The Stability of Milling of Thin-walled Workpiece

Tongyue Wang[1,2], Ning He[1], Liang Li[1] and Dahu Liu[1]
[1]*Nanjing University of Aeronautics and Astronautics, Nanjing, 210016*
[2]*Huaiyin Institute of Technology, Huaian, 223003*

Abstract
In order to control the cutting chatter in machining of thin-walled workpieces, the dynamic milling model of thin-walled workpieces is analyzed and built based on the analysis of degrees in two perpendicular directions of tool-workpiece system. In high speed milling of 2A12 aluminum alloy, the compensation method based on the modification of inertia effect was proposed and accurate cutting force coefficients were obtained. Modal parameters of tool-workpiece system were acquired via modal analysis tests. The stable lobe for high speed milling of 2A12 aluminum alloy thin-walled workpieces and limit cutting axial-depth at different cutting radial-depth were obtained. The results were verified with cutting tests. The method can be used in milling of thin-walled workpieces to select cutting parameters properly. All these work lay a reliable foundation to the further studies on the cutting chatter rules of thin-walled workpieces.
Keywords Thin-walled workpiece, Cutting chatter, Dynamic milling model, Inertia modification, Stable lobe, Limit cutting depth

1 Introduction

With the structural properties of light weight, high strength et al, thin-walled workpiece has been used widely in many fields such as aeronautics & astronautics, mold & die manufacturing. Because of the inherent poor rigidity, complicated structures, large metal allowance and bad processing properties, the milling of thin-walled components is difficult for cutting deformation and vibration.

A noted previous research on end milling of thin-walled structures was carried out in Ref.[1]. A flexible thin-walled rectangular plate was clamped on three edges (CCCF) was assumed, the effect of the deflection on the chip load and cutting geometry was not considered in the force calculation. Sutherland and Devor[2] presented an improved model to take into account the effect of the deflection on the chip load. A dynamic model for milling of a very flexible cantilever plate with rigid end mill by neglecting the time varying structural properties and the changes in the immersion boundaries was built by Altintas *et al.*[3]. Budak and Altintas went forward one-step by considering the milling of a flexible cantilever plate with slender end mills that incorporate with the mechanistic force model and finite element methods[4]. Lim *et al.* developed a mechanistic force model for predicting the machining errors caused by tool deflection[5]. Budak considered the dynamic model of milling of thin-walled workpiece as a MDOF(Multi-Degree

Of Freedom) system in[6]. Duncan *et al.* proved the FRF (Frequency Response Function) of the tip and error for measuring force coefficients had influenced the prediction accuracy of stability limit[7]. Although these models are very useful to analyze the milling of thin-walled structures, the machining of thin-walled workpiece is limited due to the non-liner dependency between the forces and the continuously changing tool immersion angle and chip thickness.

In this paper, considering the degrees in two perpendicular directions of tool-workpiece is analyzed and built. The authors proposed the compensation method based on the modification of inertia effect. Theoretical analysis to high speed milling stability of thin-walled workpiece is carried out via high speed milling tests and modal analysis tests. The stable lobe and limit cutting axial-depth at different cutting radial-depth for high speed milling of 2A12 aluminum alloy thin-walled workpieces are obtained.

2 Dynamic Milling Model of Thin-walled Workpiece

Milling of thin-walled workpiece is usually expressed by the dynamic milling model as shown in Figure 1, the cutter and the workpiece are modeled as two-degree-of freedom structures, respectively. After the first revolution, the cutter starts leaving a wavy surface behind because of the bending vibration of the cutter in the normal direction, which is the direction of radial cutting force. When the second revolution starts, the wavy surface causes the tool-workpiece system fluctuating. Hence, the resulting dynamic chip thickness is no longer constant. The general dynamic chip thickness can be divided into two parts, one is the intended static chip thickness and the other is the dynamic chip thickness produced owing to vibrations at the present time and one spindle revolution period before. It can be expressed as follows:

$$h(\phi_j) = [f_z \sin\phi_j + (\upsilon_{j,0} - \upsilon_j)]g(\phi_j) \tag{1}$$

where f_z is the feed rate per tooth (mm/rev-tooth),ϕ_j is the instantaneous angular immersion of tooth j, and $(\upsilon_{j,0} - \upsilon_j)$ are dynamic offsets of the cutter at the previous and present tooth periods, respectively. The function $g(\phi_j)$ is a unit step function, which is used to decide the cutting status of the cutter tooth. If $g(\phi_j) = 1$, the cutter is in cutting, and $g(\phi_j) = 0$, the cutter is out of cutting. Henceforth, the static component of the chip thickness $f_z \sin\phi_j$ can be dropped from the above equation because it does not contribute to the regeneration dynamic chip thickness. Then, the chip thickness can be written in terms of the fixed coordinate system x and y (see Fig.1) as follows:

$$h(\phi_j) = [\Delta x \sin\phi_j + \Delta y \cos\phi_j]g(\phi_j) \tag{2}$$

where $\Delta x = x - x_0$ and $\Delta y = y - y_0$. Here (x, y) and (x_0, y_0) represent the dynamic offsets of the cutter at the present and previous tooth periods, respectively.

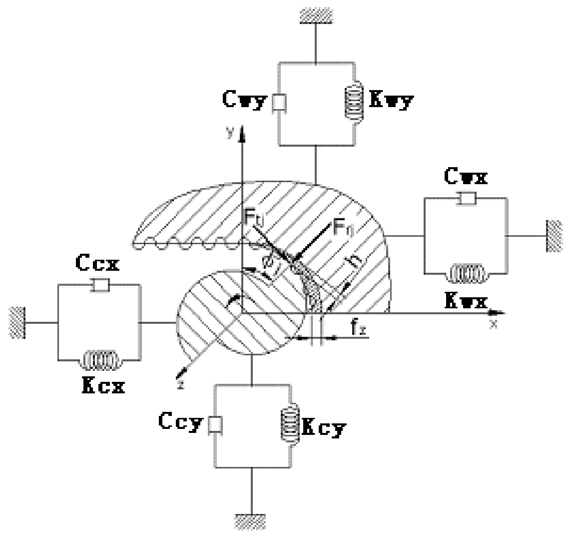


Fig.1 Dynamic milling model of thin-walled workpieces

The tangential and radial cutting forces acting on the tooth are proportional to the chip thickness and the axial depth of cut

$$F_{tj} = K_t a h(\phi_j), F_{rj} = K_r F_{tj} \tag{3}$$

where K_t is the tangential milling force coefficient which is experimentally determined for a tool-workpiece material pair,K_r is the ratio of radial force coefficient to tangential force coefficient, a is the cutting width or axial depth of cut. Resolving the cutting forces in the x and y directions and summing the cutting forces contributed by all teeth, the milling forces formulate can be built.

The dynamic milling forces can be expressed in matrix form:

$$\begin{pmatrix} F_x \\ F_y \end{pmatrix} = \frac{1}{2} a K_t \begin{pmatrix} \alpha_{xx} & \alpha_{xy} \\ \alpha_{yx} & \alpha_{yy} \end{pmatrix} \begin{pmatrix} \Delta_x \\ \Delta_y \end{pmatrix} \tag{4}$$

where time-varying directional force coefficients are given by:

$$
\begin{aligned}
\alpha_{xx} &= \sum_{j=1}^{N-1} -g_j[\sin 2\phi_j + k_r(1-\cos 2\phi_j)] \\
\alpha_{xy} &= \sum_{j=1}^{N-1} -g_j[(1+\cos 2\phi_j) + k_r \sin 2\phi_j] \\
\alpha_{xy} &= \sum_{j=1}^{N-1} -g_j[(1-\cos 2\phi_j) - k_r \sin 2\phi_j] \\
\alpha_{yy} &= \sum_{j=1}^{N-1} -g_j[\sin 2\phi_j - k_r(1+\cos 2\phi_j)]
\end{aligned} \tag{5}
$$

As the tool rotates, the directional force coefficients vary with time, they can be expanded into Fourier series. Take the average component of the Fourier series expansion, the directional force coefficients are written as:

$$
\begin{aligned}
\alpha_{xx} &= \frac{1}{2}[\cos 2\phi - 2k_r\phi + k_r \sin 2\phi]_{\phi_{st}}^{\phi_{ex}} \\
\alpha_{xy} &= \frac{1}{2}[-\sin 2\phi - 2k_r\phi + k_r \cos 2\phi]_{\phi_{st}}^{\phi_{ex}} \\
\alpha_{yx} &= \frac{1}{2}[-\sin 2\phi + 2k_r\phi + k_r \cos 2\phi]_{\phi_{st}}^{\phi_{ex}} \\
\alpha_{yy} &= \frac{1}{2}[\cos 2\phi - 2k_r\phi - k_r \sin 2\phi]_{\phi_{st}}^{\phi_{ex}}
\end{aligned} \tag{6}
$$

where N is the cutter tooth number, ϕ_{st} and ϕ_{ex} are the entry and exit angles of a tooth, respectively.

The transfer function matrix $[\Phi(iw)]$ of the milling system is:

$$
[\Phi(iw)] = \begin{pmatrix} \Phi_{xx}(iw) & \Phi_{xy}(iw) \\ \Phi_{yx}(iw) & \Phi_{yy}(iw) \end{pmatrix} \tag{7}
$$

where $\Phi_{xx}(iw)$ and $\Phi_{yy}(iw)$ are the direct transfer functions in the x and y directions, and $\Phi_{xy}(iw)$ and $\Phi_{yx}(iw)$ are the cross transfer functions. Considering the vibration at the present and previous tooth period, system equation at the chatter frequency w_c in the frequency domain can be written as:

$$
[F]e^{iw_ct} = \frac{1}{2}aK_t[1-e^{iw_ct}][A_0][\Phi(iw_c)][F]e^{iw_ct} \tag{8}
$$

where A_0 is the directional milling matrix.

3 Principle of Dynamic Force Coefficient Measurement and Inertia Modification

A force measuring chain is functionally illustrated in Fig.2. In milling, the cutting force produced between the cutter and workpiece acts on the workpiece and is transmitted to the dynamometer. The cutting force leads to a deformation of the piezozlectric sensors of the dynamometer, which produce electric charges in correspondence with the deformation of the sensors. The electric charges are processed into a data file which reflects the size of cutting force following several steps of amplifier and filter. The validity of the acquired force data depends not only on the precision of the hardware devices used and their parameter settings, but also the dynamic effect of the dynamometer itself, particularly when measuring forces in high speed milling. In high speed milling, the tooth passing frequency and/or the high harmonic frequency components induced from the impact effect of the milling cutter are often in the neighborhood of the natural frequencies of the dynamometer. So the vibration of the structure including the dynamometer is exaggerated in the output signals and the measured force signals are damaged.

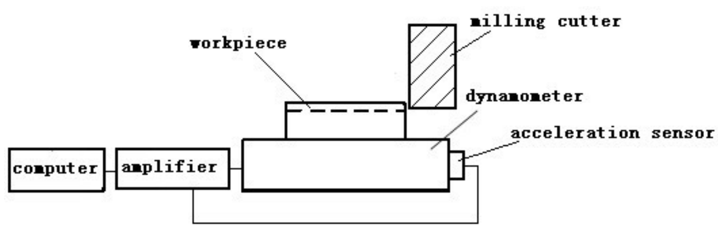


Fig.2 Diagram of force measuring principle

The structure including the dynamometer and workpiece can be treated as typical spring-damper-mass model. To obtain the accurate cutting force data in high speed milling, compensation of the inertia forces is necessary to improve the measurement results. It can be modified as follows:

$$\vec{F}=\vec{F_0}-\vec{F_i} \tag{9}$$

where $\vec{F}$ is the modified cutting force (N), $\vec{F_0}$ is the directly measured cutting force (N), $\vec{F_i}$ is the inertia force (N). The inertia force $\vec{F_i}$ can be written as $\vec{F_i}=(m_e+m_w)a$, m_e is the equivalent mass of the dynamometer (kg), m_w is the mass of workpiece (kg), and a is the measured acceleration(m/s^2).

Following the quick mechanistic method of calibrating and the above inertia modification approach, a set of high speed milling experiments are conducted at following cutting conditions:

Workpiece: 2A12 aluminum alloy

Equipment: Mikron UCP 710 high speed machining center, Kistler 9265B dynamometer, KD1001A acceleration sensor

Tool: YG813 carbide tipped tool, two flutes, diameter=20mm

Test parameters: slotting, axial depth of cut=1mm, radial depth of cut=3mm, spindle revolution=10000 rev/min, feed rate per tooth changes from 0.01mm/z to 0.055mm/z at 0.005mm/z increment

After treating the experimental results, the values of force coefficients can be worked out: K_t=4252Mpa,K_r=0.858.

4 Stability Analysis of Milling of Thin-walled Workpiece[3,4,6]

4.1 The Limit Axial Depth of Cut

After solving the characteristic equation (8) of dynamic milling system, the eigenvalue is given as:

$$\Lambda = -\frac{1}{2a_0}(a_1 \pm \sqrt{{a_1}^2 - 4a_0}) \tag{10}$$

$$where \begin{cases} a_0 = \Phi_{xx}(iw_c)\Phi_{yy}(iw_c)(\alpha_{xx}\alpha_{yy} - \alpha_{xy}\alpha_{yx}) \\ a_1 = \alpha_{xx}\Phi_{xx}(iw_c) + \alpha_{yy}\Phi_{yy}(iw_c) \end{cases} \tag{11}$$

In [3,4,6] , Altintas and Budak presented the solution of stability is in detail and provided the limit depth of cut and spindle speed, *i.e.* stability lobes as:

$$a_{lim} = -\frac{2\pi\Lambda_R}{NK_t}(1 + K^2) \tag{12}$$

$$n = -\frac{60w_c}{N(\varepsilon + 2k\pi)} \tag{13}$$

where $\kappa = \frac{\sin w_cT}{1-\cos w_cT}$, Λ_R is the real part of the eigenvalue, $\Psi = \arctan\kappa$ is the phase shift of the eigenvalue, $\varepsilon = \pi - 2\Psi$ is the phase shift between the current chatter mark and the previous chatter mark.

4.2 End Milling with a Flexible Cutter

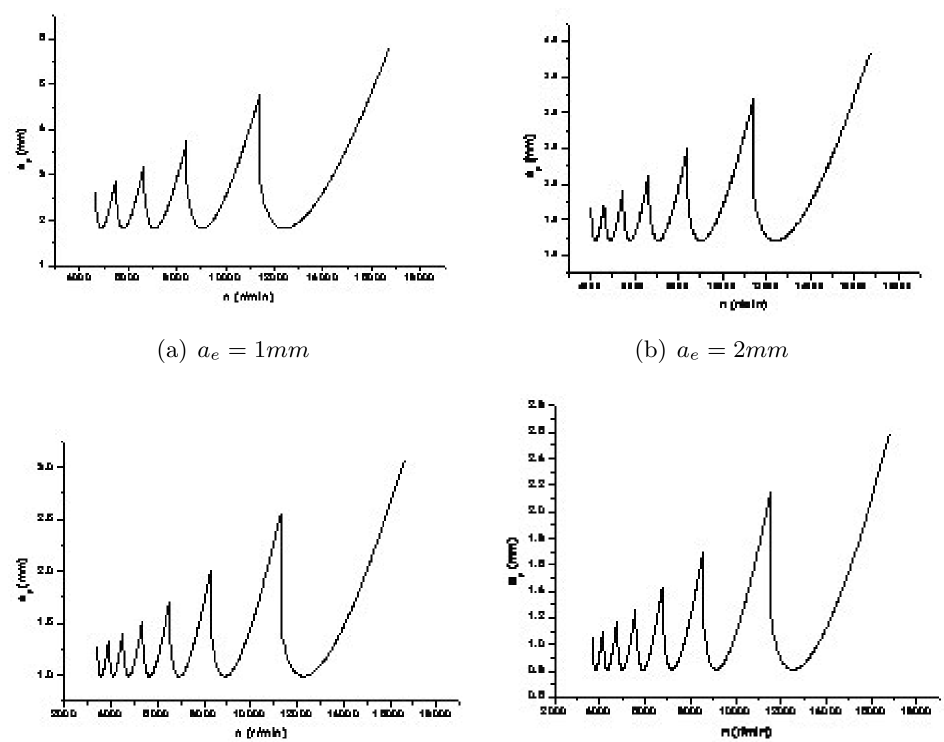
A above mentioned YG813 carbide tipped tool with 2 flutes, 20mm diameter is used in end milling of aluminum alloy 2A12. The gage distance is 150mm from the collet.

The transfer function of the cutter attached to the spindle is measured in both feed and normal directions with an impact hammer instrumented with a PCD208C02 piezoelectric force transducer and a B&W22100 acceleration sensor. The modal parameters such as modal mass, modal dampness etc are identified from modal analysis software uTekLMa and are given in Table 1.

Later the modal parameters are used to simulate the stability lobes for milling of 2A12 alloy. Four kinds of radial depth of cut are adopted. The results are given in Fig.3. The region above the curve is unstable cutting region and under the curve is stable cutting region.

Table 1 Identified modal parameters

direction	modal mass $m(Kg)$	dampness $c(N*s/m)$	stiffness $k(N/m)$	nature frequency $\omega_c(Hz)$
tangential	1.37	533.1	1.67e+7	556
radial	1.32	510.0	1.57e+7	550



(a) $a_e = 1mm$ (b) $a_e = 2mm$

(c) $a_e = 3mm$ (d) $a_e = 4mm$

Fig.3 Stability lobe

4.3 Analysis of Experimental Results of Varying Axial Depth of Cut

Two sets of varying axial depth of cut experiments are carried out to investigate the influence of axial depth of cut to cutting stability. The principle scheme is given in Fig.4. The 2A12 aluminum alloy workpiece with 3^0 inclined angles is adopted in order to increase or decrease the axial depth of cut during the milling period. The above mentioned YG813 carbide tipped tool with 2 flutes, 12mm diameter is used with the 150mm gage distance measured from the collet. The milling stability is studied by analyzing the radial cutting force, which influences the stability most. During the milling period, the 3mm radial depth of cut, 10000 rev/min spindle revolution, 0.01mm/z feed rate per tooth are kept unchanged. The axial depth of cut changes from 0.1mm to 3.98mm and from 3.98mm to 0.1mm, respectively. The radial cutting force results measured via the Kistler 9265B dynamometer are shown in Fig.5. Although the varying way is different, the radial cutting force fluctuated severely at about 2mm axial depth of cut together. The limit axial depth of cut seems to be about 2mm at 3mm radial depth of cut, 10000 rev/min spindle revolution. Obviously, the measured results are in very close agreement with the results of stability lobe (see Fig.3(c)). The stability lobes presented a reasonable range for selecting the cutting parameters.

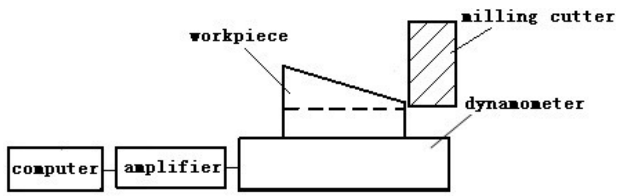
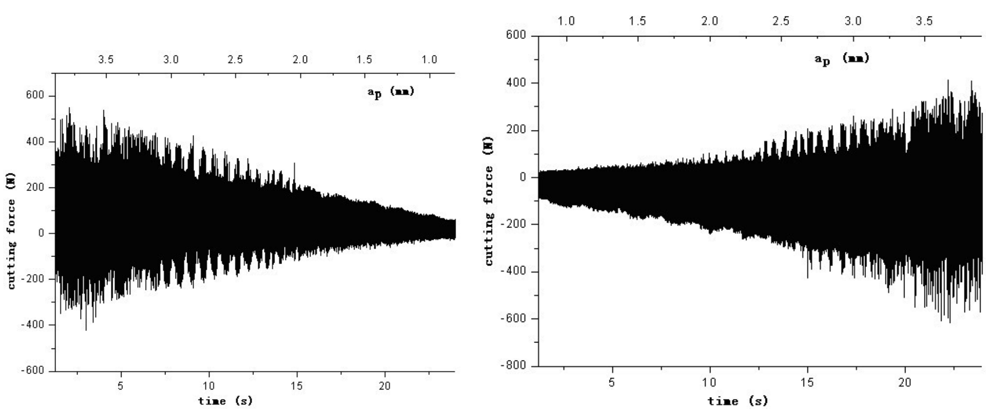


Fig.4 Diagram of varying axial depth of cut test principlee



(a) axial depth of cut, 0.1mm to 3.98mm (b) axial depth of cut, 3.98mm to 0.1mm

Fig 5 Radial cutting force of varying cutting-depth test

5 Conclusions

On the basis of analysis of degrees in two perpendicular directions of tool-workpiece system, the dynamic milling model of thin-walled workpieces is analyzed and built. The compensation method based on the modification of inertia effect is proposed and accurate cutting force coefficients are obtained through high speed milling test. Modal parameters of tool-workpiece system are acquired via modal analysis tests. The modal parameters and cutting force coefficients are used to simulate the stability lobe at 4 cutting radial depth of cut for high speed milling of 2A12 aluminum alloy thin-walled workpieces. The results are verified with varying axial depth of cut tests. All these work lay a reliable foundation to the further studies on the high speed milling stability of thin-walled workpiece.

Acknowledgements
The authors greatly appreciate the financial support provided by the National Natural Science Foundation of China (No.10477008), the Foundation of Education Department of Jiangsu Province (No.07KJB460008) and the Research Foundation of DML-HYIT(HGDML-0801).

References

[1] W.A. Kline, R.E. Devor, J.R. Lindberg. (1982), The prediction of cutting forces in end milling with application to cornering cuts, *International Journal of Machine Tool Design and Research,* Vol.22, No.1, pp.7-22.

[2] J.W. Sutherland, R.E. DeVor. (1986), An improved method for cutting force and surface error prediction in flexible end milling systems, *ASME Journal of Engineering for Industry,vol.* Vol.108, No.B-4, pp.269-279.

[3] Y. Altintas, E. Budak. (1995), Analytical prediction of stability lobes in milling, *Annals of the CIRP,* Vol.44, No.1, pp.357-362.

[4] E. Budak and Y. Altintas. (1998), Analytical prediction of chatter stability in milling. Part II:Application of the general formulation to common milling systems. Trans, ASME Journal of Dynamci Systems, *Measurement and Control,* Vol.120, No.1, pp.22-36.

[5] E.M. Lim, H. Feng, C. Menq, Z. Lin. (1995), The prediction of dimensional error for sculptured surface productions using the ball-end milling process. Part 1: chip geometry analysis and cutting force prediction, *International Journal of Machine Tools Manufacture,* Vol.35, No.8, pp.1149-1169.

[6] E. Budak. (2006), Analytical models for high performance milling. Part I: cutting forces, structural deformations and tolerance integrity, *International Journal of Machine Tools and Manufacture,* Vol.46, No.12-13, pp.1478-1488.

[7] G.S. Duncan, M. Kurd, T.L. Schmitz. (2006), Uncertainty propagation for selected analytical milling stability limit analyses, *Transactions of NAMRI/SME,* Vol.34, No.1, pp.17-24.