Rotor, Bearing and Dynamic Equations in Energy Storage Flywheels for Vehicles

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Abstract

Energy storage systems for vehicles present significant challenges for rotor and bearing design. This paper discusses rotor and bearing design technology in energy storage flywheels for vehicles, with particular emphasis on orientation of flywheel rotors, rotor geometry and magnetic bearings. Material, rotational speed and geometry are mainly factors of flywheel rotor design. In order to achieve an attractive specific energy, the rotor speed should be as high as possible. The bearings must be capable of extremely high speed, have very low friction, and be stiff to adequately constrain the rotor, have long life, and have high load capacity. These requirements frequently lead to choosing magnetic bearings for Flywheel Energy Storage System applications. A flywheel energy storage system prototype with active magnetic bearings was designed. The flywheel was suspended by the permanent magnetic bearings and stabilized by the active magnetic bearing. Finally, we deduce differential equations of the magnetic suspended flywheel for the design of control system.

Keywords Flywheel, Energy storage, Active magnetic bearing, Vehicle, Dynamic equations

1 Introduction

Traditionally, the energy storage requirement for vehicles has been satisfied by chemical batteries. However, batteries have a number of disadvantages such as limited cycle life, maintenance, conditioning requirements, and modest power densities which have been improved upon by newer technologies such as energy storage flywheels. Flywheels in particular offer very high reliability and cycle life without degradation, reduced ambient temperature concerns, and construction free of environmentally harmful materials.

The energy storage flywheel system mainly consists of flywheel rotor, motor/generator, magnetic bearings, housing and power transformation electronic system. One of the major advantages of flywheels is the ability to handle high power levels. This is a desirable quality in e.g. a vehicle, where a large peak power is necessary during acceleration and, if electrical breaks are used, a large amount of power is generated for a short while when breaking, which implies a more efficient use of energy, resulting in lower fuel consumption. Individual flywheels are capable of storing up to 500MJ and peak power ranges from kilowatts to gigawatts,with the higher powers aimed at pulsed power applications. The flywheel energy storage packed in vehicle can operate at a nearly constant, optimum speed, reducing fuel consumption, air and noise pollution, and engine maintenance requirements, and extending engine life. Short bursts of power, for climbing hills and acceleration, are taken from flywheel energy storage, which is replenished directly by the engine or by regenerative braking when the vehicle is slowed down. Unlike friction brakes, which turn kinetic energy into waste heat, regenerative braking changes it to speed up the flywheel for subsequent acceleration[1-4].

The University of Texas, Center for Electromechanics, developed and tested a high speed composite flywheel for an Advanced Technology Transit Bus. This flywheel operated at 40,000 r/min and could deliver 844 Wh at a power rating of 150 kW. Road testing of the bus revealed acceleration time to 75km/h was reduced by a factor of two with a simultaneous reduction in engine power of 25%,The design of a high performance flywheel energy storage system for use in a vehicle poses many challenges. In order to achieve an attractive specific energy (kWh/kg), it is necessary to construct the rotor from materials with high specific strength (ultimate stress/density), leading to selection of composite materials employing graphite fibers over metals. This allows a higher specific energy, and increases rotor tip speed. The high tip speed, in turn, leads to an enormous increase in parasitic windage loss. To reduce windage losses to acceptable levels requires spinning the rotor in a very tight vacuum[5-6].

This paper describes issues associated with rotor and bearing design technology in energy storage flywheels for vehicles, with particular emphasis on orientation of flywheel rotors, rotor geometries , magnetic bearings and differential equations of the magnetic suspended flywheel.

2 Orientation of Flywheel Rotors

The interaction between vehicle and flywheel dynamics produces many sources of bearing loads not present in a stationary application. The primary contributors to bearing loads are shown to be vehicle shock, vibration, maneuvering, and gyrodynamics[6].

Among these loads, gyroscopic loads occur when the rotor is precessed as the vehicle angular velocity changes, for instance cornering, driving over a hill or through a dip. There is no vehicle axis in which angular velocity is avoided so the bearings must withstand the gyroscopic loads. A spinning flywheel has a relatively large angular momentum so changing its spin axis requires significant torque, which must be produced by the bearings. The required torque is:

$$M = J\hat{\theta} \times \Omega \tag{1}$$

where J is the polar moment of inertia of flywheel rotor, is the spin speed of flywheel rotor, and $\hat{\theta}$ is the turning rate of the flywheel spin axis. On the basis

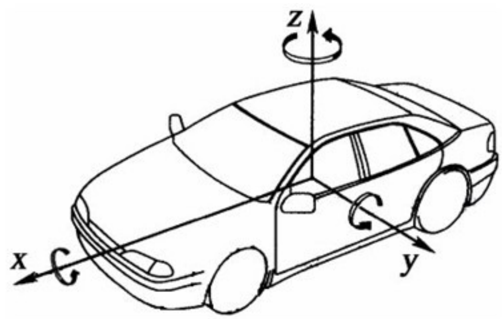


Fig.1 Coordinate system

of the coordinate system defined in Fig.1, (1) can be expressed as follow:

$$\begin{cases} M_x = J(\dot{\theta}_y \Omega_z - \dot{\theta}_z \Omega_y) \\ M_y = J(\dot{\theta}_z \Omega_x - \dot{\theta}_x \Omega_z) \\ M_z = J(\dot{\theta}_x \Omega_y - \dot{\theta}_y \Omega_x) \end{cases} \tag{2}$$

In vehicle operating conditions,yaw rate Ω_z is greater than roll rate Ω_x and pitch rate Ω_y .If the flywheel is oriented vertically, flywheel rotor and the yawing (turning) axis of the vehicle in the same direction, and $\dot{\theta}_x = \dot{\theta}_y = 0$,so

$$\begin{cases} M_x = -J\dot{\theta}_z \Omega_y \\ M_y = J\dot{\theta}_z \Omega_x \\ M_z = 0 \end{cases} \tag{3}$$

This means that the flywheel spin axis should be vertical.

In the case of a vehicle, the gyroscopic torque is too small to influence the motion of the vehicle. A way to reduce the impact is to employ two similar flywheels, each contra-rotating at the same speed. However, the torque is large enough to be a major contributor to loads on the radial bearings. In practice some mechanism or device must support the flywheel, isolating the flywheel from the motions of the bus and decrease the loads on the bearings, and allowing it to pitch and roll as freely as possible relative to the vehicle.

The usual method to rigidly mounting the flywheel housing to the vehicle is to mount it in a two-axis gimbal. Without the use of a gimbal, bearing loads induced by gyroscopic reactions to pitch and roll movements of the bus can significantly reduce bearing life and make the design impractical for long term use.

3 Rotor Design

In rotor design, there are mainly three fully-coupled design factors that have significant effect in the overall performance of flywheels, material strength, rotational speed and rotor geometry.

The kinetic energy stored in a flywheel is proportional to the mass and to the square of its angular velocity. It is given as

$$E_k = \frac{1}{2} I \omega^2 \tag{4}$$

where I is the mass moment of inertia and ω is the angular velocity. The moment of inertia for any object is a function of its shape and mass. It is obtained by the mass and geometry of the flywheel and given as,

$$I = \int x^2 dm_x \tag{5}$$

where x is the distance from rotational axis to the differential mass dm_x .

The way to increase the energy density and minimise the volume of the system is to use a flywheel with both a high rotational speed and a large inertia.
The speed limit is set by the tensile strength of the flywheel material. The stored energy density with respect to mass is given by:

$$e_m = K\sigma/\rho \tag{6}$$

where e_m is kinetic energy per unit mass, K is the shape-factor which relates the relative energy stored in the solid disk to that of a constant stress disk of infinite radius, σ is maximum stress in the flywheel and ρ is mass density.

In case of planar stress, if the height of the disk is small compared with the diameter, and a homogenous isotropic material with Poisson ratio of 0.3, i.e. s-teel, is used, the K factors are given in Table 1[7] .

In a three-dimensional flywheel there will be three-dimensional interaction of material stresses. For the flywheel design is based on a hollow cylinder and the outside radius is assumed to be large compared to the flywheel thickness, the two stresses of primary concern are the radial stress and the tangential stress. Table 2 presents characteristics for common rotor materials.

There are two basic classes of flywheels based on the material in the rotor. The first class uses a rotor made up of an advanced composite material such as carbon-fiber or graphite. These materials have very high strength to weight ratios, which give flywheels the potential of having high specific energy. The second class of flywheel uses steel as the main structural material in the rotor.

The highest tensile flywheels are not made of steel, but of fiber-reinforced composites. As well as rotating faster and storing more energy than steel flywheels, these composite flywheels are much safer if the maximum safe speed is exceeded, since they tend to delaminate and disintegrate gradually from the outer circumference rather than explode catastrophically.

The material at the outside diameter of the rotor is most effective in storing energy with energy storage of that material being proportional to square of the radius. The peripheral speed of the rotor should be as high as possible for maximum energy storage but this is limited by the stress levels the designer is willing to accept. For a chosen tip speed and rotor outer diameter, the rotational speed of the rotor is fixed and this also fixes the shape.

There are three flywheel geometries were developed to meet the energy storage and power requirement needs for vehicles, shown in Fig.2.

Table 1 Shape-factor K for different planar stress geometries

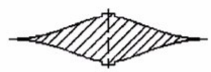
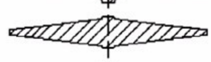
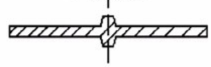
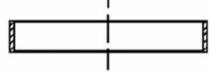
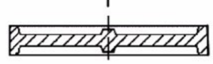
Fly wheel geometry	Cross section	Shape factor
Disc		1.000
Modified constant stress disc		0.931
Conical disc		0.806
Flat unpierced disc		0.606
Thin firm		0.500
Rim with web		0.400
Flat pierced bar		0.305

Table 2 2 Data for different rotor materials

Material	Density (kg/m^3)	Tensile strength (MPa)	Max energy density (for 1 kg)
Monolithic material 4340 Steel	7700	1520	0.19 MJ/kg = 0.05 kWh/kg
Composites			
E-glass	2000	100	0.05 MJ/kg = 0.014 kWh/kg
S2-glass	1920	1470	0.76 MJ/kg = 0.21 kWh/kg
Carbon T1000	1520	1950	1.28 MJ/kg = 0.35 kWh/kg
Carbon AS4C	1510	1650	1.1 MJ/kg = 0.30 kWh/kg

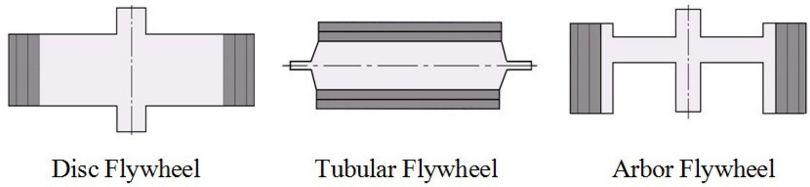


Fig.2 Flywheel geometry

Disc flywheel is based on more traditional flywheel designs. The mass of the steel hub provides the main source of inertial energy storage in the system. The profile of the steel hub is based on modified equations for a constant stress disc which include a rim section for an increased radius of gyration.

As long as the rotor speed is within acceptable limits and there are no rotor dynamics issues, a long tubular flywheel rotor is the preferred option for a number of reasons as opposed to a disc shaped flywheel. The flywheel designs were denoted by the name "PowerBeams" for practical commercial application[8-12].

An arbor incorporates an inside out topography for the motor-generator. The inside-out topography makes better use of the available space and increases the specific energy and power densities of the design. The arbor flywheel shows significant advantages with respect to system size and energy storage capability[5].

4 Bearing Design , Prototype and Dynamic Equations

4.1 Bearing Design

The spinning flywheel rotor must be supported on bearings. Initially, both mechanical bearings and magnetic bearings were considered. The important parameters in assessing the use of bearings are weight, loss, cost, lifecycle life, and low losses. They also can isolate rotor and stiffness.

If the rotor speed is within acceptable limit, mechanical bearings are ideal in that they can operate with low losses and have high life for the average load yet can accept high loads on an intermittent basis several times the average load.

Due to the high friction and short life, mechanical bearings cannot be adapted to modern high-speed flywheels. Mechanical bearings have benefited greatly from material advances such as ceramics and very hard steels. The main life issues are not material fatigue life, but rather lubricant life. Lubricant life depends primarily on temperature.

Instead magnetic bearing system is utilized, including permanent bearings, active magnetic bearings (electromagnetic bearings) and high temperature superconducting (HTS) bearings. Magnetic bearings do not have any contact with the shaft, has no moving parts, experience little wear and require no lubrication.

Active magnetic bearings present major advantages in terms of lifetime and rotational speed, and also favorably integrate into high-speed flywheel systems.

Unlike active magnetic bearings, the HTS magnetic bearing can situate the flywheel automatically without need of electricity or positioning control system. However, HTS magnets require cryogenic cooling by liquid nitrogen. It is not suitable for vehicles.

Due to the higher magnetic flux density reached by Nd-Fe-B magnets and their low cost, applications with permanent magnetic bearings have become attractive, in spite of being very unstable. Therefore, permanent magnetic bearings can be used as an auxiliary bearing to reduce the load weight of the rotor and the flywheel and to increase the stiffness of the whole bearing system.

4.2 Prototype of Flywheel

A flywheel energy storage system prototype with an arbor flywheel and a hybrid bearing set is shown in Fig.3.

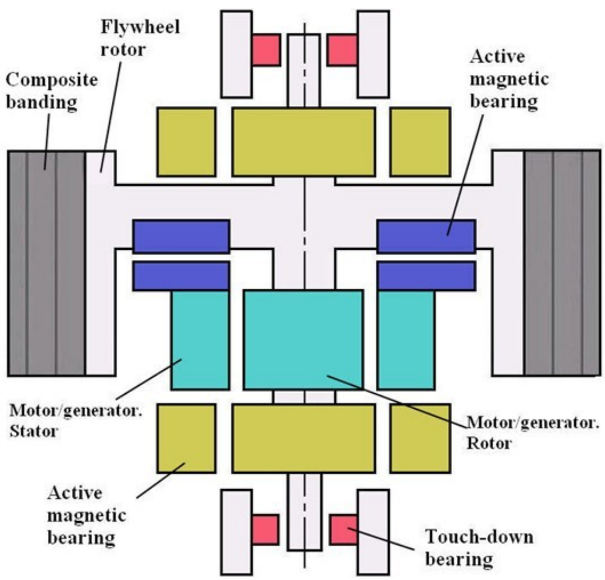


Fig.3 Flywheel geometry

According to the above discussion, high speed is desirable since the energy stored is proportional to the square of the speed but only linearly proportional to the mass. Magnetic bearings can accommodate very high spin speeds and have theoretically unlimited imbalance induced vibrations. Magnetic bearings offer very low friction enabling low internal losses during long-term storage. So, our design of a flywheel system using the magnetic bearing consists of a vertical arbor flywheel rotor, permanent magnetic bearings, and an active magnetic bearing. The flywheel axial stability is actively controlled by the active magnetic bearing while the motions in other directions are restricted by other two pairs of active magnetic bearings. A motor/generator is located in the center region of the arbor flywheel rotor.

4.3 Dynamic Equations of Flywheel

Active magnetic bearing usually use differential excitation. It uses a pair of symmetrical power amplifier circuit to drive electromagnet in differential model and get a pair of magnetic force in opposite direction. Magnetic force of rotor is the difference value between upper magnets and lower magnets. Assuming disturbance and control current are very small, in accordance with the Taylor series expansion the magnetic force in static working point can be expressed as:

$$f_i = k_y y_i + k_i i_i \tag{7}$$

Here,

$$k_y = \frac{\mu_0 A_0 N^2 {i_0}^2}{y_0^3}\ ,\ k_i = \frac{\mu_0 A_0 N^2 i_0}{y_0^2} \tag{8}$$

Where

f_i = total magnetic force, its direction and the y positive direction are consistent

k_y = displacement rigidity coefficient of magnetic bearing

k_i = current rigidity coefficient

y_i = current rigidity coefficient

i_i = displacement according to the balance position of rotor, its positive direction is upward

i_0 = offset current

y_0 = air-gap in balance position

μ_0 = air magnetic permeability

A_0 = area of electromagnet pole

N = number of coil winding turns

Equation (7) is the linear model of resultant force in small deviation range.With the increase of distance of balance point, the precision of (7) is decrease. In some limit state, such as rotor contact with stator, strong current (iron-core saturation) or weak current in winding, (7) is incongruity.

We consider the base as stationary firstly. Inertial Reference Frames and Flywheel Reference Frames were set up. Fig.4 depicts the coordinate system and forced diagram.

f_i = magnetic force of radial direction

f_z = magnetic force of radial direction

The centroid of flywheel is (x_c, y_c, z_c) .The rotation angles of rotor in yz and xz plane are θ_x and θ_y .

Differential equations of the magnetic suspended flywheel are:

$$\begin{cases} m\ddot{x}_c = f_1 + f_3 \\ m\ddot{y}_c = f_2 + f_4 \\ m\ddot{z}_c = f_z - mg \\ J\ddot{\theta}_x = -J_z\omega\dot{\theta}_y + h_1 f_4 - h_2 f_2 \\ J\ddot{\theta}_y = J_z\omega\dot{\theta}_x + h_2 f_1 - h_1 f_3 \\ T_0 = T \end{cases} \tag{8}$$

T_0 is motor torque. The above differential equations of the 5-DOF magnetic suspended flywheel can decouple into a single DOF differential equation and a 4-DOF differential equation. The two equations can be separated into axial and radial direction and solved respectively. In this way, control system in axial magnetic bearing can be considered as a single DOF magnetic suspension control system. Control system in radial magnetic bearing can be considered as a multi-DOF magnetic suspension control system.

Because θ_x and θ_y are small, the displacements may be defined as follows:

$$\begin{cases} y_1 = x_c + h_2\theta_y \\ y_2 = y_c - h_2\theta_x \\ y_3 = x_c - h_1\theta_y \\ y_4 = y_c + h_1\theta_x \end{cases} \tag{9}$$

We have

$$q_{(t)} = [x_c, y_c, \theta_x, \theta_y]'$$
$$i_{(t)} = [i_1, i_2, i_3, i_4]'$$

Carrying (7) and (9) into (8) , we obtain:

$$\begin{cases} \ddot{q} = P_1\dot{q} + P_2 q + P_3 i \\ y = P_4 q \end{cases} \tag{10}$$

in which

$$P_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{J_z}{J} \\ 0 & 0 & \frac{J_z}{J} & 0 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} \frac{2k_y}{m} & 0 & 0 & \frac{k_y h_2 - k_y h_1}{m} \\ 0 & \frac{2k_y}{m} & \frac{k_y h_1 - k_y h_2}{m} & 0 \\ 0 & \frac{k_y h_1 - k_y h_2}{J} & \frac{k_y {h_1}^2 + k_y {h_2}^2}{J} & 0 \\ \frac{k_y h_2 - k_y h_1}{J} & 0 & 0 & \frac{k_y {h_1}^2 + k_y {h_2}^2}{J} \end{bmatrix}$$

$$P_3 = \begin{bmatrix} \frac{k_i}{m} & 0 & \frac{k_i}{m} & 0 \\ 0 & \frac{k_i}{m} & 0 & \frac{k_i}{m} \\ 0 & \frac{-k_i h_2}{J} & 0 & \frac{k_i h_1}{J} \\ \frac{k_i h_2}{J} & 0 & \frac{-k_i h_1}{J} & 0 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 1 & 0 & 0 & h_2 \\ 0 & 1 & -h_2 & 0 \\ 1 & 0 & 0 & -h_1 \\ 0 & 1 & h_1 & 0 \end{bmatrix}$$

We can set $z = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$

Equation (10) then becames

$$\begin{cases} \dot{z} = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ P_2 & P_1 \end{bmatrix} z + \begin{bmatrix} 0 \\ P_3 \end{bmatrix} i \\ y = \begin{bmatrix} P_4 & 0 \end{bmatrix} z + [0]\, i \end{cases} \tag{11}$$

If flywheels is set up on vehiclesthe dynamic equations are given by the expression

$$\begin{cases} m(\ddot{x}_c + \ddot{x}_{car}) = f_1 + f_3 \\ m(\ddot{y}_c + \ddot{y}_{car}) = f_2 + f_4 \\ m(\ddot{z}_c + \ddot{z}_{car}) = f_z - mg \\ J(\ddot{\theta}_x + \ddot{\theta}_{xcar}) = -J_z\omega\dot{\theta}_y + h_1 f_4 - h_2 f_2 \\ J(\ddot{\theta}_y + \ddot{\theta}_{ycar}) = J_z\omega\dot{\theta}_x + h_2 f_1 - h_1 f_3 \\ T_0 = T \end{cases} \tag{12}$$

Where

$\ddot{x}_{car}, \ddot{y}_{car}and\ddot{z}_{car}$ are accelerations of different directions of vehicle.

$\ddot{\theta}_{xcar}$and$\ddot{\theta}_{ycar}$ are angular acceleration of x and y directions of vehicle.

$$\begin{aligned} &(m + m_{car})\ddot{x}_{car} = F_{xcar} \\ &m_{car} \gg m \\ &\ddot{x}_{car} = \frac{F_{xcar}}{m_{car}} \end{aligned}$$

And equation (11) then results in

$$\begin{cases} m\ddot{x}_c = f_1 + f_3 - \frac{m}{m_{car}} F_{xcar} \\ m\ddot{y}_c = f_2 + f_4 - \frac{m}{m_{car}} F_{ycar} \\ m\ddot{z}_c = f_z - mg - \frac{m}{m_{car}} F_{zcar} \\ J\ddot{\theta}_x = -J_z\omega\dot{\theta}_y + h_1 f_4 - h_2 f_2 - \frac{J}{J_{car}} M_{xcar} \\ J\ddot{\theta}_y = J_z\omega\dot{\theta}_x + h_2 f_1 - h_1 f_3 - \frac{J}{J_{car}} M_{ycar} \\ T_0 = T \end{cases} \tag{13}$$

Here,

M_{car} = mass of vehicle.

$F_{xcar}, F_{ycar}, F_{zcar}$ = applied forces of of different directions of vehicle, F_{zcar} is connected with road surface profile spectral excitations.

The differential equations of the magnetic suspended flywheel can be used for the design of control system.

5 Conclusion

In this paper, we presented a prototype of flywheel energy storage system for a vehicle. The prototype is viable and can achieve perfect robustness. More improvements in material, magnetic bearings and power electronics make flywheels a competitive choice for vehicle energy storage applications. The use of composite materials, Optimized rotor shape and magnetic bearings enable high rotational velocity with power density greater than that of chemical batteries. Finally, we deduce differential equations of the magnetic suspended flywheel for the design of control system.

Acknowledgements

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