Purpose and Non-Purpose Resource Use Models in Two-Level Control Systems

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Abstract
In the paper a two-level control system consisting of one element in top level and one element in bottom level is considered. Both levels have purpose use and non-purpose use interests. The model of resource allocation between purpose and non-purpose use interests for different classes of payoff functions is investigated. The model is built as a two-person game where the Stackelberg equilibrium is found. Analytical and numerical results are presented.
Keywords resource allocation, purpose use interests, non-purpose use interests, Stackelberg equilibrium

1 Introduction

Many problems of the social-economic development are solved due to the federal financing. The financing has different forms (grants, subsidies, assignments, credits) and is always strictly purpose oriented, i.e. the allocated resources should be spent for the pre-scribed needs only. There are legislative sanctions for the non-purpose use of the federal financing. Nevertheless, the non-purpose use of federal resources is widely spread and may be considered as a variety of the opportunistic behavior meeting the private interests of active agents[1].

The non-purpose resource use is closely connected with corruption, especially with so-called "returns" when federal resources in some programs are assigned in exchange of a bribe and only partly satisfy the prescribed social destination being mostly used in the private interests of bribe-givers.

It is natural to consider the problem of non-purpose resource use from the point of view of the concordance of interests in hierarchical control systems. This permits to use the mathematical formalism of the hierarchical game theory [2], the theory of incentives [3] and the theory of organizational systems [4-5]. In the same time, namely the models of resource allocation in the hierarchical systems with respect of their non-purpose use are scantily known and are analyzed by the authors' methodology [6].

In this paper the emphasis is on the dependence of the distribution of resources between the purpose and non-purpose use on different classes of payoff functions characterizing the common (social) interest and the private interests of the resource distributor and resource recipient.

In Section 2 the structure of investigation is presented. Sections 3 and 4 include analytical and numerical results respectively. Section 5 concludes.

2 Structure of Investigation

Let us consider a two-level control system which consists of one element on the top level A1 (resource distributor-she) and one element on the bottom level A2 (resource recipient-he). Without loss of generality we can equate to one the number of resources on the top level. The distributor delegates a part of her resources to the recipient for the purpose use, and the other part she keeps for the non-purpose use. In turn, the bottom level divides the received resource between the purpose use and his private non-purpose use. Both levels have their shares in the purpose-use income and have their private payoff functions (Fig.1).

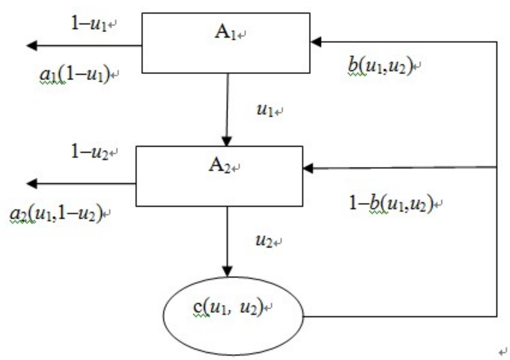

Fig.1 The structure of the modeled system

The model is built as a hierarchical two-person game in which a Stackelberg equilibrium is sought [2]. The payoff functions of both players include two terms: a non-purpose income and the respective share of the purpose-use income. So, the payoff functions are:

$$g_1(u_1, u_2) = a_1(1 - u_1) + b_1(u_1, u_2)c(u_1, u_2) \to \max_{u_1},$$

$$g_2(u_1, u_2) = a_2(u_1, 1 - u_2) + b_2(u_1, u_2)c(u_1, u_2) \to \max_{u_2}$$

subject to $0 \le u_i \le 1$.

and the following conditions for the functions a, b and c:

$$a_i \ge 0, \frac{\partial a_i}{\partial u_i} \le 0, \frac{\partial a_i}{\partial u_{j \ne i}} \ge 0, b_i \ge 0, \frac{\partial c}{\partial u_i} \ge 0, i = 1, 2.$$

Here the subscript 1 relates to the parameters of the top level (the leader), and the subscript 2 relates to the parameters of the bottom level (the follower);

u_i - a part of resources assigned by the i-th level for the purpose use (respectively the part 1- u_i leaves for the private non-purpose use);

g_i - the i-th level payoff function;

a_i - the i-th level function of his/her private interest;

b_i - a share of the purpose-use income received by the i-th level;

c - a function of the purpose-use income of the whole system (society, organization).

As functions a and c power, exponential and logarithmic functions of the variables u_1 and u_2 are considered which are cumulative ones, i.e. a_1=a_1(1-u_1), a_2=a_2(u_1 (1-u_2)), c=c(u_1u_2). In this case the share u_1u_2 of resources is assigned for the purpose use.

The relations a_1=a_1(1-u_1), a_2=a_2(u_1 (1-u_2)) reflect the system's hierarchical structure. The non-purpose income of the top level does not depend on the part of resources assigned by the bottom level for the purpose use, but the non-purpose income of the bottom level depends on the share of resources received by him from the top level.

The following distributions of the purpose-use income b are considered:

(1) uniform one, in particular, for $n = 2$

$$b_i = \frac{1}{2}, i = 1, 2.$$

(2) proportional one

$$b_1 = \frac{u_1}{u_1 + u_2}, b_2 = \frac{u_2}{u_1 + u_2}.$$

In this paper it is assumed that $b_1 + b_2 = 1$.

The strategy of a player i is a part u_i of his/her resources as-signed for the purpose use. The top-level player moves first, i.e. she chooses a value u_1 and informs about it the bottom-level player who chooses his optimal reaction u_2.

The aim of investigation is study of the influence of the relations between functions a_1, a_2, b_1, b_2, c to the solution of the game (Stackelberg equilibrium).

The following types of non-purpose use functions were used:

- power function with an exponent smaller than one ($a(x) = ax^{\alpha}, 0 < \alpha < 1, a > 0$),
- linear function ($a(x) = ax$),
- power function with an exponent greater than one ($a(x) = ax^k, k > 1, a > 0$);
- exponential function ($a(x) = a(1 - e^{-\lambda x}), \lambda > 0, a > 0$);
- logarithmic function ($a(x) = a \log_2(1 + x), a > 0$).

Almost all of the functions satisfy the conditions $\partial a/\partial x \geq 0$, $\partial^2 a/\partial x^2 \leq 0$ (except the second condition for the function $a(x) = ax^k, k > 1$). Similarly, the following types of purpose use functions were used:

- power function with an exponent smaller than one ($c(x) = cx^{\alpha}, 0 < \alpha < 1, c > 0$);

- linear function ($c(x) = cx$);
- power function with an exponent greater than one ($c(x) = cx^k, k > 1, c > 0$);
- exponential function ($c(x) = c(1 - e^{-\lambda x}), \lambda > 0, c > 0$);
- logarithmic function ($c(x) = c\log_2(1 + x), c > 0$).

Thirteen of the possible twenty five combinations of the functions a and c are studied analytically, namely:

(1) combinations of the one-type functions (both functions a and c are power, exponential, or logarithmic ones);

(2) combinations of any non-purpose use function with linear purpose-use function;

(3) combinations of any purpose-use function with non-purpose use linear function.

Six of other twelve cases are investigated numerically.

3 Analytical Investigation of the Different Classes of Models

First, let's consider the following parameterization:

$$a_1(u_1, u_2) = a_1(1 - u_1), a_2(u_1, u_2) = a_2u_1(1-u_2),$$
$$c(u_1, u_2) = c\log_2(1 + u_1u_2), b_1 = b, b_2 = 1 - b.$$

In this case the payoff functions are

$$g_1(u_1, u_2) = a_1(1 - u_1) + bc\log_2(1 + u_1u_2) \tag{1}$$

$$g_2(u_1, u_2) = a_2u_1(1-u_2) + (1 - b)c\log_2(1 + u_1u_2) \tag{2}$$

subject to $0 \leq u_i \leq 1, i = 1, 2$.

Omitting the calculations, consider each branch of the Stackelberg equilibrium separately:

I. $u = (0; 0)$, if $a_2 > (1 - b)\, c/ln2$ or $a_1 > bc/ln2$ (Fig.2), i.e. for one of the players the non-purpose resource use is much more profitable than the purpose-use activity; therefore, it is disadvantageous for him/her to assign resources for the purpose-use activity, and in this case it is also disadvantageous for the other player. The payoffs in this case are equal to: $g_1 = a_1, g_2 = 0$.

II. $u = (1; 1)$, if $a_2 < (1 - b)\, c/ln2$ and $a_1 < bc/ln2$ (Fig.2), i.e. for both players the purpose-use activity is much more profitable, and each of them assigns all resources for it. The payoffs are equal to: $g_1 = bc, g_2 = (1 - b)\, c$.

III. $u = (bc/\left(a_1 ln2\right) - 1; 1)$, if the conditions $bc/2ln2 < a_1 < bc/ln2$ and $a_2 < a_1\left(1 - b\right)/b$ are satisfied (Fig.2), i.e. for the top-level player it is profitable to assign only a part of her resources for the purpose use because her incomes from both activities are comparable, meanwhile for the bottom-level player it is profitable to assign all his resources to the purpose-use activity. The payoffs are

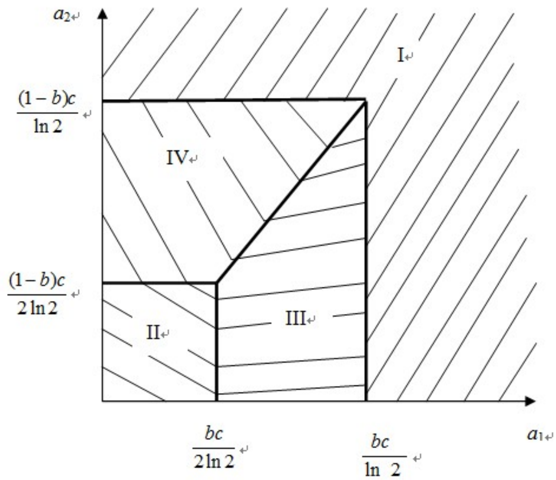

Fig.2 Equilibrium outcomes in the game(1) - (2)

equal to:

$$g_1 = 2a_1 - \frac{bc}{\ln 2} + bc\log_2\left(\frac{bc}{a_1 \ln 2}\right),$$
$$g_2 = (1-b)c\log_2\left(\frac{bc}{a_1 \ln 2}\right)$$

IV. $u = ((1-b)\,c/\,(a_2 ln2) - 1; 1)$, if $(1-b)\,c/2ln2 < a_2 < (1-b)\,c/ln2$ and $a_2 > a_1\,(1-b)\,/b$ (Fig.2), i.e. for both players it is profitable to assign only a part of their resources for the purpose-use activity because their incomes from both types of activities are comparable. But the leader gives to the follower exactly the fixed number of resources which he planned to allocate for the purpose use, therefore compel-ling him to assign all his resources for the purpose use. The players' payoffs are equal to

$$g_1 = 2a_1 - \frac{a_1(1-b)c}{a_2 \ln 2} + bc\log_2\left(\frac{(1-b)c}{a_2 \ln 2}\right),$$

$$g_2 = (1-b)c\log_2\left(\frac{(1-b)c}{a_2 \ln 2}\right).$$

Second, consider the following parameterization:

$$a_1(u_1,u_2) = a_1(1-u_1)^k, a_2(u_1,u_2) = a_2(u_1(1-u_2))^k,$$
$$c(u_1,u_2) = c(u_1 u_2), b_1 = b, b_2 = 1-b.$$

Then the payoff functions have the form

$$g_1(u_1, u_2) = a_1(1 - u_1)^k + bc(u_1 u_2) \to \max_{u_1} \quad (3)$$

$$g_2(u_1, u_2) = a_2(u_1(1-u_2))^k + (1 - b)c(u_1 u_2) \to \max_{u_2} \quad (4)$$

subject to $0 \leq u_i \leq 1, i = 1, 2$.

The Stackelberg equilibrium outcomes are the following :

$$\bar{u} = \begin{cases} (1;1), & (a_1 < bc) \,\&\, (a_2 < (1-b)c) \\ (0;0), & (a_1 > bc) \vee (a_2 > (1-b)c) \end{cases}$$

Consider the cases separately (Fig.3):

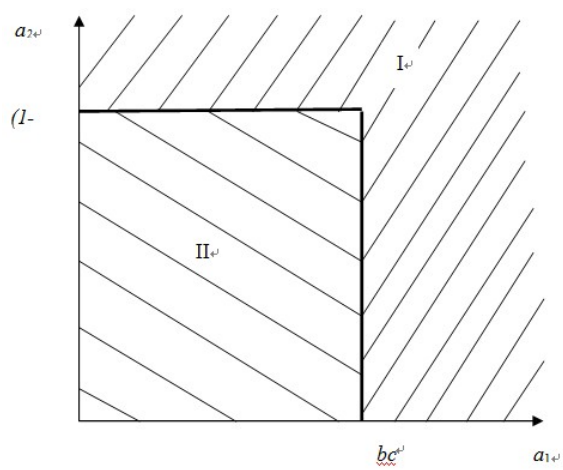

Fig.3 Equilibrium outcomes in the game(3) - (4)

I. $u = (0; 0)$, if $a_2 > (1 - b)\,c$ or $a_1 > bc$, i.e. for one of the players the non-purpose resource use is much more profitable than the purpose-use one, therefore it is disadvantageous for her/him to finance the purpose-use activity. The payoffs are: $g_1 = a_1, g_2 = 0$.

II. $u = (1; 1)$, if $a_2 < (1 - b)\,c$ and $a_1 < bc$, i.e. for both players the pur-pose-use activity is much more profitable, and each of them assigns all resources for it. The payoffs are equal to: $g_1 = bc, g_2 = (1 - b)\,c$.

When even one of the functions of purpose or non-purpose re-source use is power with an exponent greater than one (and the other function is the same or linear) then it is advantageous for both players to allocate resources or only to the purpose use (altruistic strategy), or only to the non-purpose use (egoistic strategy).

Third, consider the following case:

$$a_1(u_1,u_2)=a_1(1-e^{-\lambda(1-u_1)}), a_2(u_1,u_2)=a_2(1-e^{-\lambda u_1(1-u_2)}),$$
$$c(u_1,u_2)=c(1-e^{-\lambda u_1u_2}), b_1=b, b_2=1-b.$$

Then the payoff functions have the form

$$g_1(u_1,u_2)=a_1(1-e^{-\lambda(1-u_1)})+bc(1-e^{-\lambda u_1u_2}) \quad (5)$$
$$g_2(u_1,u_2)=a_2(1-e^{-\lambda u_1(1-u_2)})+(1-b)c(1-e^{-\lambda u_1u_2}) \quad (6)$$

subject to $0\le u_i\le 1, i=1,2.$

Let's consider all Stackelberg outcomes separately (Fig.4):

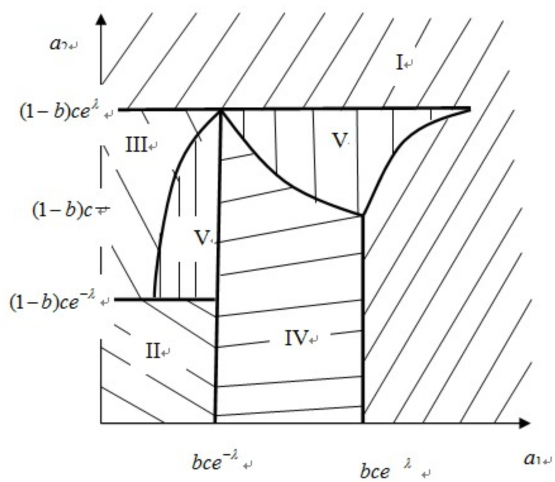

Fig.4 Equilibrium outcomes in the game(5) - (6)

I. $u=(0;0)$, if $\left(a_1>bce^{\lambda}\right)\&\left(a_1>(1-b)\,c\right)$ or $a_1>bce^{\lambda}\sqrt{\frac{a_2}{(1-b)c}}$, i.e. for one of the players the non-purpose resource use is much more profitable than the purpose-use one, therefore it is disadvantageous for her/him to finance the purpose-use activity. The payoffs are: $g_1=a_1(1-e^{-\lambda}), g_2=0$.

II. $u=(1;1)$, if $a_1<bce^{-\lambda}$ and $a_2<(1-b)\,ce^{-\lambda}$, i.e. for both players the purpose-use activity is much more profitable, and each of them assigns all resources for it. The payoffs are equal to:$g_1=bc(1-e^{-\lambda}), g_2=(1-b)c(1-e^{-\lambda})$.

III. $u=\left(1;\frac{1}{2}-\frac{1}{2\lambda}\ln\frac{a_2}{(1-b)c}\right)$, if $a_1<\frac{b}{2}\sqrt{\frac{a_2c}{1-b}}e^{-\frac{\lambda}{2}}$ and $(1-b)\,ce^{-\lambda}<a_2<(1-b)\,ce^{\lambda}$, i.e. for the top-level player it is profitable to assign all her resources to the purpose-use activity, meanwhile for the bottom-level player it is advantageous to divide his resources. Payoffs are the following:

$$g_1 = bc\left(1 - e^{-\lambda\left(\frac{1}{2} - \frac{1}{2\lambda}\ln\frac{a_2}{(1-b)c}\right)}\right) = b\left(1 - e^{-\frac{\lambda}{2}}\right)\sqrt{\frac{a_2 c}{(1-b)}},$$
$$g_2 = (1-b)\left(1 - e^{-\frac{\lambda}{2}}\right)\sqrt{\frac{a_2 c}{(1-b)}}.$$

IV. $u = \left(\frac{1}{2} - \frac{1}{2\lambda}\ln\frac{a_1}{bc}; 1\right)$, if $bce^{-\lambda} < a_1 < bce^{\lambda}$ and $a_2 < (1-b)\sqrt{\frac{bc}{a_1}}ce^{\frac{\lambda}{2}}$, i.e. the situation is opposite to the previous one. The payoffs are:

$$g_1 = a_1 - a_1 e^{-\frac{\lambda}{2}}\sqrt{\frac{bc}{a_1}} + bc - e^{-\frac{\lambda}{2}}\sqrt{bca_1},$$
$$g_2 = (1-b)c\left(1 - e^{-\frac{\lambda}{2}}\sqrt{\frac{a_1}{bc}}\right).$$

V. $u = \left(\frac{2}{3} - \frac{2}{3\lambda}\ln\frac{2a_1}{b}\sqrt{\frac{1-b}{a_2 c}}; \frac{\lambda - \ln\frac{2a_1a_2}{b(1-b)c}}{2\left(\lambda - \ln\frac{2a_1}{b}\sqrt{\frac{1-b}{a_2c}}\right)}\right)$, if the conditions $\frac{b}{2}\sqrt{\frac{a_2c}{1-b}}e^{-\frac{\lambda}{2}} < a_1 < \frac{b}{2}\sqrt{\frac{a_2c}{1-b}}e^{\lambda}$ and $(1-b)\sqrt{\frac{2a_1}{bc}}e^{-\frac{\lambda}{2}} < a_2 < \frac{bc^2}{2a_1(1-b)}e^{\lambda}$ are satisfied, i.e. for both players it is profitable to divide their resources. The payoffs are omitted due to their tediousness.

At last, consider the following case:

$$a_1(u_1,u_2) = a_1(1-u_1), a_2(u_1,u_2) = a_2\left(u_1(1-u_2)\right),$$
$$c(u_1,u_2) = c(u_1u_2)^{\alpha}, b_1 = b, b_2 = 1-b.$$

Then the payoff functions have the form

$$g_1(u_1,u_2) = a_1(1-u_1) + bc(u_1u_2)^{\alpha} \to \max_{u_1} \tag{7}$$

$$g_2(u_1,u_2) = a_2\left(u_1(1-u_2)\right) + (1-b)c(u_1u_2)^{\alpha} \to \max_{u_2} \tag{8}$$

The Stackelberg equilibrium has the form (Fig.5):

$$\bar{u} = \begin{cases} (1;1), & (a_1 < bc\alpha)\&(a_2 < (1-b)c\alpha), \\ \left(\sqrt[1-\alpha]{\frac{\alpha bc}{a_1}}; 1\right), & (a_1 > bc\alpha)\&(a_2 < a_1), \\ \left(\sqrt[1-\alpha]{\frac{(1-b)c\alpha}{a_2}}; 1\right), & (a_2 > (1-b)c\alpha)\&(a_2 > a_1). \end{cases}$$

In this case the egoistic strategy is disadvantageous for both players. Besides, the leader is always able to compel the follower to assign all his resources to the

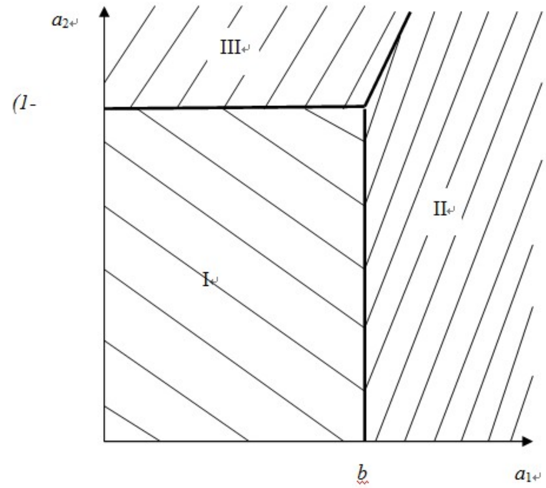

Fig.5 Equilibrium outcomes in the game(7) - (8)

purpose use. The payoffs are:

$$g_1 = a_1 - a_1\left(\frac{\alpha(1-b)}{a_2}\right)^{\frac{1}{1-\alpha}} + b\left(\frac{\alpha^\alpha(1-b)^\alpha c}{a_2{}^\alpha}\right)^{\frac{1}{1-\alpha}},$$

$$g_2 = \left(\frac{\alpha^\alpha(1-b)c}{a_2{}^\alpha}\right)^{\frac{1}{1-\alpha}}.$$

When the purpose-use function is linear, and the function of non-purpose use is power with exponent smaller than one, the altruistic strategy is profitable for the bottom-level player. The egoistic strategy is disadvantageous for both players.

The thirteen analyzed cases are grouped by the structure of equilibrium outcomes of the game:

I. One outcome when both functions of purpose and non-purpose use are power with exponent smaller than one. In this case it is profitable for both players to divide their resources between purpose and non-purpose use.

II. Two outcomes $(0;0)$ and $(1;1)$ (Fig.3) when:

• The function of non-purpose use is power with exponent smaller than one, and the function of purpose use is linear;

• Both functions are linear or power with exponent greater than one, in any combination.

III. Three outcomes (Fig.5) when the non-purpose resource use function is linear, and the purpose-use function is power with exponent smaller than one. In this the altruistic strategy is profitable even for one player.

IV. Four outcomes (Fig.2) in cases when one of the functions is linear, and the other is logarithmic.

V. Five outcomes (Fig.4) when

• Both functions are linear or exponential in any combination except the case when they are both linear.

• Both functions are logarithmic.

4 Numerical Analysis

Let's consider an example of the numerical analysis for the following parameterization:

$$a_1(u_1,u_2)=a_1(1-u_1)^\alpha, a_2(u_1,u_2)=a_2(u_1\,(1-u_2))^\alpha,$$
$$c(u_1,u_2)=c(1-e^{-\lambda u_1 u_2}), b_1=b, b_2=1-b.$$

In this case the game has the form

$$g_1(u_1,u_2)=a_1(1-u_1)^\alpha+bc(1-e^{-\lambda u_1 u_2})\to \max_{u_1} \tag{9}$$

$$g_2(u_1,u_2)=a_2(u_1\,(1-u_2))^\alpha+(1-b)c(1-e^{-\lambda u_1 u_2})\to \max_{u_2} \tag{10}$$

To find an optimal strategy of the bottom-level player let's calculate the derivative of the function g_2 with respect to the variable u_2 and equate it to zero:

$$\frac{\partial g_2}{\partial u_2}(u_1,u_2)=-\frac{a_2\alpha {u_1}^\alpha}{(1-u_2)^{1-\alpha}}+\lambda u_1(1-b)ce^{-\lambda u_1 u_2}=0 \tag{11}$$

Let's prove that the method of bisection is applicable for the solution of (11). Note that the second derivative of the function g_2 with respect to the variable u_2 is negative

$$\frac{\partial^2 g_2}{\partial {u_2}^2}(u_1,u_2)=\frac{a_2\alpha(1-\alpha){u_1}^\alpha}{(1-u_2)^{2-\alpha}}-\lambda^2 {u_1}^2(1-b)ce^{-\lambda u_1 u_2}<0,$$

and therefore the function $\partial g_2/\partial u_2$ is monotone.

Now let's calculate the signs of $\partial g_2/\partial u_2$ in the ends of the segment $[0;1]$.

$$\frac{\partial g_2}{\partial u_2}(u_1,0)=-a_2\alpha {u_1}^\alpha+\lambda u_1(1-b)c \tag{12}$$

$$\frac{\partial g_2}{\partial u_2}(u_1,u_2)\underset{u_2\to 1_-}{\to}-\frac{a_2\alpha {u_1}^\alpha}{0_+}+\lambda u_1(1-b)ce^{-\lambda u_1 u_2}\underset{u_2\to 1_-}{\to}-\infty \tag{13}$$

If (12) is positive then the equation can be solved by bisection, and the solution will be the point of maximum due to the negativity of the second derivative. If (12) is negative then the method of bisection is not applicable but the left side of the equation is monotone and therefore it is negative in the segment $[0;1]$, so the function g2 de-creases and the point of maximum is $u_2=0$. Thus,

$$u_2{}^* = \begin{cases} 0, & -a_2\alpha u_1{}^\alpha + \lambda u_1(1-b)c < 0, \\ \in (0;1), & -a_2\alpha u_1{}^\alpha + \lambda u_1(1-b)c > 0. \end{cases}$$

The top-level player can use the information to impel the bottom-level player to choose a non-zero strategy $u_2 > 0$. It is necessary for this to assure the condition $-a_2\alpha u_1{}^\alpha + \lambda u_1 (1-b) c > 0$. Solving the inequality with respect to u_1 we rewrite the condition as $u_1 > \sqrt[1-\alpha]{\frac{a_2\alpha}{\lambda(1-b)c}}$. The top-level player can satisfy the condition only if $\sqrt[1-\alpha]{\frac{a_2\alpha}{\lambda(1-b)c}} < 1$, or $a_2 < \left(\frac{\lambda(1-b)c}{\alpha}\right)^{1-\alpha}$. If the top-level player cannot choose the strategy then the bottom-level player chooses $u_2 = 0$. In this case $g_1(u_1, 0) = a_1(1-u_1)^\alpha$. As far as the function g_1 decreases with respect to u_1 we receive $u_1 = 0$.

Let's summarize:

I. If $a_2 > \left(\frac{\lambda(1-b)c}{\alpha}\right)^{1-\alpha}$ then the leader cannot influence to the follower and $u_2 = 0$, therefore $u_1 = 0$. This case takes place when the effect from non-purpose activity on the bottom level is essentially greater than the effect from his purpose-use activity.

II. If $a_2 < \left(\frac{\lambda(1-b)c}{\alpha}\right)^{1-\alpha}$ then the leader can impel the follower to assign a part of his resources for the purpose-use activity by choosing $u_1 > \sqrt[1-\alpha]{\frac{a_2\alpha}{\lambda(1-b)c}}$. This case takes place when the effect from purpose-use activity on the bottom level is essentially greater than the effect from his non-purpose use activity.

The case of parameterization

$$a_1(u_1, u_2) = a_1\log_2(2-u_1), a_2(u_1, u_2) = a_2\log_2(1+u_1(1-u_2)),$$
$$c(u_1, u_2) = c(u_1u_2)^\alpha, b_1 = b, b_2 = 1-b.$$

is considered similarly. The payoff functions have the form

$$g_1(u_1, u_2) = a_1\log_2(2-u_1) + bc(u_1u_2)^\alpha, \tag{14}$$

$$g_2(u_1, u_2) = a_2\log_2(1+u_1(1-u_2)) + (1-b)c(u_1u_2)^\alpha \tag{15}$$

The results of analysis are presented in Fig.6.

If $a_1 < \alpha bc ln2$ and $a_2 > \alpha(1-b) c ln2$ then $u_1 = \sqrt[1-\alpha]{\frac{(1-b)c\alpha\ln 2}{a_2}}, u_2 = 1$.

The payoffs of the players are:

$$g_1\left(\sqrt[1-\alpha]{\frac{(1-b)c\alpha\ln 2}{a_2}}, 1\right) = a_1\log_2\left(2 - \sqrt[1-\alpha]{\frac{(1-b)c\alpha\ln 2}{a_2}}\right) + bc\left(\sqrt[1-\alpha]{\frac{(1-b)c\alpha\ln 2}{a_2}}\right)^\alpha$$

$$g_2\left(\sqrt[1-\alpha]{\frac{(1-b)c\alpha\ln 2}{a_2}}, 1\right) = (1-b)c\left(\frac{(1-b)c\alpha\ln 2}{a_2}\right)^{\frac{\alpha}{1-\alpha}}.$$

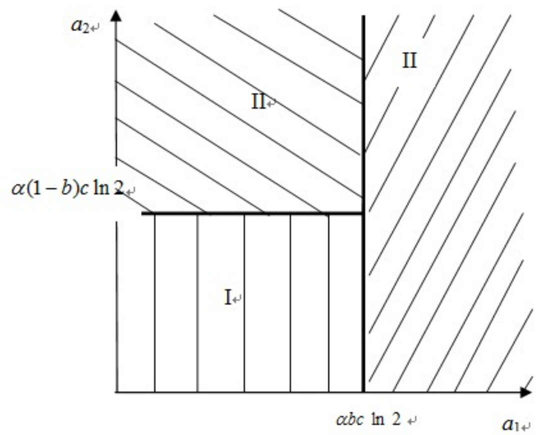

Fig.6 Equilibrium outcomes in the game(14) - (15)

If $a_1 < \alpha bcln2$ and $a_2 < \alpha\,(1-b)\,cln2$ then the altruistic strategy is advantageous for both players: $u_1 = 1, u_2 = 1$. The payoffs are:$g_1\,(1,1) = bc, g_2\,(1,1) = (1-b)\,c$.

If $a_1 > \alpha bcln2$ then it is profitable for the top-level player to allocate for the purpose use a part of her resource $u_1 \in \left(0; \min\left\{ \sqrt[1-\alpha]{\frac{(1-b)c\alpha\ln 2}{a_2}}; 1 \right\}\right)$.

5 Conclusion

In this paper the problem of non-purpose resource use is considered from the point of view of analysis and design of the control mechanisms providing the concordance of interests in the hierarchical (two-level) systems. Interests of the agents are described by their payoff functions including two terms: profit from the purpose and non-purpose resource use respectively. Different classes of the payoff functions are studied. The top-level control agent (resource distributor) is treated as leader, and the bottom-level agent (resource recipient) as follower what results in the concept of Stackelberg equilibrium. The analytical and numerical investigation allows for the following conclusions.

• When both purpose and non-purpose interests functions are power ($k < 1$) then it is advantageous for both players to invest a part of their resources to the purpose use and the other part to the non-purpose use;

• When even one of the payoff functions is power ($k > 1$), and the other is also power ($k > 1$) or linear then it is advantageous for both players or assign the resources only for the purpose use (the altruistic strategy), or only for non-purpose use (the egoistic strategy);

• In other cases the following situations are possible:

A) When the payoff from the non-purpose activity of a player is much greater than the payoff from the purpose activity then the egoistic strategy is advantageous;

B) When the payoff of non-purpose activity of both players is much smaller than the payoff from the purpose activity then the strategy of pure altruism is advantageous;

C) When the payoffs of the purpose and non-purpose activities are comparable then it is profitable to divide the resources between the purpose and non-purpose interests.

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