
$P_i^a(t), P_i^w(t)$ −agent emissions of pollutants into the atmosphere and water, respectively, in year *t*;
$v_i^a(t) + v_i^w(t)$ − the difference between the total expenditures of the region and the sum of national, national-economic, housing and communal and socio-cultural expenses (consolidated budget);
c_i^a, c_i^w − the share of neutralization of air pollutants, the share of reused resources in the total volume of water use, respectively, is used for calculating.
Cross-border interaction of municipalities is described using control variables $k_{ij}(t)$. Parameter η_i characterizes labor productivity; μ_i − resource-saving technologies; $c_i^a, c_i^w, B_{K_i}^a, B_{K_i}^w$ − environmental technology; b_i − population policy; m_i − healthcare innovation; $B_{L_i}^a, B_{L_i}^w$ − environmental awareness.
It is natural to define the agent optimality criterion in model (2.1)-(2.11) as a function of a combination of general and private interests:

$$\bar{J}_i = \sum_{t=1}^T e^{-\rho t}[c_i(t) + r_i(t)c(t)], \tag{2.15}$$

where $c(t) = \frac{C_i(t)}{L_i(t)}$ is current specific consumption of the agent, reflecting the private interests of the region;
$c(t) = \frac{\sum_{i=1}^n C_i(t)}{\sum_{i=1}^n L_i(t)}$ is specific consumption of the macroregion, which reflects the common interests of the regions in the macroregion;
ρ is a discount rate;
$r_i(t)$ reflects the region's interest in increasing the specific consumption of the macro-region (the specific gravity of the region in total consumption is used as this parameter).
The agent's interest in maximizing not only domestic specific consumption, but also all neighboring regions, and, consequently, the macro-region as a whole, means the desire to improve the general standard of living and well-being of residents. Accordingly, the smaller the difference in the consumption parameters of agents, the better the social climate of neighboring regions, the lower the level of migration of unskilled labor, the more favorable the overall economic dynamics due to the matching cost of resources on the market, which determines the costs of implementing joint projects and development programs of the macro-region.
Practical testing of the model was carried out on the basis of cross-border interaction between the subjects of the Southern Federal District (SFD) of the Russian Federation, which include the Rostov Region (the index is assigned i = 1), Volgograd Region (i = 2), Krasnodar Region

(i = 3), Republic of Adygea (i = 4), Astrakhan region (i = 5), Republic of Kalmykia (i = 6), Republic of Crimea (i = 7).

Simulation scenarios in model (2.1)-(2.11) include defining trajectories of control variables from vector (2.13).

To study the cross-border interactions of regions within the Southern Federal District, it is necessary to identify the parameters of the Z_i vectors for each region, and also to create a program for calculating the main indicators, specifically $Y_i, C_i, I_i, K_i, L_i, R_i$ for the time period from 2017 to 2022.

3. PARAMETER IDENTIFICATION

The model parameters $Z_i = (\alpha_i, \eta_i, b_i, m_i, \mu_i, c_i^a, c_i^w, B_{K_i}^a, B_{K_i}^w, B_{L_i}^a, B_{L_i}^w)$ and the $A_i(t)$ coefficient were identified according to the data of the Federal State Statistics Service of the Russian Federation for 2005, 2010, 2015-2017 [19]. These time intervals are determined on the basis of criteria for stable positive dynamics of macroeconomic growth and in order to exclude the extreme impact of global pre-crisis economic trends in 2008 and the systemic consequences of the socio-political Russian crisis of 2013.

The model parameters for each agent are distinctive characteristics that show the level of development of individual areas of each region as part of the macro-region. They are formed in retrospective dynamics and reflect the historical factors of the sectoral structure of the economy.

1) The parameter α_i of the Cobb-Douglas production function is not directly reflected in the statistical data. A formula to determine the elasticity of output $Y_i(t)$ by fixed asset was used to find α_i:

$$\alpha_i = \frac{K}{Y} \cdot \frac{\partial Y}{\partial K}. \tag{3.1}$$

Calculations according to the formula (3.1) were made on the basis of information characterizing in dynamics the values of the volume of fixed assets $K_i(t)$ and GRP of each region $Y_i(t)$. After determining the elasticity coefficients α_i the arithmetic mean values of these coefficients are calculated for each of the studied regions (Table 3.1).

Table 3.1. Arithmetic mean values of elasticity coefficients of output $Y_i(t)$ by volume of FA in the region's economy

Region	Arithmetic mean values of elasticity coefficients	Value
Rostov Region	α_1	0,214360
Volgograd Region	α_2	0,107626
Krasnodar Region	α_3	0,144864
Republic of Adygea	α_4	0,236457
Astrakhan Region	α_5	0,078457
Republic of Kalmykia	α_6	0,145812
Republic of Crimea	α_7	0,150498

2) To calculate the difference between the reproduction rate and the labor retirement rate ($b_i - m_i$), we used the data of natural growth by region for the selected periods, from which the arithmetic average was formed. As a result, the following parameter values are obtained (Table 3.2).

Table 3.2. Parameter Values (b_i- m_i) for the regions of the Southern Federal District

Region	Parameter (b_i- m_i)	Value
Rostov Region	$b_1 - m_1$	-0,0005
Volgograd Region	$b_2 - m_2$	-0,01775
Krasnodar Region	$b_3 - m_3$	0,00575
Republic of Adygea	$b_4 - m_4$	-0,008
Astrakhan Region	$b_5 - m_5$	0,00375
Republic of Kalmykia	$b_6 - m_6$	-0,00375
Republic of Crimea	$b_7 - m_7$	-0,007

3) To determine the parameter of labor efficiency η_i dynamics we use data on labor productivity $R_i(t)$ for the corresponding periods, which is determined by the ratio of the gross product value $Y_i(t)$ and the size of the able-bodied population $L_i(t)$. Then the annual growth is calculated. To determine the growth in the five-year interval 2005-2010 and 2010-2015 we use the formula $\eta_i = \sqrt[5]{\frac{R_i(t+5)}{R_i(t)}}$-1, and at further annual intervals $\eta_i = \frac{R_i(t+1)}{R_i(t)}$-1. Further, all the obtained parameters are averaged over the period (Table 3.3).

Table 3.3. Values of the parameters of labor efficiency for the regions of the Southern Federal District dynamics

Region	The parameters of the labor efficiency dynamics	Value
Rostov Region	η_1	0,009251
Volgograd Region	η_2	-0,017337
Krasnodar Region	η_3	0,024048
Republic of Adygea	η_4	0,001221
Astrakhan Region	η_5	0,027663
Republic of Kalmykia	η_6	0,002532
Republic of Crimea	η_7	0,017375

4) Depreciation ratio of fixed assets μ_i is taken from the statistics in direct form for each year, followed by averaging (Table 3.4).

Table 3.4. The values of the coefficient of depreciation of fixed assets for the regions of the Southern Federal District

Region	Depreciation ratio of fixed assets	Value
Rostov Region	μ_1	0,4064
Volgograd Region	μ_2	0,4064
Krasnodar Region	μ_3	0,511
Republic of Adygea	μ_4	0,511
Astrakhan Region	μ_5	0,4064
Republic of Kalmykia	μ_6	0,511
Republic of Crimea	μ_7	0,711667

5) Specific emissions of pollutants from industrial activities into the atmosphere $B^{\alpha}_{K_i}$and water $B^{w}_{K_i}$ are the parameters of environmental technology that are regulated by the state. Specific

emissions of pollutants $B_{L_i}^{\alpha}, B_{L_i}^{w}$ from human sources of activity are not controlled. To determine this data, we use the values of the region's discharge of pollutants into the atmosphere and water $P_i^{\alpha}(t), P_i^{w}(t)$ for the indicated time periods, as well as the percentage of pollution of air and water, respectively, as a result of industrial activity and life population. We denote them by $\text{prK}^{a}_{i}(t)$, $\text{prL}^{a}_{i}(t)$, $\text{prK}^{w}_{i}(t)$ and $\text{prL}^{w}_{i}(t)$, where $\text{prK}^{a}_{i}(t) + \text{prL}^{a}_{i}(t) = 1$ and $\text{prK}^{w}_{i}(t) + \text{prL}^{w}_{i}(t) = 1$.

All these parameters are determined on the basis of the data of the Federal State Statistics Service of the Russian Federation for 2005, 2010, 2015-2017 [19].

The previously identified data on the volume of fixed assets $K_i(t)$ and the able-bodied population $L_i(t)$ were also used for calculations. The desired values are calculated as follows:

$$B_{K_i}^{\alpha} = \frac{P_i^{\alpha}(t)\cdot\frac{\text{prKa}_i(\text{t})}{100}}{K_i(t)}, B_{L_i}^{\alpha} = \frac{P_i^{\alpha}(t)\cdot\frac{\text{prLa}_i(\text{t})}{100}}{L_i(t)}, B_{K_i}^{w} = \frac{P_i^{w}(t)\cdot\frac{\text{prKw}_i(\text{t})}{100}}{K_i(t)}, B_{L_i}^{w} = \frac{P_i^{w}(t)\cdot\frac{\text{prLw}_i(\text{t})}{100}}{L_i(t)}.$$

Further, each parameter is reduced to the arithmetic mean value. As a result, the following values were obtained for the regions of the Southern Federal District (Table 3.5).

Table 3.5. The values of environmental technology parameters for the regions of the Southern Federal District

Region	Environmental Technology Parameters	Value
Rostov Region	$B_{K_1}^{\alpha}$ $B_{K_1}^{w}$ $B_{L_1}^{\alpha}$ $B_{L_1}^{w}$	8,34E-05 5,365E-05 0,0232252 0,0756389
Volgograd Region	$B_{K_2}^{\alpha}$ $B_{K_2}^{w}$ $B_{L_2}^{\alpha}$ $B_{L_2}^{w}$	9,585E-05 6,303E-05 0,0687909 0,0617981
Krasnodar Region	$B_{K_3}^{\alpha}$ $B_{K_3}^{w}$ $B_{L_3}^{\alpha}$ $B_{L_3}^{w}$	4,2518E-05 0,00013121 0,02400843 0,22684937
Republic of Adygea	$B_{K_4}^{\alpha}$ $B_{K_4}^{w}$ $B_{L_4}^{\alpha}$ $B_{L_4}^{w}$	2,35727E-05 8,18161E-05 0,028614278 0,10440849
Astrakhan Region	$B_{K_5}^{\alpha}$ $B_{K_5}^{w}$ $B_{L_5}^{\alpha}$ $B_{L_5}^{w}$	5,63E-05 2,27E-05 0,185544 0,085229
Republic of Kalmykia	$B_{K_6}^{\alpha}$ $B_{K_6}^{w}$ $B_{L_6}^{\alpha}$ $B_{L_6}^{w}$	2,222E-06 0,0001432 0,0220744 0,0896685
Republic of Crimea	$B_{K_7}^{\alpha}$ $B_{K_7}^{w}$ $B_{L_7}^{\alpha}$ $B_{L_7}^{w}$	3,64224E-06 9,41469E-07 0,02335533 0,006049158

6) To calculate the efficiency coefficients of environmental protection allocations $c_i^{\alpha}$and c_i^{α}, respectively, with the index a for the atmosphere, and with the index w for water, data on the share of the neutralization of pollutants are used. Direct statistics are taken for the air environment, and the proportion of reused water is used to estimate the costs of protecting the aquatic environment. We denote these quantities $do^a_i(t)$ for air and $do^w_i(t)$ for water. Previously determined values of the volumes of pollutant discharges into the atmosphere and water $P_i^a(t), P_i^w(t)$are used for calculations.

The amount of expenses for compensation of environmental damage from pollutants is determined on the basis of available official statistics as the difference between the total expenditures of the region and the sum of national, national-economic, housing and communal and socio-cultural (consolidated budget). We denote these costs by $v_i(t)$. Then, the ratio of environmental costs to total costs $v_i^a(t) + v_i^w(t)$ is calculated, i.e.$v_i^a(t) + v_i^w(t) = \frac{v_i(t)}{I_i(t)}$), which gives a share of the cost of eliminating pollution. We assume that the costs of water and air purification are proportional to the volume of pollutant discharges, i.e.

$$v_i^a(t) = \frac{p_i^a(t)}{p_i^a(t)+p_i^w(t)}\left(v_i^a(t) + v_i^w(t)\right), v_i^w(t) = \frac{p_i^w(t)}{p_i^a(t)+p_i^w(t)}\left(v_i^a(t) + v_i^w(t)\right).$$

Then we calculate the desired values from (2.7) - (2.8) by the formulas

$$c_i^a(t) = \frac{1-\frac{do_i^a(t)}{100}}{v_i^a(t)I_i\ (t)}, c_i^w(t) = \frac{1-\frac{do_i^w(t)}{100}}{v_i^w(t)I_i\ (t)}.$$

Each value is reduced to the arithmetic mean value.

As a result, the following initial data were obtained for the regions of the Southern Federal District (Table 3.6).

Table 3.6. The values of the efficiency ratios of environmental appropriations for the regions of the Southern Federal District

Region	Environmental appropriations effectiveness ratios	Value
Rostov Region	c_1^{α} c_1^{w}	0,000393 0,000104
Volgograd Region	c_2^{α} c_2^{w}	0,000497 0,000528
Krasnodar Region	c_3^{α} c_3^{w}	0,000635 9,42E-05
Republic of Adygea	c_4^{α} c_4^{w}	0,016904 0,002975
Astrakhan Region	c_5^{α} c_5^{w}	0,00085 0,001107
Republic of Kalmykia	c_6^{α} c_6^{w}	0,029967 0,002841
Republic of Crimea	c_7^{α} c_7^{w}	0,000772 0,001659

7) The agent's interest rate in increasing the specific consumption in the macro-region $r_i(t)$ is calculated as a share of the region's consumption in the total consumption in accordance with the formula (3.2).

$$r_i = \frac{C_i(t)}{\sum_{i=1}^{n} C_i(t)}. \quad (3.2)$$

With subsequent reduction to the arithmetic mean value, we obtain the following values of the desired parameter (Table 3.7).

Table 3.7. The values of the coefficients of the region's interest in increasing the specific consumption in the macro-region

Region	Coefficients of the region's interest in increasing the specific consumption in the macro-region	Value
Rostov Region	r_1	0,278
Volgograd Region	r_2	0,18
Krasnodar Region	r_3	0,438
Republic of Adygea	r_4	0,05695
Astrakhan Region	r_5	0,05695
Republic of Kalmykia	r_6	0,00899
Republic of Crimea	r_7	0,01

8) We understand the totality of all phenomena that lead to an increase in production without the use of additional resources by innovative activity in this work. We believe that innovation affects the results of modeling through the introduction of more advanced equipment and technologies into production, or through special measures to improve the skills of the workforce. These improvements are set externally and take the form of a time-dependent function.
Indices of the innovative activity impact in the regions of the Russian Federation for 2005, 2010, 2015 were obtained as a result of previous research by D.S. Lozovitskaya and E.I. Lazareva based on official data from Federal State Statistics Service of the Russian Federation [19] on the following indicators: the degree of depreciation of fixed assets, investment in fixed assets per capita, developed advanced production technologies (inventive activity rate). The index is built on the basis of regression models for each period (2005, 2010, 2015), in which these indicators are defined as statistically significant [3, 13].
The values of the calculated coefficients are given in Table 3.8.

Table 3.8. The values of the indices of the impact of innovation on the production of the final product of the region

Year	Value of index
2005	0,308
2010	0,222
2015	0,27

By interpolating the available values, the following type of index of the influence of innovation activity is obtained:

$$A_i(t) = 0{,}00268 \cdot t^2 - 10{,}7774 \cdot t + 10835{,}33.$$

9) The initial values of the parameters of the model $K_i(0)$, $L_i(0)$ and $R_i(0)$ are determined according to the official data of Federal State Statistics Service of the Russian Federation [19] on the volume of fixed assets and the number of working population, as well as finding the ratio of production / GRP to the number of labor resources in the region for 2017 year. The obtained parameter values are presented in the Table 3.9.

Table 3.9. Initial values of model parameters

Region	Model parameters	Value
Rostov Region	$K_1(0)$ $L_1(0)$	2786870 1958,1

	$R_1(0)$	687,984679
Volgograd Region	$K_2(0)$ $L_2(0)$ $R_2(0)$	2180917 1124,6 685,9694113
Krasnodar Region	$K_3(0)$ $L_3(0)$ $R_3(0)$	5937791 2599,1 856,4186449
Republic of Adygea	$K_4(0)$ $L_4(0)$ $R_4(0)$	202111 152,1 653,556213
Astrakhan Region	$K_5(0)$ $L_5(0)$ $R_5(0)$	1498692 487,6 863,3328548
Republic of Kalmykia	$K_6(0)$ $L_6(0)$ $R_6(0)$	203657 111,1 598,6642664
Republic of Crimea	$K_7(0)$ $L_7(0)$ $R_7(0)$	2212391 840,4 427,3089005

10) The discount factor ρ based on the average refinancing rate of the Central Bank of the Russian Federation for the period under consideration is taken equal ρ=0,094.

4. NASH EQUILIBRIUM

Regional strategies are characterized by the following parameters:
Percentage $s_i(t)$ of GRP spent on production;
Shares $v_i^a(t), v_i^w(t)$ from industrial investments that are used to eliminate the effects of air and water pollution;
The shares $k_{ij}(t)$ from production investments that are spent both on the general development of the macroregion and on their own development (for i = j). The share $k_{i0}(t)$is also allocated by the region to goals external to the Southern Federal District (federal and inter-district programs and projects).
When finding Nash equilibrium, you need to find a set of agent strategies $\{s_i(t), v_i^a(t), v_i^w(t), (k_{ij}(t))_{j=0}^n\}_{-i}$, delivering maximum of their objective function with fixed strategies of other regions.
Using simulation, Nash equilibrium is sought by the method proposed in [16]:
Step 1. In the loop, we sort through all the agents and for each agent:

$$\{s_i(t), v_i^a(t), v_i^w(t),\ (k_{ij}(t))_{j=0}^n\}_{-i};$$

2) For a fixed strategy of other agents, we sort through the strategies of this agent according to a certain principle and remember the one that delivers the maximum of its objective function, we denote this strategy NE_i;
Step 2. The resulting NE_i strategies form the equilibrium outcome of the game $NE=(NE_1, NE_2, \ldots, NE_n)$.
For the model described and investigated in the article, fixing the strategies of the environment of the agent i is as follows.
If j<i then the optimal strategy NE_i has already been found. It should be defined as a fixed strategy of the agent i.
If j>i then the optimal strategy NE_i has not yet been found, therefore, we fix a strategy in which the available resources are evenly distributed by the agent for all goals and tasks: i.e. $s_i(t) =$

0,5 (equal parts go to investments in production and consumption), $v_i^a(t) = 0{,}1, v_i^w(t) = 0{,}1$, $k_{ij}(t) = 0{,}1$ for j=0,1,…,n.

Point 2) of step 1 causes significant difficulties, since for n = 7, T = 5, even if each strategy consisted of only two possible values, it is necessary to sort out 23+n+1=211 values in only one moment in time, and when T = 5 the number of iterations will be (211)T=(211)5=255. But really, there are much more than two options for each strategy, and to sort through strategies with a step of 0.1, 1155 operations are required.

To reduce the number of operations, we apply the following unidirectional iteration method. We sequentially sort through each of the eleven agent strategies with the remaining ten strategies fixed. Agent strategies are sorted in the following order:

- in the first place, $s_i(t)$ is moved, since in fact the volume of required resources for the development of regions and environmental costs depend on it; further strategies are a fraction of this value;
- then the shares $v_i^a(t), v_i^w(t)$ from production investments that go to the elimination of air and water pollution are moved;
- then goes each of the $k_{ij}(t)$ strategies.

This reduces the number of steps to 11 * 5 * 11 = 605 for each value in increments of 0.1.

Simulation modeling showed that the scenarios with small values of $s_i(t)$ gave large values of the payoff function of the agent, compared with large $s_i(t)$.

This can be explained as follows. Since the values of specific consumption in each region and in the macro-region as a whole are subject to maximization, in fact, maximization also occurs in relation to the volume of regional consumption. And since the final product is distributed between production and consumption, while maximizing the volume of consumption, it is necessary to minimize the volume of production. Therefore, the optimal value is $s_i(t) = 0$. But since resources are not spent on production, all cost items, including the development of own production and the general production programs of neighboring regions, as well as environmental costs for environmental protection and the elimination of air and water pollution, are indifferent. Therefore, the dominant strategy of each agent is $s_i(t) = 0$.

At first glance, it is surprising that the region's dominant strategy is the lack of investment in production. Indeed, in this case, the fixed assets are not replenished, but only retired, therefore, by virtue of (2.1), the final product is formed, but in a smaller quantity, i.e. production decreases over time. But the level of pollution of natural resources is becoming less. However, to maximize the volume of consumption, it is necessary to increase the volume of production, which in modern conditions is due to extensive forms of management and, accordingly, is highly dependent on the volume of fixed assets. The latter, in turn, require constant replenishment due to natural and economic depreciation. Under such conditions, the behavior of each region can be interpreted as waiting for the help of neighboring agents.

However, as can be seen from the calculation results, all regions choose similar strategies, reasoning rationally and not wanting to invest resources in the development of material production. As a result, the replenishment of fixed assets does not occur in any region, since all agents choose a production stagnation strategy. Nash equilibrium is formed by the strategies $\{s_i(t) = 0, i = 1, \dots, n\}$ with indifferent values of other strategies of all agents. This outcome of the game, when all agents choose a free-rider strategy of economic inaction based on the production activity of neighboring regions, inevitably leads to a decrease in production due to a decrease in available resources and should, as it seemed, reduce the value of the target function of the regions. But, as calculations showed, this mode of behavior continues to be beneficial to the regions (Rostov, Volgograd, Astrakhan, Krasnodar), but at the same time it ceases to be beneficial to the republics (Republic of Adygea, Republic of Kalmykia, Republic of Crimea).

A strategy to reduce production and increase consumption is not advisable for any of the regions in real economic conditions. The presented results show the need for further theoretical studies of the model, the introduction of additional parameters and limitations. In particular, we assume the importance of taking into account external financial flows from the federal center, the introduction of control variables to reflect the process of harmonizing common and private interests in the field of socio-economic development of regions.

5. CONCLUSION

The article presents the results of the initial stage of studies of a dynamic model of combining the common and private interests of regions in the macro-region using the example of the Southern Federal District. Maximizing the region's specific consumption is considered as the region's private interests, and maximizing the specific consumption of the entire macro-region as general interests. Each region of the Southern Federal District can invest in its own development, as well as in the development of other regions as part of the macro-region. It was revealed that the optimal strategy of each region in the current conditions is a complete rejection of investments in the development of production. Calculations showed that the republics of the Southern Federal District immediately become unprofitable for the general reduction of production, while the regions of the Southern Federal District continue to benefit from this strategy in the future. It is not possible to recommend the obtained strategies to present specific practical recommendations in real economic conditions. Further practical studies of the model and the introduction of additional parameters and limitations are required for practical reasons.

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