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Dynamic Fracture Tests Data Analysis Based on the Randomized Approach

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Abstract: Methods of measurement and prediction of materials dynamic strength are complicated and not standardized. Usually dynamic tests are labour-consuming and each assay demands a lot of time in contrast to static experiments. The way of experimental data treatment based on incubation time criterion and randomized approach of sign-perturbed sums is considered. It is shown that a few experimental points are enough to determine strength parameter with accuracy proper for engineering. The lack of real experimental data is compensated in SPS-algorithm by carrying out a large number of virtual random tests in which the Bernulli signs are randomly generated in this method.

Keywords: sign-perturbed sums, incubation time, randomized approach, data analysis

1. INTRODUCTION

One of the most important problems in mechanical engineering is measuring the strength and rheological parameters of materials. Determination of the static strength or Young modulus is a common procedure, because these values result from direct observation. There are standard measurement techniques to obtain the average value of a required parameter with a certain degree of accuracy, for example $\sigma_c \in [\sigma_{c-}; \sigma_c{}^{+}]$. Also direct observations stipulate that the physical sense of the measured parameter is established and well-known, and all researchers have a unified conception about the final result.

The more complicated situation is observed in dynamics, where material strength cannot be characterized by one parameter of the critical stress. Experimental test results demonstrate that under intensive impacts, specimens can resist to stress level significantly higher than their static strength σ_c and stress value at fracture moment σ_* depends on the rate of loading and the shape of breaking pulse [1–7]. These strain-rate dependencies are interpreted as passport specifications of materials in certain theories. The main difficulty of these approaches is that the variety of strain-rate curves is infinite due to strong influence of impact conditions to σ_d value. Also it is impossible to describe material strength in the same way when fracture is initiated by threshold impacts. In this case the stress level at the breaking moment can be less even than the static strength, this phenomenon is called the fracture delay effect.

The structure-temporal approach based on the incubation time criterion of fracture [8,9] is considered at the present work. The main idea of this criterion is that fracture does not occur instantly and there is some characteristic time for every transient process. Implementation of only one additional strength parameter as the incubation time τ permits to predict fracture

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stress value and calculate strain-rate dependencies for all types of impacts. This structure-temporal approach was successfully applied for many different problems of dynamic strength estimation for different materials and condensed matters e.g. dynamic fracture of rocks and concretes, dynamic yielding of metals, acoustic ultrasonic cavitation of liquids, etc. [9–12].

The tests measuring incubation time directly are not realized, and now its value could be obtained only implicitly. The simplest way is to select an appropriate value of τ ensuring good correspondence between a model curve and a scatter of the experimental data. Any method could be applied for fitting the model curve, e.g. least mean squares (LMS) method, but it gives only one value of the incubation time and there is no any estimation of inaccuracy when we have an absence of any adequate variability in observations. Standard estimation algorithms use typically the condition of persistent excitation in data. However, this condition is difficult to ensure in the considered problem since dynamic tests are very complicated and labour-consuming, so usually there are no many experimental points to treat. This often results in a degenerate observation data and complicated identification problems. The well-known *set-membership approach* to identification uses no statistical properties of the noise, but assumes some known upper bounds on uncertain system components instead. The purpose of the approach is typically to compute some upper or lower set estimates on the set of data-consistent parameters and no convergence of this set estimates to the true unknown parameter vector can be achieved without any significant additional assumptions (see, e.g. [13, 14]). In the context of the considered problem the robust estimates of min & max methods give a very wide interval for τ with high inaccuracy and this result is very conservative. Randomness of the experimental values stipulates to use stochastic methods for treatment algorithms [15,16].

New SPS (Sign-Perturbed Sums) procedure proposed in [17] provides rigorously guaranteed non-asymptotic confidence regions for the unknown parameters τ of a linear dynamical control plant in the small-sample setting. In this paper we adopt this approach to the problem of the incubation time evaluation. This method is applicable to the problem because it can provide a confidence interval of τ with the rate of accuracy admissible for engineering. Approximately ten data points are enough to determine an average value of incubation time with $\varepsilon = 20 - 35\%$. The applicability of the new algorithm to the incubation time approach will be illustrated on few dynamic fracture tests of different types of rocks [18].

2. INCUBATION TIME APPROACH

The main features of the structure-temporal approach are the following. The general form of the incubation time criterion is

$$\frac{1}{\tau}\int_{t-\tau}^{t}\left(\frac{\sigma(t')}{\sigma_c}\right)^{\alpha}dt' \leq 1 \tag{2.1}$$

where $\sigma(t')$ is the loading stress function, τ is the incubation time of the fracture, α is a dimensionless parameter, for most brittle materials $\alpha = 1$. According to (2.1) the fracture does not occur while left part of the criterion is less than one and the fracture moment t_* corresponds to equality.

Experimental dynamic tests are often carried out in overthreshold regimes hence stresses grow linearly until the fracture. Then the shape function of the loading impacts can be determined by strain-rate of load $\dot{\epsilon}$ and elastic modulus k:

$$\sigma(t) = h(t)k\dot{\epsilon}t, \tag{2.2}$$

where $h(t)$ is Heaviside step function. Substitution function (2.2) to criterion (2.1) leads to the following equation for fracture time t_*

$$h(t_*)\left(\frac{t_*}{\tau}\right)^{\alpha+1} - h(t_* - \tau)\left(\frac{t_*}{\tau} - 1\right)^{\alpha+1} = s \tag{2.3}$$

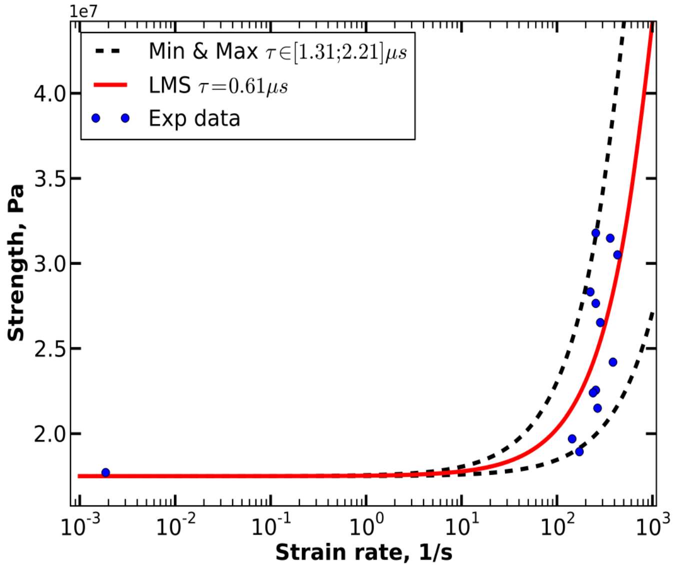

Fig. 1. Dynamic fracture of gabbro-diabase [18]. Blue points are experimental data. Lines are theoretical curves plotted by the incubation time fracture criterion: solid line - least mean squares method; dashed lines - min & max robust method

where $s = (\alpha+1)\big(\sigma_c/(k\dot{\epsilon}\tau)\big)^{\alpha}$ is a dimensionless parameter, which value depends on strain-rate of load impacts. Fracture time t_* could not be negative, hence $h(t_*) = 1$ and there are two cases: when $t_* > \tau$ or $t_* \le \tau$. From expression (2.3) it follows that $s = 1$ for $t_* = \tau$, hence the final equation set determining fracture moment t_* is

$$\begin{cases} (t_*/\tau)^{\alpha+1} = s, & s < 1, \\ (t_*/\tau)^{\alpha+1} - (t_*/\tau - 1)^{\alpha+1} = s, & s \ge 1. \end{cases} \tag{2.4}$$

Thus, incubation time approach is able to predict strain-rate dependence for the dynamic threshold of fracture $\sigma_*(\dot{\epsilon}) = k\dot{\epsilon}t_*$. In most cases parameter α equals to 1 during calculation of dynamic strength of brittle materials such as rocks. Then roots of equation (2.4) could be expressed in an explicit form and the critical fracture stress is the following:

$$\sigma_*(\dot{\epsilon}) = \varphi(\tau, \dot{\epsilon}) = \begin{cases} \sigma_c + \frac{\tau}{2}k\dot{\epsilon}, & \dot{\epsilon} \le 2\sigma_c/k\tau, \\ \sqrt{2\sigma_c\tau k\dot{\epsilon}}, & \dot{\epsilon} > 2\sigma_c/k\tau. \end{cases} \tag{2.5}$$

An application of the incubation time criterion is demonstrated on the example of an impact fracture test on gabbro-diabase and marble. The theoretical curve in comparison with experimental [18] points is shown in Fig. 1. The value of $\tau = 0.61$ µs (solid line) is calculated by LMS method, as it is mentioned above this result does not provide any information about inaccuracy of this value. Another min & max method gives very wide interval for incubation time value $\tau \in [1.31; 2.21]$ µs, and there is no information about properties, that new experimental points would lie in this interval. Hence, it is necessary to apply some method of data analysis, which gives, at first, a more accurate interval for possible values of τ and, at second, the degree of confidence in it.

3. PROBLEM DESCRIPTION

In dynamic test experiments we can choose acting factor $\dot{\epsilon}$ and to get the correspondent observation σ_*. Dynamic test data are fitted using the following model of N noisy observations:

$$\sigma_{*i} = \varphi(\tau, \dot{\epsilon}_i) + v_i, \quad i = 1, 2, \ldots, N \tag{3.6}$$

where v_i is a random noise (an inaccuracy) with symmetrical distribution. If the strain-rate dependence satisfies the principles of structure-temporal approach, then function $\varphi(\tau, \dot{\epsilon}_i)$ is

$$\varphi(\tau, \dot{\epsilon}_i) = k\dot{\epsilon}t_*(\tau), \tag{3.7}$$

where t_* is the fracture time predicted by the incubation time criterion by solving equation (2.4).

Equation (3.7) allows to calculate fracture stress for different τ, then the LMS method gives the best fitted value of the incubation time, for that follow sum has a minimum value

$$\sum_{i=1}^{N} \Big(\varphi(\tau, \dot{\epsilon}_i) - \sigma_{*i}\Big)^2 \to \min_{\tau}.$$

However, we are not able to get a sufficiently good confidence interval for unknown τ without significant restrictions for the noise v_i when N is small.

The objective is to construct confidence regions for unknown τ that have guaranteed user-chosen confidence probabilities for finite, and possibly small number of data points. It must be defined by the observations of outputs $\{\sigma_{*i}\}_{i=1}^{N}$ and known acting factors $\{\dot{\epsilon}_i\}_{i=1}^{N}$ which may be chosen. The constructed regions are almost distribution-free, as the only assumption is the noise to have a property of symmetry. This is important since in practice the knowledge about the noise distribution is limited. Additionally, the confidence regions should contain the least-squares point estimate.

4. SPS PROCEDURE FOR CONSTRUCTING OF CONFIDENCE REGIONS

For a finite number of observations we can use the following procedure which is similar to SPS procedure from [17]:

The LMS estimate is obtained as the solution of the equation

$$H_0(\tau) = \sum_{i=1}^{N} (\sigma_{*i} - \varphi(\tau, \dot{\epsilon}_i)) \frac{d\varphi(\tau, \dot{\epsilon}_i)}{d\tau} = 0,$$

where

$$\frac{d\varphi(\tau, \dot{\epsilon})}{d\tau} = \begin{cases} \frac{1}{2}k\dot{\epsilon}, & \dot{\epsilon} \leq 2\sigma_c/k\tau, \\ \frac{1}{\sqrt{2\tau}}\sqrt{\sigma_c k\dot{\epsilon}}, & \dot{\epsilon} > 2\sigma_c/k\tau. \end{cases}$$

We will try to exploit the information in the data as much as possible while assuming minimal prior statistical knowledge about the noise. Our core assumption is the symmetry of the noise. For some $M > 0$ we generate $N(M-1)$ Bernoulli random values $\beta_{ij} = \pm 1$ with probability of $1/2$, and introduce $M-1$ sign-perturbed sums

$$H_j(\tau) = \sum_{i=1}^{N} \beta_{ij} (\sigma_{*i} - \varphi(\tau, \dot{\epsilon}_i)) \frac{d\varphi(\tau, \dot{\epsilon}_i)}{d\tau},$$

$j = 1, 2, \ldots, M-1$.

If $\tau^\star$ is a nominal value of τ then $H_0(\tau^\star)$ and $H_j(\tau^\star)$ have the same distribution since $\{v_i\}$ are symmetric. Therefore, there is no reason why a particular $|H_j(\tau^\star)|$ should be bigger or smaller than another $|H_{j'}(\tau^\star)|$ and the probability that a particular $|H_j(\tau^\star)|$ is the m-th largest one in the ordering of $\{|H_j(\tau^\star)|\}_{j=0}^{M-1}$ will be the same for all j, including $j = 0$ (the case where there are no sign-perturbations). As it can take different values, this probability is exactly $1/M$.

Algorithm:

1. Given a (rational) confidence probability $p \in (0, 1)$, set integers $M > q > 0$ such that $p = 1 - q/M$.
2. Generate $N(M-1)$ i.i.d. random signs $\{\beta_{ij}\}$ with $\mathrm{Prob}\{\beta_{ij} = 1\} =$ $= \mathrm{Prob}\{\beta_{ij} = -1\} = 1/2$ for $i = 1, 2, \dots, N$ and $j = 1, 2, \dots, M-1$.
3. Set
$$\mathcal{T} := \{\tau : \mathit{SPS\_Indicator}(\tau) = \mathit{1}\}.$$

Procedure: $SPS\_Indicator(\tau)$

1. For the given τ compute the prediction error for $i = 1, 2, \dots, N$
$$\delta_i(\tau) = \sigma_{*i} - \varphi(\tau, \dot{\epsilon}_i).$$
2. Evaluate
$$\begin{aligned} H_0(\tau) &= \sum_{i=1}^{N} \delta_i(\tau) \frac{d\varphi(\tau, \dot{\epsilon}_i)}{d\tau}, \\ H_j(\tau) &= \sum_{i=1}^{N} \beta_{ij} \delta_i(\tau) \frac{d\varphi(\tau, \dot{\epsilon}_i)}{d\tau}, \end{aligned}$$
for $j = 1, 2, \dots, M-1$.
3. Order scalars $|H_j(\tau)|$ from smallest to biggest.
4. Compute the rank $\mathcal{R}(\tau)$ of $|H_0(\tau)|$ in the ordering, where $\mathcal{R}(\tau) = 1$ if $|H_0(\tau)|$ is the smallest in the ordering, $\mathcal{R}(\tau) = 2$ if $|H_0(\tau)|$ is the second smallest, and so on.
5. Return 1 if $\mathcal{R}(\tau) \leq M - q$, otherwise Return 0.

Note that the LMS estimate $\hat{\tau}$ has by definition the property that $H_0(\hat{\tau}) = 0$. Therefore, the LMS estimate $\hat{\tau}$ is included in the SPS confidence region. The probability that $\tau^\star$ belongs to $\mathcal{T}$ is given in the following Theorem.

Theorem 1:
If the observation noise in (3.6) *is independent and it has a property of symmetry then for nominal value $\tau^\star$ of the incubation time we have*
$$\mathrm{Prob}\{\tau^\star \in \mathcal{T}\} = 1 - q/M,$$
where M, q, $\mathcal{T}$ from Steps 1 - 3 *of the algorithm described above.*

The proof of Theorem 1 is similar to the correspondent one in [17] and it is provided at the expanded version of this paper [19]. The main difference is the nonlinearity of function $\varphi(\cdot, \cdot)$ from (2.5), but it is a convex function with monotonic property.

5. EXPERIMENTS

Results of SPS procedure applied to experimental data of dynamic fracture tests for different types of rocks are shown in Fig. 2-4. There are different amount of data points for each material, but it does not influence the applicability of SPS algorithm. It provides values of the incubation time with the proper degree of accuracy approximately $20 - 40\%$.

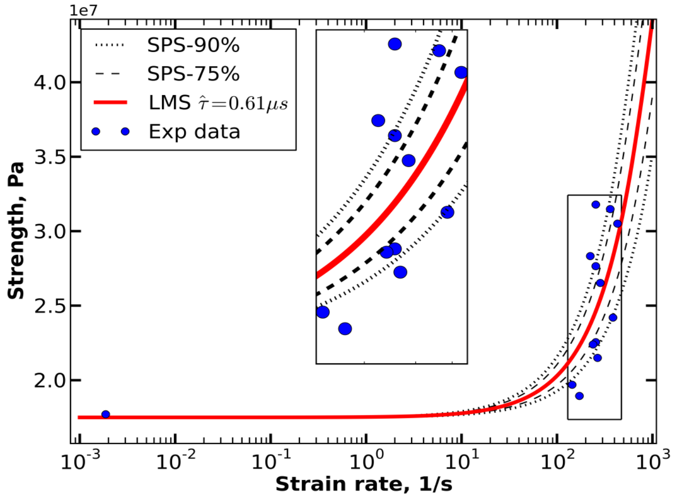

Fig. 2. Dynamic fracture of gabbro-diabase [18]. Red line is plotted by LMS method; dotted lines are plotted by SPS procedure for $\tau_{0.9} \in [0.39; 0.89]$ µs; Dashed lines - $\tau_{0.75} \in [0.48; 0.78]$ µs

In the case of gabbro-diabase (see Fig. 2) two values of confidence probabilities 90% and 75% were achieved by the following values of SPS procedure parameters $M = 200$, $q = 20$ and correspondingly $M = 200$, $q = 50$. This gives us two intervals of the incubation fracture time $\tau_{0.9} \in [0.39; 0.89]$ µs and $\tau_{0.75} \in [0.48; 0.78]$ µs, while LMS method provides $\tau = 0.61$ µs. Thus, SPS procedure allows us to calculate the incubation time value with relatively small inaccuracy 25% and with sufficiently high degree of confidence 75%.

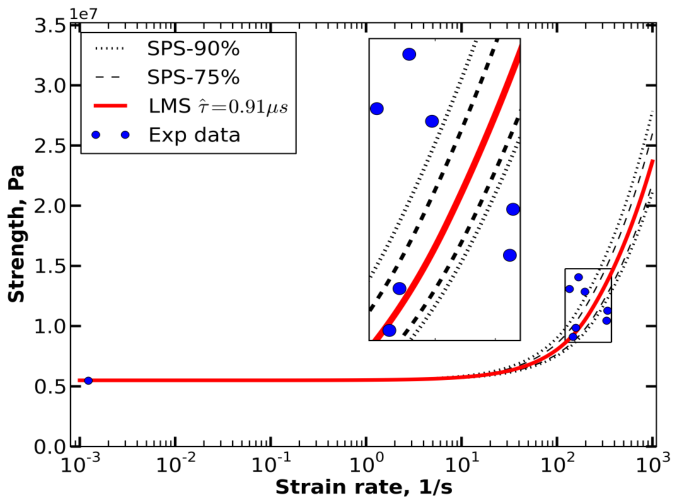

Fig. 3. Dynamic fracture of Coelga marble [18]. Red line is plotted by LMS method; dotted lines are plotted by SPS procedure for $\tau_{0.9} \in [0.39; 0.89]$ µs; Dashed lines - $\tau_{0.75} \in [0.48; 0.78]$ µs

In the case of Coelga marble (see Fig. 3) we have similar situation for following values of the SPS procedure parameters: $M = 100$, $q = 10$ for 90% and $M = 100$, $q = 25$ for 75%. The inaccuracy does not exceed acceptable for engineering 30% for both confident intervals of the incubation time $\tau_{0.9} \in [0.72; 1.26]$ µs and $\tau_{0.75} \in [0.78; 1.1]$ µs, since LMS method gives us $\tau = 0.91$ µs. SPS algorithm also provides good results for Pervouralsky marble (see Fig. 4) despite there are only five data points and LMS method provides $\tau = 1.81\mu s$.

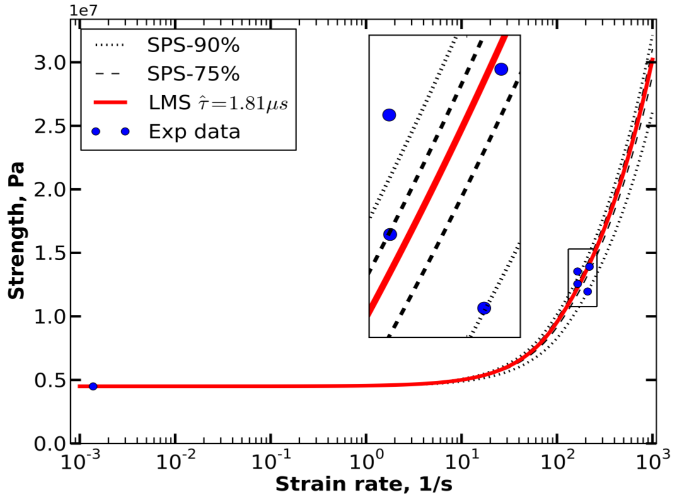

Fig. 4. Dynamic fracture of Pervouralsky marble [18]. Red line is plotted by LMS method; dotted lines are plotted by SPS procedure for $\tau_{0.9} \in [0.39; 0.89]$ µs; Dashed lines - $\tau_{0.75} \in [0.48; 0.78]$ µs

This number of points leads to relatively small number of iterations $M = 20$, so proper confidences are achieved by follows $q = 2$ and $q = 5$. These parameters give us incubation time intervals $\tau_{0.9} \in [1.35; 2.05]$ µs and $\tau_{0.75} \in [1.66; 1.91]$ µs with inaccuracy less than 20%. It should be noted, that 90%-confident and min & max intervals for Pervouralsky marble are approximately the same. The advantage of the SPS procedure in comparison with robust estimation is that it gives additional information about inaccuracy of the determined interval.

6. CONCLUSION

The incubation time approach proved itself as a good instrument to estimate and predict the limit parameters of loading impacts. The main problem is that there is no standard procedure to determine the incubation time value τ with estimation of its inaccuracy ε. The main difficulty is that the incubation time could be measured only implicitly and standard methods of data analysis do not work. The proposed application of SPS procedure demonstrates how to calculate the incubation time with certain inaccuracy ε under limited number of experimental points. The advantage of this method is that it allows us to predetermine the confidence of the target intervals for parameter τ. The incubation time approach complemented by SPS procedure for data analysis can become a universal instrument of engineering to determine material strength in dynamics. The further researches are aimed to improve developed method in case of arbitrary noises like in [20, 21]

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