Static Models of Corruption in Hierarchical Systems

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Abstract
The principles of modeling and control of corruption in the hierarchical systems are formulated. A system of the theoretical static models of administrative and economic corruption as well as models of corruption in real estate development is built. The dependence of corrupted behavior on model parameters is investigated, and the analytical conditions in which corruption is not profitable for the agent or can be controlled by the principal for ensuring sustainable development of the system are received.
Keywords corruption, hierarchical systems, game theory, optimization, sustainable development

1 Introduction

The pioneering work on the mathematical modeling of corruption is Susan Rose-Ackerman's paper [1] with the consequent development of the proposed approach in her monograph [2]. The monograph specifies the ideas by Gary Becker [3] concerning the modeling of arbitrary crimes and punishments.

Later the static models of corruption were developed in two principal directions. The first one includes mathematical models that investigate corruption inside an organization (internal corruption) and between organizations (external corruption). Within the framework of this class of models the problems of interaction of competence and corruption, non-optimal resource distribution, the principal's impact to the model equilibrium and some other ones are studied. The paper [4] may be taken as an example in which the problems of federal resources distribution by bureaucrats and the forms of impact of the competence between both the bureaucrats and the agents on the bribery scope are investigated. Models of corruption in tax authorities also belong to this class, for example, [5-6].

The second research direction is connected with modeling of corruption in social and political life (particularly the corruption patterns presented in the electoral process). Models of the principal - agent type that analyze such problems as anti-corruption incentives development, building of decision making rules that facilitate the indication of bribery facts, providing the hierarchical structure and control mechanisms for the principal that compel or impel the agent to abstain from corruption. One of the principal papers in this research domain is [7] that describes the social costs from corruption in the bureaucratic chain of an arbitrary length with consideration of the competence between agents throughout their hierarchy. The paper by M.Bac [8] continues the investigations started by F.Kofman and J.Lawarree [9] and targeted to the analysis of corruption in the hierarchy principal - controller - agent. As an extension of the basic model M.Bac has built a derivative model in which a hierarchy of homogeneous agents is considered instead of one agent. The papers [10-12] also refer to this class.

This research direction includes also the works dealing with studies of the political-economic corruption, especially in the voting systems [13-15]. Problems of endemic corruption are discussed in the papers [16-17]. In [18] a game theoretic model predicts that resource rents tend to an increase in corruption if the quality of the democratic institutions is relatively poor, but not otherwise. The paper [19] investigates the role of guilt aversion in public administration.

The authors' approach to the modeling of corruption is represented in [20-21]. Its essence consists in the following principles.

2 Principles of Modeling

Twelve principles are used in building of the models of corruption.

1). The basic modeling pattern is a hierarchical structure "principal - supervisor - agent - object" in different modifications and its investigation by means of the optimization theory and Stackelberg games theory. In the dynamic models the state of the object is described explicitly while in the static models only the impact of the agent to the object is considered. The supervisor may be corrupted, while the principal is not corrupted and controls corruption. So the elements of the above structure are bribe-controller, bribe-taker, bribe-giver and object of impact respectively.

2). Certain requirements of the sustainable development of the controlled system (object) are supposed to be known. In the dynamic models they are formulated in terms of the object's state while in the static ones - in terms of the agent's impact to the object. If the requirements of sustainable development are satisfied then the principal's control problem is assumed to be solved even if corruption exists.

3). Both pairs "principal - supervisor" and "supervisor - agent" are in the relations "leader - follower". The leading player on any level (principal or supervisor respectively) uses methods of compulsion (administrative or legislative impacts) or impulsion (economic impacts) for achievement of his/her objectives. The mathematical formalization of compulsion means an impact of the leader to the set of admissible strategies of the follower (as a rule, without a feed back), and impulsion means the impact to the follower's payoff function (as a rule, with a feed back) [20].

4). The cases of administrative corruption when administrative requirements or constraints are weakened for a bribe and the economic corruption when the economic ones are weakened by corruption are differentiated. The model of administrative corruption describes compulsion of the agent by the supervisor with a feed back on bribe, and the model of economic corruption describes impulsion in that pair with an additional feed back on bribe.

5). Corruption threatens to the object's sustainable development because it is profitable for the bribe-taker to weaken the requirements of sustainable development in exchange for a bribe. From the other side, corruption is a specific form of a feedback in the hierarchical systems subject to which the control variables become the functions of bribe.

6). Corruption exists in the forms of capture and extortion. In the case of capture a basic set of administrative or economic services is guaranteed while additional indulgences are provided for a bribe. In the case of extortion a bribe is required already for the basic set of services, otherwise the requirements are enhanced.

7). The bribe-taker's behavior is characterized by tractability (a willingness to weaken the administrative or economic constraints in exchange for a bribe) and greed (a price of the weakening). A set of quantitative indicators of tractability and greed is developed.

8). The bribe may represent a part of the payoff received by the agent subject to the administrative or economic indulgences or an absolute sum. In both cases it is convenient to suppose the bribe variable to be a part of the corrupted payoff or the whole agent's income respectively.

9). For studying corruption in hierarchical control systems with consideration of the requirements of sustainable development both descriptive and normative approaches are applicable. In the case of descriptive approach the functions of administrative and economic corruption are given, and the main problem is to identify their parameters on statistical data. In the case of normative approach the corruption (bribery) function is found as the solution of an optimization or game theoretic problem.

10). The investigation of corruption in the system "principal-supervisor-agent-object" is possible from three points of view. If the bribery function is known then from the point of view of the agent the corruption can be described by an optimization model. From the supervisor's point of view a hierarchical parametrical game of the class Γ_2 arises which solution in the form of bribery function with a feedback on the value of bribe is known in a general form [22, see Appendix]. From the point of view of the principal the problem of corruption control consists in seeking of such values of control variables that the found optimal strategy of the supervisor satisfies the requirements of sustainable development.

11). The identification problem in this domain is not at all trivial and requires special investigations and expertises. Each data set determines a specific social, economic and political system exposed to corruption.

12). It makes sense to build "genetic" series of sequentially complicated models that more and more precisely describe the real phenomena of corruption in hierarchical control systems. The principal logical pattern of this sequential complication has a form “optimization models - hierarchical two-person games - hierarchical three-person games”. With consideration of the possible modifications of the models of each type the “series” become the “genetic networks”.

It is the last principle that determines the rest of the paper.

3 System of Theoretical Models of Corruption

The principles formulated above are used to build a system of static models of corruption in hierarchical control systems.

3.1 Static Models of Economic Corruption

The basic optimization model of economic corruption has the form

$$g(b) = b + r(b) \to min \tag{1}$$

$$0 \leq b \leq 1 \tag{2}$$

where b is a part of the bribe, $r(b) \in [0, 1]$ is a given function of the economic corruption (for example, a real diminishing of the tax rate, i.e. absence of sanctions in case of non-payment for the bribe). Thus, the function $g(b)$ means the total costs for tax payments and bribe that are to be minimized by the agent. In case of the linear parameterization $r(b) = r_0 - Ab$ the model (1) takes the form

$$g(b) = r_0 + (1 - A)b \to min, 0 \leq b \leq 1 \tag{3}$$

Here r_0 is an official tax rate ($0 \leq r_0 \leq 1$), A is a model parameter. Considering that the function of economic corruption $r(b) = r_0 - Ab$ monotonically decreases when $0 \leq b \leq 1$ then $A > 0$. From the other side, the total costs $g(b)$ are non-negative, therefore $A \leq 1 + r_0$. Thus $0 < A \leq 1 + r_0$. The parameter determines the qualitative characteristics of the bribe-taker’s behavior. If $A = 0$ then corruption is completely absent. As the value of A increases, the bribe-taker’s tractability also increases and his greed decreases. The threshold value is $A = r_0$: in this case $r(1) = 0$, i.e. the maximal greed ensures the maximal tractability. If $A < r_0$ then the greed is over-limited and the tractability does not reach the maximal value (i.e. a positive tax is paid for any bribe). When $A > r_0$ the agent can avoid the tax payments completely in exchange for a moderate bribe (maximal tractability and small greed).

Return to the solution of the optimization problem (1). Having that $\frac{dg(b)}{db} = 1 - A$ the function monotonically increases when $0 < A < 1$ and its minimal value is reached in the left end of the admissible range: $g_{min} = g(0) = r_0$. Respectively when $1 < A < 1+r_0$ the function g monotonically decreases and its minimal value is reached in the right end: $g_{min} = g(1) = 1 + r_0 - A < r_0$. In the degenerate case $A = 1$ we get $g(b) \equiv r_0$(the bribe is useless and corruption is absent).

So, the parameter A again plays the key role and determines two qualitatively different behavior strategies of the agent. If $0 < A < 1$ then the total costs of the agent $g(b)$ increase, it is rational to abandon from bribe and to pay taxes equal to the legislative rate r_0. If $1 < A < 1 + r_0$ then the costs $g(b)$ diminish and an economic incentive arises to give the bribe and to pay in total $1 + r_0 - A < r_0$.

Now consider the function of economic corruption in a more general form

$$g(b) = b + r(b) = b + r_0 - Ab^k (k > 0).$$

It is true that

$$\frac{dg}{db} = 1 - kAb^{k-1} = 0 \Rightarrow b^* = (kA)^{\frac{1}{1-k}};$$

$$\frac{d^2g}{db^2} = k(1-k)Ab^{k-2}; \frac{d^2g(b^*)}{db^2} = (1-k)(kA)^{\frac{1}{k-1}}.$$

Therefore,

$$\frac{d^2g(b^*)}{db^2} \begin{cases} > 0, 0 < k < 1 \Rightarrow b^* \ is\ the\ point\ of\ min\ imun; \\ < 0, k > 1 \Rightarrow b^* \ is\ the\ point\ of\ max\ imun. \end{cases}$$

If b^* is a point of maximum then

$$\min_b\ g(b) = \begin{cases} g(0) = r_0, 0 < A < 1, \\ g(1) = r_0 + 1 - A, 1 < A < 1 + r_0. \end{cases}$$

Finally

$$b_{min} \begin{cases} (kA)^{\frac{1}{1-k}}, 0 < k < 1; \\ 0, k > 1 \wedge 0 < A < 1; \\ 1, k > 1 \wedge 1 < A < 1 + r_0; \end{cases} \qquad (the\ case\ k = 1\ is\ studied\ separately).$$

So, if $0 < k < 1$ then it is always profitable for the agent to give a bribe; if $k > 1$ then the reason to pay the bribe depends on the parameter .

Now consider a hierarchical game supervisor - agent in the form

$$g_S(r, b) = b + pr \to \max, \quad 0 \leq r \leq r_0; \tag{4}$$

$$g_A(r, b) = b + r \to \min, \quad 0 \leq b \leq 1. \tag{5}$$

Here the parameter p designates a part of the collected tax payments transferred by the principal (considered in the model implicitly) to the supervisor as a reward. In this model the function $r = r(b)$ is not given and is found as an optimal strategy of the leader in the game (4)-(5). Using Germeyer's theorem (see Appendix), we find the ε-optimal strategy in the form

$$\tilde{r}^{\varepsilon}(b) = \begin{cases} 0, \ b = r_0 - \varepsilon \wedge p < 1 - \frac{\varepsilon}{r_0}, \\ r_0, \ otherwise. \end{cases}$$

So, it is almost always profitable for the agent to give the bribe $b = r_0 - \varepsilon$ and to receive an arbitrary small but positive tax economy ε. The condition of effectiveness of the economic control of corruption is given by the inequality $p \geq 1 - \frac{\varepsilon}{r_0}$. However, it is hardly possible because in this case almost all tax payments should be assigned to the supervisors reward.

Thus let's consider a principal's problem of the administrative (compulsive) control of corruption as a hierarchical three-person game in the form

$$g_p = C(q) + K(r_0 - r) \to min, 0 \leq q \leq r_0 \tag{6}$$

$$g_s = b + pr \to max, q \leq r \leq r_0 \tag{7}$$

$$g_A = b + r \to min, 0 \leq b \leq 1 \tag{8}$$

Here q is a variable of the principal's administrative control that constraints from below the ability of supervisor's corrupted behavior; $C(q)$ is an increasing convex principal's control cost function, $C(0) = 0, C(r_0) = \infty$ is a parameter of the penalty charged on the supervisor if the condition of sustainable development $r = r_0$ is violated.

Now the solution of the hierarchical game (7)-(8) takes the form

$$\tilde{r}^{\varepsilon}(b) = \begin{cases} q, \ b = r_0 - \varepsilon \wedge p < 1 - \frac{\varepsilon}{r_0}, \\ r_0, \ otherwise. \end{cases}$$

The first-order condition for the problem (6) gives $\hat{q} = (C')^{-1}(K)$. The values of the objective function in this point and in the ends of the admissible segment are equal to $g_p(\hat{q}) = C'(K) + K(r_0 - r), g_p(0) = Kr_0, g_p(r_0) = C(r_0)$. Having that $K \gg 1$ or even $K \to \infty$, i.e. the condition of sustainable development is unalterable for the principal, we get that his objective function reaches its maximum when $q = r_0$ Therefore the supervisor's optimal strategy is identically equal to r_0, the condition of sustainable development is satisfied and corruption is absent. Thus, in this model the administrative control of corruption is more effective than the economic one.

3.2 Static Models of Administrative Corruption

The basic optimization model of administrative corruption has the initial form

$$g_A(u, b) = (1 - b)f(u) \to \max \tag{9}$$

$$0 \le u \le s(b), 0 \le b \le 1 \tag{10}$$

where b is a part of the bribe, u is the agent's action, $f(u)$ is the agent (bribe-giver)/s production function, $s(b)$ is a quota that constraints the agent's action from above and may be extended for the bribe. Having that the production function increases its maximum is always reached in the right end of the admissible segment. Therefore the model (9)-(10) can be represented as an optimization problem with one variable

$$g(b) = (1-b)f(s(b)) \to max, 0 \le b \le 1 \tag{11}$$

In case of the linear parameterization of the function of administrative corruption $s(b) = s_0 + Ab$, where s_0 is the official value of quota (notice that the function monotonically increases when $0 \le b \le 1$ because it describes the quota extension in exchange for the bribe) and the linear production function $f(x) = x$ the model (11) takes the form

$$g(b) = (1-b)(s_0 + Ab) \to max, 0 \le b \le 1 \tag{12}$$

As in the case of economic corruption, the parameter determines the qualitative characteristics of the bribe-taker's behavior. If $= 0$ then corruption is completely absent. As the value of increases, the bribe-taker's tractability also increases and his greed decreases. The threshold value is $A = 1 - s_0$: in this case $s(1) = 1$, i.e. the maximal greed ensures the maximal tractability. If $A < 1 - s_0$ then the greed is over-limited and the tractability does not reach the maximal value (i.e. any bribe nevertheless requires to obey a quota strictly less than 1). When $A > 1 - s_0$, the agent can ignore the quota completely in exchange for a moderate bribe (maximal tractability and small greed).

Return to the solution of the problem (12). We have

$$g(0) = s_0, g(1) = 0, \frac{dg(b)}{db} = A - s_0 - 2Ab, \frac{d^2g(b)}{db^2} = -2A < 0,$$

therefore $b^* = \frac{A-s_0}{2A}$ is the point of maximum, $g(b^*) = \frac{(A+s_0)^2}{4A} \ge g(0)$. Notice that

$$b^* \begin{cases} > 0, A > s_0, \\ < 0, A < s_0, \end{cases} , and\ g_{max} = \begin{cases} g(b^*), A > s_0, \\ g(0), A < s_0. \end{cases}$$

So, the parameter again plays the key role and determines two qualitatively different behavior strategies of the agent. If $A < s_0$ then there is no reason to give a bribe because the agent's payoff is maximal when $b = 0$ and is equal to s_0. However, if $A > s_0$ then the optimal part of bribe is equal to $b^* = \frac{A-s_0}{2A}$ that leads to the agent's payoff $\frac{(A+s_0)^2}{4A} \ge s_0$.

In more general case $g(b) = (1-b)(s_0+Ab)^k, k \leq 1$, the maximal agent's payoff is equal to

$$g_{max} = \begin{cases} g(0), A \leq \frac{s_0}{k}, \\ g(b^*), A > \frac{s_0}{k}, \end{cases} \quad where\ b^* = \frac{kA - s_0}{(1+k)A}.$$

So, it is disadvantageously to give a bribe when $s_0 \geq kA$.

Adding of a supervisor leads to the two-person hierarchical game in the form

$$g_s(s,u,b) = bf(u) \to max, s_0 \leq s \leq 1;$$

$$g_A(s,u,b) = (1-b)f(u) \to max, 0 \leq u \leq s, 0 \leq b \leq 1.$$

Taking the agent's production function in the form $f(u) = au^m$ and considering that its maximum is reached in the right end of the admissible segment, we get the following game:

$$g_s(s,b) = abs^m \to max, s_0 \leq s \leq 1; \tag{13}$$

$$g_A(s,b) = a(1-b)s^m \to max, 0 \leq b \leq 1; \tag{14}$$

Using Germeyers theorem (see Appendix) and considering that interests of the players coincide in s, we get the solution of the game (13)-(14) in the form

$$\tilde{s}^{\varepsilon}(b) = \begin{cases} 1, & b = 1 - \varepsilon - {s_0}^m, \\ s_0, & otherwise. \end{cases}$$

So, it is profitable for the agent to give the bribe and to avoid quota completely. To prevent corruption it is necessary to add to the game (13)-(14) a principal with the control problem

$$g_p = C(q) + K(q - s_0) \to min, s_0 \leq q \leq 1; \tag{15}$$

that is similar to the problem (6) in the model of economic corruption; the supervisor's problem (13) takes the form

$$g_s(s,b) = abs^m \to max, s_0 \leq s \leq q; \tag{16}$$

i.e. the supervisor's ability of corrupted behavior is restricted from above by the variable of non-corrupted principal's administrative control q. Then the solution of the game (16), (14) is

$$\tilde{s}^{\varepsilon}(b) = \begin{cases} q, & b = 1 - \varepsilon - \left(\frac{s_0}{q}\right)^m, \\ s_0, & otherwise. \end{cases}$$

and the solution of the problem (15) has the form

$$q^* = \begin{cases} s_0, C(s_0) < C(1) + K(1 - s_0) \\ 1, otherwise. \end{cases}$$

Considering that as in the model of economic corruption $K \gg 1$ or even $K \to \infty$, we get $q^* = s_0$, i.e. the principal ensures the condition of sustainable development $s = s_0$, and corruption is absent.

4 Models of Corruption in Real Estate Development

In a general form an optimization model of the economic corruption in real estate development may be represented as

$$g(b) = (1 - b)[r(b) + \xi(1 - r(b))] \to max, 0 \le b \le 1, \tag{17}$$

where $g(b)$ is the agent (developer)'s payoff function having the sense of his income from a real estate development project considering a bribe cost; b is an economic bribe as a part of the agent's income; $r(b)$ is a part of the social real estate redeemed with guarantee by the state at a fixed price (the price may be augmented for a bribe); ξ is a part of the social real estate that can be sold by the agent himself.

The function of economic corruption $r(b)$ is supposed to be known according to the descriptive approach. According to the economic sense it increases monotonically in the segment [0,1] and in the considered case of capture (extortion is analyzed similarly) $r(0) = r_0$, where r_0 is the legislative value of r (it is supposed that $r = r_0$ is the condition of sustainable development of the real estate project). It is natural to use the function of economic corruption in the form

$$r(b) = r_0 + Ab^k \ (A > 0), 0 \le r(b) \le 1, \tag{18}$$

Restrict ourselves by the linear parameterization ($k = 1$). Considering (18) we get $r(b) = min\{r_0 + Ab, 1\}, A > 0$.

The parameter A determines the qualitative characteristics of the bribe-taker's behavior. If $A = 0$ then corruption is completely absent. As the value of increases, the bribe-taker's tractability also increases and his greed decreases. The threshold value is $A = 1 - r_0$: in this case $r(1) = 1$, i.e. the maximal greed ensures the maximal tractability. If $A < 1 - r_0$ then the greed is over-limited and the tractability does not reach the maximal value (i.e. the agent should sell a part of the social real estate himself with any bribe). When $A > 1 - r_0$ the agent can sell the total amount of the social real estate to the state at a fixed price in exchange for a moderate bribe (maximal tractability and small greed).

The optimization problem (17) takes the form

$$g(b) = (1 - b)[r_0 + Ab + \xi(1 - (r_0 + Ab))] \to max, 0 \le b \le 1,$$

we have $g(0) = r_0 + \xi(1 - r_0), g(1) = 0, \dfrac{dg(b)}{db} = -2A(1-\xi)b + A - A\xi - r_0 - \xi + \xi r_0.$

From the first-order condition $\frac{dg(b)}{db} = 0$,we find $b^* = \frac{(A-r_0)(1-\xi)-\xi}{2A(1-\xi)}$ that is the point of maximum because $\frac{dg^2(b^*)}{db^2} = -2A(1-\xi) < 0$. By the structure

$$g_{max} = \begin{cases} g(0) = r_0 + \xi(1 - r_0), A < \frac{r_0(1-\xi)+\xi}{1-\xi}. \\ g(b^*) > g(0), otherwise. \end{cases} \tag{19}$$

Thus, the agent's optimal strategy is determined by the parameters A, r_0, ξ subject to (19): in the first case it is the zero bribe, in the second one the bribe $b^* > 0$ giving the payoff $g(b^*) > g(0)$.

Now consider the models of administrative corruption in real estate development. In a general case the optimization problem has the form

$$G(b, u) = (1 - b)[ru + \xi(1 - r)u + \gamma\eta(\gamma)(1 - u)] \to max, s(b) \leq u \leq 1, 0 \leq b \leq 1, \tag{20}$$

where $G(b, u)$ is the agent(developer)'s payoff function that means his income from selling all types of the real estate considering corruption; b is a part of the administrative bribe; u is a part of the social type in the total amount of real estate; r – a part of the social real estate redeemed by the state at a fixed price with a guarantee; ξ – an estimated part of the social real estate that can be sold by the agent himself at the same price; $\gamma > 1$ – an increasing factor of the price of more expensive types of the real estate; $\eta(\gamma)$ – an estimated part of the agent's own sale of more expensive types of the real estate; $s(b)$ – an obligatory minimal quota of the part of social real estate in the total amount (can be diminished for a bribe).

Notice that in the optimization problem (20) the payoff function depends on two variables. Investigate the problem:

$$\frac{\partial G}{\partial b} = -[(r + \xi(1 - r) - \gamma\eta)u + \gamma\eta];$$

$$\frac{\partial G}{\partial b} = 0 \Rightarrow \mu^* = \frac{\gamma\eta}{\gamma\eta - (r + \xi(1 - r))} > 0, r + \xi(1 - r) < \gamma\eta;$$

$$\frac{\partial^2 G}{\partial b^2} = 0, \frac{\partial^2 G}{\partial b \partial u} = \gamma\eta - (r + \xi(1 - r));$$

$$\frac{\partial G}{\partial u} = (1 - b)(r + \xi(1 - r) - \gamma\eta); \frac{\partial G}{\partial u} = 0 \Rightarrow b^* = 1;$$

$$\frac{\partial^2 G}{\partial u^2} = 0; \frac{\partial^2 G}{\partial b \partial u} = \gamma\eta - (r + \xi(1 - r)).$$

The Hesse matrix has the form

$$H = \left\| \begin{matrix} 0 & \gamma\eta - (r + \xi(1-r)) \\ \gamma\eta - (r + \xi(1-r)) & 0 \end{matrix} \right\|, its\ determinant\ is\ equal\ to$$

$|H| = -[\gamma\eta - (r+\xi(1-r))]^2 < 0$. Therefore (b^*, u^*) is a saddle point and maximum of the function $G(b,u)$ is reached in the boundary of the set of admissible values. We have

$$G(1,u) = 0; G(0,1) = r + \xi(1-r) - \gamma\eta;$$

$$G(0,s_0) = r + \xi(1-r) + (1-s_0)\gamma\eta > G(0,1); G(0,s_0) > 0.$$

Thus in any case maximum of the function $G(b,u)$ is reached when $u = s(b)$. Then one-variable optimization problem arises in the form

$$g(b) = (1-b)[(r + \xi(1-r) - \gamma\eta)s(b) + \gamma\eta] \to max, 0 \leq b \leq 1.$$

Denote $C = \gamma\eta > 0, D = \gamma\eta - (r + \xi(1-r))$. Notice that if $D < 0$ then it is more profitable to sell the social real estate than more expensive one, and to give a bribe is not rational. Therefore the agents payoff function can be written in the form $g(b) = (1-b)(C - Ds(b)), D > 0$.

As earlier, represent a function of administrative corruption in the form

$$s(b) = s_0 - Ab^k \ (A > 0), 0 \leq s(b) \leq 1,$$

where s_0 is the legislative value of quota, and restrict ourselves by the case of linear ($k = 1$) parameterization of the function.

$$s(b) = s_0 - Ab \ (A > 0),$$

$$g(b) = (1-b)(C - Ds_0 + ADb) = C - Ds_0 + (D(A+s_0) - C)b - ADb^2;$$

$$\frac{\partial g}{\partial b} = AD(1-2b) = 0 \Rightarrow b^* = 1/2; \frac{\partial^2 g}{\partial b^2} = -2AD < 0 \Rightarrow b^* is\ the\ point\ of\ maximum;$$

$$g(0) = C - Ds_0; g(1) = 0; g(b^*) = \frac{2C + D(A - 2s_0)}{4}.$$

Thus,

$$g^{max} = \begin{cases} g(b^*), D(A + 2s_0) > 2C, \\ g(0), otherwise \end{cases}$$

i.e. the advantageousness of giving a bribe depends on the relation between the model parameters. Notice that in the case $b = b^*$ the agent's payoff is positive if $s_0 < \frac{2C+AD}{2D}$, i.e. an obligatory quota of the social real estate is not very big.

Now consider a game theoretic model of the administrative corruption in real estate development in the form

$$g_p(q,s) = p_1(s - s_0) - \frac{q}{s_0 - q} \to max, 0 \le q \le s_0.$$
$$g_s(s,b) = p_2(s - s_0) + bc[(r + \xi(c)(1-r))u + \gamma\eta(\gamma)(1-u)] \to max, q \le s \le s_0.$$
$$g_A(b,u) = c(1-b)[(r + \xi(c)(1-r))u + \gamma\eta(\gamma)(1-u)] \to max, s \le u \le 1, 0 \le b \le 1. \tag{21}$$

Here b is a part of the administrative bribe; u is a part of the social type in the total amount of real estate; c – a price of the social real estate; r – a part of the social real estate redeemed by the state at the fixed price c with a guarantee; $\xi(c)$ – an estimated part of the social real estate that can be sold by the agent himself at the same price; $\gamma > 1$ – an increasing factor of the price of more expensive types of the real estate; $\eta(\gamma)$ – an estimated part of the agent's own sale of more expensive types of the real estate; $s(b)$ – an obligatory minimal quota of the part of social real estate in the total amount (can be diminished for a bribe); q – a parameter of the principal's administrative control; $p_1, p_2 > 0$ – penalty factors charged on the principal and the supervisor respectively if the quota (condition of sustainable development $s \ge s_0$) is violated. It is supposed that $r = const, \tilde{s} = s(b)$.

The problem (21) is solved using a heuristic two-stage algorithm. On the first stage the hierarchical game between the supervisor and the agent is considered. Notice that the function g_A achieves its maximum in u independently from b, therefore subject to the linearity of the function g_A in u the solution of the agent's problem has the form

$$u^* = \begin{cases} 1, r + \xi(c)(1-r) \ge \gamma\eta(\gamma), \\ s, otherwise \end{cases}$$

Consider these two cases separately.
1). $r + \xi(c)(1-r) \ge \gamma\eta(\gamma)$, i.e. it is more profitable to sell the social real estate. In this case a game does not arise because when $u = 1$ then the function g_A does not depend on s. An evident solution of the optimization problem is $b = 0$, i.e. corruption is absent.
2). $r + \xi(c)(1-r) < \gamma\eta(\gamma)$, i.e. it is more profitable to sell the expensive real estate. Then a standard hierarchical two-person game arises in the form

$$g_s(s,b) = p_2(s - s_0) + cb[r + \xi(c)(1-r) - \gamma\eta(\gamma)] \to max, q \le s \le s_0;$$

$$g_A(s,b) = c(1-b)[(r + \xi(c)(1-r) - \gamma\eta(\gamma))s + \gamma\eta(\gamma)] \to max, 0 \le b \le 1.$$

Application of Germeyer's theorem (see Appendix) gives

$$s^P(b) \equiv s_0; s^D(b) = \begin{cases} s_0, p_2 > bc(r + \xi(c)(1-r) - \gamma\eta(\gamma)), \\ q, otherwise; \end{cases}$$

$$L_A = c[(r + \xi(c)(1-r) - \gamma\eta(\gamma))s_0 + \gamma\eta(\gamma)];$$

$$E_A = \{b = 0\}; D_A = \{(s,b) : g_A(s,b) > L_A\} = \emptyset, s.t.;$$

$$(1-b)[(r + \xi(c)(1-r) - \gamma\eta(\gamma))s + \gamma\eta(\gamma)] \le (r + \xi(c)(1-r) - \gamma\eta(\gamma))s_0 + \gamma\eta(\gamma);$$

when $0 \le b \le 1, q \le s \le s0$. It is assumed in this case that $K_1 = -\infty < K_2$, thus

$$\tilde{s}^*(b) = \begin{cases} s^D(b), & b = 0, \\ s_0, & otherwise, \end{cases} = \begin{cases} q, & p_2 < bc(r + \xi(c)(1-r) - \gamma\eta(\gamma)) \wedge b = 0, \\ s_0, & otherwise. \end{cases}$$

However the conditions $p_2 < bc(r + \xi(c)(1-r) - \gamma\eta(\gamma)) and\ b = 0$ are disjoint, therefore $\tilde{s}^*(b) \equiv s_0$.

So, the solution of the agent's optimization problem has the form $b^* = 0$ (in fact, corruption is absent), and $u^* = s_0$, therefore it is not required to involve the principal to ensure the condition of sustainable development. Respectively, in the second stage it is sufficient for the principal to solve the control costs minimization problem $\frac{q}{s_0 - q} \to min, 0 \le q \le s_0$,the trivial solution of which has the form $q^* = 0$.

At last, consider a model of the economic corruption in the three-level system of real estate development

$$g_P = H(c) + M(r_0 - r) \to min, 0 \le c \le 1;$$

$$g_s = f(1)(b + cr) \to max, 0 \le r \le r_0;$$

$$g_A = f(1)(1 - b - r) \to max, 0 \le b \le 1;$$

where $H(c)$ – an increasing convex principal's cost function; M – a penalty factor (the penalty is charged if the condition of sustainable development $r = r_0$ is violated). The supervisor's optimal guaranteeing strategy has the form

$$\tilde{r}^*(b) = \begin{cases} 0, & b = r_0 - \varepsilon \wedge c < 1 - \frac{\varepsilon}{r_0}, \\ r_0, & otherwise, \end{cases}$$

If $b \ne r_0 - \varepsilon$ then $r \equiv r_0$ and the evident solution of the principal's optimization problem is $c = 0$. If $b = r_0 - \varepsilon$ then the principal can ensure the condition of sustainable development only by choosing $c = 1$. Therefore the solution of his optimization problem has the form

$$c^* = \begin{cases} 1, H(1) < Mr_0, \\ 0, otherwise, \end{cases}$$

i.e. the principal is obliged to compare the penalty for the violation of sustainable development condition and the costs required for its control.

5 Conclusion

The principles of modeling and control of corruption in the hierarchical systems are formulated. Particularly, a problem of corruption control is set as the problem of realization of certain requirements to the state of controlled system (conditions of sustainable development). If the conditions are satisfied then the corruption control problem is supposes to be solved even if corruption in the system exists. This setting differs from the proposed by G.Becker especially economic approach based on commensurability of the damage caused by corruption and its control costs [3].

Another important methodical principle is building of "genetic" series of sequentially complicated models that more and more precisely describe the real phenomena of corruption in hierarchical control systems. This principle is realized in the paper by building of series of the static theoretical models of administrative and economic corruption as well as models of corruption in real estate development. The dependence of corrupted behavior on model parameters is investigated, and the analytical conditions in which corruption is not profitable for the agent or can be controlled by the principal for ensuring sustainable development of the system are received.

Appendix

Germeyer's Theorem [22]. Suppose that functions $M_1(x_1, x_2), M_2(x_1, x_2)$ are continuous on the compacts X_1, X_2. Introduce the following functions: a punishment strategy $x_1^P(x_2)$ by the rule $M_2(x_1^P, x_2) = \underset{x_1 \in X_1}{min} M_2(x_1, x_2)$, and the dominance strategy $x_1^D(x_2)$ that satisfies the condition $M_1(x_1^D, x_2) = \underset{x_1 \in X_1}{max} M(x_1, x_2)$. Besides, the following quantities and sets are introduced:

$$L_2 = \underset{x_2 \in X_2}{max} M_2(x_1^P, x_2); E_2 = \{x_2 \in X_2 : M_2(x_1^P, x_2) = L_2\};$$

$$D_2 = \{(x_1, x_2) \in X_1 \times X_2 : M_2(x_1, x_2) > L_2\};$$

$$K_1 = \sup_{(x_1,x_2) \in D_2} M_1(x_1, x_2) \leq M_1(x_1^\varepsilon, x_2^\varepsilon) + \varepsilon \quad (D_2 = \emptyset \Rightarrow K_1 = -\infty);$$

$$K_2 = \min_{x_2 \in E_2} \max_{x_1 \in X_1} M_1(x_1, x_2).$$

Then the guaranteed payoff of the player 1 (leader) in the game Γ_2 is equal to $\omega_1 = max(K_1, K_2)$, and the respective ε-optimal strategy has the form

$$\tilde{x}_1^\varepsilon(x_2) = \begin{cases} x_1^\varepsilon, & x_2 = x_2^\varepsilon,\ K_1 > K_2, \\ x_1^D(x_2), & x_2 \in D_2,\ K_1 \le K_2, \\ x_1^P(x_2), & \text{otherwise.} \end{cases}$$

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