The Theory of Parametric Control of Macroeconomic Systems and Its Applications(I)

A. Ashimov, Zh. Adilov, R. Alshanov, Yu. Borovskiy and B. Sultanov
Kazakh National Technical University named after K. Satpayev, 22 Satpaev Street, Almaty, 05x0013, Kazakhstan

Abstract
This work consists of three parts and presents the recent results of development of the theory of parametric control of macroeconomic systems and some its applications for solving a number of concrete problems.
Keywords Mathematical Model, Structural Stability, Parametrical Identification, Parametric Control

Introduction

Development of adequate methods on the basis of mathematical models for macroeconomic analysis and of evaluating optimal values of parameters (economic policy tools) for macroeconomic systems control on the level of national economies, regional economic unions and world economic system is urgent problem, sharply necessity in solving which was emphasized by the latest global crisis.

Nowadays, mathematical models of corresponding macroeconomic systems without comprehensive testing for possibility of their application are widely used for macroeconomic analysis (including scenario analysis) and evaluating optimal parameter values of economic policy for controlof macroeconomic systems evolution [1-15].

This paper is devoted to the development of the theory of macroeconomic analysis and evaluating optimal parameter values of economic policy for control of macroeconomic systems on the basis of the corresponding mathematical models, tested for possibility of their application, and it consists of three parts.

The first part describes the components of the parametric control theory and its algorithmic foundations. The second part describes mathematical foundations of the parametric control theory, and the third part-the developed theory applications for solving a number of applied problems on the basis of some mathematical models of macroeconomic systems.

Part 1. Components of the parametric control theory and its algorithmic foundations

1.1 Components of the Parametric Control Theory of Macroeconomic Systems

Given the following facts:

- solution of either continuous or discrete dynamical system [which can include both controllable parameter vectors-state policy tools (μ), and uncontrollable parameter vectors (a)] depends on initial condition vectors and parameters (coefficients) of this system;

- solution of static system (for instance, static model of small open economy) depends on parameters (coefficients) of this system;

- for judging by the study results of dynamical system about an object, described by it, an existence of structural stability (or robustness) of this system is required [16];

- for judging by the study results of (static or dynamic) model about an object, described by it, an existence of stability of mapping, defined by this model, is required [17];

- and also a condition of macroeconomic model (presented by one of dynamic or static system) stability is required at small perturbations of the initial statistical data for parametric identification of the model (input parameters) and the following components of the parametric control theory are proposed [18-19].

1. The methods for forming the set (library) of macroeconomic mathematical models. These methods are oriented towards the description of various specific socio-economic situations.

2. The methods for estimating the conditions for robustness (structural stability) of the dynamical mathematical models, the methods for estimating the stability indicators and the methods for estimating stability of mappings, set by models of national economic system from the library (without parametric control).

3. The methods for adjusting the structural instable dynamical mathematical model to obtain its structural stability (methods for attenuation of structural instability). Choosing (or synthesizing) the algorithms for attenuation of structural instability for the mathematical model of macroeconomic system.

4. The methods for choosing and synthesizing the laws of parametric control of macroeconomic system based on its dynamical mathematical models. The methods for setting and solving the parametric control problems in terms of corresponding mathematical programming problems on the basis of static mathematical models of macroeconomic systems.

5. The methods for estimating the robustness (structural stability) of dynamical mathematical model. The methods for estimating the stability and the methods for estimating the stability of mappings, set by models of macroeconomic systems (with parametric control).

6. The methods for adjusting the constraints on parametric control of macroeconomic system in the case of structural instability of its mathematical model with parametric control. Specification of constraints on the parametric control of macroeconomic system.

7. The methods for studying the effects of uncontrollable parameters and

functions (uncontrollable factors) on the results ofsolving of variational calculus problems of synthesis and choice (among given finite algorithms set) of parametric control laws. Study of bifurcation points of extremals of variational calculus problems of choosing optimal laws of parametric control. The methods for studying the effects of uncontrollable factors variance on the solution results of mathematical programming problems based on static mathematical models.

8. Approach for choosing recommendations on evaluating political rules in the frame of implementing the laws of parametric control of macroeconomic system on the base of the analysis of dependences of optimal criteria values of corresponding parametric control problems on uncontrollable factor values.

This paper presents the general results of component-specific development of the parametric control theory (its mathematical and algorithmic foundations).

- Within the framework of the methods for forming the set (library) of macroeconomic mathematical models, it is proposed an algorithm for parametric identification of large-scale macroeconomic models, which uses jointly two identification criteria.

- Within the framework of the methods for examining mathematical models stability, it is proposed numerical algorithms for stability indicators estimation and numerical algorithms for estimation stability of mappings, set by model (in terms of the theory of differentiated mappings singularities);

- Within the framework of the methods for examining the weak structural stability, it is described the proposed numerical algorithm based on the Robinson theorem about sufficient conditions of weak structural stability of dynamical mathematical models.

- Within the framework of the methods for choosing and synthesizing parametric control of national economy, based on continuous and discrete non-autonomous dynamical systems, as well as discrete dynamical systems with additive noise, there are formulated and proved the corresponding theorems about conditions for the existence of solutions of variational calculus problems on synthesis and choice (among given finite algorithms set) of optimal parametric control laws.

- Within the framework of the methods for studying the effects of uncontrollable factors variance on the solution results of variational calculus problems on synthesis and choice of optimal parametric control laws, there are formulated and proved the theorems about conditions for the continuous dependence of optimal criteria values of variational calculus problems on uncontrollable parameters (uncontrollable function values).

- Within the framework of studying the bifurcations of extremals of the variational calculus problem of choosing the optimal parametric control laws, there are formulated and proved the theorems about sufficient conditions for the existence of appropriately defined bifurcational point of extremals of the variational calculus problem;

- there is proposed an approach for choosing the recommendations on evaluating political rules within the framework of implementation of appropriate economic tools for adjusting national economy on the base of analyzing dependences of optimal criteria values of corresponding parametric control problems on uncontrollable factors values.

1.2 Algorithm for the Parametric Identification of Large-scale Macroeconomic Models

The following algorithm for the parametric identification of large-scale macroeconomic models is proposed within the framework of elaborating the 1st component of the parametric control theory [19].

The parametric identification problem for discrete dynamical macroeconomic model is finding the estimates of unknown values of its parameters (to which belong unknown values of exogenous functions of model and unknown initial values of its dynamical equations), at those one can obtain the minimum of the objective, characterizing the deviations of output variables values of the model from corresponding observed values (of known statistical data for the period $t = t_1, t_1 + 1, ..., t_2$). This problem comes to finding the minimum of the multi variable function (parameters) in some closed domain D of Euclidean space with constraints, overlaying both on endogenous variables values of the model (E constraints)and on initial parameter values (F constraints). In the case of large number of dimensions N of this domain, the standard methods for finding function extrema are often ineffective because of presence of several local minima of the objective. Below we present the algorithm, allowing for features of the parametric identification problems for macroeconomic models and allowing passing over the mentioned problem of "local extrema".

Constraints E are formed by economic meaning of endogenous variables of model (for example, by their non-negativity). Domain of type $D = \prod_{i-1}^{N_i}[a^i, b^i]$, where $[a^i, b^i]$ is segments of possible values of the parameter p^i; $i = 1, ..., N$ was considered as a range, defined by F constraints for evaluating possible values of exogenous parameters.Herewith, parameter estimates, for which had observed values, were searched either within $[a^i, b^i]$ segments with centers at corresponding observed values (in the case of one such value) or within some segments, covering observed values (in the case of several such values). Other $[a^i, b^i]$ segments for search of the parameters were chosen using indirect estimates of their possible values.Nedler-Mead algorithm of directed search was used in calculating experiments for finding the minimal values for continuous function K: D $\rightarrow$ R of several variables [20]. Use of this algorithm for initial point $p_1 \in D$ can be interpreted in terms of (converged to local minimum $p_0 = argminK$ of criterion K) sequencep_1, p_2, ..., where $K(p_j + 1) \leq K(p_j)$, $p_j \in D$, j = 1, 2,... We will assume that point can be found accurately enough, when we describe the following algorithm.

For solving the parametric identification problem of model in question on the base of obvious assumption about divergence (in general case) of minimum points of two different functions, two criteria of the following type were proposed:

$$K_A(P) = \sqrt{\frac{1}{n_\mu(t_2 - t_1 + 1)} \sum_{t=t_1}^{t_2} \sum_{i=1}^{n^A} \alpha_i \left(\frac{y^i(t) - y^{i^*}(t)}{y^{i^*}(t)} \right)^2}$$

$$K_B(P) = \sqrt{\frac{1}{n_\beta(t_2 - t_1 + 1)} \sum_{t=t_1}^{t_2} \sum_{i=1}^{n^\beta} \alpha_i \left(\frac{y^i(t) - y^{i^*}(t)}{y^{i^*}(t)} \right)^2}$$

Here $\{t_1, ..., t_2\}$ is identification period;$y^i(t)$, $y^{i^*}(t)$-correspondingly computed and observed values of model output variables, $K_A(P)$-subsidiary criterion, $K_B(P)$-basic criterion; $n_b > n_a$; $\alpha_i > 0$ and $\beta_i > 0$ are some weight coefficients, values of which are defined during solution of parametric identification problem for dynamical system; $\sum_{i=1}^{n_A} \alpha_i = n_\alpha$, $\sum_{i=1}^{n_B} \beta_i = n_\beta$.

The minimization problems based on the model of corresponding criterion (K_A and K_B) in the domain D, we will call the Problem A and the Problem B.Theaggregate algorithm for solving the parametric identification problem of model was chosen in terms of the following steps:

1. For some vector of initial values of parameter $p_1 \in D$, solve problems A and B simultaneously. Then, find the minimum points p_{A0} and p_{B0} of criteria K_A and K_B, respectively.

2. If $K_B(p_{B0}) < \varepsilon$ for some sufficiently small number ε, then the model parametric identification problem is solved.

3. Otherwise, choose the point p_{B0} as the initial point p_1, solve problem A and, choosing the point p_{A0} as the initial point p_1, solve problem B. Go to step 2.

After sufficiently large number of iterations of stages 2 and 3, initial values of the parameters might leave neighborhoods of the non-global minima in one criterion with help of the other and thereby solve the parametric identification problem.

The following methods for evaluating the stability indicators and the structural stability of mathematical models are proposed within the framework of elaborating the 2 component of the parametric control theory.

1.3 Methods for Evaluating the Stability of Mathematical Models of Macroeconomic Systems

1.3.1 Methods for Evaluating the Weak Structural Stability of Dynamical Models

The methods of analysis of the robustness (structural stability) of mathematical model of national economic system are based on:

- Fundamental results on dynamical systems theory in the plane;

- Methods of verification of mathematical models belonging to certain classes of structurally stable systems (classes of Morse-Smale systems, Ω-robust systems, -systems, systems with weak structural stability).

At present, the theory of parametric control of market economic development has available a number of theorems about structural stability of specific mathematical models (the model of the neoclassical theory of optimal growth; model of national economic system taking into consideration the influence of the share of public expenses and of the interest rate of governmental loans on economic growth; model of national economic systems taking into consideration the influence of international trade and currency exchanges on economic growth; and others) formulated and proved on the basis of the aforementioned fundamental results.

Along with analysis of the structural stability of specific mathematical models (both with and without parametric control), based on results of the theory of dynamical systems, one can consider approaches to the analysis of structural stability of mathematical models of national economic system by means of computer simulations.

We shall consider below the construction of a computational algorithm for estimating the structural stability of mathematical models of national economic system on the basis of Robinson theorem (Theorem A) on weak structural stability [21].

Theorem.*Let N' be some manifold, and N a compact subset in N' such that the closure of the interior of N is N. Let some vector field be given in a neighborhood of the set N in N'. This field defines the C^1-flux f in this neighborhood. Let $R(f,N)$ denote the chain-recurrent set of the flux f on N.*

Let $R(f,N)$ be contained in the interior of N. Let it have a hyperbolic structure. Moreover, let the flux f upon $R(f,N)$ also satisfy the transversability conditions of stable and unstable manifolds. Then the flux f on N is weakly structurally stable. In particular if $R(f,N)$- an empty set, then the flux f is weakly structurally stable on N. A similar result is also correct for the discrete-time dynamical system (cascade) specified by the homeomorphism (with image) $f: N \to N'$.

Therefore, one can estimate the weak structural stability of the flux (or cascade) f via numerical algorithms based on this Theorem by means of numerical estimation of the chain-recurrent set $R(f,N)$ for some compact region N of the phase space of the considered dynamical system.

Let us further propose an algorithm of localization of the chain-recurrent set for a compact subset of the phase space of the dynamical system described by a system of ordinary differential (or difference) equations and algebraic system. The proposed algorithm is based on the algorithm of construction of the symbolic image [22]. A directed graph (symbolic image), being a discretization of the shift mapping along the trajectories defined by this dynamical system, is used for computer simulation of the chain-recurrent subset.

Suppose an estimate of the chain-recurrent set $R(f, N)$ of some dynamical system in the compact set N of its phase space has been found. For a specific mathematical model of the economic system, one can consider, for instance, some parallelepiped of its phase space including all possible trajectories of the economic system evolution for the considered time interval as the compact set N.

The localization algorithm for the chain-recurrent set consists of the following:

1. Define the mapping f defined on N and given by the shift along the trajectories of the dynamical system for the fixed time interval.

2. Construct the partition C of the compact set N into cells N_i. Assign the directed graph G with graph nodes corresponding to the cells and branches between the cells N_i and N_j corresponding to the conditions of the intersection of the image of one cell $f(N_i)$ with another cell N_j.

3. Find all recurrent nodes (nodes belonging to cycles) of the graph G. If the set of such nodes is empty, then $R(f, N)$ is empty, and the process of its localization ceases. One can draw a conclusion about the weak structural stability of the dynamical system.

4. The cells corresponding to the recurrent nodes of the graph G are partitioned into cells of lower size, from which a new directed graph G is constructed (see item 2 of the algorithm).

5. Go to item 3.

Items 3, 4, 5 must be repeated until the diameters of the partition cells become less than some given number ε.

The last set of cells is the estimate of the chain-recurrent set $R(f, N)$.

The method of estimating the chain-recurrent set for a compact subset of the phase space of a dynamical system developed here allows one, in the case in which the obtained chain-recurrent set $R(f, N)$ is empty, to draw a conclusion about the weak structural stability of the dynamical system.

In the case that the considered discrete-time dynamical system is a priori the semi-cascade f, one should verify the invertibility of the mapping f defined on N (since in this case, the semi-cascade defined by f is the cascade) before applying Robinson's theorem A for estimating its weak structural stability.

Let us give a numerical algorithm for estimating the invertibility of the differentiable mapping $f : N \to N'$, where some closed neighborhood of the discrete-time trajectory $\{f'(x_0), t = 0, ..., T\}$ in the phase space of the dynamical system is used as N. Suppose that N contains a continuous curve L,which sequentially connects the points $\{f'(x_0), t = 0, ..., T\}$. One can choose as such curve a piecewise linear curve with nodes at the points of the above mentioned discrete-time trajectory of the semi-cascade.

An invertibility test for the mapping $f : N \to N'$ can be implemented in the following two stages:

1. An invertibility test for the restriction of the mapping $f : N \to N'$ to the curve L, namely, $f : L \to f(L)$. This test reduces to the ascertainment of the fact that the curve $f(L)$ does not have points of self-crossing, that is, $(x_1 \neq x_2) \Rightarrow (f(x_1) \neq (f(x_2))$. For instance, one can determine the absence of self-crossing points by means of testing monotonicity of the limitation of the mapping f onto L along any coordinate of the phase space of the semi-cascade f.

Let us choose sufficiently large set of points like $x_i = (x_i^1, x_i^2, ..., x_i^n) \in L$, $y_i = f(x_i), y_i = (y_i^1, y_i^2, ..., y_i^n)$ and coordinate number of these points (j). If for all $x_i^j, i = 1, ..., n$ at $x_{i_1}^j < x_{i_2}^j$ theine quality $y_{i_1}^j < y_{i_2}^j$ is met (or at $x_{i_1}^j < x_{i_2}^j$ the inequality $y_{i_1}^j > y_{i_2}^j$ is met), then $f : L \to f(L)$ mapping is estimatedas invertible.

2. An invertibility test for the mapping f in neighborhoods of the points of curve L (local invertibility). Based on the inverse function theorem, such a test can be carried out as follows: For a sufficiently large number of chosen points $x \in L$ one can estimate the Jacobians of the mapping f using the difference derivations: $J(x) = det(\frac{\partial f_i}{\partial x_j}(x)), x, j = 1, ..., n$. Here i, j are the coordinates of the vectors, and n is the dimension of the phase space of the dynamical system. If all the obtained estimates of Jacobians are nonzero and have the same sign, one can conclude that $J(x) = 0$ for all $x \in L$ and, hence, that the mapping f is invertible in some neighborhood of each point $x \in L$.

An aggregate algorithm for estimating the weak structural stability of the discrete-time dynamical system (semi-cascade defined by the mapping f) with phase space $N' \in R''$ defined by the continuously differentiable mapping f can be formulated as follows:

1. Find the discrete-time trajectory $\{f'(x_0), t = 0, ..., T\}$ and curve L in a closed neighborhood N which is required to estimate the weak structural stability of the dynamical system.

2. Test the invertibility of the mapping f in a neighborhood of the curve L using the algorithm described above.

3. Estimate (localize) the chain-recurrent set $R(f, N)$. By virtue of the evident inclusion $R(f, N_1) \subseteq R(f, N_2)$ for $N_1 \subset N_2 \subset N'$, one can use any parallelepiped belonging to and containing L as the compact set N.

4. If $R(f, N) = \Phi$, draw a conclusion about the weak structural stability of the considered dynamical system in N.

This aggregate algorithm can be also applied for estimating the weak structural stability of a continuous-time dynamical system (the flux f), if the trajectory $L = \{f'(x_0), 0 \leq t \leq T\}$ of the dynamical system is considered as the curve L. In this case, item 2 of the aggregate algorithm is omitted. The mapping f^t for some fixed $t (t > 0)$ can be accepted as the mapping f in item 3.

1.3.2 Methods for Evaluating the Weak Structural Stability of Dynamical Models

By definition of Orlov [18], the mathematical model of an economic system in general view is some mapping

$$f : A \to B$$

transferring values of initial (exogenous) data $p \in A$ to solutions (values of endogenous variables) $y \in B$.

After constructing a mathematical model of some real-life phenomena or process and defining some actual values of the point p by known measured data or solving the parametric identification problem, the question about adequacy of the analyzed model arises. The condition of model stability relative to admissible perturbations of the initial data [16] is a one of the conditions of the model adequacy. In case of such stability, small perturbations of the model's initial data results in small changes of its solution. In the mentioned monograph, the definitions of the basic stability indicators are introduced (these definitions are presented below). Monograph, however, does not propose any algorithm for computing the considered indicators of the mathematical model stability.

Below we present the developed algorithms for evaluating the mathematical model stability indicators which characterize stability of solutions of the mathematical model relative to initial data perturbations. At that, all of the model parameters and variables must be made dimensionless beforehand.

Let $X = (X^1, X^2, ..., X^k)$ be some vector of values of the model exogenous parameters for the time interval $t \in \{0, ..., T\}$. Let $X_0 = (X_0^1, X_0^2, ..., X_0^k)$ denote the respective vector of base values for the same time interval. The vector that incorporates the values of parameters and initial values of the variables of differential (or difference) equations is considered as the vector X. The vector of measured statistical data used for finding the model equation coefficients is considered as vector X for the econometric models.

Let $p = (p^1, p^2, ..., p^k)$ be a vector of the normalized input data of the mathematical model where $p^i = \frac{x^i}{x_0^i}, i = 0, ..., k$. The vector $p_0 = (1, 1, ..., 1)$.

Let be a space of the normalized input data vectors which includes all admissible sets p, $A \subset R^k$ is a metric space with the Euclidean metric defined by the space $R^k, p_0 \in A$.

Let $Y = Y(p) = (Y^1, Y^2, ..., Y^k)$ be a selected vector of the values of endogenous variables for some chosen interval (or moment) of time obtained for the selected values of p. The vector that incorporates the values of some selected set of the model endogenous variables for the aforesaid interval (or moment) of time is considered as vector Y for the dynamical models. The vector of coefficients of the model equations or vector of values of some selected set of the model endogenous variables for the aforesaid interval (or moment) of time is considered as vector Y for the econometric models.

In particular, with $p = p_0$, introduce the notation $Y_0 = Y_0(p) = (Y_0^1, Y_0^2, ..., Y_0^k)$. The normalized vector of values of the endogenous variables for the moment of time T_1 is denoted by $y = y(p) = (\frac{Y^1}{Y_0^1}, \frac{Y^2}{Y_0^2}, ..., \frac{Y^n}{Y_0^n})$; $y_0 = y(p_0) = (1, 1, ..., 1)$.

Let $B \subset R^n$ be a region which contains all possible output values y for $p \in A$ with the Euclidean metric of space $R^n, y_0 \in B$. The considered model defines the mapping f of set A into set B.

For the selected point $p \in A$ and number $\alpha > 0$, let $U_\alpha(p)$ denote the intersection of a neighborhood of the point p with radius α with set A:

$$U_\alpha(p) = \{p_1 \in A : \rho(p_1, p) \leq \alpha\}$$

Here and below, $\rho(.,.)$ denotes the Euclidean distance between two points of the Euclidean space.

For some subset $B_1 \subset B$, let $d(B_1)$ denote the diameter of set V_1, that is

$$d(B_1) = sup(\rho(y_1, y_2) : y_1, y_2 \in B_1)$$

Definition1.1. The number $\beta(p, \alpha) = d(f(U_\alpha(p)))$ is defined as the stability indicator of the econometric model at the point for ¿0.

Algorithm1.1 for evaluating the model stability indicator $\beta(p, \alpha)$ by the Monte Carlo method is as follows:

2. Define the vector of normalized input data $p = (p^1, p^2, ..., p^k)$, number $\alpha > 0$, and set $U_\alpha(p)$.

3. Generate a set of sufficiently large number M of pseudo-random points $(p_1, p_2, ..., p_M)$ uniformly distributed in $\beta(p, \alpha)$.

For this purpose, consecutively generate the coordinates $p_j^i (i = 1, ..., k; j = 1, ..., M)$ of the point p_j in numerical segments $[p^i - \alpha, p^i + \alpha]$ covering $U_\alpha(p)$ using a generator of uniformly distributed pseudo-random numbers. If the inequality $\sum_{i=1}^{k} (p_j^i - p^i)^2 \leq \alpha^2$ holds (i.e. $x_j \in U_\alpha(p)$), this point is added to the created set.

4. For each point p_j of the set, define point $y_j = f(p_j), j = 1, ..., M$, by simulation.

5. Evaluate $\beta = max(\rho(y_i, y_j) : i, j = 1, ..., M)$.

6. Stop.

With $\alpha = 0.01$, the obtained number $\beta/2$ characterizes the (maximum) percentage change of values of the model output variables under the perturbed input data by 1%.

Definition1.2. The number $\beta(x) = \inf_{0 \leq \alpha \leq \alpha_0} \beta(p, \alpha)$ is called the absolute stability indicator of the econometric model at point $x \in A$. Here, α_0 is the maximal admissible relative deviation of values of the model input data.

Algorithm 1.2 for evaluating the absolute stability indicator $\beta(p)$ of the e-conometric model is as follows:

For the selected value α_0 and numbers j = 0, 1, 2, consecutively find (by Algorithm 1.1) numbers $\beta_j = \beta(p, \alpha_0/2^j)$, and then evaluate the number

$$\beta(p) = \underset{j=0,1,2,...}{inf} \beta_j$$

If $\beta(p)$ turns out to be less than some a priori given small number (i.e. $\beta(p)$ is considered to be approximately zero), then the mapping f defined by the analyzed model is evaluated at point p continuously depending on the input values.

Definition1.3. The number

$$\gamma = \sup_{p \in A} \beta(p)$$

is called the maximal absolute stability indicator of the model for region A.

Algorithm1.3 for evaluating the maximal absolute stability indicator of the model by the Monte Carlo method is as follows:

1. Generate the set of sufficiently large number M of pseudo-random points $(p_1, p_2, ..., p_M)$ uniformly distributed in .
2. For each point p_j in the set and chosen ,$\alpha_0 > 0$ find the numbers $\beta(p_j)$ by Algorithm 1.2.
3. Determine the number $\gamma = \underset{j=1,...,M}{max} \beta(p_j)$.
4. Stop.

If the number turns out to be less than some a priori given small number ε(i.e. is considered to be approximately zero), then the mapping f defined by the analyzed model is evaluated in set A continuously depending on the input values.

The developed algorithms were applied for evaluating econometric model of a small open economy and computable general equilibrium model of economic branches.

1.3.3 Methods for Evaluating the Stability of Mappings, Defined by Models, in Terms of the Theory of Differentiated Mappings Singularities

This section describes the methods for evaluating in sense of definition the stability of smooth mappings $F : D \to E$, defined by statical or dynamical model [17]. As domains (D) of the mappings in question are used corresponding domains of possible values of uncontrollable, controllable parameters, coefficients of the model econometric equations in question, and also domain of possible values of observed (statistical) data, used for functions building, which set models econometric equations of the model. Range of values (E) of the mapping F contains set of possible values of endogenous variables of the model.

Existence of such stability property indicates preservation of qualitative properties of mapping, using which the model is described, at small variances of this mapping. When real economic phenomena are described adequately using the mathematical model, stability (or instability) of the mapping, presented by the model, may indicate stability (or instability) of corresponding dependencies of possible values of economic indicators on external (controllable or uncontrollable) factors at small variances of these dependencies. Instability of mapping, set by the model, may also indicate inadequacy of the model in question [17].

An algorithm for estimating critical point set of the mappings in question is presented within the framework of this study of the given mappings stability. This algorithm, in particular, allow sestimating the maximality of Jacobian matrix rank of mapping at all points of its domain, i.e. to check whether either the investigated map is immersion or submersion. For the immersion case, an algorithm for estimating injectivity of the mapping is proposed.

There are formulated and proved the statements, which allow to estimate the stability (and in some case ratios of the image dimension to the counter image dimension-instability) of the mapping in question, when conditions of immersion and injectivity or condition of submersion are satisfied for the mapping in question.

There are presented the statements about stability conditions for the mapping in question in the case if this mapping is submersion with fold [23]. There is presented the algorithm, which allows to estimate the mapping as submersion with fold and estimate the stability of the mapping in question in this case.

1.3.3.1 Algorithm for Estimating the Critical Point Set of the Mappings, Set by the Model

Hereinafter, we will imply the mapping, defined by the mathematical model when we use the mapping

$$F : D \to E \tag{1}$$

Let's denote the mapping arguments vector (1) through $p = (p^1, ..., p^n) \in D$, and corresponding p point image - the model solutions vector denote through $y = y(p) = y(p^1, ..., p^v) \in E$, ($D \subset R^n$ and $E \subset R^v$ are some regions). In this case, Jacobian matrix with size vn for the mapping (1) at the point p will be written in the following form:

$$J(p) = \left(\frac{\partial y^i}{\partial p^j}(p) \right)_{i=1,...,v; j=1,2,...,n} \tag{2}$$

We will also denote the Jacobian matrix estimate (2) at some point $p \in D$, obtained by numerical differentiation from (1), through $J(p)$.

Within the framework of the solution of F mapping stability studying problem, we will present an algorithm, which allows to estimate the $J(p)$ matrix rank maximality for $p \in D$, that is the algorithm for condition estimate

$$rank((J(p))) = min(\upsilon, n), p \in D \tag{3}$$

When this condition is satisfied, the mapping F will noth ave any critical point in the domain D [17].

For (3) condition estimate to find any nonsingular minor (of the matrix $J(p)$ of order $min(\upsilon, n)$) for each $p \in D$ is enough, taking into account that total amount of maximal-order minors in J is $l = C_\upsilon^n = \frac{\upsilon!}{n!(\upsilon-n)!}$ if $n < \upsilon$ and $l = C_n^\upsilon = \frac{n!}{\upsilon!(n-\upsilon)!}$, if $n \geq \upsilon$. We will denote the determinant value estimate of such minor of order $min(\upsilon, n)$) in $J(p)$ for $p \in D$ through $|M_i(p)|, i = 1, ..., l$.

Algorithm1.4. The aggregate algorithm for estimating condition(1.3)for estimation of mapping critical point set.

1) Domain D divides to sufficiently large amount of (elementary) parallelepipeds D^k of the same size, and define the net P from N points, those are vertices of chosen parallelepipeds: $p = \{p_j : j = 1, ..., N\}$.

2) Compute the values of all $J(p_j)$ matrix elements for $j = 1, ..., N$.

3) For $i = 1, ..., l$, $j = 1, ..., N$, compute the determinants $|M_i(p_j)|$.

4) For each $i = 1, ..., l$ the set $D(i)$ is defined in the following way. $D(i)$ isa sumof all (closed) parallelepipeds D^k, which have property that not all values of $|M_i(p_j)|$ at D^k vertices have the same sign.

5) Find the set $\tilde{D} = \bigcap_{i=1}^{l} D(i)$.

6) If the set $\tilde{D}$ is empty, then condition (3) is evaluated as satisfied. Stop.

7) Otherwise,the steps 1) - 6) of present algorithm are performed substituting domain D by $\tilde{D}$ and diminishing sizes of parallelepipeds, participating in partitioning of $\tilde{D}$.

Sufficiently large amount iteration of the steps of the presented above algorithm allow either to estimate satisfaction of the condition (3) or to obtain the estimate (using set $\tilde{D}$)the set of critical points of F mapping.

1.3.3.2 Algorithm for Estimating the Nonlocal Injectivity of Mapping,Set by Model

In this section it assumes that $n < \upsilon$ (D domain dimension of mapping (1) is less than Erange of values dimension). This section presents an algorithm for estimating conditions of nonlocal injectivity (absence of non-close points in D, having equal images at mapping F) of mapping (1), set by model in domain D. Satisfaction of mentioned nonlocal injectivity condition and condition (3), ensuring local injectivity in neighborhood of each point means an existence of inverseto Fmapping, determined in set $F(D)$.

Fix sufficiently small number $\varepsilon > 0$. For each point $p_j \in P$ through $d(p_j)$

denote set of all points $p_k \in P$ which have $|p_j - p_k| > \varepsilon$. Here $|\cdot|$ is magnitude of vector.

Algorithm1.5 for estimating the condition of the nonlocal injectivity is presented in terms of the following steps.

1) Compute the numbers

$$m_j = \min_{p_k \in d(p_j)} |F(p_k) - F(p_j)|$$

for each point $p_j \in P$.

2) Compute $m == min_{p_j \in P} m_j$ and determine the points $p_j, p_k \in P$ such that $|F(p_k) - F(p_j)| = m$.

3) Repeat steps 1) and 2) of this algorithm substituting the net P by net P_1 being the vertices of less size parallelepipeds and containing all points of the net P, being distant from one of p_j, p_k points by the distance not exceeding 2ε.

There are two possible cases given sufficiently large amount of iterations of steps of the presented above algorithm.

a) The sequence of obtained values are diminishing about proportional to the P_1 net step. In this case,F mapping is estimated as non-injective (in other words, $F(D)$ set is estimated as self-crossing).

b) Condition $m > \varepsilon$ is met for all nets in question with sufficiently small step. In this case, F mapping is estimated as injective (in other words, $F(D)$ set is estimated as non-self-crossing).

1.3.3.3 Estimating the Stability of Mappings Set by the Model

This section presents propositions about sufficient conditions for F mapping stability in open domain $D^0 = D \setminus \gamma(D)$, where $\gamma(D)$ is boundary of set D, within the framework of determining stable mapping [17]. It also presents an aggregate algorithm for estimating mapping $F : D \to E$, as stable submersion with fold.

According to stability theorem of Mazer [17], mapping F is stable in manifold D^0, if it is infinitesimally stable in D^0. Condition of infinitesimal stability of mapping $h(p)$ in $k(y)$ is formulated in terms of solubility in relative to mappings and of the following homologous equation [17].

$$\mu(p) = -J(p)h(p) + k(F(p)) \tag{4}$$

Here $\mu(p)$ is an arbitrary infinitesimal deformation of F mapping, which is presented in terms of smooth correspondence to each point $p \in D^0$ of tangent vector to manifold E atpoint $F(p)$; $h(p)$ is the smooth vector field in D^0; $k(y)$ is the smooth vector field in E.

Consider two possible cases.

1. Let $n < v$ Let condition (3) be satisfied, that is F mapping is the immersion

in D. According to the theorem about inverse function (when F mapping injectivity condition is satisfied, which checks by the algorithm described in item 2) in set $F(D^0)$ the smooth mapping $F^{-1}: F(D^0) \to D^0$, inverse to F is defined. If in Equation (4) input $h(p) = 0$, and $k(y) = \mu(F^{-1}(y))$, then this equation become an identity $\mu(p) = 0 + \mu(F^{-1}(F(p)))$. This means that the mentioned mappings, $h(p)$ and $k(y)$, are solutions of homologous equation (4), that is this equation is soluble. Consequently, the first statement of the following proposition is true.

Proposition 1.3.1. Let given $n < \upsilon$ for all points of chosen set D,condition (3)for injective mapping(1.1)is satisfied.Then mapping(1) is stable in domain $D^0 = D \setminus \gamma(D)$. If condition(3) is not satisfied for any point $p \in D^0$, then mapping(1) is not stable in its domain.

The second statement of the proposition 1.3.1 results from the sentences 2.4 and 3.12 [23].Formulate them.

Statement 2.4. Let X is compact and $dimY \geq 2dimX+1$. Mapping $f : X \to Y$ is stable, if and only if f isone-to-one immersion.

Statement3.12. Let X is compact manifold and $dimY = 2dimX$. Mapping $f : X \to Y$ is stable, if and only if it is an immersion with normal crossings.

From these sentences results that when condition $dimY \geq 2dimX$ is satisfied, the immersion property is a requirement for stability, that is when (3)is not satisfied, mapping f cannot be stable.

Note that although in formulations of mentioned sentences X manifold compactness condition is used, proofs of immersion properties for stable mappings given $dimY \geq 2dimX + 1$(or $dimY = 2dimX$) are based on the theorems 5.6 and 5.7 [23], which do not require X manifold compactness. Proposition 1.3.1 is fully proved.

2. Let $n \geq \mu$ Let condition (3) be satisfied, that is F mapping is a submersion in D^0. Test, in this case, the solubility of homologous equation (4).

Consider first the case, when some v-order minor determinant $M_i(p)$ of Jacobian matrix has constant sign for $p \in D^0$, that is, $|M_i(p)| \geq \varepsilon \geq 0$ in D^0. Let, for determinacy, such anon singular minor $M_i(p)$ consists of first six columns of matrix $J(p)$. For arbitrary deformation $\mu(p)$, written in terms of υ-dimensional column vector in equation (4) determine $h(p)$ in the following way. First υ coordinates of n-dimensional column vector $h(p)$ set using column $-(M_i(p))^{-1}\mu(p)$, and all other coordinates of vector $h(p)$ assume to be zero. If put $k(y) = 0$, then equation (4) becomes the identity:

$$\mu(p) = -J(p)h(p) + 0 = M_i(p)(M_i(p))^{-1}\mu(p) \tag{5}$$

Consider the general case of satisfaction of condition (3). Let for each point $p \in D$ (and some its neighborhood) be found its nonsingular minor $M_i(p)$. Choose from such neighborhoods U_j finite domain cover: $D \subset U_{j=1}^{s} U_j$. Make subject to this cover the partition of unity into domains D in terms of s smooth functions

$\varphi_j(p) \geq 0$ [23], where $\varphi_j(p) = 0$ at $p \in R^n \backslash U_j (j = 1, ..., s)$ and $\sum_{j=1}^{s} \varphi_j(p) = 1$ for $p \in D$. Consider arbitrary deformation $\mu(p)$ in equation (4). Then, for each neighborhood U_j and its corresponding nonsingular minor $M_i(p)$, make (according to the method proposed in the previous paragraph) the vector field $h_j(p)$ in U_j, which is the solution of equation (4) in U_j at $k(y) = 0$. Determine the vector field $h(p)$ in D^0 using the formula:

$$h(p) = \sum_{j=1}^{s} \varphi_j(p) h_j(p)$$

Thus, from (4) (where $h(p)$ should be substitute $h_j(p)$) results that the vector fields $h(p)$ and $k(y) = 0$ are the solutions of (4):

$$-J(p)h(p) + k(F(p)) = -J(p) \sum_{j=1}^{s} \varphi_j(p) h_j(p) + 0 = \sum_{j=1}^{s} \varphi_j(p)[-J(p)h_j(p)]$$

$$= \sum_{j=1}^{s} \varphi_j(p)\mu(p) = \mu(p) \sum_{j=1}^{s} \varphi_j(p) = \mu(p)$$

This means that the equation (4) is soluble. Therefore, the following proposition is true.

Proposition 1.3.2. Let given $n \geq \upsilon$ for chosen set D condition(3)be satisfied for mapping (1).Then mapping(1) is stable in domain $D^0 = D \setminus \gamma(D)$

3. Now consider the case, when given $n \geq \upsilon$ condition (3) is not satisfied for some points of domain D^0 (that is, the case, when domain D^0 contains the critical points of mapping F). Denote by $S_1(F)$ set of points of domain D^0, in which Jacobian matrix of mapping Frank is less by unit than the maximal one, that is

$$S_1(F) = \{P \in D : rank(J(p) = \upsilon - 1\}$$

It is known that when the additional condition ($j^1 F \pitchfork S_1$, where $j^1 F$ is 1-stream of mapping F, S_1 is submanifold in the space of 1-streams $J^1(D^0, E)$ consisting of streams with 1 co-rank, $\pitchfork$ is the sign of transversality), is satisfied, the set $S_1(F)$ is a submanifold in D^0 with dimension $\upsilon - 1$ [23].

Definition. Let the mapping $F : D^0 \to E$ satisfy condition $j^1 F \pitchfork S_1$. Point $p \in S_1(F)$ is called as fold point, if sum of tangent space to $S_1(F)$ and kernel of tangent mapping dF at this point has dimension n, that is, if

$$T_p S_1(F) + Ker(dF)_p = T_p D \tag{6}$$

Since sum of dimensions of summands of LHS is equal to the dimension of RHS $(\upsilon - 1) + (n - \upsilon + 1) = n$, then condition (6) is equivalent to that these summands have the only common point - origin of coordinates and

$$cos\angle(T_pS_1(F), Ker(dF)_p) \neq 1 \tag{7}$$

Definition. Mapping $F : D^0 \to E$ is called a submersion with folds, if eachits singular pointis a fold point. In this case submanyfold $S_1(F)$ is called as a fold.

It is known that if $F : D^0 \to E$ is submersion with fold, then F mapping constraint on $S_1(F)$ fold is immersion.

The following theorem is valid [23].

Consequence 1.3.4. If $F : D^0 \to E$ is submersion with fold and $F|_{S_1(F)}$ is injective, then mapping $F : D^0 \to E$ is stable.

We present an algorithm for estimating the satisfaction of these conditions of this consequence.

Algorithm 1.6. Aggregate algorithm for estimating mapping $F : D^0 \to E$ as stable submersion with fold.

Let given $n \geq \upsilon$, after use of algorithm 1.4, the estimate of singular points set of the mapping Fin terms of non-empty set $\tilde{D} \subset D$ and set of $\tilde{P}$ vertices of elementary parallelepipeds, containing $\tilde{D}$, be found. Algorithm steps for testing transversality condition ($j^1F \bowtie S_1$) is not presented here because of unhandiness. We will assume that condition $j^1F \bowtie S_1$ is estimated as satisfied.

1. For each primary parallelepiped $D^k \subset \tilde{D}$, satisfaction of condition $rank(J(p)) = \upsilon - 1$ is estimated in the following way.

Let $\{p_j^k\}_{j=1}^{2^n}$ be set of parallelepiped vertices D^k; $\{|M_i(p_j^k)|\}_{i=1}^{l}$ is set of values of $\upsilon - 1$-order minor determinants of Jacobian matrix $J(p_j^k)$ at point p_j^k; $l = C_n^{\upsilon-1}n = \frac{n!n}{(\upsilon-1)!(n-\upsilon+1)!}$. If for chosen D^k one can find such number i, that all determinants $|M_i(p_j^k)|$ for $j = 1, ..., 2^n$ have the same sign, then $rank(J(p))$ is estimated by number $\upsilon - 1$ in parallelepiped D^k. If $rank(J(p))$ is estimated by number $\upsilon - 1$ for all $D^k \subset \tilde{D}$, then set $\tilde{D}$ is considered to be the estimate of submanifold $S_1(F)$. Otherwise, if one can find such parallelepiped $D^k \subset \tilde{D}$, that for each chosen $i = 1, ..., l$ numbers $|M_i(p_j^k)|$ for $j = 1, ..., 2^n$ have different signs, then subset $\tilde{D}$ is not estimated as a fold. Stop.

2 The next steps 3, 4, 5 are performed for each point of the net $\tilde{P}$.

3. For $p \in \tilde{P}$ basis vectors $(e_1, ..., e_{\upsilon-1})$ of tangent space $T_pS_1(F)$ are estimated in the following way. Choose M (where $M \gg \upsilon - 1$) close (except this point itself) to p points of the net $\tilde{P} : \{p_1, ..., p_M\}$. Identify the following set of M vectors $\{f_i = p_i - p : i = 1, ..., M\}$. Here the points p_i and p are consider edasradius-vectors. Linear envelope of arbitrary set of n-dimensional vectors $(e_1, ..., e_{\upsilon-1})$ denote by $T = T(e_1, ..., e_{\upsilon-1})$. The distance from point f_i up to the plane T

denote by $d(f_i, T)$. Sum of squares of distances from points f_i to T denote by

$$S(e_1, ..., e_{\upsilon-1}) = \sum_{i=1}^{M} (d(f_i, T))^2 \tag{8}$$

Coordinates of required $e_1, ..., e_{\upsilon-1}$ vectors are determined by the least squares method from the condition of $S(e_1, ..., e_{\upsilon-1})$ function minimum.

4. $(g_1, ..., g_{\upsilon-n+1})$ basic vectors of $Ker(dF)_p$ kernel for $p \in \tilde{P}$ are estimated in the following way.

4.1. Since mapping matrix $(dF)_p$ with theoretical rank $\upsilon - 1$ is estimated by numerically found Jacobian matrix $J(p)$, all $\upsilon - 1$-order minors of which have determinants close to zero, we firstly determine the row close to linear combination of other rows of the matrix $J(p)$. In case, if matrix rank is less by unit than the number of its rows, according to the theorem about principal minor, one of matrix rows is a linear combination of its other rows and its elimination does not change the kernel of the linear operator, appropriate to this matrix.

Let $\{J^1, J^2, ..., J^\upsilon\}$ be the set of all normalized (the elements of each row divide to the magnitudeof this row, if magnitude of any row is zero, then the problem in item 4.1 is solved) row of $J(p)$ matrix. Let $P^i, (i = 1, ..., \upsilon)$ be linear envelope of all rows of mentioned set, except the row J^i, which is considered as plane in the space R^n. Let m^i be the distance from the point $J^i \in R^n$ to plane $P^i : m^i = d(J^i, P^i)$. m^i value can be found by finding minimum of function of $\upsilon - 1$ variable $(\alpha_1, ..., \alpha_{i-1}, \alpha_{i+1}, \alpha_\upsilon)$:

$$D^i(\alpha_1, ..., \alpha_{i-1}, ..., \alpha_{i+1}, \alpha_\upsilon) = |J^2 - \sum_{j=1, j\neq i}^{\upsilon} \alpha^i J^j|^2$$

Choose number i relevant to minimal number m^i. Denote the matrix $J(p)$ of size $(\upsilon - 1) \times n$ with removed i-row by $\tilde{J}(p)$.

4.2. Solve the linear homogeneous system from $(\upsilon - 1)$ equation with n unknowns and with matrix of system $\tilde{J}(p)$ by Gauss method. Herewith, basis of its decision space $(g_1, ..., g_{\upsilon-n+1})$ is found.

5. For $p \in \tilde{P}$ estimate angle cosine $(cos\varphi_p)$ between the planes $T_pS_1(F)$ and $Ker(dF)_p$ in R^n in the following way.

Let $(e_1, ..., e_{\upsilon-1})$ and $(g_1, ..., g_{\upsilon-n+1})$ be the found above estimates of bases of these planes; $(\alpha_1, ..., \alpha_{\upsilon-1})$ and $(\beta_1, ..., \beta_{\upsilon-n+1})$ be variables sets. Let the vector $e = \sum_{i=1}^{\upsilon-1} \alpha_i e_i$ be arbitrary vector of plane $T_pS_1(F)$, vector $g = \sum_{i=1}^{\upsilon-1} \beta_i g_i$ be arbitrary vector of plane $Ker(dF)_p$. Define function f (expressing angle cosine between the vectors e and g) on n variables in terms of $(\alpha_1, ..., \alpha_{i-1}, \beta_1, ..., \beta_{n-\upsilon+1})$

$$y = f(\alpha_1, ..., \alpha_{i-1}, \beta_1, ..., \beta_{n-\upsilon+1}) = \frac{fg}{|f||g|}$$

Its maximal value takes as the value of required φ_p.

6. Choose small number $\varepsilon > 0$. If for all $p \in \tilde{P}$ condition $cos\varphi_p < 1-\varepsilon$ is met, then based on the definition, $\tilde{D}$ set is the fold estimate and mapping $F : D^0 \to E$ is estimated as the submersion with fold.

Otherwise, if $p \in \tilde{P}$ is found, for which $cos\varphi_p \geq 1 - \varepsilon$, then $\tilde{D}$ set is not estimated as fold. Stop.

7. Estimate the injectivity of the mapping F on fold $\tilde{D}$. Since F mapping constraint on fold is immersion then local injectivity of this constraint is guaranteed. Nonlocal injectivity is estimated by algorithm 1.5, in which one should substitute the net P by the net $\tilde{P}$. Number ε, used in this algorithm, should exceed doubled diameter of elementary parallelepipeds of $\tilde{D}$ set. Two cases are possible given sufficiently large amount of step iterations of algorithm 1.5.

a) The sequence of obtained m values are diminishing about proportional to the $\tilde{P}_1$ net step. In this case, F mapping constraint on the fold $\tilde{D}$ is estimated as non-injective (in other words, $F(\tilde{D})$ set is estimated as self-crossing). Additional study of $F(\tilde{D})$ self-crossing points for normality is required.

b) Condition $m > \varepsilon$ is met for all nets in question with sufficiently small step. In this case, F mapping constraint on the fold $\tilde{D}$ is estimated as injective (in other words, $F(\tilde{D})$ set is estimated as non-self-crossing). Based on the consequence 1.3.4, the mapping $F : D^0 \to E$, in this case, is estimated as stable.

References

[1] R.A. Mundell. (2000), "A reconsideration of the twentieth century", *The American Economic Review*, Vol.90, No.3.

[2] J.A. Frenkel, A. Razin. (1987), "The mundell-fleming model: a quarter century later", *National Bureau of economic research, Working paper* , Cambridge, Massachusetts, No.2321.

[3] C. Erceg, L. Guerrieri, C. Gust. (2005), "SIGMA: a New open economy model for policy analysis", *Board of Governors of the Federal Reserve System, International Finance Discussion Papers*, July, No.835.

[4] J. Andrés, P. Burriel, Á Estrada. (2006), "BEMOD: a DSGE model for the spanish economy and the rest of the euro Area", *Documentos de Trabajo Banco de España*, No.0631.

[5] N. Stähler, C. Thomas. (2011), "FiMod-a DSGE model for fiscal policy simulations", *Deutsche Bundesbank Discussion Paper. Series 1: Economic Studies*, No.06.

[6] S. Paltsev, J.M. Reilly, H.D. Jacoby, R.S. Eckaus, J. McFarland, M. Sarofim, M. Asadoorian, M.H. Babiker. (2005), "The MIT e-

missions prediction and policy analysis (EPPA) model: version 4", http://globalchange.mit.edu/research/publications/697, Report No.125.

[7] P.D. Dixon, M.T. Rimmer. (1998), "Forecasting and policy analysis with a dynamic CGE model of australia", *Preliminary Working Paper*,No.OP-90, http://www.monash.edu.au/policy/elecpapr/op-90.htm.

[8] V.L. Makarov, A.R. Bahtizin, and S.S. Sulakshin. (2007), "Application of computable models in state administration", *Moscow: Nauchniy Expert*, Russian.

[9] G. Gandolfo. (2014), "International trade theory and policy", *Springer Heidelberg New York Dordrecht London*.

[10] K. Farmer, M. Schelnast. (2013), "Growth and international trade. An introduction to the overlapping generations approach", *Springer Heidelberg New York Dordrecht London*.

[11] R.C. (2004), "Fair, estimating how the macroeconomy works", *Cambridge, Massachusetts: Harvard University Press, London, England*.

[12] S. Murchison, A. Rennison. (2006), "To TEM: the bank of Canada's new quarterly projection model", *Bank of Canada Technical Report*, No.97.

[13] M. Adolfson, S. Laséen, J. Lindé and L.Svensson. (2011), "Optimal monetary policy in an operational medium sized DSGE model", *Journal of Money, Credit and Banking, Blackwell Publishing*, Vol.43(7), pp.1287-1331.

[14] D. Acemoğlu. (2008), "Introduction to modern economic growth", *Princeton University Press*.

[15] F.J. André, M.A. Cardenete, and C. Romero. (2010), "Designing public policies. An approach based on multi-criteria analysis and computable general equilibrium modeling", *Lecture Notes in Economics and Mathematical Systems, 1st ed*, Vol. 642, Springer.

[16] V.I. Arnold. (1988), "Geometrical methods in the theory of ordinary differential equations", Springer-Verlag, New York.

[17] V.I. Arnold, S.M. Gusein-Zade, A.N. Varchenko. (1985), "Singularities of differential maps. Volume I", *Basel, Stuttgard: Birkhauser*, Boston.

[18] A.I. Orlov, Econometrics. (2002), "A Textbook", *Ekzamen*, Moskow, Russian.

[19] A.A. Ashimov, B.T. Sultanov, Zh.M. Adilov, Yu.V. Borovskiy, D.A. Novikov, R.A. Alshanov, As.A. Ashimov. (2013), "Macroeconomic analysis and parametrical control of a national economy", Springer, New York.

[20] J.A. Nelder, R. Mead. (1965), "A simplex method for function minimization", *The Computer Journal*, No.7, pp.308-313.

[21] Robinson C. (1980), "Structural stability on manifolds with boundary", *Journal of Differential Equations*, No.37, pp.1-11.

[22] E.I. Petrenko. (2006), "Development and realization of the algorithms for constructing the symbolic set", *Differential Equations and Control Processes (Electronic Journal)*, No.3, pp.55-96.

[23] . Golubitsky, GueilleminV. (1973), "Stable mappings and their singularities", *New York, Heidelberg, Berlin: Springer-Verlag.*

Corresponding author
A. Ashimov can be contacted at: ashimov37@mail.ru