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Estimates of the Relative Deviation for the Equity Home Bias

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Abstract
There are a variety of explanations of equity home bias, the underlying difference being that different factors are emphasized, such as trade costs that impede international diversification, and the possibility that terms of trade responses to supply shocks provide international risk sharing. We found that with the ratio of world consumption of Home vs. Foreign goods and the ratio of world demand for Home vs. Foreign goods used for physical investment, we have three different estimates of the relative deviation about terms of trade, and then figured out three expressions representing these estimates. What we concluded helps to uncover the puzzle of the equity home bias.
Keywords International equity and bond portfolios, Capital flows, Current account

1 Introduction

Though international capital flows have rallied with the liberalization of the global capital market two decades ago, equity home bias still exits in all industrial economies. Abundant researches have been contributed to solve this puzzle. There are two major explanations of the continuously sizable equity home bias. The first one emphasizes on transaction costs and information barriers in cross-border financial transactions and suggests that international risk sharing is insufficient. The second one emphasized on the possibility that terms of trade changes in response to supply shocks may provide international insurance against these shocks, so that even a portfolio with home bias delivers efficient international risk sharing. Seen from these very different opinions, where we start the work can make a big difference to what we conclude about the equity home bias.

Much published research made use of statistical methods and empirical analysis to interpret the home bias under capital account liberalization by transaction costs, information barriers and financial openness (recently from behavioural finance, cultural factors and legal system perspectives). Ferreira and Miguel, Bekaert and Wang and Mondria and Wu attributes much of home bias to information, familiarity and capital market openness [1-3].

Another class of literature tried to decide home bias determinants through characterization of steady-state equilibrium portfolios in a general equilibrium model. Engel and Matsumoto analysed international equity portfolio choices in a model with money, sticky prices and trade in bonds [4-5]. Under price stickiness, the short-run output is fixed, so that a positive productivity shock leads to a fall in employment and labor income, but an increase in profits. Ownership of local equity is thus an effective hedge against labor income risk. Heathcote and Perri investigated the importance of physical investment for equity portfolios for the first time [6]. The HP model only generates realistic equity home bias if the terms of trade respond strongly to Total Factor Productivity shocks. Since the empirical evidence concerning the response of the terms of trade to technology shocks is mixed, it is important that our model does not require strong terms of trade effects of productivity shocksnevertheless, there is sizable equity home bias.

The main contribution of this paper is that with the ratio of world consumption of Home vs. Foreign goods and the ratio of world demand for Home vs. Foreign goods used for physical investment, we have three different estimates of the relative deviation about terms of trade, and then figured out three expressions representing these estimates.

2 Model

We consider two symmetric countries, Home (H) and Foreign (F), each with a representative household. Country $i = H, F$ produces one good using labor and capital. Goods and financial assets (stocks and bonds) are traded in perfectly competitive markets. Country i is inhabited by a representative household who lives in periods $t = 0, 1, 2....$ The household has the following life-time utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{i,t}^{1-\sigma}}{1-\sigma} - \frac{l_{i,t}^{1+\omega}}{1+\omega} \right) \tag{1}$$

with $w > 0$. $C_{i,t}$ is Country i's aggregate consumption in period t and $l_{i,t}$ is labor input. Like much of the macroeconomics and finance literature, we take the coefficient of relative risk aversion to be greater than one: $\sigma > 1$.

$C_{i,t}$ is a composite good given by:

$$C_{i,t} = \left[a^{1/\varphi} \left(c_{i,t}^{i} \right)^{(\varphi-1)/\varphi} + (1-a)^{1/\varphi} \left(c_{j,t}^{i} \right)^{(\varphi-1)/\varphi} \right]^{\varphi/(\varphi-1)}, j \neq i \tag{2}$$

where $C_{j,t}^{i}$ is country i's consumption of the good produced by country j at time t. $\phi > 0$ is the elasticity of substitution between the two goods. In the (symmetric) deterministic steady state, a is the share of consumption spending devoted to the local good. We assume a preference bias for local goods, $1/2 < a < 1$.

The welfare-based consumer price index that corresponds to these preferences is:

$$P_{i,t} = \left[a (p_{i,t})^{1-\varphi} + (1-a)(p_{j,t})^{1-\varphi} \right]^{1/(1-\varphi)}, j \neq i \tag{3}$$

where $P_{i,t}$ is the price of good i.

Likewise, the associated investment price index is:

$$P_{i,t}^{I}=\left[a_I(p_{i,t})^{1-\varphi_I}+(1-a_I)(p_{j,t})^{1-\varphi_I}\right]^{1/(1-\varphi_I)},j\neq i \tag{4}$$

3 Relative Deviation Estimation

Coeurdacier, Kollman and Martin (2010) introduced a concept in their research on equity home bias, *relative deviation.* $\overset{\wedge}{z_t}=(z_t-z)/z$ denotes the relative deviation of a variable z_t from its steady state value Z. Here, variables without a time subscript refer to the steady state.

$$\begin{aligned}c_{H,t}^{H}+c_{H,t}^{F}&=p_{H,t}^{-\varphi}\left[aC_{H,t}P_{H,t}^{\varphi}+(1-a)C_{F,t}P_{F,t}^{\varphi}\right],\\ c_{F,t}^{F}+c_{F,t}^{H}&=p_{F,t}^{-\varphi}\left[aC_{F,t}P_{F,t}^{\varphi}+(1-a)C_{H,t}P_{H,t}^{\varphi}\right]\end{aligned}$$

Coeurdacier et al. (2010) derived from the above first-oder conditions that:

$$y_{C,t}\equiv\frac{c_{H,t}^{H}+c_{H,t}^{F}}{c_{F,t}^{F}+c_{F,t}^{H}}=q_t^{-\phi}\Omega_a\left[\left(\frac{P_{F,t}}{P_{H,t}}\right)^{\phi}\frac{C_{F,t}}{C_{H,t}}\right],with\ \Omega_Z(x)\equiv\frac{1+x\left(\frac{1-z}{z}\right)}{x+\left(\frac{1-z}{z}\right)} \tag{5}$$

where $y_{C,t}$ is the ratio of world consumption of Home goods over world consumption of Foreign goods, while $q_t\equiv p_{H,t}/p_{F,t}$ denotes the country H terms of trade. The ratio of world demand for Home vs. Foreign goods used for physical investment $y_{I,t}\equiv\frac{i_{H,t}^{H}+i_{H,t}^{F}}{i_{F,t}^{F}+i_{F,t}^{H}}$ can similarly be expressed as:

$$y_{I,t}\equiv q_t^{-\phi_I}\Omega_{a_I}\left[\left(\frac{P_{F,t}^{I}}{P_{H,t}^{I}}\right)^{\phi_I}\frac{I_{F,t}}{I_{H,t}}\right] \tag{6}$$

Coeurdacier et al. (2010) found a zero-order portfolio such that the ratio of Home to Foreign marginal utilities of aggregate consumption $(C_{H,t}^{-\sigma}/C_{F,t}^{-\sigma})$ is equated to the consumption-based real exchange rate $(RER_t\equiv P_{H,t}/P_{F,t})$, up to the following first-order condition:

$$-\sigma\left(\hat{C}_{H,t}-\hat{C}_{F,t}\right)=R\hat{E}R_t \tag{7}$$

which is a linearized version of a risk sharing condition that holds under complete markets.

It follows from the definition of Home and Foreign CPI indices (see Equation 3) that:

$$R\hat{E}R_t=\hat{P}_{H,t}-\hat{P}_{F,t}=(2a-1)\overset{\wedge}{q_t} \tag{8}$$

Due to consumption home bias ($a > 1/2$), an improvement of the Home terms of trade leads to an appreciation of the Home real exchange rate.

Lemma 1 If α, β are non-zero constants and $\alpha+\beta \neq 0$, then we have for continues functions f_1 and f_2 that

$$(\alpha f_1(q_t) \overset{\wedge}{+} \beta f_2(q_t)) = \frac{\alpha f_1(q)}{\alpha f_1(q) + \beta f_2(q)} \overset{\wedge}{f_1}(q_t) + \frac{\beta f_2(q)}{\alpha f_1(q) + \beta f_2(q)} \overset{\wedge}{f_2}(q_t) \tag{9}$$

$$\begin{aligned} proof: \quad (\alpha f_1(q_t) \overset{\wedge}{+} \beta f_2(q_t)) &= \frac{\alpha f_1(q_t) + \beta f_2(q_t) - (\alpha f_1(q) + \beta f_2(q))}{\alpha f_1(q) + \beta f_2(q)} \\ &= \frac{\alpha\left(f_1(q_t) - f_1(q)\right) + \beta\left(f_2(q_t) - f_2(q)\right)}{\alpha f_1(q) + \beta f_2(q)} \\ &= \frac{\alpha f_1(q) \cdot f_1\overset{\wedge}{(q_t)} + \beta f_2(q) \cdot f_2\overset{\wedge}{(q_t)}}{\alpha f_1(q) + \beta f_2(q)} \end{aligned}$$

Lemma 2 For continues functions f_1 and f_1, we have that

$$(f_1(q_t) \overset{\wedge}{\cdot} f_2(q_t)) = \overset{\wedge}{f_1}(q_t) + \overset{\wedge}{f_2}(q_t) + \overset{\wedge}{f_1}(q_t) \cdot \overset{\wedge}{f_2}(q_t) \tag{10}$$

$$\begin{aligned} proof: \quad (f_1(q_t) \overset{\wedge}{\cdot} f_2(q_t)) &= \frac{f_1(q_t) f_2(q_t) - f_1(q) f_2(q)}{f_1(q) f_2(q)} \\ &= \frac{\left(f_1(q_t) - f_1(q)\right) f_2(q_t) + f_1(q) f_2(q_t) - f_1(q) f_2(q)}{f_1(q) f_2(q)} \\ &= \frac{f_1(q_t) - f_1(q)}{f_1(q)} \cdot \frac{f_2(q_t)}{f_2(q)} + \frac{f_2(q_t) - f_2(q)}{f_2(q)} \\ &= \overset{\wedge}{f_1}(q_t) \left(1 + \overset{\wedge}{f_2}(q_t)\right) + \overset{\wedge}{f_2}(q_t) \\ &= \overset{\wedge}{f_1}(q_t) + \overset{\wedge}{f_2}(q_t) + \overset{\wedge}{f_1}(q_t) \cdot \overset{\wedge}{f_2}(q_t) \end{aligned}$$

Lemma 3 For continues functions f_1 and f_1, we have that

$$\left(\frac{f_1\overset{\wedge}{(q_t)}}{f_2(q_t)}\right) = \frac{\overset{\wedge}{f_1}(q_t) - \overset{\wedge}{f_2}(q_t)}{1 + \overset{\wedge}{f_2}(q_t)} \tag{11}$$

$$proof: \quad \left(\frac{f_1(q_t)}{f_2(q_t)}\right)^{\wedge} = \frac{\frac{f_1(q_t)}{f_2(q_t)} - \frac{f_1(q)}{f_2(q)}}{\frac{f_1(q)}{f_2(q)}}$$
$$= \frac{f_1(q_t)f_2(q) - f_1(q)f_2(q_t)}{f_1(q)f_2(q_t)}$$
$$= \frac{\left(f_1(q_t) - f_1(q)\right)f_2(q) + f_1(q)f_2(q) - f_1(q)f_2(q_t)}{f_1(q)f_2(q_t)}$$
$$= \overset{\wedge}{f_1}(q_t) \cdot \frac{f_2(q)}{f_2(q_t)} - \frac{f_2(q_t) - f_2(q)}{f_2(q)} \cdot \frac{f_2(q)}{f_2(q_t)}$$
$$= \frac{\overset{\wedge}{f_1}(q_t) - \overset{\wedge}{f_2}(q_t)}{1 + \overset{\wedge}{f_2}(q_t)}$$

In order to improve the expression of Equation 8, we need to modify Equation 9, 10 and 11 and present as follows:

Lemma 4 For continues functions f_1 and f_1, we have that

$$(\alpha f_1(q_t) \overset{\wedge}{+} \beta f_2(q_t)) = \frac{\alpha \overset{\wedge}{f_1}(q_t) + \beta \overset{\wedge}{f_2}(q_t)}{\alpha + \beta} \tag{12}$$

Lemma 5 For continues functions f_1 and f_2, we have that

$$(f_1(q_t) \overset{\wedge}{\cdot} f_2(q_t)) = \overset{\wedge}{f_1}(q_t) + \overset{\wedge}{f_2}(q_t) \tag{13}$$

Lemma 6 For continues functions f_1 and f_1, we have that

$$\left(\frac{f_1(q_t)}{f_2(q_t)}\right)^{\wedge} = \overset{\wedge}{f_1}(q_t) - \overset{\wedge}{f_2}(q_t) \tag{14}$$

Obviously, Lemma 4 is unique when $f_1(q)$ equals to $f_2(q)$ as is implied by ex-ante symmetry of two countries in our model, and Lemma 5 and Lemma 6 is one special case when ignoring second-order relative deviations and $\overset{\wedge}{f_2}(q_t)$ in the dominator, respectively.

Theorem 1 When Equation 5 holds, then the relative world consumption demand for the Home good obeys

$$\hat{y}_{C,t} = -\left[(1-(2a-1)^2)\varphi + (2a-1)^2\frac{1}{\sigma}\right]\hat{q}_t \tag{15.a}$$

proof:Since $\overset{\wedge}{y_{C,t}} = q_t^{-\phi}\Omega_a\left[\overset{\wedge}{\left(\frac{P_{F,t}}{P_{H,t}}\right)^{\phi}\frac{C_{F,t}}{C_{H,t}}}\right] = q_t^{-\phi}\overset{\wedge}{\Omega_a}(x)$, we derive from Lemma 4-6 that:

$$\begin{aligned}\overset{\wedge}{\Omega_a}(x) &= \left(\frac{1+\overset{\wedge}{x\left(\frac{1-a}{a}\right)}}{x+\left(\frac{1-a}{a}\right)}\right)\\ &= \left(1+\overset{\wedge}{x\left(\frac{1-a}{a}\right)}\right)-\left(x+\overset{\wedge}{\left(\frac{1-a}{a}\right)}\right)\\ &= (1-a)\hat{x} - a\hat{x}\\ &= (1-2a)\hat{x}\end{aligned}$$

Then,

$$\begin{aligned}\overset{\wedge}{y_{C,t}} = \overset{\wedge}{q_t^{-\phi}} + \overset{\wedge}{\Omega_a}(x) = -\phi\overset{\wedge}{q_t} + (1-2a)\left(-\phi+\frac{1}{\sigma}\right)(2a-1)\overset{\wedge}{q_t}\\ = -\left[(1-(2a-1)^2)\varphi + (2a-1)^2\frac{1}{\sigma}\right]\overset{\wedge}{q_t}\end{aligned}$$

And we can similarly prove for Expression 2 and 3.

Theorem 2 When Equation 5 holds, then the relative world consumption demand for the Home good obeys

$$\overset{\wedge}{y_{C,t}} = -\left[(1-\frac{(2a-1)^2}{a})\varphi + \frac{(2a-1)^2}{a}\frac{1}{\sigma}\right]\overset{\wedge}{q_t} \tag{15.b}$$

Theorem 3 When Equation 5 holds, then the relative world consumption demand for the Home good obeys

$$\overset{\wedge}{y_{C,t}} = -\left[(1-\frac{(2a-1)^2}{1-a})\varphi + \frac{(2a-1)^2}{1-a}\frac{1}{\sigma}\right]\overset{\wedge}{q_t} \tag{15.c}$$

Note that $\lambda > 0$(as $1/2 < a < 1$ implies $0 < 1-(2a-1)^2$). Thus, an improvement in the Home terms of trade lowers worldwide relative consumption of the Home good.

Introduce the following signs, whose expressions are illustrated in Fig.1:

$$\begin{gathered}\lambda_{11} = 1-(2a-1)^2, \lambda_{12} = (2a-1)^2, \lambda_{21} = 1-\frac{(2a-1)^2}{a}\\ \lambda_{22} = \frac{(2a-1)^2}{a}, \lambda_{31} = 1-\frac{(2a-1)^2}{1-a}, \lambda_{32} = \frac{(2a-1)^2}{1-a}\end{gathered}$$

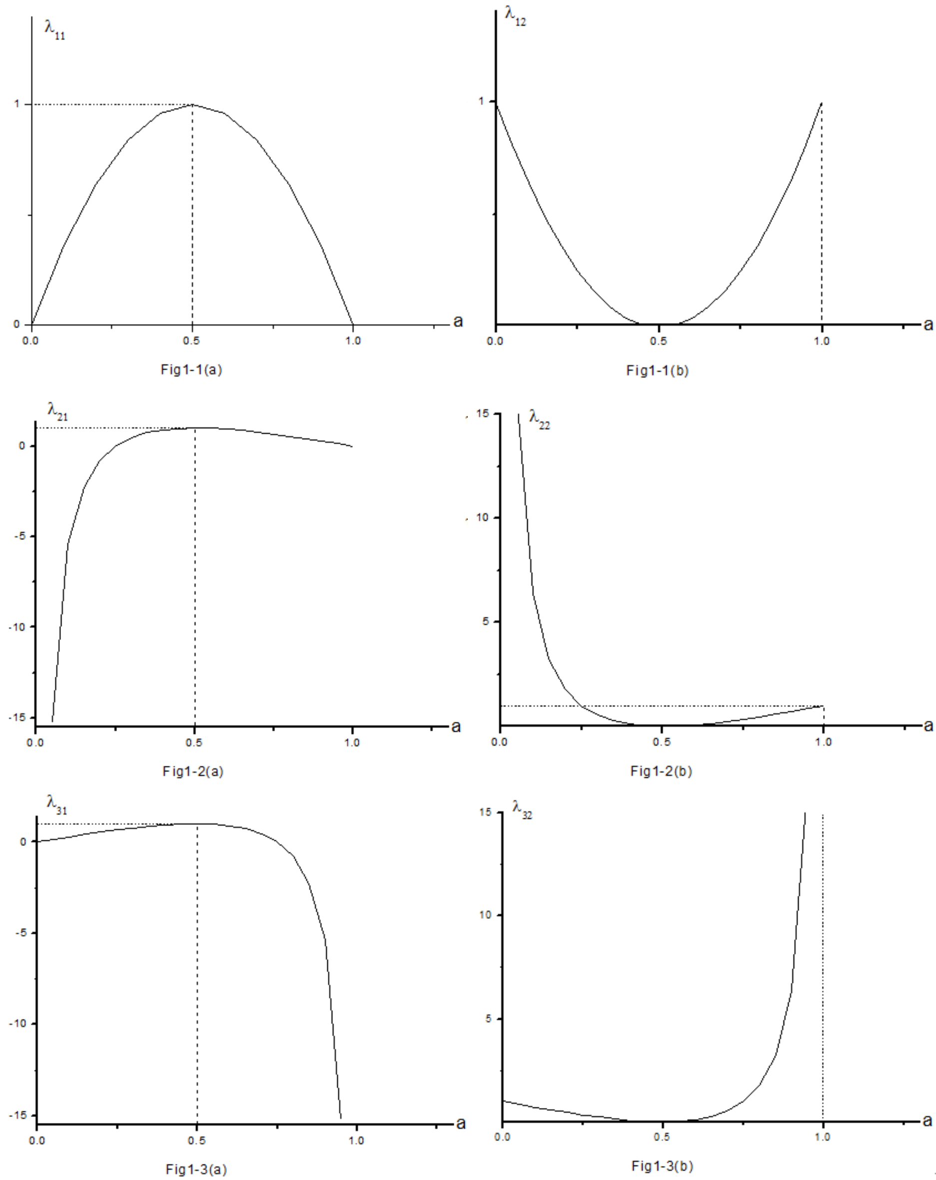

Fig.1 Home terms of trade lowers worldwide relative consumption of the home good.

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