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Interaction between Incumbent while Entrenched and Modern while Agile Companies at a Freezing Moment of Time

Jeffrey Yi-Lin Forrest [1*], Diane D. Galbraith[1], Yong Liu[2], Rafael Pla-Lopez[3]

[1)] School of Business, Slippery Rock University, Slippery Rock, PA 16057, U.S.A.
E-mail: Jeffrey.forrest@sru.edu; diane.galbraith@sru.edu
[2)] School of Business, Jiangnan University, Wuxi 214122, Jiangsu, China
E-mail: clly1985528@163.com
[3)] Department of Applied Mathematics, Universtat de Valencia, Valencia 46010, Spain
E-mail: Rafael.Pla@uv.es
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Abstract: This paper aims at designing a game-theoretical model – without any unrealistic assumptions – to study the competition between a company that is operating under an old paradigm that continuously exploits its established competitive advantage, and a modern, agile company that anticipates and adapts to market changes. By looking at the 2×2 competition of the companies, established are several results on how the incumbent while entrenched company loses its competition to the modern company. Then by including a customer into the interplay of the two companies, analyzed is the dynamics of competition between the two companies and why the old-fashioned company has to become modern by adopting the philosophy of transient competitive advantages. Specifically, this paper shows when the old-fashioned company could invest in raising the entry barrier in order to protect its customer base, and when such investment would be useless. Practically, this work provides useful guidelines for when firms that are incumbent in their markets should start to prepare to ride turbulent waves of a paradigm change, and what modern firms should constantly scan with respect to their environments for the next breakthroughs in order to design and to implement their next competitive advantages.

Keywords**:** competition, game, hybrid product, Nash equilibrium, technology, market invasion

1. INTRODUCTION

With the increasing globalization of the present world of business, many well-established companies have either disappeared or become irrelevant due to various internal or external reasons or a combination of both, as competitive forces reshape the strategies companies employ (Porter, 1979). When an organization is unable to forecast and/or adjust to changing trends or paradigm shifts, it will definitely exit the market soon as in the case of Kodak, Xerox, and Motorola's one-time dominance in the analog cellular telephone business (Barker, 1993). In today's world, clinging to established competitive advantages is no longer viable (Forrest & Tallapally, 2018). For a leadership to be futuristic and visionary, it generally requires confidence and narcissism (Navis & Volkan, 2016). Leadership is essential for the creation of transient competitive advantage in order to stay abreast of the speed of business. The staying power of a singular competitive advantage has been shortened substantially by technology and automation (Collins, 2001). An example of this is the rivalry between the cellular tech-giants,

[*] Corresponding author: Jeffrey.forrest@sru.edu

AT&T and Verizon. Even in the 1990's and the turn of the century, as one of the authors actually experienced, when one company launched a new product, service, phone, data or pricing plan, another company would surpass the competition in a matter of weeks or months. In other words, companies that cannot successfully ride the waves of transient advantages become victims of quickly shifting business landscapes (McGrath, 2013). That is because there are very few blue oceans, as explained by (Kim & Mauborgne, 2005), whereby there is little to no competition. This realization, for example, was deployed by Cirque du Soleil when they transformed the picture of a circus from animals to acrobatic, nimble human performers.

Considering the fact that in the modern world of business, some companies still hold on to the once-championed philosophy and practice – spend time to find a market niche, develop a particular advantage, and then reap in the profits by exploiting the occupied market share, while some other companies are preparing themselves by shaping their organizational cultures to face the challenge of transient competitive advantages, because for these companies to remain ahead of the fierce competition, they need to be super vigilant in their collective pursuit of the next trend or transient advantage (Chan & Chan, 2010). To this end, a natural question arises: How do companies of the former type actually lose their shirts in the competition with companies of the latter type? In other words, how can a company really tell when it is losing its market competition so that it needs to disengage itself from the current competitive advantage and adopt another competitive advantage in order to stay viable? Other than theoretical value, investigations of this question are extremely important in practice, because, for example, in the field of management a commonly accepted exercise is to implement 'best practices,' identified out of the successes of most noticeable companies (McGrath, 2013; Porter, 1979), without much theoretical reasoning. In other words, beyond its academic value (see explanations below), this work brings forward a way of thinking for decision-making managers and entrepreneurs in their daily operations.

Aiming at addressing this question, this paper considers markets as systems (Lin, 1987; 1999; Senge, 1990), and organizations also systems that are comprised of smaller component parts, they must be agile and adaptable to respond quickly to continuous market and technological changes (Johnson, 1998). In fact, Morgan (2006, p. 44) also treats organizations as systems that are "comprised of human resources, people who are organisms, open to their environment… (they) are open systems with interrelated subsystems… these organic subsystems continuously interact with their environment in a mutual state of dependence." In particular, this paper employs respectively the methods of 2×2 and 3×2 games to model the direct competition between two companies, named A and B, where Company A is of the old-fashioned type of sustainable advantages, while Company B the modern type of transient advantages. The appropriateness of summarizing a real-life scenario of business competition as that between only two companies is completely adequate in terms of the question this paper addresses. For example, when we look at a market occupied by n incumbent firms, for a natural number n, we can theoretically divide these incumbents into two theoretical firms: one focal firm of our interest, the other the aggregate of the other $n - 1$ firms. If the focal firm desires to occupy a leading position in its competition against of the other firms, it needs to compete itself way ahead of the rest, which in practice is seen as one loosely identified entity.

In particular, based on the 2×2 game theoretical modeling, among others, the following results are shown: (1) If expansion is expected for Company B to generate profits, which are more than the cost for Company A to take defensive actions, then Company A would move first and take defensive actions against Company B although the result is uncertain; and (2) If Company B's decision on whether or not to design and produce its hybrid product is based on the existence of an expanding market beyond that of Company A, then Company B will definitively introduce its hybrid product.

Because it is consumers that ultimately determine the fortune and fate of companies, in the 3×2 game theoretical modeling, we include a customer as the third player who interacts with the two companies. Based on this model, among others, we develop the following results: (1) If Company B's revenue is greater than its cost and its revenue from outside the market base of Company A is smaller than its cost, and Company A does not take any defensive actions to protect itself, then the probability for the customer to purchase Company A's product is inversely proportional to Company B's activities of designing and producing its hybrid product; and when the revenue Company B earns from the market base of Company A is equal to its fixed cost of designing and producing its hybrid product minus the revenue generated from beyond the customer base of Company A, then the probability for the customer to purchase Company B's hybrid product is equal to 1. (2) If Company B's revenue is greater than its cost and its revenue from outside the market base of Company A is smaller than its cost, and Company A does take defensive actions to protect itself, then the higher the difference between the cost for Company B to design and produce its hybrid product and the revenue generated beyond the customer base of Company A, the lower the probability for Company A's customer to purchase Company B's hybrid. (3) If the profits of Company B's expansion come mainly from Company A's market share, then Company A can successfully delay the deterioration of its territory as long as less than 2/3 of its maximum expected revenue is taken by Company B.

The rest of the paper is organized as follows. Section 2 reviews the relevant literatures. Section 3 details the game-theoretic models to be employed in the rest of this work. Section 4 derives the results by looking at a two-player game and a three-player game, respectively. Section 5 concludes this presentation.

2. LITERATURE REVIEW

Although what is studied in this paper is somewhat relevant to the vast literatures on market entry (see, for example, (Zachary, et al., 2015) and references listed there) and R & D races to develop new products, which might be used to create new market spaces and reduce market scope from competitors (see, for example, (Jindal, et al., 2016)), the main focus here is on the interaction between firms that practice the philosophy of sustainable competitive advantages and those that ride the wave of transient advantages.

In the literature, a lot of attention has been given to the question of how to develop a competitive advantage. For example, Purkayastha and Sharma (2016) inductively analyze three firms that develop a competitive advantage by shaping their business model. Koller (2016) introduces the concept of adaptive advantage and its implementation in an organization. Bashir and Verma (2017) highlight how business model innovation can serve as a competitive advantage. Christensen, Suárez and Utterback (1998) confirm that firms that target new market segments with an architectural innovation tend to be successful. And, Chan and Chan (2010) reveal that in the fast-changing fashion market, being flexible and adaptive is a key to survival.

In terms of adapting organizational cultures and strategies to the constantly changing environment, Tushman and O'Reilly III (1996) conclude that managers must create an ambidextrous organization that is capable of simultaneously pursuing both incremental and discontinuous innovation. In terms of how to achieve and sustain market-driven orientation, Day (1994) finds that the emerging capabilities approach to strategic management, coupled with total quality management, offers a rich array of ways to design change programs that will enhance a market orientation with increased capabilities in market sensing and customer linking. In terms of transient competitive advantage readiness, Kaharuddin, Handaru, Sardan, and Mohammed (2017) use hotels, cafes, and fashion retail industry in Bandung, Indonesia, to study the relevant measurement. In terms of competing in a world of transient advantages, Leavy (2016) considers what dynamic capabilities will be needed. In terms of how important innovation is, Dobni and Klassen (2015) look at whether or not the U.S.A., the world's largest economy, is still a leading nation in innovation. In terms of using patents as a competitive advantage, Harrigan and DiGuardo (2016) empirically show that the duration of patent-based advantages appears to wane with time in the highly-dynamic U.S. communications-services industry during 1998 – 2012, when technological changes occurred rapidly.

In terms of the long-standing game-theoretical studies of duopoly competition, the literature is quite large. For example, Puu (1991) investigates the problem of chaos in duopoly pricing, Agliari and Puu (2002) look at a similar problem by using a Cournot model with a bounded inverse demand function. Zhang et al (2009) report the dynamics of Bertrand model with bounded rational players. Dubiel-teleszyński (2011) examines a Bertrand duopoly with heterogeneous (bounded rational and adaptive) players engaged in manufacturing differentiated products. Fan et al (2012) study the dynamics of a Cournot duopoly with one player being a bounded rational player and the competitor following naive expectations. Wang and Ma (2013) employ the same combination of expectations to study the dynamics of Cournot-Bertrand game. Ma and Pu (2013) employ bounded rationality as the expectation of firms to investigate the dynamics of a Cournot-Bertrand duopoly model. Ma and Xie (2016) investigate the dynamic pricing game of the duopoly air conditioner market with disturbances in demand. Zhu et al (2014) investigate the stability of Nash equilibrium in a dynamic nonlinear mixed Cournot model, where the competing firms are a semi-public firm that is interested in maximizing both social welfare and profit, and a private firm that is only interested in profit maximization. Zhang and Ma (2016) consider two distinct pricing policies in a dual-channel supply chain with a fairly caring retailer by examining the complex nonlinear dynamics. Huang et al (2016) examine the influence of parameters on the stability of two manufacturers dealing with complimentary products by considering three games based on the channel power of the manufacturers. Pecora and Sodini (2018) look at a Cournot duopoly in a continuous time framework by considering a system of differential equations, where the competitors are assumed to be heterogeneous in determining output decision.

By employing nonlinear dynamics, Sice et al (2000) assess and simulate interactions in duopoly competition for quality between two consumer durable products. These authors find that when the speed of adaptation to customer demand reaches a certain value, a Hopf bifurcation occurs and the duopoly interaction converges into a limit cycle, and that further increasing the speed leads to quasi-periodicity and chaos. Lotfi and Sarkar (2016) study price competition in a duopoly with an arbitrary number of buyers with specific setting that captures a secondary spectrum access network, a non-neutral Internet, or a micro-grid network in which unused spectrum bands, resources of ISPs, and excess power units constitute the respective commodities of sale. They identify a set of necessary and sufficient properties for the Nash Equilibrium. By considering market forces affecting switching costs, Villas-Boas (2015) investigates the effects of firms being forward-looking, consumers being forward-looking, degree of stability of consumer preferences, and market time horizon, respectively. By analyzing a game theoretic model of duopoly competition, Xin and Choudhary (2019) show, among other results, that IT implementation can fail and such failure creates a possibility of cost-based differentiation and mitigates competition, while a higher probability of implementation failure can lead to lower investment risks and higher expected profits. Through considering both competitive interactions in general, and the potential price effects of a merger between the two largest players in particular by using a unique route-level price data set of the recently deregulated German interurban bus industry, Dürr et al (2016) find that route-level average prices depend not only on the number of competitors, but also on the composition of firms operating on a particular route. As motivated by real-life practices, Huang et al (2019) investigate in their duopoly competition setting a government's optimal subsidies for energy-efficient products in a market with two competing firms that differ in their production costs and compete on their product prices and product energy efficiencies. These authors find that very low subsidy might make the low-cost firm produce an uncertified product while the high-cost firm a certified product, and that minimizing the average energy consumption helps to sustain product variety/duopoly outcome.

In particular, the differentiated duopoly models, such as those mentioned above or in (Singh & Vives, 1984; Zanchettin, 2006), could be potentially useful for our study here, considering the fact that this work addresses issues related to two firms selling differentiated products to a representative consumer. However, none of these prior models actually fits our needs due to the fact that these models violate some or all of the following problems. First, these models start with the optimization of the representative customer's utility function. Based on the knowledge of behavioral economics, this very practice is extremely questionable if we aim at producing practically applicable results (Minton & Kahle, 2013). In other words, customers are predictably irrational in ways that defy economic theory developed on optimizing utility functions. Second, the representative customer's utility function used in these models is of a particularly chosen type in order to produce the desired outcomes. For example, to produce theoretically 'nice' results, particular quadratic utility functions are used in order to generate linear demand functions. In fact, when studying real-life events and processes, linearity only stands for (very) local, (extremely) special cases with nonlinearity being the norm of interactions in the universe (Lin & OuYang, 2010). Third, the cost function is assumed, see, for example, (Zanchettin, 2006), to be linear in the quantity of the product produced with constant marginal cost. This is simply untrue in real life especially in the economy of information age, for the reason why please go to Proposition A.1 in Appendix. And fourth, either the price competition or the quantity competition is assumed to be linear except a few recent works. As a matter of fact, as soon as such basic setup is taken to be linear, all the practically useful conclusions of realistic nonlinear interactions are assumed away. For details, see (Liu, 2013; Forrest & OuYang, 2010).

Considering all the listed and unlisted issues with the previously established models for duopoly competition, for the purpose of producing practically useful conclusions, we have no choice but develop a different and straightforward game-theoretical model based on a few very realistic and intuitive assumptions. And instead of optimizing the representative customer's utility function, we maximize companies' profits, which is most likely the practice of relevant companies when they try to decide their courses of action.

So, comparing with the literature, such as those listed above, including particularly (Christensen, Suárez & Utterback, 1998; Porter, 1979; McGrath, 2013), this work enriches the relevant knowledge at the height of theoretical abstraction with a much wider range of real-life applicability. And beyond addressing the question posed at the previous section, the general question this paper attempts to address is to show that other than inductive reasoning, deductive reasoning should be employed to produce scientifically sound theories and conclusions. Here, inductive reasoning is the exclusively used logic of thinking in the literature in areas related to this work, where anecdotes and data mining, as well as models developed on particular and/or unrealistic assumptions, are employed to draw general conclusions. However, such conclusions are known in science to be generally not reliable. To this end, this paper establishes a theoretical model for how market competition should develop and evolve by using game theory without imposing any unrealistic condition and then provides scientifically sound managerial suggestions. As a side contribution, this work is the first to show that the literature of duopoly competition urgently needs to be enriched through developing new models that are beyond those established under unrealistic conditions – such as linear price and quantity interactions, linear demand and cost functions, etc. – and processes – such as optimizing customers' utilities.

Another important note, appropriate at this junction, is the question of whether or not the approach and setup employed in this paper are relevant to today's world of business. In particular, this paper looks at two competing companies that produce products or services that are similar but can be differentiated from each other. It is the differences of the products that allow the companies to compete for a similar customer base. Examples of such setup are seemingly endless in the business world. For example, if a person needs a haircut, there are likely several salons in his/her area that could satisfy the need. They are all quite similar in terms of service and quality. So, each of the salons' business goals is to entice the person to use its business instead of those of its competitors. Now, the situation of competition can be treated both theoretically and practically as one salon competes against the aggregate of the rest, which falls within the setup of our model. In this example, hair salons can be seen as dummy entities that can be specified as health insurance companies, video streaming services, etc.

3. GAME-THEROETICAL MODELING

This section describes the base game-theoretic models on which we derive our conclusions.

3.1. Competition without involving customers

Consider two companies, named A and B, under two different business models. Assume that A is a successful, old fashioned company that enjoys a full line of well received products based on its time-honored sustainable competitive advantage developed over a long period of time. For the sake of convenience of communication, let us assume that Company A produces only one successful product. On the other hand, although company B also enjoys its traditional success with its unique product(s), as before, assume this company produces only one product, its leadership adopts the philosophy of transient competitive advantages. That is, Company B is market- and technology-driven (Day, 1994) and fully ready to embrace next fleeting advantage (Kaharuddin, et al., 2017). So, the management of Company B considers the question of whether or not to design and produce a new product that improves the functionality, user friendliness, and the features of the two products produced respectively by A and B. That is, the new product will be better and more advanced than any of the existing products of A and B. If it decides "yes, it will design and produce the imagined hybrid product," then the production of the company's old product will be terminated, and all its loyal customers will move on to enjoy the hybrid (better) product at the same price as that of the retired product. At the same time, a portion of Company A' customers will switch to buy this new product, which, in this case, would be considered cheaper and better than the existing product of A due to the enhanced functionality, user friendliness, and combined features. (Note: Our setup here is different from those studies touched on in the afore-mentioned literature of the previous section on how firms profit from short-term and long-term opportunities. In particular, the time variable does not enter our setup.)

So, we have the two-player game in Table 3.1, where S_A stands for the total sales revenue of Company A without Company B's invasion of A's territory, S_B the sales revenue of Company B generated from the market share of Company A by producing the imagined hybrid product, *C* the cost of Company A for it to take actions, such as raising the barrier of entry, in order to deter the invasion of Company B, R_0 (respectively, R_1) the cost of Company B, including risk and opportunity costs, for designing and producing the hybrid product without (respectively, with) Company A taking defensive actions.

Table 3.1 The payoffs of the 2 × 2 pure strategy game

		Company B	
		Design/produce	Don't design/produce
Company A	Take defense	$S_A - C - S_B, S_B - R_1$	$S_A - C$, 0
	Don't take defense	$S_A - S_B, S_B - R_0$	S_A, 0

In the setup of this game, the assumptions need to be read with generality in order to make the results of this work practically useful. In particular, what is assumed represents a momentary 'camera' snapshot of the interaction between Companies A and B with the time variable frozen. In other words, if time is considered, with the given assumptions Company A could still be monitoring what its competitor B is doing and trying to expand its existing advantage by identifying and investing in resources and capabilities that strengthen its current market position and potentially open up future opportunities. It could still be systematically using these strategic resources and capabilities to respond to the competitive pressures created by Company B. That is, what is assumed does not mean that the design and production of the product of A are static and never changing over time; instead, it means that at the particular time moment of the 'camera' snapshot of the model Company A's identification of what resources and capabilities to invest in is hindered by its existing culture, philosophy and value system. For example, historically the rivalry between Montgomery Ward and Sears had been in a game situation as described here at almost any chosen moment of time since the late 1800s. When Sears introduced hybrid methods of sales and innovative methods of managerial operations, Montgomery Ward monitored and analyzed what Sears was doing and planning on doing, and decided to develop itself and strengthen and improve its competitive advantages in a totally different direction. As a matter of fact, the very idea beyond what Sears was doing came from one of the senior leaders of Montgomery Ward and was rejected as not an acceptable strategy by the company's leadership (Sobel, 1999). Another good example is the rivalry between the Eastman Kodak and Fuji Films. The former surely identified and invested in resources and capabilities while monitored and took actions against what Fuji Films was doing in order to fence off the competition of the latter. However, its culture, philosophy and value system led the Eastman Kodak to a wrong direction (McGrath, 2013). On the other hand, by the philosophy of transient competitive advantages, we mean that the leadership of Company B, as well as the company's culture and value system, is more in sync with the technological changes and desires of consumers than A so that it knows how to design its hybrid product to excite consumers. A good example that illustrates the point here is the evolving designs and productions of Apple and Lenovo laptop computers – one continuously improves its functionality and user friendliness, such as the appearance, while the other keeps its look the same. What is just discussed here simply means that when a company decides to design and produce a hybrid product, in real-life there are still many different ways to do it. In general, when a company is in sync with the technological changes and desires of consumers, its hybrid product will more likely excite the market than the hybrid products of other companies. Once again, this end is very well demonstrated by the case of Kodak vs Fuji Photo.

A second point of notice regarding this very simply game model is that Company A, although it is described as one firm, really stands for the aggregate of all incumbent firms of the marketplace that is in a state of mutual forbearance. In other words, incumbent firms mitigate rivalry by dividing markets in proportion to firm strength (Bernheim & Whinston, 1990). They cede dominance to their stronger competitors in those market segments where they are less efficient, while in exchange the latter do the same in segments where the former are more efficient (Li & Greenwood, 2004). The firms' codependence gradually motivates them to de-escalate rivalry (Yu & Cannella, 2012). Eventually, the rates of entry and exit in the market decrease (Fuentelsaz & Gómez, 2006), and interfirm hostility declines (Haveman &

Nonnemaker, 2000). On the other hand, Company B also represents the aggregate of all firms, either incumbent or not, which always think about designing and introducing newer and better products or services.

A third point of notice is that it might seem 'logical' that if the additional profits of B come from the loss of A's market share, then Company A has an incentive to spend an amount equal to this potential loss to create entry barriers; so in equilibrium A would always spend an amount equal to S_B to eliminate any incentive for B to introduce a new hybrid product. As a matter of fact, this seemingly logical feeling is incorrect. First, in practice the magnitude of S_B cannot be estimated in advance or at the chosen moment of time the model is developed and is changing over time depending on how much the hybrid product of B excites the market. Second, although S_B could potentially turn out to be zero or very close to zero, the theorem established in (Forrest, Amatucci & Markman, 2017) shows that Company B still has to introduce its hybrid product to just maintain its current customer base. Because of these reasons, the amount C Company A spends on building its entry barriers is not a function in S_B, which is unknown at the moment of time when the model – a snapshot of the interaction between A and B – is fastened, and fluctuates over time. As a matter of fact, a theorem, established by Forrest, Buttermore and Wajda (2017), shows that S_B (less cost) could potentially be greater than S_A (less cost). So, if the afore-mentioned 'logic' holds true, then Company A has to simply close its door down because it has to spend all of its revenue and some additional capital on building the entry barrier.

3.2. Competition involving customers

In real life, no matter how Companies A and B compete with each other, the ultimate fortune and fate of the companies are really determined by the consumers. So, in this second situation, let us include a fictitious customer in the interplay of the competition of the companies. In this case, Company A takes an initiative to offer a product in the market, and Company B correspondingly provides its improved hybrid product, while the customer takes the final action of either to purchase or not to purchase. The customer decides which product to buy and significantly influences the operations of these companies in the market. As a practitioner of the philosophy of transient competitive advantages, Company B observes the initiatives of Company A and then acts accordingly, while as a believer of sustainable competitive advantages, Company A continues to identify and invest in its set of resources and capabilities that will not only maintain its current market position but also extrapolate it into the future by exploring how these resource and capabilities could be bettered in order to respond to competitive pressures of the market. In other words, Company B follows the vibration of the market and the changing desires of the consumers by either rejuvenating its existing advantages or establishing new ones through absorbing those of its competitors, while Company A continues its improvement and expansion of its proven advantages, believing that continuously improvement will strengthen and expand its market success. (At this junction, Montgomery Ward and Kodak were good examples of Company A described here.) So, we assume that although the customer is from the customer base of Company A, she would try the new product designed and produced by Company B if such a product is available. In other words, initiatives of Company A are closely monitored and followed by Company B, while the customer follows the actions of both companies and would try out the new product, as exactly what happens in real life. So, Company A has to increase the consumer confidence on its product while deciding on whether or not to take defensive actions to deter Company B's possible invasion of its territory. That is, we have established the following game theory modeling:

Company A chooses either to take defensive actions (with cost C) or not to take any defensive action (with cost 0); Company B elects between design/produce hybrid product (with fixed cost D) and not design/produce the imagined hybrid product (with cost 0). If Company

A takes defensive actions to deter Company B's following, assume that Company B will spend additional αC to design/produce its hybrid product, where $0 < \alpha < 1$. This end is due to the fact that to follow up with Company A's defensive actions, Company B has to push its organization's knowledge envelope outward in order to incorporate unknown technologies in its effort of introducing the imagined hybrid product (Harrigan & DiGuardo, 2016).

Ultimately the customer decides to purchase or no to purchase. If she purchases Company A's product, her utility is $(-u)$, because after purchasing the product she feels being exploited by a product that is not as innovative as what the market is expected to provide. In other words, instead of felling happy at least slightly from the consumption, she actually feels bad from the purchase. Such situations occur in real life quite frequently and almost surely when the market is controlled by a monopolist. If she purchases Company B's hybrid product, her utility is u, because Company B's hybrid innovatively reflects the features of both Company A's product and the old product of Company B and more. When the customer decides to purchase, if there is no hybrid product available in the market, Company A earns revenue S_A. In this case, although the customer does not feel good from the purchase, she does not really have a choice. However, if there is hybrid product in the market, then Company B grabs revenue S_B from Company A by selling its hybrid to the customer. So, it follows that $S_A \geq S_B$. When the customer decides not to purchase, the revenues of Company A and B will be both 0.

Table 3.2 Payoffs of customer and Company B when Company A takes defense

		Company B	
		Design/produce (p)	Don't design/produce ($1 - p$)
Customer	Buy (τ)	u, $[(S_B + S_{B^-}) - D - \alpha C]$	$-u$, 0
	Don't buy ($1 - \tau$)	0, $[(S_{B^-}) - D - \alpha C]$	0, 0

Table 3.3 Payoffs of customer and Company B when Company A does not take any defense

		Company B	
		Design/produce (q)	Don't design/produce ($1 - q$)
Customer	Buy (ε)	u, $[(S_B + S_{B^-}) - D]$	$-u$, 0
	Don't buy ($1 - \varepsilon$)	0, $[(S_{B^-}) - D]$	0, 0

So, the payoffs of Company B and the customer are respectively given in Table 3.2 for the case when Company A takes defensive actions and in Table 3.3 for the case when Company A does not take any defensive action, where S_{B^-} represents the revenue Company B generates from the market of the hybrid product beyond the market share of Company A and its expected revenue from its old product.

As the conclusion of this section on game-theoretic modeling, we like to see to what practical scenarios this work can be potentially employed in order to produce tangible economic benefits. Indeed, our modeling only focuses theoretically on the interaction between two companies A and B, characterized by that A's success is based on its time-honored sustainable competitive advantage developed over a long period of time, while B is market-, technology-driven, and embraces next fleeting advantages. In practically applications, the results of this work can be beautifully applied to analyzing the strategic management and planning of any focal firm that competes against a host of other firms within a market, as long as the focal firm desires to introduce its hybrid product that is better than all the substitutable but differentiated products offered by the other firms. In particular, this focal firm will be seen as Company B in our game-theoretic models, while the aggregate of all other firms as Company A.

4. RESULTS

In this section, we look at the competition of Companies A and B in two cases as described in the previous section: (i) two-player competitions without involving customers; and (ii) three-player competition involving a fictitious customer.

4.1. Two-player competitions

Based on the setup given in the previous section without involving any customer, we have the following implications. (1) For both companies, each unit of their product produced is successfully sold. (2) When the sales revenue of Company B satisfies $S_B > 0$, the sales revenue of Company A is $S_A - S_B$. That is, S_B stands for the portion of sales revenue Company B takes away from Company A, assuming that that is the total increased sale of Company B. (3) When Company A takes actions to deter Company B from invading its territory, the total cost for Company B to design and produce the hybrid product goes higher. That is, $R_1 > R_0$. In the rest of this paper, we assume that all players establish their best responses by playing the Nash equilibrium through pure self-analyses.

In the following, let us analyze this game in different cases.

Case 1: $S_B \leq R_0$, that is, the increased sales revenue of Company B is less than the sum of the cost of designing and producing the hybrid product and the risk and opportunity cost. So, it is logical that Company B does not pursue after the strategy of the hybrid product, a losing proposition. And, Company A does not need to take any action to deter Company B from invading its territory. Indeed, in this case, (don’t take defense, don’t design/product) is the Nash equilibrium, although in real life such a situation does not really exist. It is because at least some of the features of Company A’s product can be combined with those of Company B’s product without breaking the budget by adopting appropriate technologies.

Case 2: $R_0 < S_B < R_1$ and $S_B < C$. So, (Don’t take defense, Design/produce) is the Nash equilibrium. In other words, if Company A takes defensive actions against the potential entry of Company B into its territory, A has to spend more than its loss in terms of revenue. Consequently, due to the high costs, Company A does not take any defensive action; and Company B will enter the territory of Company A.

Next, if $R_0 < S_B < R_1$ and $S_B > C$, then Company A would decide which strategy to use to maximize its revenue, assuming this company chooses its strategy first before Company B. In this case, A will do better by taking defense and by expelling B from the market, because the strategy profile (Take defense, Don’t design/produce) will give A the maximum revenue. That is, in the subgame for A to take defensive actions, (Take defense, Don’t design/produce) is the Nash equilibrium.

Table 4.1 A 2×2 mixed strategy game

		Company B	
		Design/produce	Don’t design/produce
Company A	Take defense (p)	$S_A - C - S_B, S_B - R_1$	$S_A - C, 0$
	Don’t take defense ($1 - p$)	$S_A - S_B, S_B - R_0$	$S_A, 0$

To make the decision making process more dynamic, instead of using pure strategies, let us assume that Company A assigns probability p for the event of itself taking defensive actions. It is because at the time moment when the model is established, Company B does not really know whether or not Company A will take defensive actions. So, we have the payoff matrix in Table 4.1. Analyzing this game leads to the following practically useful results:

Proposition 4.1:

Assume that the revenue of Company B from the territory of A is greater than its costs when A does not take defense, while less than the costs when A does take defense, and greater than the defense costs of A and that all other conditions are held constant. Then the probability for Company A to take defensive actions and the sales revenue of Company B generated from the market share of Company A by producing its hybrid product are directionally proportional to each other.

What this result means practically is that the more Company A expects to lose its revenue, the higher probability Company A would take defensive actions against Company B. And on the other hand, the higher probability Company A would take defensive actions, the more revenue Company B is expected to earn from the share of Company A.

Proposition 4.2:

Assume that the revenue of Company B from the territory of A is greater than its costs when A does not take defense, while less than the costs when A does take defense, and greater than the defense costs of A and that all other conditions are held constant. Then the probability for Company A to take defensive actions and the cost for Company B to design and produce its hybrid product with Company A taking defensive actions are inversely proportional to each other.

Proof of Propositions 2.1 and 2.2. From Table 4.1, for Company B, its indifference condition is

$$p(S_B - R_1) + (1-p)(S_B - R_0) = 0,$$

from which we have

$$p = \frac{S_B - R_0}{R_1 - R_0},$$

where $R_1 > S_B > R_0$ is assumed for the analysis here. And when $R_1 > 0$, R_0 will be a nonexistent value. In this case, we assume $R_0 = 0$. □

In practice, Proposition 4.2 says that the higher the cost for Company B to design and produce its hybrid product, the smaller probability for Company A to take any defensive actions. And on the other hand, the smaller probability for Company A to take any defensive actions, the higher cost will be for Company B to successfully design and produce its hybrid product. The defensive actions Company A could take might include ways to tighten the control of its sales network, additional advertisement campaigns, etc. So, the cost for Company A to design and produce and especially sales of its product will increase. As a result, the chance for Company B to move ahead to design and produce its imagined hybrid product will be lower. However, a lower degree for Company A to take defensive actions against the potential invasion of Company B, the lower level of costs will be for Company B to design and produce its hybrid product. Therefore, each initiative Company A takes to minimize the chance of potential invasion will create a situation with increased costs for Company B to crack the entry barrier and to improve its design of its hybrid product. Hence, in practice, Company A has to continuously improve its product in order to retain and even grow its market share. Other the other hand, if Company A does not implement strong barriers for entry, then various hybrid products could more easily appear in the marketplace and will sooner or later push Company A out of the competition (This result follows from Corollary 1 in (Forrest & Anderson, 2017), which says that: When the competition of a market grows with an increasing number of firms entering the market, the base of loyal customers for each incumbent firm will gradually diminish). More specifically, when the probability for Company A to take defensive actions is $p = S_B/R_1$, from the assumption that $R_1 > S_B$ and $S_B > C$, it means that A will take defense and B will not design and produce its imagined hybrid product. In other words, Company B will have to consider its design and production of a different hybrid product with the output of another company. In such a case, the expected revenue of Company A will be maximized at $S_A - pC$.

Next, let us consider the case that when deciding on designing and producing its hybrid product, other than the possibility of taking a portion of Company A's revenue, Company B also knows the fact that its new product will appeal to other customers who were not within the individual customer bases of Company A and B. For example, in 1979 although Tom Whiteley, the head of Kodak's Emulsion Research Division, observed presentations of overhead projector slides generated on Xerox Alto, an early version of a personal computer, he somehow did not see the future of the new technology. Instead, he was very much engulfed in how well-entrenched the Kodak technology was. So, Kodak continued to exploit and strengthen its long-standing advantages in film, while Fuji Photo moved into a future without film (McGrath, 2013). Here, Fuji could be identified with our Company B – other than potentially taking market shares of Company A (Kodak), it also competes within a new market where Company A is absent.

To study how the incumbent firms in this new market would raise entry barriers to keep new comers, especially Company B in our case, from entering, assume that the market the imagined hybrid product will enter competitively is occupied by m incumbent firms, m = 1, 2, … These firms provide customers with mutually substitutable products; and each of the firms enjoys the backing of its loyal customers who only purchase the product of their firms provided that the price is no more than their reservation value, which is set to 1. Assume that these firms use the strategy of raising the entry barriers for Company B by competing over those customers who switch from the product of one firm to that of another firm with adjustable prices. To reflect the fact that these incumbent firms work together to prevent Company B from entering, assume that these firms are well aware of the pricing strategies of each other, and that these firms have established their best responses by playing the Nash equilibrium through pure self-analyses. Then, we have the following result, motivated by Forrest, Zhao and Shao (2018).

Proposition 4.3:

In the Nash equilibrium, the following two statements are equivalent:

1) *Company B can profitably enter the pre-described market, as a competitor of the incumbent firms; and*
2) *The size of the market segment of switchers is greater than zero.*

Proof. Let α stand for the portion of the market of the loyal customers of the m firms and $\beta = 1 - \alpha$ be the market segment of switchers, where $0 \leq \alpha, \beta \leq 1$.

For the sake of convenience of communication, assume that the constant marginal costs of the incumbent firms and Company B are set to zero without loss of generality – so in the rest of this proof, revenues are the same as profits. And consider the aggregate of the incumbent firms as one firm, because these m firms are in a state of mutual forbearance (Bernheim & Whinston, 1990). So, the market share of this aggregate firm is α so that $\beta = 1 - \alpha$ represents the market size of switchers who base their purchase decision on which price is lower.

(⇒) Suppose that Company B enters profitably into the oligopoly market of m firms. Then, the size of the market switchers or customer surplus must satisfy $\beta = 1 - \alpha > 0$.

(⇐) Assume that the customer surplus satisfies $\beta = 1 - \alpha > 0$. Let α_0 be a real number satisfying $\beta = 1 - \alpha > \alpha_0 > 0$ and $\alpha = \ell\alpha_0$, where ℓ is a large natural number, indicating that the market has been largely taken by the incumbent firms.

Imagine that the aggregate firm is divided into ℓ many identical "firms", named i, $i = 1, 2, \ldots, \ell$, each of which provides customers with identical products and enjoys the market share $\alpha_0 = \alpha/\ell$ of loyal customers. These imaginary firms compete over the switchers with adjustable prices. Because these imaginary firms are really equal partitions of the same aggregate firm, they have the same constant marginal cost, which is set to zero without loss of generality, the managements of these firms are fully aware of the pricing strategies used by all the firms (because the firms are managed by the same administrative unit), and they establish their best, identical responses by playing the Nash equilibrium through their unified self-analyses.

These ℓ imaginary firms do not have any symmetric pure strategy Nash equilibrium. Because the reservation price of the loyal customers is set to be 1, all the prices (or pure strategies) are values in the interval $[0,1]$. (For the setup here, there is no need to consider asymmetric pure strategy Nash equilibrium, because all these imaginary firms take identical actions). In fact, for any symmetric pure strategy portfolio $(x_1, x_2, \ldots, x_\ell)$, where $x_i = x_j$, for $i, j = 1, 2, \ldots, \ell$, a randomly chosen Firm j ($\in \{1, 2, \ldots, \ell\}$) can slightly lower its price from x_j to x_j' to produce additional profits for all the firms as long as $x_j'\beta > (x_j - x_j')\alpha$, which is possible to do by adjusting x_j' sufficiently close to x_j. So, $(x_1, x_2, \ldots, x_\ell)$ is not an Nash equilibrium. Even so, (Forrest, Buttermore, & Wajda, 2017) shows that these ℓ firms do have a symmetric mixed-strategy Nash equilibrium.

For the rest of this proof, it suffices to show that Company B will be expected to profit by entering this market through uniformly randomizing its price strategy over the interval [0,1].

Let $F(P)$ be the price distribution of Firm j, one of the imaginary firms of the aggregate firm. The aggregate firm or equivalently each of the ℓ imaginary forms sets its price after taking into account the price of Company B and those of all other imaginary firms. Hence, the profits for Firm j from its loyal customers is $\alpha_0 P$ and those from its share of the switchers is

$$\beta P(1-P)\prod_{i\neq j}^{\ell}[1-F(P)] = \beta P(1-P)[1-F(P)]^{\ell-1},$$

where $(1-P)$ stands for the portion of the switchers not taken by Company B, and $[1-F(P)]$ the portion of the switchers not taken by one of the other imaginary and identical firms. Hence, the profits Π Firm j generates when the firm sells its product at price P are $\alpha_0 P + \beta P(1-P)[1-F(P)]^{\ell-1}$ and the objective function of Firm j is

$$\begin{aligned} max_{F(P)} E(\Pi) &= \int_{-\infty}^{+\infty} \{\alpha_0 P + \beta P(1-P)[1-F(P)]^{\ell-1}\} dF(P) \\ &= \int_{0}^{1} \{\alpha_0 P + \beta P(1-P)[1-F(P)]^{\ell-1}\} dF(P) \end{aligned}$$

where $E(\Pi)$ stands for Firm j's expected profits for all possible prices, and the objective for Firm j is to maximize its expected profits by choosing its price distribution $F(P)$. The reason why the upper and lower limits of the integral are changed respectively from $+\infty$ and $-\infty$ to 1 and 0 is because when $P < 0$ or when $P > 1$, the profits are zero.

The equilibrium indifference condition of Firm j is

$$\alpha_0 \times P + \beta \times P(1-P)[1-F(P)]^{\ell-1} = \alpha_0 \times 1 \tag{4.1}$$

So, for the ℓ imaginary firms, solving equation (4.1) leads to their symmetric equilibrium pricing strategy as follows:

$$F(P) = 1 - \left(\frac{\alpha_0}{\beta P}\right)^{\frac{1}{\ell - 1}} \tag{4.2}$$

From $\beta > \alpha_0$, it follows that $\alpha_0/\beta < 1$. So, for any Price P, satisfying $1 \geq P \geq \alpha_0/\beta$, equation (4.2) is a well-defined probability distribution. This end implies that for the ℓ imaginary firms, or equivalently, the aggregate firm, the lowest allowed price is α_0/β.

From $lim_{P \to 1^-} F(P) = 1 - (\alpha_0/\beta)^{1/(\ell - 1)} \neq F(1) = 1$, it follows that the cumulative price distribution function $F(P)$ has a jump discontinuity at the reservation value $P = 1$, where the amount of jump is $(\alpha_0/\beta)^{1/(\ell - 1)}$. That is, $F(P)$ has a mass point of size $(\alpha_0/\beta)^{1/(\ell - 1)}$ at the reservation price $P = 1$. So, the expected profits of Company B are the following:

$$E(\Pi) = \int_0^{\alpha_0/\beta} \beta P dP + \int_{\alpha_0/\beta}^{+\infty} \beta P [1 - F(P)]^{\ell} dP \tag{4.3}$$

$$= \int_0^{\alpha_0/\beta} \beta P dP + \int_{\alpha_0/\beta}^{1} \beta P [1 - F(P)]^{\ell} dP + \beta \left(\frac{\alpha_0}{\beta}\right)^{\ell/(\ell - 1)} \tag{4.4}$$

where the first term in the right-hand side of equation (4.3) stands for the expected profits of Company B when it charges the lowest price in the marketplace and captures the entire segment of the switchers, and the second term is Company B's expected profits when it is in direct competition with the ℓ incumbent firms.

It can be readily seen from equation (4.4) that the expected profits $E(\Pi)$ of Company B is greater than zero. So, if the customer surplus satisfies $\beta = 1 - \alpha > 0$, Company B can profitably enter the market to compete with the incumbent firms. □

The assumption of Proposition 4.3 generally means that the technology involved and the relevant business operations of the new market have been standardized. From the assumption that Company B's leadership adopts the philosophy of transient competitive advantages and that it is R&D-driven and fully ready to embrace next fleeting advantage, it is reasonable to assume that Company B has come up with a more efficient technology and/or operation system that can greatly reduce the overall business expenditure. For example, Christensen, et al., (1998) confirm empirically that technological and market strategies of a new entrant are highly interrelated and that their joint effect plays an important role in a firm's probability of survival. And Friedman (2005) maintains that technology is a critical factor and force in shaping and flattening the world and balancing the global field. So, each competitive advantage is fleeting, since technology continues to change the way how people do business as it becomes cheaper, more available, lighter, smaller, etc.

What the proof of Proposition 4.3 says is that it will be difficult and even impossible for the incumbent firms to stop Company B from entering because to counter the pricing strategy of the incumbents Company B randomizes its selling price between the unit cost and the reservation price the incumbents charge their loyal consumers. At the same time, this proof also implies that Company B could potentially take the entire segment of switchers, which in turn means that if this segment of the market is larger than any of the royal-customer bases of the incumbents, then Company B could make more profits than any of the incumbents.

And from Proposition 4.3, it follows that Company B expects to expand its market share by taking first a portion of Company A's customers and second a portion of the greater market of the hybrid product. In other words, it is practically possible that the additional revenue

Company B expects to generate from its hybrid product will be more than S_B, the sales revenue of B from the market share of Company A, by producing the imagined hybrid product. A good example here is Fuji Photo vs Kodak. Let S_{B^-} represent the revenue Company B generates from the market beyond the market share of Company A, as described in Proposition 4.3. Then, we naturally have the following case.

Case 3: $S_B + S_{B^-} \geq \max\{R_1, C\}$. That is, the expanded sales revenue of Company B from producing the imagined hybrid product is at least the larger of Company B's cost (R_1) of designing and producing its hybrid product and the cost (C) of Company A for it to take defensive actions. If, additionally, $S_B > C$, then Company A takes defense and the Nash equilibrium is (Take defense, design/produce). If also $S_B + S_{B^-} = R_1$, then the Nash equilibrium is (Take defense, don't design/produce). On the other hand, if $S_B + S_{B^-} > R_1$ and $S_B > C$, then (Don't take defense, Design/produce) is the pure strategy Nash equilibrium with the corresponding payoffs (S_A – S_B, S_B – R_0), see Table 3.1 for details. In other words, the market share of Company A erodes and is gradually taken by Company B's hybrid product while Company A cannot do anything about it. That is, Company A will soon disappear from existence when more such companies as B enter the territory of A.

By summarizing what is established above, we have the following result:

Proposition 4.4:

Assume that Company A exploits its sustainable competitive advantages, while Company B looks forward to adopting transient competitive advantages (McGrath, 2013), as described above. Then the following hold true:

1. *If expansion is expected for Company B to generate profits, which are more than the cost for Company A to take defensive actions, then Company A would move first and take defensive actions against Company B;*
2. *If Company B's decision of whether or not to design and produce its hybrid product is based on the existence of an expanding market beyond that of Company A, then Company B will introduce its hybrid product.*

The second result comes from Proposition 4.3 and Theorem 5 in (Forrest, Buttermore & Wajda, 2017), where the expanding market means that the segment of switchers exists in this market, and is also expanding with the overall increasing market scale.

4.2. Three-player competitions

In this subsection, we focus on the competition between the companies while a representative customer is involved.

Proposition 4.5:

Assume that the additional revenue of Company B from beyond its original territory is less than its cost of designing and producing its hybrid product. Then the pure strategy Nash equilibrium is reached when Company A does not take defense, Company B does not design/produce its hybrid product, and the customer does not make any purchase. In this Nash equilibrium, none of the companies earns any revenue while the customer does not spend any money.

Proposition 4.6:

Assume that the additional revenue of Company B from beyond its original territory is greater than its cost of designing and producing its hybrid product. Then the pure strategy

Nash equilibrium is reached when Company A does not take defense, Company B designs and produces its hybrid product, and the customer makes purchase. In this Nash equilibrium, Company A collects no revenue, Company B makes profits, while the customer enjoys her utility u. *And, if, moreover, the additional revenue of B from A's market territory is greater than the cost for A to take defense, then when A moves first, the Nash equilibrium is (Take defense, design/produce).*

Proof of Propositions 3.1 and 3.2. This result follows from the payoffs of Company A that are corresponding to the scenarios that Company A either takes defensive actions or does not take any defensive actions (either Table 3.2 or Table 3.3). For details see Tables 3.3 and 3.4, respectively. □

Table 4.2 The payoffs of Company A when it takes defensive actions

		Company B	
		Design/produce (p)	Don't design/produce (1 – p)
Customer	Buy (τ)	$-C$	$S_A - C$
	Don't buy ($1-\tau$)	$-C$	$-C$

Table 4.3 The payoffs of Company A when it does not take any defensive actions

		Company B	
		Design/produce (q)	Don't design/produce (1 – q)
Customer	Buy (ε)	0	S_A
	Don't buy ($1-\varepsilon$)	0	0

What Proposition 4.5 indicates is that when the additional revenue Company B expects to generate from its hybrid product is less than the total cost of designing and producing the hybrid product, then it is foolish for Company B to design and produce its hybrid product. So, consequently, Company A does not need to take any defensive actions; and if the customer decides to purchase, she has no choice but purchase Company A's product. On the other hand, what Proposition 4.6 implies is that when the additional revenue Company B expects to generate from its hybrid product is more than the total cost of designing and producing the hybrid product, then Company B will move ahead with its design and production of the hybrid product; in this case, because any defensive action is useless, Company A does not even attempt to take any action. However, considering the assumption that Company A is a diehard believer and practitioner of the old school of sustainable competitive advantages, such as Montgomery Ward and Kodak mentioned above, it in really life would most likely not passively watch for its base of customers to deteriorate without trying to save it and/or even trying to expand it. Because of this reason, let us next consider mixed strategies under the condition of $S_B + S_{B^-} > D + \alpha C$, that is, the additional revenue of Company B from beyond its original territory is greater than the cost of designing and producing its hybrid product. In this case, we also assume that $S_{B^-} < D + \alpha C$ (that is, the revenue Company B earns outside the market base of Company A is less than B's cost), because otherwise Company B will definitely design and produce its hybrid product.

Assume that when Company A takes defensive actions, the probability for Company B to design/produce hybrid product is p, (because before making the decision of what to do, Company B has to firstly sense, secondly seize and transform itself, and then acquire the critical capabilities for successful organizational adaptation (Day & Schoemaker, 2016)), and the probability for the customer to purchase is τ. On the other hand, when Company A does not take any defensive action, the probability for Company B to design/produce hybrid product is q, and the probability for the customer to purchase ε. Solving for mixed strategy Nash equilibrium provides $p = q = 1/2$, and

$$\tau = \frac{D + \alpha C - S_{B^-}}{S_B} \tag{4.5}$$

$$\varepsilon = \frac{D - S_{B^-}}{S_B} \tag{4.6}$$

Proposition 4.7:

If Company B's additional revenue from beyond its original territory is greater than its cost of designing and producing its hybrid product and its additional revenue from outside the market base of Company A is smaller than the cost, and Company A does not take any defensive actions to protect itself, then

1. *The probability for the customer to purchase Company A's product is inversely proportional to Company B's activities of designing and producing its hybrid product; and*
2. *When the revenue Company B earns from the market base of Company A is equal to its fixed cost of designing and producing its hybrid product minus the revenue generated from beyond the customer base of Company A, then the probability for the customer to purchase Company B's hybrid product is equal to 1.*

Proposition 4.8:

If Company B's additional revenue from beyond its original territory is greater than its cost of designing and producing the hybrid product and its additional revenue from outside the market base of Company A is smaller than the cost, and Company A does take defensive actions to protect itself, then

1. *The higher the difference between the cost for Company B to design and produce its hybrid product and the additional revenue generated from beyond the customer base of Company A, the higher the probability for Company A's customer to purchase Company B's hybrid; and*
2. *If, additionally, the α-value is fixed, then the higher investment Company A makes in its effort to raise the barriers for Company B to invade its territory, the lower probability its customer will buy Company B's hybrid.*

This result means that the defensive actions Company A takes actually work and can help slow down the deterioration of Company A's customer base.

Proof of Propositions 3.3 and 3.4. To solve for the mixed strategy Nash equilibrium, we solve the following indifference equations:

$$pu + (1 - p)(-u) = 0$$

$$qu + (1 - q)(-u) = 0$$

$$\tau[(S_B + S_{B^-}) - D - \alpha C] + (1 - \tau)[(S_{B^-}) - D - \alpha C] = 0$$

and

$$\varepsilon[(S_B + S_{B^-}) - D] + (1 - \varepsilon)[(S_{B^-}) - D] = 0$$

So, we have $p = q = 1/2$ and equations (4.5) and (4.6). Now, Proposition 4.7 follows from equation (4.6), while Proposition 4.8 follows from equation (4.5).

Proof of Proposition 4.9: Let π_A represent the expected revenue of Company A. Then we have

$$\pi_{A,defense} = p\tau(S_A - C - S_B) + p(1-\tau)(-C) + (1-p)\tau(S_A - C) + (1-p)(1-\tau)(-C) \tag{4.7}$$

and

$$\pi_{A,don't\ defense} = q\varepsilon(S_A - S_B) + (1-q)\varepsilon S_A \tag{4.8}$$

So, by substituting the values of $p = q = 0.5$, τ and ε from equations (4.5) and (4.6) into equations (4.7) and (4.8), we have

$$\pi_{A,defense} = \frac{S_A(D + \alpha C - S_{B^-})}{S_B} - \frac{1}{2}(D + \alpha C - S_{B^-}) - C \tag{4.9}$$

and

$$\pi_{A,don't\ defense} = \frac{1}{2}\frac{D - S_{B^-}}{S_B}(2S_A - S_B) \tag{4.10}$$

Because

$$\frac{\partial \pi_{A,defense}}{\partial C} = \frac{\alpha S_A}{S_B} - \frac{\alpha}{2} - 1 = \alpha\left(\frac{S_A}{S_B} - \frac{1}{2}\right) - 1 \tag{4.11}$$

and α is a constant between 0 and 1, it follows that when $S_A < \left(\frac{1}{\alpha} + \frac{1}{2}\right) S_B$, we have $\frac{\partial \pi_{A,\text{defense}}}{\partial C} < 0$. That is, when $S_A < \left(\frac{1}{\alpha} + \frac{1}{2}\right) S_B$, the expected revenue $\pi_{A,\text{defense}}$ of Company A becomes a decreasing function in variable C. In other words, as soon as the revenue Company B generates from Company A's customer base is greater than Company A's revenue when A does not experience any challenge from Company B divided by $\left(\frac{1}{\alpha} + \frac{1}{2}\right)$, then Company A can no longer exploit its sustainable competitive advantages by continuously raising its investment in raising the entry barrier in order to prevent Company B from invading its territory.

On the other hand, equation (4.10) indicates that when $S_{B^-} > D$, the expected revenue $\pi_{A,\text{don't defend}}$ of Company A will be a loss. In other words, if Company A decides not to defend itself against Company B's challenge and the expected revenue Company B generates from the market except Company A's territory is greater than the fixed cost of designing and producing the hybrid product, then Company A's expected revenue will be a loss. □

4.3. COST-BENEFIT ANALYSIS

Under the assumption that Company B's additional revenue from beyond its original territory is greater than its cost of designing and producing the hybrid product and its revenue from outside the market base of Company A is smaller than its cost (or $S_B + S_{B^-} > D + \alpha C$ and $S_{B^-} < D + \alpha C$), the previous discussion indicates that to slow down the deterioration of Company A' customer base the most significant decision is to raise the investment on building a costly barrier for Company B's entry into Company A's territory and to increase the customer retention. As part of the most important investment, Company A could constantly make the features and functionalities of its product design better, which makes the design and production of Company B's hybrid difficult to follow up. For the comparison of the deterrence cost C of Company A and its revenue, we have:

Proposition 4.9.

Assume that Company B's additional revenue from beyond its original territory is greater than its cost of designing and producing the hybrid product and its additional revenue from outside the market base of Company A is smaller than the cost. Then the following hold true:

1. *If Company A decides to defend its territory by raising entry barriers for Company B, then it can successfully delay the deterioration of its territory as long as less than* $\left(\frac{1}{\alpha}+\frac{1}{2}\right)$ *of its maximum expected revenue is taken by Company B; and*
2. *If Company A decides not to take any defensive actions against Company B's possible invasion through designing and producing hybrid product, then it can expect to make positive revenues. However, this positive revenue decreases with increasing revenue Company B generates from outside Company A's territory.*

What Proposition 4.9 indicates is that in order to continuously stay in business, Company A has no choice except also adopt the philosophy of transient competitive advantages. That is, as pointed out by Dobni and Klassen (2015), when the company matures and becomes slow in growth, innovation is a key driver in maintaining the company's market advantage.

Another fact the proof of Proposition 4.9 reveals is that with the constant improvement of technology and growing convenient availability of information, the fixed cost D and the cost coefficient α for Company B to design and produce its hybrid product are getting lower with time, while the cost C for Company A to raise the entry barrier is getting higher. So, the expected revenue of Company A will be crashed by Company B quickly unless Company A also starts to improve its product by designing and producing hybrid product for the purpose of expanding its market share, as what Company B tries to do throughout this presentation. However, Tushman and O'Reilly III (1996) confirm this end as a real challenge for the managers of Company A, just as the cases of Montgomery Ward and Kodak as discussed previously.

5. SOME FINAL WORDS

McGrath (2013) demonstrates vividly by using anecdotes and data that failure to respond and adapt to the ever-evolving market conditions by not focusing on multiple or transient competitive advantages may place a firm in an unenviable deficit with a commensurate serious loss in market share and/or dominance. The competitive landscape will continue to fluctuate and change. Organizations can no longer remain at the pinnacle of their respective industries through sustaining main competitive differentiators or advantages. The acceptance that any advantage will be short-lived will be critical in a continuous learning and innovative company. That is, the once sustainable competitive advantages have become transient in the modern world of business, although they were indeed seemed to be long-lasting in the past. So, theoretically, there is a need to show why any company that plans on continuously exploiting its competitive advantages for the long term will disappear from the market place or soon become irrelevant when more and more of its competitors are adopting and practicing the philosophy of transient competitive advantages.

This paper employs a very intuitive, straightforward game-theoretic model to clearly show how unresponsively the company that plans on continuously reaping in its profits by exploiting its sustainable competitive advantages is losing its future to those that adopt and practice transient competitive advantages. To make the conclusion definite, this paper looks at the scenarios of both pure strategies and mixed strategies. Other than avoiding the weaknesses, such as deriving unreliable conclusions from maximizing the representative customer's utility function, developing specific results from particularly chosen utility functions, and creating beliefs from imposed unrealistic linearities, of the previously developed models – say, for example, those by Singh and Vives (1984) and Zanchettin (2006) – for duopoly competition, this work establishes several thought provoking results, which can be practically applied, based on the mathematical formulation of the problem by providing the feasible solutions that incorporate the differences in actions between two companies, one of which exploits its sustainable competitive advantage, while the other rides the wave of market conditions. The most important conclusion this study produces is that in order to avoid becoming history, any company should adopt the philosophy of transient competitive advantages.

Because the model developed in this paper does not involve the time variable, all the established conclusions only describe aspects of the momentary situation of the competition between Companies A and B. So, what is both theoretically and practically significant is to see what would follow after Company A realizes what it needs to do at the next time moment in the competition with Company B. Another important issue this paper does not address is that in practical applications, how will a host of incumbent firms interact with each other when more than one of them are market-, technology-driven, and embrace next fleeting advantages. These and other relevant issues will be theoretical interesting and practically significant research topics for future works.

The managerial implications of this study include, but not limited to, the following:

1. If the potential loss to an emerging competition is more than the cost of taking defensive actions, then the incumbent firm should proactively approach the forthcoming challenge by taking defense although the chance of success is uncertain;
2. If a new product has the potential to penetrate several markets simultaneously, although the expected success in each of these markets is limited initially, the product has a great promise for market success;
3. Although a business expansion into several markets simultaneously seems to be only break-even in terms of cost-benefit analysis, the company should jump ahead to expand;
4. If less than 2/3 of its maximum expected revenue could be potentially taken by a competitor who has trouble entering any other market, then the incumbent firm can successfully delay the deterioration of its territory by taking defensive actions.

Based on what the literature shows (e.g., McGrath, 2013 and references found there), this study is the first of its kind that employs a rigorous approach to address, either through confirming or disapproving, issues related to many overriding conclusions – drawn mostly on anecdotes, or data mining, or models developed on limiting or unrealistic conditions (for example, Kaharuddin, et al., 2017; Jindal, et al., 2016; Purkayastha & Sharma, 2016; Zachary, et al., 2015; Zanchettin, 2006; Tushman & O'Reilly III, 1996; Singh & Vives, 1984). Because of such an approach employed in this paper, we are able to provide insights into the practically significant question: How do companies of the philosophy of sustainable competitive advantages actually lose their shirts in the competition with companies that ride the current wave of transient advantages (e.g., Bashir & Verma, 2017; Koller, 2016; Purkayastha & Sharma, 2016; Chan & Chan, 2010)? At the same time, the model developed in this paper surely enriches the game-theoretical literature of duopoly competition (e.g., Pecora & Sodini, 2018; Villas-Boas, 2015; Dubiel-teleszyński, 2011; Zhang, et al., 2009; Zanchettin, 2006; Agliari & Puu, 2002; Sice, et al., 2000) through avoiding some of the major weaknesses of the models previously established for studying either Cournot competition or Bertrand competition (for related references, see those listed in the Literature Review section on why existing models of duopoly do not fit our need of this research project).

APPENDIX

Proposition A.1:

The cost function of a company that produces consumer goods cannot be generally assumed to be linear in the quantity of the product produced and sold with constant marginal cost, as given in (Zanchettin, 2006).

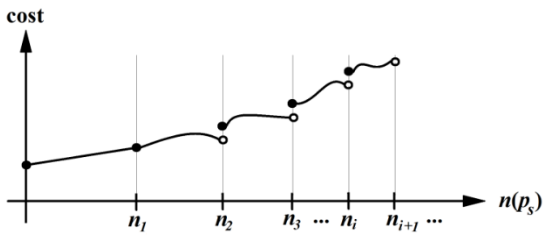

Figure A.1. The cost function

As a matter of fact, in the framework of fast technological progress, the marginal cost cannot be constant. Instead, the cost function in reality is a piecewise defined function in number $n(p_s)$ of units of product produced and sold, because when the employed technology is fixed, by slightly improving the managerial operation of the program – such as changing the efficiency of the promotion, making the usage of resources and technology more effective, etc. – without or only slightly increasing the cost, the production can be increased majorly. That is, the total cost function should be a piecewise defined function, as shown in Figure A.1. Here, within each interval $[n_j, n_{j+1}], j = 1, 2, \ldots$, the cost is relatively flat while the productivity rises through improving efficiency in terms of the usage of resources, deployment of technology, managerial strategies, etc. The decreasing lengths of the intervals $[n_j, n_{j+1}]$ represent the fact that with time technology needs to be updated more frequently, where at each location $n_j, i = 1, 2, \ldots$, a large expenditure occurs due to investments in capital assets, human resources, etc.

As an empirical illustration for the previous general argument, one can simply look at the upgrading of office computers. When new computers replace the existing ones, the productivity subsequently increases, while the cost for the increased productivity jumps drastically or nonlinearly due to the one-time expenditure on the new equipment. Now, with time people will become more and more efficient regarding how they operate the new equipment. Such gradual increasing efficiency can help improve productivity without additional expenditure. □

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