Robust Design and Its Challenge for Manufacturing System

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Abstract
Robust performance is one of the most important concerns in design of any system in manufacturing industry. This performance can be achieved by the robust design. The paper will provide a brief overview of the robust design. It includes discussions about how to account for design uncertainty, and how to measure and evaluate robustness for both static and dynamic systems. By reviewing the strengths and weaknesses of different design methods, the challenges in this area will be discussed.
Keywords robust design, sensitivity analysis, stability, uncertainty

1 Introduction

In order to design and manufacture high quality productions at a minimal cost, increasingly accurate systems are required in practical industry. One serious problem in these systems is the inconsistent performance due to uncontrollable variations existing in the real-world, including manufacturing operations, variations in material properties and the operating environment. If these variations are not considered, they will degrade the performance and may result in a failure in practice [1]. Thus, robust performance is one of the most important concerns in design of any system.

The concept of the robust design was introduced by Taguchi. The fundamental principle in the robust design is to improve the quality of a product by minimizing the effects of variations without eliminating the causes [2]. By making a design more robust to variations, it is possible to improve number of the eligible parts or use less experiment [3]. In past decades, much effort has been dedicated to the robust design. In general, robust design may be classified two groups, static model based design and dynamic model based design.

- Static model based robust design is relatively simple since it is irrelevant with time and only considers the effect of the static variations on performance;
- Dynamic model based robust design is complex because it must consider the stability, flexibility and robustness of the system in the whole operating process.

The aim of this paper is to review and compare the applicable methods, so that their underlined philosophy can be clearly shown and easily understood. Though the review, the challenges in this area can be discussed.

2 Uncertainty

In a robust design problem, the system includes three kinds of variables :
- Design variables (or control variable) ÍC since their nominal values can be selected between the range of upper and lower bounds, they are controllable ;
- Uncertainties - that can not be adjusted by the designer, so they are uncontrollable ;
- Performances - that are the objective of design, they depend on the system model, design variables and uncertainties.

These uncertainties, which should be first identified to obtain the robust performance, mainly include :
- Noise - it is caused by changes of operating conditions, such as, environmental temperature, pressure, humidity and material change, etc. Thus, it is of stochastic nature.

In a robust design problem, the system includes three kinds of variables :
- Model parameters - it is often caused by manufacture error. Parameters of a product can only be realized to a certain degree of accuracy due to machinery limitation. It could be either stochastic or non-stochastic.
- Model uncertainty - it is often caused by approximations in the modelling process.

Basically, these uncertainties can have following three different natures :
- The deterministic type defines the domains in which the uncertainties can vary ;
- The probabilistic type defines probability measure describing the likelihood by which a certain event occurs ;
- The possibilistic type defines fuzzy measures describing the possibility or membership grade by which a certain event can be plausible or believable.

These three different types of uncertainties are usually modelled by crisp sets, probability distributions and fuzzy sets, respectively.

3 Robust design

The robust design framework is shown in Figure 1. The key issue in this framework is the robust design strategy, which optimizes the design variables to make the system less robust to uncertainties.

All existing robust design methods depend on the characteristics of a system given in Table 1 and Table 2. If a system model is unknown and design parameters are non-probabilistic type, then no robust design method is applicable to it. The indirect method for this case is data-based modelling methods that transfer it into the case of the known model.

3.1 Static model based robust design

This robust design includes three different approaches : deterministic robust design, probabilistic robust design, and fuzzy analysis.

Table 1 Existing robust design methods for static system

System	Static model			
model	known		Unknown	
Uncertainty	Non-probabilistic type	Probabilistic type	Non-probabilistic type	Probabilistic type
Robust design methods	•Deterministic robust methods •Fuzzy analysis	•Monte Carlo simulation •First and second order moment methods •Probabilistic sensitivity analysis • Performance index	N/A	•Taguchi method •Monte Carlo simulation

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3.1.1 Deterministic robust design

The design parameters are deterministic.

(1) Euclidean norm method and Conditional number method

The methods obtain the system robustness by analytically measuring the sensitivity of a design using the gradient information of parameters. The sensitivity analysis is based on the Taylor-series expansion of the performance. The Euclidean norm method [1, 3] is to minimize the largest singular value $\sigma_{max}$of the sensitivity matrix. The condition number method [1, 4] is to minimize the condition number $\frac{\sigma_{max}}{\sigma_{min}}$ of the sensitivity matrix. Caro, et al. [1] has compared these two methods when designing the damper, and shown that the Euclidean norm is more suitable as the robust index than the condition number.

(2) The sensitivity region measures method

Gunawan & Azarm [5] proposed the sensitivity region measures for the single objective robust design optimization. This method first projects the desirable objective function space into the design parameter space, where the sensitivity region is constructed. Then the most sensitivity direction in the sensitivity region can be found. This most sensitivity direction is actually a measurement of a robust performance.

Li, Azarm & Boyars [6] proposed another sensitivity region measures for robust optimization. This method first projects the design parameter space into the desirable objective variations domain, where the objective sensitivity region is constructed. Then the maximum performance variation in the sensitivity region is found. This variation is used as a measurement of a robust performance. Moreover, the comparison with the Gunawanąŕs method is carried out.

3.1.2 Fuzzy analysis

The possibility methods are proposed to apply in areas where it is not possible to obtain accurate statistical data dueto restriction of resource or conditions. Its foundation is possibility theory. The extension principle calculates the possibility distribution of the fuzzy response from the possibility distribution of the fuzzy input variables.

Recently, this method was applied to the robust optimal design to deal with the epistemic uncertainty [7]. Also, this method was applied for the modeling of tolerances and clearances in the mechanism analysis [8]. The comparison of probability and possibility for design against catastrophic failure under uncertainty was presented by [9]. The review of this application can be found in He & Qu [10] and Beyer & Sendhoff [11].

3.1.3 Probabilistic robust design

This probabilistic robust design will be less conservative than the deterministic robust design since it makes use of the probabilistic information of parameters.

(1) Monte Carlo simulation

Monte Carlo methods are a class of computational algorithms that rely on repeated random sampling from probabilistic density function of each parameter to compute their results. Then, an experiment is executed under the generated samples and the process data is collected. Finally, the statistical method is used to estimate its probability density function from the process data.

(2) First and Second order moment methods

The most widely used non-statistical uncertainty analysis method is moment methods. A Taylor series expansion is employed to estimate response variance based on variance of model parameters Xas

$$\sigma_{first_order}Y = (\frac{\partial y}{\partial x}) \mid_{\mu X}{}^2 \sigma(X) \tag{3.1}$$

$$\sigma_{second_order}Y = (\frac{\partial y}{\partial x}) \mid_{\mu X}{}^2 \sigma(X) \tag{3.2}$$

(3) Probabilistic sensitivity analysis
This method evaluates the effect of design variables on performances using the sensitivity information. Probabilistic sensitivity analysis methods have been developed to provide insight into the probabilistic behavior of a model, which can be used to identify those non-significant variables and reduce the dimension of random design space. A review about probabilistic sensitivity analysis was presented by [12]. The sensitivity analysis includes : variance-based methods [13], probabilistic sensitivity coefficients [14], Kullback-Leibler Entropy based probabilistic sensitivity analysis (PSA) method [15].
(4) Performance index
Various performance indexes are constructed to measure the performance and capability of a system.

- *Suhąŕs information content*

The information axiom is used to evaluate quality of designs so that an appropriate design can be chosen from available design alternatives. According to the information axiom proposed by Suh [16], a design candidate that has minimum information content should be selected. Thus, the information content is regarded as a robust index.

- *Design capability indices*

The six standard deviations($\pm 3\sigma$)is commonly a measure of process capability, which compares the variation of a process to the customer specifications through the equation

$$C_{dl} = \frac{USL - LSL}{6\sigma} \tag{3.3}$$

Where USL is the upper specification limit, LSL is the lower specification limit, is the standard deviation of the process. We hope that Cp is greater than one so that the process variation is less than the specification limits and the performance can satisfy the requirement.
Chen, et al. [18] extended the concept of measuring process capability to measure the approximate degree between the mean of the process and the target value. They proposed design capability indices (DCI) as metrics for system performance and robustness. The indices Cdu, Cdl and Cdk means that smaller is better, larger

is better and target is better respectively, and are defined as

$$C_p = \frac{USL - LSL}{6\sigma}, C_{du} = \frac{URL - \mu}{3\sigma}, C_{dk} = \min(C_{dl}, C_{du}) \tag{3.4}$$

Where URL and LRL are upper and lower requirement limits.
The index is expected to be greater than unity so that the design will meet the requirement satisfactorily. Forcing the index larger to unity is achieved by reducing performance deviation and/or locating the mean of performance deviation farther from requirement limits.

(5) Taguchi method

Taguchi robust design, also known as parameter design, is an approach to identify design variable values that satisfy a set of performance requirements despite variation in noise factor [2]. Since 1960, Taguchi methods have obtained a great success in improving the quality of products and design robustness. This method is based on experimental data and includes three parts : experimental design, quality loss function and signal-to-noise ratio.

Taguchi robust design approach for the variable design starts from the experimental design, where the orthogonal array is used for design. Control factor resides in an inner array and noise factors conditions in an outer array. The experimental results in all combinations of control factors and noise factors are recorded. Then, Taguchi proposed a signal-to-noise ratio for measuring sensitivity analysis of response to variation of noise factors. Based on the signal-to-noise ratio, the robust design is obtained.

Although Taguchiąŕs method has obtained great success, there are certain assumptions and limitations associated with his methods. Use of the Taguchi method will not yield an accurate solution for design problems that embody highly nonlinear behavior [19]. The Taguchi method has been criticized by the statistical community [20]. Many of Taguchiąŕs statistical methods, e.g., orthogonal arrays, linear graphs and accumulation analysis, are not statistically efficient [20]. Shoemaker et al., [21] presented a combined single array for both control and noise factors instead of orthogonal array.

3.2 Dynamic model based robust design

These uncertainties are, in general, also dynamic in nature and correspond to variations in either external variables or internal process parameters [22].

3.2.1 Stability based design

The recent integration of the steady-state design and the dynamic stability is to explicitly consider dynamic elements in the process design by use of the eigenvalue theory. Blanco and Bandoni [23] proposed the multi-period program to solve the Lyapunovąŕs stability matrix equality that can guarantee the system stability under uncertainty and disturbance. However, the accuracy of this method will depend on the degree of discretization. Since Lyapuova's criteria for asymptotic stability can not be easily implemented within a design optimization framework, Mohideen, Perkins & Pistikopoulos [24] and Kokossis & Floudas [25] proposed an alternative robust stability criteria based on the matrix measures, which can avoid the tedious calculation of all eigenvalues. Matrix measures can provide a single upper bound for all eigenvalues. However, this bound should not be used because it is typically not tight, therefore, may result in an overestimation of the stability boundary. Monnigmann & Marquardt [26] and Grosch, Monnigmann & Marquardt [27] proposed the stability design in the steady-state process optimization. This design employed manifolds method to figure out the bound of the parameter variations that guaranteed all eigenvlues of the process smaller than zero when the parameter variations were limited in this bound.

3.2.2 Flexibility analysis

The flexibility, which defines the ability to maintain feasible operation over a range of uncertain conditions, is a vital important characteristic for the operation of these plants. As the state of Dimitriadis & Pistikopoulos [22], the flexibility analysis problem generally consists of two tasks which are complementary to each other.

- The first task is to determine if a given design can feasibly operate over the range of uncertainty considered. This problem is known as the flexibility test problem.
- The second task is to calculate a measure to quantify the ability of the design to operate in the presence of uncertainty. This is known as the flexibility index problem and is usually tackled by establishing the maximum parameter range over which the design can operate feasible.

The flexibility measure is often used to select the suitable design by comparing different design alternatives with respect to their flexible operation [28]. Swaney and Grossmann [29] defined the flexibility index for measuring the flexibility of steady-state processes where the uncertain parameters are described by bounds of a specified range of operation. This approach was also extended to the analysis of dynamic systems under time-varying uncertainties [22]. The stochastic flexibility index is a metric for quantifying the ability of a process to maintain feasible operation in the face of stochastic uncertainties [30].

3.2.3 Robustness index

Various robustness indexes can be designed to measure the degree to which a system can meet its design objectives despite external disturbance and uncertainties in its design parameters. Since adequate robust index is a necessary part of the optimal process design, it is desirable to consider process resiliency assessment when determining the process structure and establishing the operating range

*The resiliency index*measures the effect of the disturbance on the control input u. Skogestad & morari [31] and Lewin [32] considered the ratio of the control input u to disturbance d as the resiliency index based on the linear process model. Cao, Rossiter and Owens [33] applied it to select the control inputs. Solovyev & Lewin [34] extended this resiliency index into the nonlinear system.
The condition number is defined as the ratio between the maximum and minimum singular values. The condition number provides a direct measure of the directionality of the system. A large condition number indicates that the gain of the plant changes significantly with the input direction and that the system is sensitive to input uncertainty [35].
*The disturbance condition number*is a measure of the input magnitude which is needed to reject a disturbance in the given direction, relative to rejecting a disturbance with the same magnitude, but in the direction requiring the least control effort. A small disturbance condition number are most effective for disturbance rejection [36].
*The relative gain matrix (RGA)*was originally proposed by Bristol [37]. Its objective is to provide a measure of interactions for multivariable square systems [31, 35]. If the plant has large RGA elements within the frequency range where effective control is desired, then it is not possible to achieve good reference tracking with feedforward control because of strong sensitivity to diagonal input uncertainty. Manousiouthakis et al. [38] generalized the concept of the RGA to block relative gain which is capable of handling partially decentralized control systems. Chen and Yu [39] extended this method to non-square multivariable systems for selection of square subsystem from non-square system. A dynamic relative gain was proposed by Avoy, et al. [40].

3.2.4 Operability index

The operability measure can quantify the inherent ability of the process to move from one steady state to another and to reject any of the expected disturbances. The operability index is defined by Vinson and Georgakis [41] to effectively capture the inherent operability of continuous processes. Vison and Georgakis [41] applied this index to analyze the steady state of the linear system. The technique has also been proven to be effective for nonlinear processes [42]. It was also extended to dynamic systems by UztÍźrk and Georgakis [43]. A brief survey paper about the operability index was presented by Georgakis, et al. [44].

4 Challenge

Any method has its strength and weakness. The fundamental difficulties in robust design are related to model/parameter uncertainties and nonlinearity of the system. Most of the existing methods can deal with linear system with the known model, and are not capable to handle model uncertainty or nonlinearity that in turn generates extra uncertainties to the system. The weakness of the existing design methods poses the challenges to unsolved problems as shown in Table 3.

4.1 Robust design for static system

1) There is still no method that can consider the model uncertainties in the deterministic robust design. The accurate model is needed for Euclidean norm method and Conditional number method to obtain the gradient information. The sensitivity region measure methods are based on the projection between parameter space and performance space, which requires the system model.

2) The model uncertainties still can not be handled in the probabilistic robust design, where all the existing methods can be classified two categories : model-based robust design and data-based robust design.

The data-based robust design includes Monte Carlo method and Taguchi method.

- Since Monte Carlo method is a simulation method, all system knowledge, which has to be known beforehand, need to be translated into computer code. Thus, the accurate system model is critical. Moreover, it costs huge computational time that will limit its application.
- Taguchi method obtains the system robustness based on the experiment data. However, it is only suitable for the stochastic environment and not suitable to minimize the effect of parameter variations.

The model-based robust design includes first- and second- order moment methods. Since these methods are based on the Taylor series expansion, which requires the accurate system model. Thus, the model uncertainty will lead to the significant performance degradation. So far, there is still no solution.

3) The membership function in fuzzy analysis is defined according to human experience, which could be too subjective and causes uncertainties. It would be a challenge if the experimental data can be used to reduce subjective uncertainties of the membership.

4.2 Robust design for dynamic system

In difference to the static system, another problem for the dynamic system is that the design optimization becomes extremely difficult when the system has both continuous and discrete design variables.

1) The current stability design can work for the linear system, where its eigenvalues are critical to the system stability. The weakness of the existing methods have not considered two influences : one is model uncertainties on the eigenvalues and their variations, and the other is parameter perturbation on the eigenvaluesąŕ variations.

2) The stochastic flexibility index is only applicable to the linear dynamic system. So far, there is still no study on its application to the nonlinear dynamic system.

4.3 Potential solutions

For the static system, a novel model-based robust design is proposed to design the system using the nominal model. The system-model mismatch, model uncertainties, can be properly considered in the design.
For the dynamic system, a novel stability based robust design is proposed to guarantee the robust performance as well as the system stability, so that the method can also be applied to the weak nonlinear system.

5 Conclusion

This paper presents a brief overview about advances in robust design. Different approaches in robust design are reviewed and compared, upon which challenges have been proposed to the unsolved problems. Two novel approaches are proposed to the unsolved problems.

Table 3 Summary of evaluation of existing methods

Design classification		Existing method	Challenges for unsolved problems
Static robust design	Deterministic system	Deterministic robust design	These methods need an accurate model and can not handle the model uncertainty.
	Probabilistic system	Data-based methods	All these methods can not handle model uncertainties properly, furthermore,
		• Monte Carlo	• Monte Carlo simulation costs huge computational time, and requires the accurate model;
		• Taguchi method	• Taguchi method is not effective to parameter variations;
		Model-based methods	• Model-based methods require the accurate model;
	Fuzzy system	• Fuzzy analysis	Membership function strongly depends on human experience that causes subjective uncertainties.
Dynamic robust design		• Stability based design	All existing methods can not work for the system with model uncertainties, and
		• Feasibility design	hybrid system withboth continuous and discrete design variables, furthermore,
		• Other design	• Stability design has not been applied to the nonlinear system.
			• Feasibility design has not been applied to nonlinear dynamic system.

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