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Identification of nonlinear systems having hard function

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Abstract: In this work, an identification approach of nonlinear systems is studied. Presently, the nonlinear system can be described by Wiener-Hammerstein model. This latter is composed of a nonlinearity surrounded by two linear blocks. The linear blocks can be nonparametric. The nonlinear element is allowed to be discontinuous, specifically it can be of hard element (e.g. Preload, Coulomb friction, Dead-zone). Roughly, it is very difficult to model these types of nonlinearities by orthogonal decompositions (e.g. polynomial).

Keywords: Nonlinear systems, linear system, static and dynamic systems, control system, system identification, discontinuous nonlinearity, hard element.

1. INTRODUCTION

Nonlinear systems exist widely in industry and science applications [2,5-7,13], among which the Wiener-Hammerstein model (Fig. 1.1) is one of the most typical cases [1,3,9,15-16]. The Wiener-Hammerstein models consist of a nonlinear block surrounded by two linear elements (Fig.1.1). The identification problem have been paid considerable attention due to their benefits such as control [7,11-12]. The Wiener-Hammerstein like systems are used in a wide range of applications such as identification of skeletal muscle [1]. Note that, Wiener and Hammerstein nonlinear systems can be modeled by Wiener-Hammerstein systems.

The solution of this problem identification can be dealt using several method. The available methods have been developed following three main approaches i.e. iterative nonlinear optimization procedures [19]; stochastic methods e.g. [3,21];

In [27], an approach based on the standard SVM (support vector machines) for regression was presented. The quite poor results obtained in that work highlighted some of the limitations of the method. In particular, only a NFIR (nonlinear finite impulse response) model structure was taken into account, which did not perform well since the considered system has a long impulse response. Another problem was given by the high computational time and memory usage, which made it difficult to work with a large amount of data.

Several SVM-like approaches [18], based on the least squares SVM (LS-SVM), are characterized by a very high number of parameters. Many approaches use the BLA, or a similar correlation analysis, as a starting point for the algorithm (e.g. [24-26]). Then, the user does not have to take order decisions needed to parametrize the BLA (or the QBLA).

In [23], a nonparametric approach to separate the front and back dynamics starting from the best linear approximation (BLA) is proposed. In [16], a recursive identification method of Wiener-Hammerstein system with internal noises is developed. In [17], a recursive least-squares identification method of wiener-Hammerstein system is proposed. This approach is treated in the case where the system nonlinearity is dead-zone.

Then, a wide multiplicity of approaches is currently under study. These contain parametric and non-parametric methods. There exist different class of non-parametric

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solutions. Among these concepts the frequency modeling, for which control and output are relying using periodic signals [4,8,10,14,20,22]. Depending on the nature of the control (the input excitation) to the system, several estimators have been developed.

In this paper, a solution based on analytical geometry (frequency study) is proposed. Presently, the linear elements are allowed to be nonparametric. The nonlinear block can be discontinuous or of hard nature. Examples of popular hard functions are shown by Figs. 1.2a-b. For convenience, Fig. 1.2a illustrates Coulomb friction nonlinearity with dead-zone and Fig. 1.2b shows example of viscous friction function. A hard function is often known as being a nonlinearity having several discontinuities and it is an affine nonlinearity elsewhere [9,14].

On the other hand, the modeling of hard function using orthogonal polynomials decomposition often remains a challenge. It is very complicated to approximate these types of nonlinearities using any orthogonal basis approximation (e.g. polynomial decomposition). For convenience, Fig. 1.3 illustrates example of orthogonal polynomial decomposition of hard function. This example shows that remarkable modeling errors are thus induced especially around discontinuity points even though the troncature degree *m* is too big. Presently, the nonlinear element is allowed to be discontinuous and not necessarily affine function between two consecutive discontinuities. Except in a small interval where it is supposed to be parametric function (e.g. polynomial nonlinearity).

Then, recall that the identification method is based only on control signal $u(t)$ and observed output signal $y(t)$ (i.e. all inner signals are not accessible). In this work, the linear blocks are not necessarily parametric. Accordingly, because of these last difficulties, it is not surprising to notice that they are very rare papers dealing the identification of Wiener-Hammerstein models having hard function.

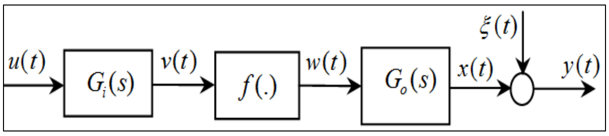

Fig. 1.1. Wiener-Hammerstein model

The paper is organized as follows: the identification problem is formulated in Section 2, which also introduces the problem of multiplicity of identification solutions; Section 3 is devoted to the determination of nonlinear system parameters (linear block and nonlinear element); the performances of the identification method are illustrated by simulation in Section 4.

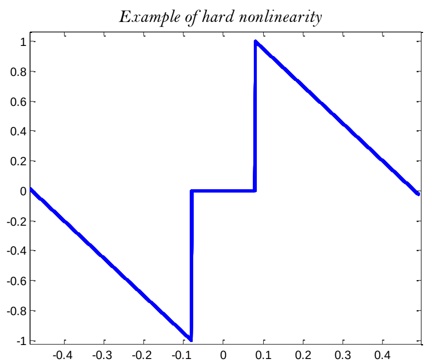

Fig. 1.2a. Coulomb friction nonlinearity with dead-zone

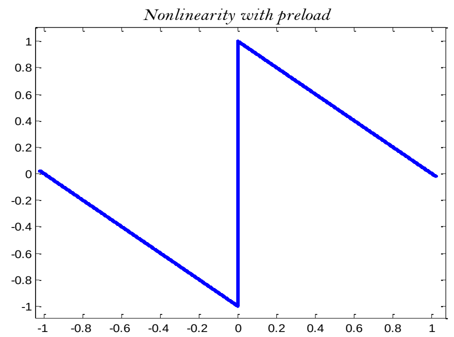

Fig. 1.2b. Viscous friction function

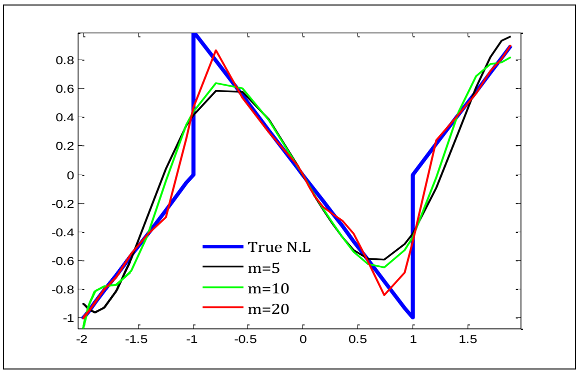

Fig. 1.3. Decomposition of Hard NL with series of polynomial basis

2. PROBLEM FORMULATION

Presently, the problem of determination of nonlinear system parameters is discussed. The considered nonlinear system is structured by Wiener-Hammerstein model (Fig. 1.1). Let $G_i(s)$ and $G_o(s)$ denote the transfer function of linear blocks. The associated impulse responses (or the inverse Laplace transforms) are respectively denoted $g_i(t) = L^{-1}\left(G_i(s)\right)$ and $g_o(t) = L^{-1}\left(G_o(s)\right)$.

The control signal $u(t)$ and the inner signal $v(t)$ are thus related by the following relation:

$$v(t) = g_i(t) * u(t) \tag{2.1}$$

where the notation " * " designates the convolution product. Then, the inner signals $v(t)$ and $w(t)$ undergo the following equation:

$$w(t) = f\left(v(t)\right) = f\left(g_i(t) * u(t)\right) \tag{2.2}$$

As far as that goes, the inner signal $w(t)$ and the undisturbed output $x(t)$ are related as follows:

$$x(t) = g_o(t) * w(t) \tag{2.3}$$

Finally, it follows from (2.3) and Fig. 1.1 that, the system output can be expressed as:

$$\begin{aligned} y(t) &= g_o(t) * w(t) + \xi(t) \\ &= g_o(t) * f\left(v(t)\right) + \xi(t) \end{aligned} \tag{2.4}$$

where the extra input $\xi(t)$ accounts for measurement noise and other modelling effects.

On the other hand, note that this identification problem does not have a unique solution [6,12]. Indeed, if the triplet $\left(G_i(s), f(v), G_o(s)\right)$ is solution of this Wiener-Hammerstein identification problem, then any set of form $\left\{\left(k_i G_i(s)\,,\ f(v/k_i)/k_o\,,\ k_o G_o(s)\right);\ for\ k_i \neq 0 \text{ and } k_o \neq 0\right\}$ is also solution of this problem (by distributing nonzero constants between the blocks of system).

In this respect, the question that arises is how to choose a solution to this problem?

The answer to this will be dealt with in the next section.

The considered Wiener-Hammerstein nonlinear system is completed by the following assumptions:

Assumptions 2.1:

- *The nonlinear element is hard function having multiple discontinuities. Except in a small interval where it is supposed to be parametric or polynomial function of degree increased by a finite integer n.*
- *The linear elements have a nonzero static gains, i.e.* $G_i(0) \neq 0$ *and* $G_o(0) \neq 0$.
- *The extra input* $\xi(t)$ *is supposed to be zero-mean ergodic and uncorrelated with the control input u(t).* □

Except of these assumptions, the system is arbitrary. In particular, the nonlinear element is allowed to be discontinuous and it is not necessarily an affine function between two consecutive discontinuities. Then, the linear elements are nonparametric.

3. SYSTEM PARAMETERS ESTIMATION

3.1. Nonlinear element estimation

The goal presently is to develop a solution allowing to give the estimate of a set of points belonging to nonlinear function *f*(.). The question: from the plurality problem discussed in section 2, what is the system to be estimate?

A key idea is to get benefit from this model plurality to make the identification problem more tractable. In this respect, the following selection of the free scalars (k_i, k_o) will prove to be judicious:

$$k_i = \frac{1}{G_i(0)} \quad \text{and} \quad k_o = \frac{1}{G_o(0)} \tag{3.1}$$

Without loss of generality, it is readily follows from (3.1) and model plurality that, the linear elements of system to be determined check the following property:

$$G_i(0) = G_o(0) = 1 \tag{3.2}$$

On the other hand, let excite the system by a constant value:

$$u(t) = U_1\,, \quad t \in \begin{bmatrix}0 & T\end{bmatrix} \tag{3.3}$$

where T is more superior to the system rise time t_r. Then, it follows from (2.1), (3.2) and (3.3) that, the inner signal *v*(*t*) boils down (in steady state) to:

$$v(t) = U_1 \tag{3.4}$$

One immediately gets from (2.2) and (3.4) that (after transient regime):

$$w(t) = f\left(U_1\right) \tag{3.5}$$

Accordingly, in view of (2.3), (3.2) and (3.5), it follows that *x*(*t*) is written as follows (after transient regime):

$$x(t) = X_1 = f\left(U_1\right) \tag{3.6}$$

Then, it is readily seen from (2.4), (3.2) and (3.6) that the output $y(t)$ undergoes (after transient regime) the following expressions:

$$y(t) = f\left(U_1\right) + \xi(t) \tag{3.7}$$

At this point, it is worth emphasizing that, the signal $y(t)$ is constant up to noise. Furthermore, it readily seen (in steady state) that:

$$\left(u,\, x\right) = \left(U_1,\, X_1\right) = \left(U_1,\; f(U_1)\right) \tag{3.8}$$

is a point belonging to the nonlinearity $f(.)$. One difficulty with the considered identification problem is that, the output of the system $y(t)$ is infected by the disturbance $\xi(t)$ whose stochastic law is not known. Then, it follows from the assumption on the noise $\xi(t)$, just as suggested in [6,12] the following estimator for $f(U_1)$ is proposed:

$$\hat{f}(U_1) = \hat{X}_1 = \frac{1}{T - t_r}\sum_{t=t_r}^{T} y(t) \tag{3.9}$$

Indeed, one has from (3.7) and (3.9) that:

$$\hat{f}(U_1) = f\left(U_1\right) + \frac{1}{T - t_r}\sum_{t=t_r}^{T} \xi(t) \tag{3.10}$$

Bearing in mind that, the noise $\xi(t)$ is zero-mean ergodic stochastic sequence, the last term in (3.10) converges (with probability 1) to zero. This implies that:

$$\hat{f}(U_1) \underset{T\to\infty}{\longrightarrow} f\left(U_1\right) \quad \text{w.p.1} \tag{3.11}$$

This result shows that, an accurate estimate of a point belonging to the function $f(.)$ can be obtained. Accordingly, the estimate of set of points belonging to $f(.)$ can be achieved using the same procedure, i.e. applying the control (input) sequence:

$$u(t) = U_k\,,\;\; t \in \left[(k-1)T \;\;\, kT\right] \text{ for } \; k = 1 \ldots N \tag{3.12}$$

Then, $\hat{f}(U_k)$ ($k = 1 \ldots N$) can be obtained using the estimator (3.9):

$$\hat{f}(U_k) = \frac{1}{T - t_r}\sum_{t=(k-1)T+t_r}^{kT} y(t) \quad \text{for} \;\; k = 1 \ldots N \tag{3.13}$$

Remark 3.1:

Consider the nonlinear system (Wiener-Hammerstein) described by (2.1)-(2.4). Then, the system nonlinearity estimator, given by (3.9), enjoys the consistency property for any value U_k. Specifically, one has the following result:

$$\hat{f}(U_k) \underset{T\to\infty}{\longrightarrow} f\left(U_k\right) \quad \text{w.p.1} \tag{3.14}$$

The proof of this can be found by combining (3.9)-(3.10) and the property of $\xi(t)$ (zero-mean ergodic stochastic). □

Remark 3.2:

- *The identification method can be easily applied in the case where f(.) is parametric (e.g. polynomial function of degree n). Then, a number of points* $N = n+1$ *(arbitrarily chosen by the user) is largely sufficient to determine the nonlinearity over the entire working range.*
- *In the case where f(.) is hard element, it is thus not necessarily a parametric function and can be discontinuous. Furthermore, in a small interval (e.g. between two consecutive discontinuities), f(.) is supposed to be parametric or polynomial function of degree increased by an integer n. Then, using the set of estimated points* $\left\{\left(U_k, \hat{f}(U_k)\right); k=1\ldots N\right\}$*, choose a candidate interval and verify if the nonlinear element can be decomposed with orthogonal polynomials approximation. To this end, we can excite the system by other inputs in this interval. Then, find out if we can approximate the set of points* $\left(U_k, \hat{f}(U_k)\right)$ *with a polynomial function of degree n.*
- *The last statement can be checked using a sine control in the chosen interval. Accordingly, by observing the spectrum of the output signal if it does not contain harmonics of higher rank than n. For further information, please see the following subsection « The linear blocks determination ». In this interval, the output of nonlinear block can thus be expressed as:*

$$w(t) = f\left(v(t)\right) = \sum_{k=0}^{n} c_k v^k \tag{3.15}$$

where $C = \left[c_0 \ldots c_n\right]^T$ *is the parameters vector corresponding to the nonlinear element.*□

3.2. The linear blocks determination

Presently, the aim is to present an identification approach permitting to provide the estimate of linear blocks parameters. For convenience, the nonlinear system (2.1)-(2.4) is excited by the following control signal:

$$u(t) = U\left(\alpha + \cos(\omega t)\right) \tag{3.16}$$

where the parameter α is adjusted such that $u(t)$ belongs to the chosen interval using remark 3.2 for any small amplitude U. This result can be practically depicted by observing the spectrum of the output signal. Let $\varphi_i(\omega)$ and $\varphi_o(\omega)$ designate the phases (argument) of linear elements $G_i(j\omega)$ and $G_o(j\omega)$, for any frequency ω, respectively. Then, it readily follows from (2.1), (3.2) and (3.16) that, the inner signal $v(t)$ (in the steady state) is given by:

$$v(t) = U\left(\alpha + \left|G_i(j\omega)\right|\cos(\omega t + \varphi_i(\omega))\right) \tag{3.17}$$

One has then using (3.15) and (3.17) that:

$$w(t) = \sum_{k=0}^{n} c_k U^k \left(\alpha + \left|G_i(j\omega)\right|\cos(\omega t + \varphi_i(\omega))\right)^k \tag{3.18}$$

Bearing in mind that:

$$\left(\alpha + \left|G_i(j\omega)\right|\cos(\omega t + \varphi_i(\omega))\right)^k = \sum_{l=0}^{k} C_l^k \alpha^l \left(\left|G_i(j\omega)\right|\cos(\omega t + \varphi_i(\omega))\right)^{k-l} \tag{3.19}$$

where:

$$C_l^k = \frac{k!}{l!(k-l)!} \tag{3.20}$$

Then, it immediately follows from (3.18)-(3.20) that:

$$w(t) = \sum_{k=0}^{n} c_k U^k \sum_{l=0}^{k} C_l^k \alpha^l \left(|G_i(j\omega)| \cos(\omega t + \varphi_i(\omega)) \right)^{k-l} \tag{3.21}$$

For convenience, the formulas of power identities of types $\cos^{2m}\lambda$ and $\cos^{2m+1}\lambda$ can also be given analytically as:

$$\cos^{2m}\lambda = \frac{1}{2^{2m}} C_m^{2m} + \frac{1}{2^{2m-1}} \sum_{p=0}^{m-1} C_p^{2m} \cos\left(2(m-p)\lambda\right) \tag{3.22a}$$

$$\cos^{2m+1}\lambda = \frac{1}{4^m} \sum_{p=0}^{m} C_p^{2m+1} \cos\left((2m+1-2p)\lambda\right) \tag{3.22a}$$

In view of (3.22a-b) and grouping the components having the same harmonics, (3.21) becomes (for any limited integer *n*):

$$w(t) = A_0\left(|G_i(j\omega)|\right) + \sum_{k=1}^{n} A_k\left(|G_i(j\omega)|\right) \cos\left(k\omega t + \beta_k\left(\varphi_i(\omega)\right)\right) \tag{3.23}$$

where the unknown variables in the amplitude $A_k\left(|G_i(j\omega)|\right)$ $(k = 0 \ldots n)$ and the phases $\beta_k\left(\varphi_i(\omega)\right)$ $(k = 1 \ldots n)$ are the parameters of linear element $G_i(j\omega)$ (i.e. the modulus gain $|G_i(j\omega)|$ and the phase $\varphi_i(\omega)$). Therefore, one immediately gets using (2.3), (3.2) and (3.23):

$$\begin{aligned} x(t) = {} & A_0\left(|G_i(j\omega)|\right) + \\ & \sum_{k=1}^{n} A_k\left(|G_i(j\omega)|\right) |G_o(jk\omega)| \cos\left(k\omega t + \beta_k\left(\varphi_i(\omega)\right) + \varphi_o(k\omega)\right) \end{aligned} \tag{3.24}$$

It readily seen that, the unknown parameters in the expression of undisturbed output signal are $\left(|G_i(j\omega)|, \varphi_i(\omega)\right)$ and $\left\{\left(|G_o(jk\omega)|, \varphi_o(k\omega)\right); k = 1 \ldots n\right\}$. In this respect, note that the inner signal *x*(*t*) is equivalent to sum of *n* sinusoidal signals and DC component. Specifically, having the spectrum of *x*(*t*), the equation (3.24) leads to a system of $(2n+1)$ equations and $(2n+2)$ unknowns, i.e. more unknowns than equations. This problem can be overcome by repeating the same experiment using the input signal (3.19) having the frequency 2ω, 3ω, ….

The difficulty that arises is that, the signal *x*(*t*) is not accessible to measurement. Fortunately, an estimate of *x*(*t*) can be established using the fact that this latter is periodic of the same period $T = 2\pi/\omega$ of control signal *u*(*t*). This property suggests the following estimator:

$$\hat{x}(t) = \frac{1}{L} \sum_{p=1}^{L} y(t + pT) \quad \text{for } t \in [0, T) \tag{3.25a}$$

$$\hat{x}(t + kT) = \hat{x}(t) \quad \text{for any integer } k \tag{3.25b}$$

where *L* is any integer preferably large. Indeed, one immediately gets from (2.4) and (3.25a-b) that:

$$\begin{aligned}\hat{x}(t) &= \frac{1}{L}\sum_{p=1}^{L} x(t+pT) + \frac{1}{L}\sum_{p=1}^{L}\xi(t+pT) \\ &= x(t) + \frac{1}{L}\sum_{p=1}^{L}\xi(t+pT)\end{aligned} \tag{3.26}$$

The periodic stationarity of $\xi(t)$ means that $E\left(\xi(t+pT)\right) = E(\xi(t))$, for any p and t. Then, the zero-mean ergodicity of $\xi(t)$ means that:

$$\frac{1}{L}\sum_{p=1}^{L}\xi(t+pT) \underset{L\to\infty}{\to} 0 \tag{3.27}$$

This implies that (using (3.26)-(3.27)):

$$\hat{x}(t) \underset{L\to\infty}{\to} x(t) \tag{3.28}$$

4. SIMULATION

Presently, the system (2.1)-(2.4) is characterized by the linear elements of transfer functions:

$$G_i(s) = \frac{0.1}{(0.4+s)(0.1+s)} \tag{4.1a}$$

$$G_o(s) = \frac{1}{(0.2+s)(0.5+s)} \tag{4.1b}$$

The curve of nonlinear block $f(.)$, used in simulation, is shown by Fig. 4.1. The noise signal $\xi(t)$ is a sequence of random numbers, with zero-mean and standard deviation $\sigma = 0.5$.

In the first stage, we apply to the input of system the sequence plotted in Fig. 4.2. The collected system output is illustrated in Fig. 4.3a. Then, using the estimator (3.9) or (3.13), the estimate values $\hat{f}(U_k)$ ($k = 1 \ldots N$) are also given by Fig. 4.3a. For convenience, a zoom of these results is given by Fig. 4.3b.

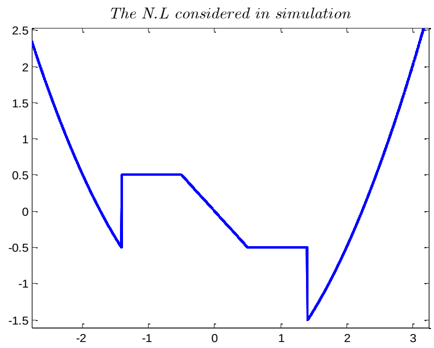

Fig. 4.1. Shape of the function *f*(.) considered in simulation

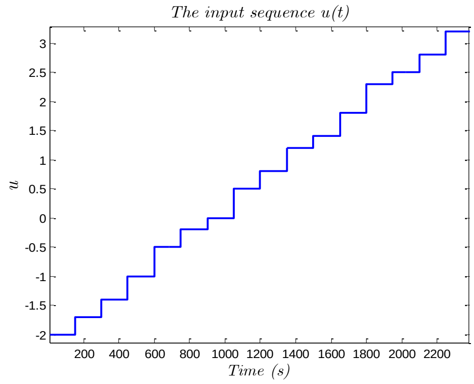

Fig. 4.2. The applied control sequence

Then, to compare the estimated and the true nonlinearities, Fig. 4.4 shows the set of points $\left\{\left(U_k, \hat{f}(U_k)\right); k=1\ldots16\right\}$ and the true function $f(.)$ (of rescaled nonlinear system using (3.1)-(3.2)). These results show that the estimated points $\left\{\left(U_k, \hat{f}(U_k)\right); k=1\ldots16\right\}$ are very close to their true values.

It follows that the function does not seem to be of hard type in the interval $\left[1 \quad 3\right]$. In this respect, the system can be excited with other inputs within this interval and comparing the interpolation of these points with a polynomial of degree *n*.

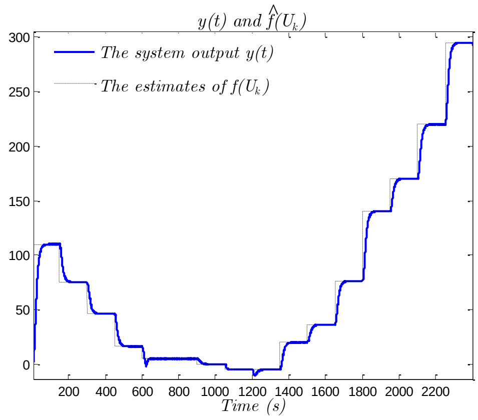

Fig. 4.3a. The system output signal and the estimate of $x(t)$

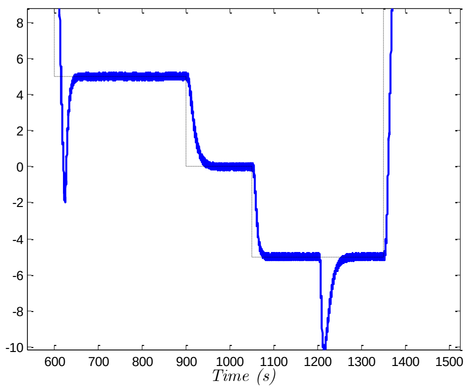

Fig. 4.3b. Zoom of the signal $y(t)$ and the estimate of $x(t)$

In the second stage, we apply to the system input the sine signal (3.16) where α is chosen such that $u(t)$ belongs to the interval $\left[1 \quad 3\right]$. Taking e.g. $\alpha = 2$, the resulting system output signal $y(t)$ for an amplitude $U = 1$ and a frequency $\omega = 0.02\,(rd\,/\,s)$ is illustrated by Fig. 4.5.

The measured signal $y(t)$ is collected on a sufficiently large interval. Then, the collected sample is used to generate the undisturbed output $\hat{x}(t)$ using (3.25a-b). The obtained estimate is plotted in Fig. 4.6 over one period of time.

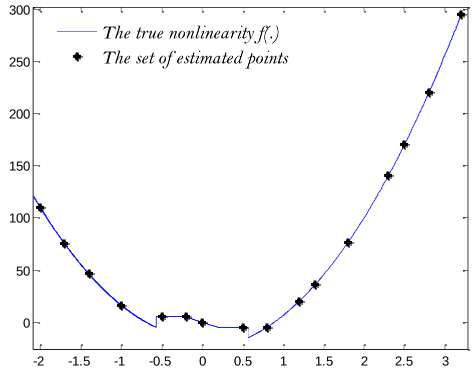

Fig. 4.4. Comparison between the true and estimated nonlinearities

Accordingly, it readily follows from the results of first stage and using an orthogonal polynomial decomposition of degree $n=3$ that, the obtained expression of $\hat{f}(.)$ within the interval $[1 \quad 3]$ is given as:

$$\hat{f}(v) = -0.003v^3 + 31.24v^2 + 0.02v - 24.8 \tag{4.2a}$$

While the expression of the true nonlinearity is as follows:

$$f(v) = 31.25v^2 - 25 \tag{4.2b}$$

Let us consider:

$$\hat{c}_3 = 0.003\,;\ \hat{c}_2 = 31.24\,;\ \hat{c}_1 = 0.02\,;\ \hat{c}_0 = -24.8\,; \tag{4.3}$$

denote the coefficients of the estimated nonlinearity. On the other hand, it readily follows from (3.18)-(3.23) and (4.2a) $(U=1)$ that, the inner signal $w(t)$ can be rewritten as:

$$w(t) = A_0\left(|G_i(j\omega)|\right) + \sum_{k=1}^{3} A_k\left(|G_i(j\omega)|\right)\cos\left(k\omega t + \beta_k\left(\varphi_i(\omega)\right)\right) \tag{4.4a}$$

where:

$$\begin{aligned}
&A_0\left(|G_i(j\omega)|\right) = \hat{c}_0 + \alpha\left(\hat{c}_1 + \alpha\hat{c}_2 + \alpha^2\hat{c}_3\right) + \frac{|G_i(j\omega)|^2}{2}\left(\hat{c}_2 + 3\alpha\hat{c}_3\right);\\
&A_1\left(|G_i(j\omega)|\right) = \left(\hat{c}_1 + 2\alpha\hat{c}_2 + 3\hat{c}_3\left(\alpha^2 + \frac{|G_i(j\omega)|^2}{4}\right)\right)|G_i(j\omega)|;\\
&A_2\left(|G_i(j\omega)|\right) = \frac{1}{2}\left(\hat{c}_2 + 3\alpha\hat{c}_3\right)|G_i(j\omega)|^2;\\
&A_3\left(|G_i(j\omega)|\right) = \frac{1}{4}\hat{c}_3|G_i(j\omega)|^3;
\end{aligned} \tag{4.4b}$$

and:

$$\beta_1\left(\varphi_i(\omega)\right) = \varphi_i(\omega);\ \ \beta_2\left(\varphi_i(\omega)\right) = 2\varphi_i(\omega);\ \ \beta_3\left(\varphi_i(\omega)\right) = 3\varphi_i(\omega); \tag{4.4c}$$

Then, one immediately gets using (3.2), (3.24) and (4.4a-c):

$$\begin{aligned} x(t) = {} & A_0\left(\left|G_i(j\omega)\right|\right) + A_1\left(\left|G_i(j\omega)\right|\right)\left|G_o(j\omega)\right|\cos\left(\omega t + \varphi_i(\omega) + \varphi_o(\omega)\right) \\ & + A_2\left(\left|G_i(j\omega)\right|\right)\left|G_o(j2\omega)\right|\cos\left(2\omega t + 2\varphi_i(\omega) + \varphi_o(2\omega)\right) \\ & + A_3\left(\left|G_i(j\omega)\right|\right)\left|G_o(j3\omega)\right|\cos\left(3\omega t + 3\varphi_i(\omega) + \varphi_o(3\omega)\right) \end{aligned} \tag{4.5}$$

where des parameters $A_k\left(\left|G_i(j\omega)\right|\right)$ ($k = 0 \ldots 3$) are given by (4.4b). Further, note that the inner signal $x(t)$ is periodic of the same period of $u(t)$ (i.e. $T = 2\pi/\omega$). Accordingly, $x(t)$ can be expanded in series Fourier, where its parameters (the amplitudes and arguments) can be easily generated using the estimate $\hat{x}(t)$. Then, it readily follows from (4.5) and $\hat{x}(t)$ that, 7 equations are provided. Specifically, using the estimate of DC component A_0, the value of harmonic amplitudes A_k ($k = 1 \ldots 3$), and the estimate of harmonic phases.

Accordingly, it readily seen from (4.4b) and (4.5) that, the modulus gain $\left|G_i(j\omega)\right|$ can be determined using the DC component of $\hat{x}(t)$. Then, the modulus gains $\left|G_o(jk\omega)\right|$, for $k = 1 \ldots 3$, can be immediately estimated using the amplitude of the first 3 harmonics (three unknowns and three equations). Furthermore, using the argument of the fundamental component and that of the first two harmonics, one has 4 unknowns (i.e. $\varphi_i(\omega)$, $\varphi_o(\omega)$, $\varphi_o(2\omega)$, and $\varphi_o(3\omega)$) and 3 equations (see (4.5)). We have more unknowns than equations. The nonlinear system is thus excited with the sine input (3.16) with the frequencies 2ω and 3ω. It readily follows that, these experiments generate 9 equations (from the phases) involving 9 unknowns ($\varphi_i(\omega)$, $\varphi_i(2\omega)$, $\varphi_i(3\omega)$, $\varphi_o(\omega)$, $\varphi_o(2\omega)$, $\varphi_o(3\omega)$, $\varphi_o(4\omega)$, $\varphi_o(6\omega)$ and $\varphi_o(9\omega)$). Finally, an estimate of $\varphi_i(l\omega)$ and $\varphi_o(kl\omega)$, for $l = 1 \ldots 3$ and $k = 1 \ldots 3$, can be obtained.

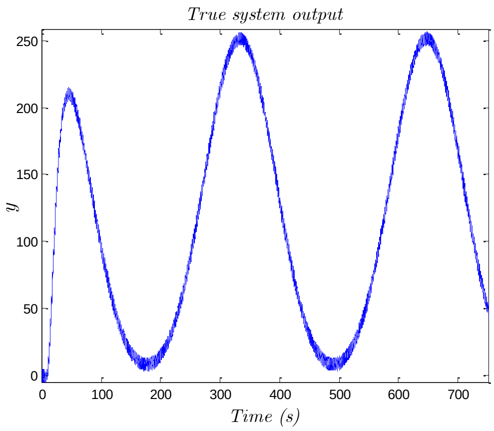

Fig. 4.5. The system output $y(t)$

Then, it follows from these three experiments that, for any frequency ω, the estimate of modulus gains $\left|G_i(jl\omega)\right|$ and $\left|G_o(jkl\omega)\right|$ ($l = 1 \ldots 3$ and $k = 1 \ldots 3$) can also be given.

On the other hand, in the case where the linear elements are parametric, the coefficients of transfer functions (numerators and denominators) can easily be determined using these estimates [6,12].

Among the advantages of the proposed method, the estimate of the gains $G_i(jl\omega)$ ($\left|G_i(jl\omega)\right|$ and $\varphi_i(l\omega)$) and $G_o(jkl\omega)$ ($\left|G_o(jkl\omega)\right|$ and $\varphi_o(kl\omega)$), for $l = 1 \ldots 3$ and $k = 1 \ldots 3$, can be obtained using only 3 experiments. Then, repeating this steps for $\omega = 0.04\,(rd/s)$ and $\omega = 0.06\,(rd/s)$, i.e.: Excite the system by the control (3.16) with these frequencies and estimate the corresponding $\hat{x}(t)$. This latter allows us to get the Fourier parameters. Finally, the gains $G_i(jl\omega)$ and $G_o(jkl\omega)$ ($l = 1 \ldots 3$ and $k = 1 \ldots 3$) can easily be determined using (4.4b) and (4.5). Table 4.1 gives the estimated numerical values. The obtained results show that the estimated parameter values are very close to their true values.

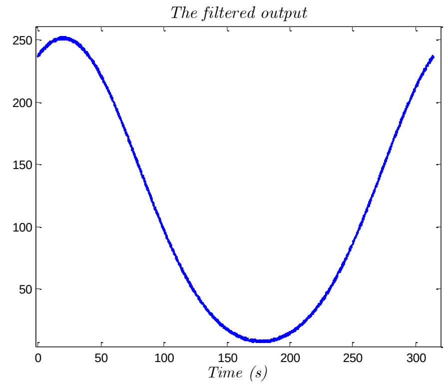

Fig. 4.6. The stimated signal $\hat{x}(t)$ over one period of time

Table 2.1. Estimate of linear elements and their true values

$\omega(rd/s)$	0.02	0.04	0.06
$(\lvert G_i(j\omega)\rvert, \varphi_i(\omega))$	(0.97, -0.25)	(0.92, -0.48)	(0.85, -0.69)
$(\lvert G_o(j\omega)\rvert, \varphi_o(\omega))$	(0.99, -0.14)	(0.98, -0.27)	(0.95, -0.41)
$(\lvert G_o(j2\omega)\rvert, \varphi_o(2\omega))$	(0.98, -0.27)	(0.91, -0.54)	(0.83, -0.77)
$(\lvert G_o(j3\omega)\rvert, \varphi_o(3\omega))$	(0.95, -0.41)	(0.83, -0.77)	(0.7, -1.08)
$(\lvert \hat{G}_i(j\omega)\rvert, \hat{\varphi}_i(\omega))$	(0.98, -0.24)	(0.9, -0.46)	(0.87, -0.67)
$(\lvert \hat{G}_o(j\omega)\rvert, \hat{\varphi}_o(\omega))$	(1.01, -0.15)	(0.96, -0.25)	(0.98, -0.39)
$(\lvert \hat{G}_o(j2\omega)\rvert, \hat{\varphi}_o(2\omega))$	(0.96, -0.25)	(0.93, -0.57)	(0.80, -0.81)
$(\lvert \hat{G}_o(j3\omega)\rvert, \hat{\varphi}_o(3\omega))$	(0.98, -0.39)	(0.80, -0.81)	(0.74, -1.04)

5. CONCLUSION

In this paper, an identification method of nonlinear systems is proposed. Presently, the nonlinear system can be described by Wiener-Hammerstein model. The nonlinear element is allowed to be discontinuous or of hard shape. It is interesting to point that, it is very complicate to decompose or approximate these types of nonlinearities using polynomial decomposition.

The estimation of system parameters is done using two stage. Firstly, the nonlinear block is determined using a simple sequence of constant controls. In the second stage, the linear elements parameters are estimated using sine signal.

The identification method also features the fact that the linear elements identification is made decoupled from the nonlinear element identification.

To the author's knowledge very few previous studies have been dealt with discontinuous or hard nonlinearity and nonparametric linear elements.

REFERENCES

[1] Bai, E.W., Cai, Z., Dudley-Javorosk, S., & Shields R.K. (2009). Identification of a modified Wiener-Hammerstein system and its application in electrically stimulated paralyzed skeletal muscle modeling. Automat. Contr., 45(3), 736-743.

[2] Benyassi, M. & Brouri, A. (2017). Identification of Nonlinear Systems Having Nonlinearities at Input and Output. European Conf. on Elec. Eng. and Comp. Sc. (EECS), Bern, 311-313.

[3] Bershad, N.J., Celka, P., & McLaughlin, S. (2001). Analysis of stochastic gradient identification of Wiener-Hammerstein systems for nonlinearities with Hermite polynomial expansions. IEEE Trans. Signal Process., 49, 1060-1071.

[4] Brouri, A. (2016). Frequency Identification Of Nonlinear Systems. LAP LAMBERT Academic Publishing, ISBN (978-3-659-94991-3).

[5] Brouri, A. (2016). Wiener-Hammerstein Models Identification. Int. Journal of Math. Mod. & Meth. in Applied Sc., 10, 244-250.

[6] Brouri, A. (2017). Frequency identification of Hammerstein-Wiener systems with Backlash input nonlinearity. W. Trans. on Syst. & Contr., 12, 82-94.

[7] Brouri, A. (2017). Identification of Nonlinear Systems. AIP Conference Proceeding, ICAMCS, Rome, 1836(1), https://doi.org/10.1063/1.4981971.

[8] Brouri, A., Chaoui, F.Z., Amdouri, O., & Giri, F. (2014). Frequency Identification of Hammerstein-Wiener Systems with Piecewise Affine Input Nonlinearity. 19th IFAC World Congress, Cape Town, 10030-10035, https://doi.org/10.3182/20140824-6-ZA-1003.00303.

[9] Brouri, A. & Giri, F. (2012). Identification d’un modèle de Wiener-Hammerstein comprenant une non-linéarité affine par morceaux. CIFA, Grenoble, France.

[10] Brouri, A., Giri, F., Ikhouane, F., Chaoui, F.Z., & Amdouri, O. (2014). Identification of Hammerstein-Wiener systems with Backlash input nonlinearity bordered by straight lines. 19th IFAC World Congress, Cape Town, 475-480, https://doi.org/10.3182/20140824-6-ZA-1003.00678.

[11] Brouri, A. & Kadi, L. (2018). Contribution on the Identification of Nonlinear Systems. CodIT'18 Conference, Thessaloniki, 605-610.

[12] Brouri, A., Kadi, L., & Slassi, S. (2017). Frequency identification of Hammerstein-Wiener systems with Backlash input nonlinearity. Int. J. of Control, Automation & Systems, 15(5), 2222-2232.

[13] Brouri, A., Kadi, L., & Slassi, S. (2017). Identification of Nonlinear Systems. European Conf. on Elec. Eng. and Comp. Sc. (EECS), Bern, 286-288.

[14] Brouri, A., Rabyi, T., & Ouannou, A. (2018). Identification of nonlinear systems with hard nonlinearity. CodIT'18 Conference, Thessaloniki, 506-511, https://ieeexplore.ieee.org/document/8394834/.

[15] Brouri, A. & Slassi, S. (2016). Identification of Nonlinear Systems Structured by Wiener-Hammerstein Model. Intern. J. of Elec. and Comp. Eng., 6(1), 167-176.

[16] Falck, T., Pelckmans, K., Suykens, J., & De Moor, B. (2009). Identification of Wiener-Hammerstein systems using LS-SVMs. 15th IFAC symposium on system identification, Saint-Malo, https://doi.org/10.3182/20090706-3-FR-2004.00136.

[17] Li, L. & Ren, X. (2017). Decomposition-based recursive least-squares parameter estimation algorithm for Wiener-Hammerstein systems with dead-zone nonlinearity. Inter. Jour. of Syst. Sc., 48(11), 2405-2414.

[18] Marconato, A. & Schoukens, J. (2009). Identification of Wiener-Hammerstein benchmark data by means of support vector machines. 15th IFAC symposium on system identification, Saint-Malo, https://doi.org/10.3182/20090706-3-FR-2004.00135.

[19] Marconato, A., Sjoberg, J., & Schoukens, J. (2012). Initialization of nonlinear state-space models applied to the Wiener-Hammerstein benchmark. Control Engineering Practice, 20, 1126-1132.

[20] Mu, B.Q. & Chen, H.F. (2016). Recursive identification of Wiener-Hammerstein systems. SIAM J. Contr. Opt., 50(5), 2621–2658.

[21] Pillonetto, G., Chiuso, A., & Nicolao, G.D. (2011). Prediction error identification of linear systems: a nonparametric Gaussian regression approach. Automatica, 47(2), 291-305.

[22] Pintelon, R., Guillaume, P., Rolain, Y., Schoukens, J., & Van Hamme, H. (1994). Parametric Identification of Transfer Functions in the Frequency Domain-A Survey. IEEE Trans. on Aut. Cont., 39(11), 2245-2260.

[23] Schoukens, M., Pintelon, R., & Rolain, Y. (2014). Identification of Wiener-Hammerstein systems by a nonparametric separation of the best linear approximation. Automatica, 50(2), 628-634.

[24] Sjöberg, J. & Schoukens, J. (2012). Initializing Wiener-Hammerstein models based on partitioning of the best linear approximation. Automatica, 48(2), 353-359.

[25] Sjöberg, J., Lauwers, L., & Schoukens, J. (2012). Identification of Wiener-Hammerstein models: Two algorithms based on the best split of a linear model applied to the SYSID'09 benchmark problem. Control Engineering Practice, 20, 1119-1125.

[26] Westwick, D.T. & Schoukens, J. (2012). Initial estimates of the linear subsystems of Wiener-Hammerstein models, Automatica, 48, 2931-2936.

[27] Wills, A. & Ninness, B. (2009). Estimation of generalised Wiener-Hammerstein systems. 15th IFAC symposium on system identification, Saint-Malo, 1104-1109, https://doi.org/10.3182/20090706-3-FR-2004.00183.

