The Theory of Parametric Control of Macroeconomic Systems and Its Applications(III)

A. Ashimov, Zh. Adilov, R. Alshanov, Yu. Borovskiy and B. Sultanov
Kazakh National Technical University named after K. Satpayev, 22 Satpaev Street, Almaty, 050013, Kazakhstan

Abstract
This work consists of three parts and presents the recent results of development of the theory of parametric control of macroeconomic systems and some its applications for solving a number of concrete problems.
Keywords Mathematical Model, Structural Stability, Parametrical Identification, Parametric Control

Part 3. Applications of the Theory of Parametric Control of Macroeconomic Systems

3.1 Macroeconomic Analysis and Parametric Control of Macroeconomic System Based on an Econometric Model of Small Open Economy

3.1.1 Building an Econometric Model of a Small Open Economy

General view of the model for small open economy of Kazakhstan, that describes equilibrium conditions in macroeconomic markets of goods, money, labor and of capital, taking into account its interaction with the Russian Federation and the rest of the world, is presented by the following relations [1-2].

Equilibrium in the goods market of the Republic of Kazakhstan is presented by the formula :

$$Y^D = Y^S \tag{1}$$

where Y^S - real supply of goods in the Republic of Kazakhstan, in billions of tenge (national currency of the Republic of Kazakhstan) ; $Y^D = C + I + G + NE^{FULL}$-real demand for goods in the Republic of Kazakhstan, in billions of tenge ; $NE^{FULL} = NE + NE^{RU}$ - real volume of goods net export from the Republic of Kazakhstan, in billions of tenge ; $NE^{RU} = Q_{ex}^{RU} - e_r Q_{im}^{RU}$- real net export of goods from the Republic of Kazakhstan to the Russian Federation, in billions of tenge (indicator, that takes into account the terms of cooperation of the Republic of Kazakhstan within the regional Customs union) ; Q_{ex} - real volume of exports of goods from the Republic of Kazakhstan to the Russian Federation, in billions of tenge ; Q_{im}^{RU} - real volume of imports of goods to the Republic of Kazakhstan from Russian Federation, in billions of tenge ; $e_r = eP^Z/P$ - real exchange rate in the Republic of Kazakhstan, tenge/US dollar ; e - exchange rate of national currency in the Republic of Kazakshtan, tenge/US dollar ; P^Z - the general price level in the outside world ; P - the general price level in the Republic of Kazakstan ; $NE = Q_{ex}^W - e_r Q_{im}^W$ - real net export of goods from the Republic of Kazakshtan to the rest of the world, in billions of tenge (indicator that takes into account the terms of cooperation between the Republic of Kazakhstan and the rest of the world) ; Q_{ex}^{W} - real volume of exports of goods from the Republic of Kazakhstan to the rest of the world, in billions of tenge ; Q_{im}^{W} - real volume of imports of goods to the Republic of Kazakhstan from the rest of the world, in billions of US dollars ; G - real volume of government expenditures in the Republic of Kazakhstan, in billions of tenge ; I - real volume of the Republic of Kazakshtan investments to the basic capital, in billions of tenge ; C - real volume of consumption by households in the Republic of Kazakhstan, in billions of tenge. All real data are presented for the year of 2000.

Equilibrium in the money market of the Republic of Kazakhstan is presented by the following relation :

$$M/P = L \tag{2}$$

where L is real cash balances in the Republic of Kazakhstan, in billions of tenge ; M - nominal money supply in the Republic of Kazakhstan (in billions of tenge).

Equilibrium in the labor market of the Republic of Kazakhstan :

$$PdY/dN = W \tag{3}$$

where W - nominal wage rate in the Republic of Kazakhstan, in thousands of tenge ; dY/dN - marginal productivity of labor in the Republic of Kazakhstan ; Y - real gross domestic product (hereinafter GDP) in the republic of Kazakhstan, in billions of tenge ; N - number of employed people in the Republic of Kazakhstan, in thousands of persons.

Equilibrium in the capital market :

$$PNE^{FULL} = NKE \tag{4}$$

where NKE - net export nominal volume from the Republic of Kazakhstan, in billions of tenge.

Let's introduce additional notations for economic indicators used in development of a model : M^{RU} - real money supply in Russian Federation (in billions of rubles) and G^{RU} - real volume of government expenditures in Russian Federation (in billions of rubles), indicators allowing for conditions of operating of a country as part of regional customs union ; i - the average interest rate of banks for loans in the Republic of Kazakhstan ; i^{Z} - interest rate of the outside world (Market yield on US, Treasury securities at 1-year constant maturity, quoted on investment basis) ; $P_{avg} = 0.6P + 0.4eP^{Z}/e_{2000}$ - weighted average price level in the Republic of Kazakhstan ; - expected exchange rate in the Republic of Kazakhstan (tenge/US dollar) ; $\hat{e}^{e} = (e^{e} - e)/e$ - expected growth rate of the exchange rate in the Republic of Kazakhstan ; P^{oil} - average oil price (in thousands of tenge for a barrel) ; Δ - operator of first difference for the series : $\Delta X = X - X_{-1}; X_{-1}$- lag variable.

Preliminary econometric analysis showed the possibility for evaluation of macroeconomic indicators *C, L, W, I, Y*, Q_{ex}^{RU}, Q_{im}^{RU}, Q_{ex}^{W}, Q_{im}^{W}, *NKE* of equilibrium conditions in macroeconomic markets as a function of the regression on the basis of the following set of time series : *Y, C, L, i*, NE^{FULL}, *W, N*, P_{avg}, *I*, Q_{ex}^{RU}, P^{oil}, M^{RU}, G^{RU}, Q_{im}^{RU}, Q_{ex}^{W}, e_r, Q_{im}^{W}, *NKE*, i^Z, $\hat{e}^e$ for the years of 2000-2011 according to statistical data of national economies of the Republic of Kazakhstan and Russian Federation.

In order to build non-spurious regression functions the considered sets were checked for stationary with the help of Augmented Dickey-Fuller method (ADF)and were decomposed [3].

According to the results of a check-up for stationarity of time series :

- regression functions of the following type $C = C(Y)$, $L = L(Y, i, NE^{FULL})$, $W = W(N, P_{avg})$, $NKE = NKE(i^Z, \hat{e}^e, i)$, $I = I(i)$, $Y = Y(N)$, $Q_{im}^{RU} = Q_{im}^{RU}(Y, G^{RU})$, $Q_{im}^{W} = Q_{im}^{W}(Y, e_r)$ were built by the method of ordinary least squares on the basis of the following non-stationary series and the results of their analysis on the statistical significance are presented respectively in table 1 :

Table 1 Regression functions for non-stationary series

Consumption of domestic products in the : Republic of Kazakhstan(R^2=0.99)	C= 558.3 + 0.38Y (0.00) (0.00)
Demand for real cash balances in the Republic of Kazakhstan(R^2=0.92) :	L=0.7Y-82.6 i-0.8 NE^{FULL} (0.00) (0.02) (0.09)
Price of labor supply in the Republic of Kazakhstan, where $P_m = 0.6P (R^2 = 0.98)$:	W=-0.15 N + 1174.8 P_m+0.4$e^Z/_{2000}$ (0.00) (0.00)
Net capital export from the Republic of Kazakhstan ($R^2 = 0.65$) :	$NKE = 291i^z - 6090\tilde{e}^e - 25.6i$ (0.03) (0.02) (0.35)
Investments of the Republic of Kazakhstan to the basic capital ($R^2 = 0.33$) :	I = 5885.5-291.5 i (0.01) (0.05)
Production function in the Republic of Kazakhstan (R^2=0,95) :	Y=-25255+4.27N (0.00) (0.00)
Import of goods from the Russian Federation (R^2=0.89) :	$Q_{im}^{RU} = 0.07Y + 0.89\Delta G^{RU}$ (0.00) (0.00)
Import of goods from the rest of the world ($R^2 = 0.79$) :	$Q_{im}^{W} = 0.22Y + 3.75e_r$ (0.00) (0.01)

- regression functions of the type $Q_{ex}^{RU} = Q_{ex}^{RU}(e_r, P^{oil}, M^{RU}, G^{RU})$, $Q_{ex}^{W} = Q_{ex}^{W}(e_r, P^{oil})$ were built by the method of least squares on the basis of corresponding stationary series or stationary with respect to determined trends and the results of the analysis for statistical significance are presented respectively in table 2 :

Table 2 Regression functions for stationary series

Import of goods from the Republic of : Kazakhstaninto the Russian Federation (R^2=0.72) :	$Q_{ex}^{RU} = 4.4\Delta e_r + 31.4P^{oil}$ $-0.066M^{RU} + 0.128G^{RU}$ (0.03) (0.04) (0.02) (0.00)
Exportof goods from the Republic of Kazakhstan to the rest of the world (R^2=0.99) :	$Q_{ex}^{W} = -11.2e_r + 278.0P^{oil} + 1830.4$ (0.01) (0.00) (0.01)

Regressions functions (where corresponding time series are stationary with respect to the determined trends) were checked on spuriousness by the T-test [4]. Spurious regressions functions (where time series were non-stationary) were checked on spuriousness by the Engle-Granger cointegration test [5].

The model of small open economy of the Republic of Kazakhstan based on the equilibrium conditions in macroeconomic markets of goods, money, labor and capital (1)-(4) and based on the built regression functions (tables 1, 2) has the following form :

$$\begin{cases} Y^D = 3201.8 + 0.84\dfrac{M}{P} - 17.63\dfrac{eP^Z}{P} + 4.01\dfrac{e_{-1}P^Z_{-1}}{P_{-1}} + 281.8P^{oil} \\ \quad - 0.06M^{RU} - 0.694G^{RU} + 0.811G^{RU}_{-1} \\ Y^S = -25525 + 19967.4P + 13392.7P^Z\dfrac{e}{e_{2000}} \\ Y^{ZB0} = 6311.72 + 1066.9P^{oil} - 66.72\dfrac{eP^Z}{P} + 15.17\dfrac{e_{-1}P^Z_{-1}}{P_{-1}} \\ \quad - 0.2276M^{RU} + 3.07G^{RU}_{-1} - 1003.45\dfrac{i^z}{P} + 21000\dfrac{e^e}{e} - 19369.8\dfrac{1}{P} \\ \quad - 0.221\dfrac{M}{P^2} - 1.04\dfrac{eP^Z}{P^2} + 0.205\dfrac{e_{-1}P^Z_{-1}}{PP_{-1}} + 19.77\dfrac{P^{oil}}{P} \\ \quad - 0.034\dfrac{M^{RU}}{P} - 0.04\dfrac{G^{RU}}{P} + 0.0478\dfrac{G^{RU}_{-1}}{P} \\ Y^S = Y^D = Y^{ZB0} \\ i = 18.5 - 0.0025\dfrac{M}{P} - 0.0117\dfrac{eP^2}{P} + 0.0023\dfrac{e_{-1}P^Z_{-1}}{PP_{-1}} + 0.19P^{oil} \\ \quad - 0.00003M^{RU} + 0.0005G^{RU}_{-1} \end{cases} \tag{5}$$

Here Y^{ZB0} is the function of zero balance of payments of the Republic of Kazakhstan.

3.1.2 Estimating Stability Indicators of the Model for a Small Open Economy

The quality of the researched econometric model of the small open economy is estimated by the stability indicator β (Section 1.3.2), which characterizes a change in the equilibrium model solutions by small deviations of the input parameters used. If the stability indicator of the model takes a small value for a small deviation of input variables used, it is considered that the model is qualitative in the sense of stability indicator.

Estimation of stability indicators is made by the algorithm 5, where the vector $X = \{M, G, P^Z, i^Z, e^e, P^{oil}, M^{RU}\}$ has been considered as the vector of input parameters, and the vector $z = \{Y, e, i\}$ has been considered as the vector of output variables.

Conducted computing experiments show that deviations of equilibrium solutions up to 1 % correspond to the deviations of input factors within 1%. This confirms the fact that the considered econometric model (5) is a qualitative model in sense of stability indicator β.

3.1.3 Parametric Control of the Country's Export Depending on Uncontrollable Factors

Based on the fact of dependence of the solution of algebraic equations on its coefficients, we propose an approach to parametric control of national economy evolution taking into account the requirements for equilibrium on macroeconomic markets, which comes down to making recommendations based on the optimal values of economic tools in the form of solutions of mathematical programming problems based on the econometric model of a small open economy.

Let us consider the possibility of estimating the optimal values of M and G tools of economic policy for the given values of the uncontrollable input parameters P^{oil}, i^Z, P^Z, e^e, M^{RU} and G^{RU} that represent the values of these factors in the framework of the Customs Union by example of one country and the rest of the world in 2011 within the framework of the model IS-LM- ZB0 (built on statistics for 2000-2011) in sense of the maximum criterion :

$$Q_{ex} = Q_{ex}^W + Q_{ex}^{RU} \to max \tag{6}$$

Here Q_{ex} is the function of total exports of goods.

The stated estimate can be obtained by solving the following problem of mathematical programming.

Problem 3.1. Based on the mathematical model (5) find values *(M, G)*, that provide maximum to the criterion (6) under the constraints (7)

Here M^* and G^* are accepted values of money supply and government expenditures respectively, for the years of 2008-2011 ; Y^*, P^*, e^*, i^* - basic equilibrium solutions of the system (5) ; Y, P, e, i - optimal equilibrium solutions of the system (5).

$$\begin{cases} |M - M^*| \leq 0.1M^*, \\ |G - G^*| \leq 0.1G^*, \\ |P - P^*| \leq 0.1P^*, \\ |e - e^*| \leq 0.1e^*, \\ |i - i^*| \leq 0.1i^*, \\ |Y - Y^*| \leq 0.1Y^*, \end{cases} \tag{7}$$

The proposed approach to the parametric control of national economy evolution consists in realization of the following algorithm :

1. Choice of mathematical model based on statistical analysis of the regression functions and estimation of stability indicator of the econometric model of economic general equilibrium for the open economy of the Republic of Kazakhstan ;
2. Statement of the mathematical programming problem ;
3. Prediction of uncontrollable factors P^Z, i^Z, P^{oil}, M^{RU}, G^{RU} and e^e for the period of choosing the recommendations on economic policy ;
4. Solution to the mathematical programming problem based on the selected mathematical model to forecast values of uncontrollable factors ;
5. Making recommendations on values of the M and G tools based on the analysis of the results of the mathematical programming problem for predicted values of the uncontrollable factors and possible additional information on the economic conjuncture.

Below we present an illustration of the proposed approach of parametric control of the national economy evolution for 2012.

1. Let the model of a small open economy be a mathematical model selected on the basis of estimation of stability indicators (less than 1%) in 2011.
2. As a statement of the optimization problem for the model of a small open economy in 2011 we take the statement of the problem 3.1.
3. Forecasted values of uncontrollable factors, obtained on the basis of the models built taking into consideration the results of time series decomposition into components, took the following values for 2012 : $P^Z = 1.32$; $P^{oil} = 14.75$ thousandtenge per barrel ; $i^Z = 0.17\%$; $e^e = 148.68$ tenge for one US dollar ; $M^{RU} = 27949.1$ billion rubles and G^{RU} =10898.3 billion rubles.
4. Solution to the mathematical programming problem on the basis of the model for a small open economy by example of the Republic of Kazakhstan and the predicted values of uncontrollable factors for 2012 are : $M = 6733.0$ billion tenge ; $G = 1444.4$ billion tenge ; the value of the criterion $\max(Q_W^{ex} + Q_{RU}^{ex}) = 3465.1 + 335.6 = 3800.7$ billion tenge ;
5. The following can be proposed as a recommendation : solutions obtained during the experiment $M = 6733.0$ billion tenge and $G = 1444.4$ billion tenge or

some correcting values, those can be obtained on the basis of the additional data analysis on economic conjuncture.

3.2 Macroeconomic Analysis and Parametric Control of Cyclical Dynamics

The major section of modern macroeconomic theory is propositions on market cycles, in which the factors generating them are considered and different mathematical models for their analysis are proposed [1,6-8]. Suppression of market cycles is the major field of stabilization policy [9-10].

3.2.1 Macroeconomic Analysis and Parametric Control of Cyclical Dynamics Based on Kondratiev Cycle Model

Model description

This model combines descriptions of non-equilibrium economic growth and non-uniform scientific and technological advancement [11]. The model is described by the following system of equations, including two differential and one algebraic equation :

$$\begin{cases} n(t) = Ay(t)^a, \\ dx/dt = x(t)(x(t)-1)(y_0 n_0 - y(t)n(t)), \\ dy/dt = n(t)(1-n(t))y(t)^2(x(t) - 2 + \dfrac{\mu + l_0}{n_0 y_0}), \\ n_0 = Ay_0^{\alpha}. \end{cases} \tag{8}$$

Here t is the time (in months) ; x is the efficiency of innovations ; y is the capital productivity ratio ; y_0 is the capital productivity ratio corresponding to the equilibrium trajectory ; n is the rate of savings ; n_0 is the rate of saving corresponding to the equilibrium trajectory ; μ is the coefficient of withdrawal of funds ; l_0 is the job growth rate corresponding to the equilibrium trajectory ; A and a are some model constants.

Estimation of the model parameters is carried out based on statistical information from the Republic of Kazakhstan for the years 2001-2005 [12]. The deviations in the observed statistical data and the calculated data do not exceed 1.9% within the considered period.

As a result of solving the problem of parametric identification, the following values of the exogenous parameters are obtained : $\alpha = -0.0046235$, $y_0 = 0.081173$, $n_0 = 0.29317$, $\mu = 0.00070886$, $l_0 = 0.00032161$, $x(0) = 1.911144$.

A retrospective prediction for 2006 and 2007 are characterized by errors equal to 6.1% and 12.1%, respectively, for the capital productivity ratio, and 2.3% and 11%, respectively, for the rate of savings.

The respective cyclic phase trajectory of the Kondratiev cycle model is presented in Fig.1. The period of cyclic trajectory corresponding to the statistical information of the Republic of Kazakhstan for the given years is estimated to be

232 months.

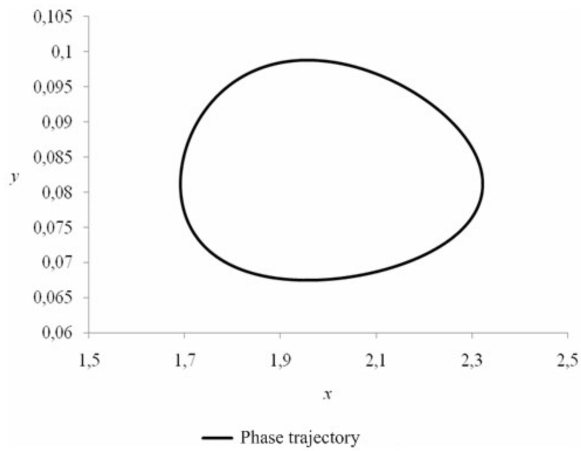

Fig.1 Cyclic phase trajectory of the Kondratiev cycle model

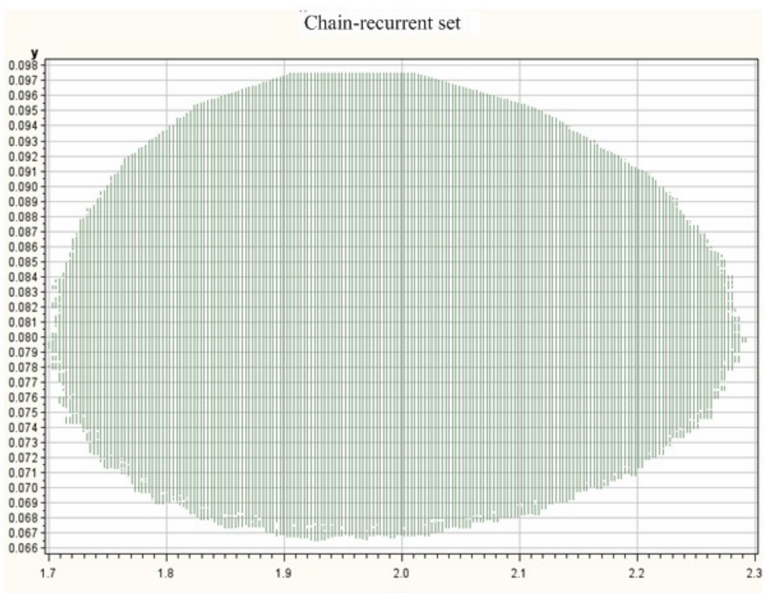

Fig.2 Chain-recurrent set for the Kondratiev cycle model

Estimating the robustness of the Kondratiev cycle model without parametric control

The estimation of structural stability (robustness) of the mathematical model is carried out according to the 4th component of the parametric control theory (Section 1.1) in the chosen compact set of the model phase space.

Fig.2 presents an estimate of the chain-recurrent set $R(f, N)$ obtained by the application of the chain-recurrent set estimation algorithm for the region $N = [1.7; 2.3] \times [0.066; 0.098]$ of the phase plane O_{xy} of system (8). Since the set $R(f, N)$ is not empty, one can draw no conclusion about the weak structural stability of the Kondratiev cycle model in N on the basis of Robinson's theorem. However, since there is a non-hyperbolic singular point in N, namely, the center ($x_0 = 2 - \frac{\mu+l_0}{n_0 y_0}, y_0$), then system (8) is not weakly structurally stable in N.

Parametric control of the evolution of economic system based on the Kondratiev cycle model

Choosing the optimal parametric control laws is carried out in the environment of the following four relations :

$$\begin{cases} 1)\ n_0(t) = {n_0}^* + k_1 \dfrac{y(t) - y(0)}{y(0)}; \\ 2)\ n_0(t) = {n_0}^* - k_2 \dfrac{y(t) - y(0)}{y(0)}; \\ 3)\ n_0(t) = {n_0}^* + k_3 \dfrac{x(t) - x(0)}{x(0)}; \\ 4)\ n_0(t) = {n_0}^* - k_4 \dfrac{x(t) - x(0)}{x(0)}. \end{cases} \tag{9}$$

Here k_i is the scenario coefficient ; n_0^* is the value of the exogenous parameter n_0 obtained as a result of the estimation of parameters.

The problem of choosing the optimal law of parametric control at the level of the econometric parameter n_0 can be formulated as follows.

On the basis of mathematical model (8), find the optimal parametric control law in the environment of the set of algorithms (9), ensuring reach of optimal values of the following criterion :

$$K = \frac{1}{T} \sum_{t=1}^{T} \left(\left(\frac{x(t) - x_0}{x_0} \right)^2 + \left(\frac{y(t) - y_0}{y_0} \right)^2 \right) \to \min \tag{10}$$

(here $T = 232$ is the period of the cycle) under the constraints

$$0 \le y(t) \le 1, 0 \le n(t) \le 1, 0 \le x(t) \tag{11}$$

The base value of the criterion (without parametric control) is as follows : $K = 0.0307$.

The value of criterion $K = 0.007273$ for the control law, that is optimal in

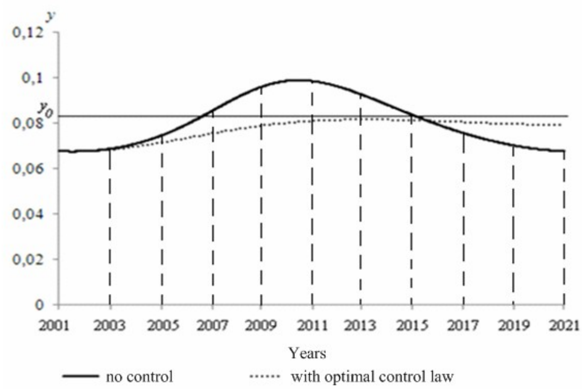

Fig.3 Capital productivity ratio without parametric control and with use of law 4, optimal in the sense of criterion K

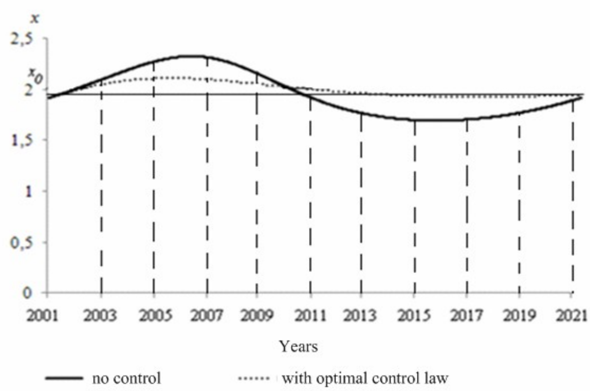

Fig.4 Efficiency of innovations without parametric control and with use of law 4, optimal in the sense of criterion K

the sense of the criterion (10) of the 4^{th} law, from the set (9) represented before is obtained by solving the problem formulated above through application of the parametric control approach to the evolution of the economic system. Corresponding value of adjustable coefficient of this law is . The values of the model's endogenous variables without applying parametric control and with use of the optimal parametric control law for criterion K are presented below in graphic form (Fig.3 and Fig.4).

Estimating the structural stability of the Kondratiev cycle mathematical model with parametric control

To carry out this analysis, the expressions for optimal parametric control laws *(11)* with the obtained values of the adjustable coefficients are substituted into the right-hand side of the second and third equations of system (1) for the parameter n_0. Then, by using a numerical algorithm for estimating the weak structural stability of the discrete-time dynamical system for the chosen compact set N determined by the inequalities $1.7 \leq x \leq 2.3$, in the state space of the variables (x, y), the estimation of the chain-recurrent set $R(f, N)$ as the empty (or one-point) set is obtained. This means that the Kondratiev cycle mathematical model with optimal parametric control law is estimated as weakly structurally stable in the compact set N.

Analysis of the dependence of the optimal value of criterion K on the parameter for the variational calculus problem based on the Kondratiev cycle mathematical model

Let us analyze the dependence of the optimal value of criterion K on the exogenous parameters μ (share of withdrawal of capital production assets per month) and a for parametric control laws (11) with the obtained optimal values of the adjusted coefficients k_i, where the values of the parameters (μ, a) belong to the rectangle $A = [0.00063; 0.00147] \times [-0.01; 0.71]$ in the plane.

Plots of dependencies of the optimal value of criterion K (for parametric control laws 0 and 2, yielding the maximum criterion values) on the uncontrollable parameters (see Fig.5) were obtained by computational experimentation. The projection of the intersection line of the two surfaces in the plane (μ, α) consists of the bifurcation points of the extremals of the given variational calculus problem.

3.2.2 Macroeconomic Analysis and Parametric Control of Cyclical Dynamics Based on Dynamic Stochastic General Equilibrium Model for the Economy of Kazakhstan

In nonlinear dynamical stochastic general equilibrium (DSGE) model is presented on the base of given composition and behavior of agents, their inter action in stochastic conditions and of taking the principle of rational expectations [13]. This nonlinear DSGE model of the economy consists of :

- first-order aggregate conditions of optimization problems of agents (house hold sand intermediate product producers) [14] ;
- description of government activity rules ; and
- rules of shocks specifying in terms of either first-order auto regression s or Gaussian white noises.

First-order aggregate conditions involve equilibrium conditions in market of labor, capital, intermediate and final goods. The nonlinear DSGE model that was built involves both the model of actual economy, and the model of potential economy. The model of potential economy is similar to the model of actual economy by its composition, except that potential economy functions under flexible prices and wages (in the model of actual economy the se prices are not flexible), and also

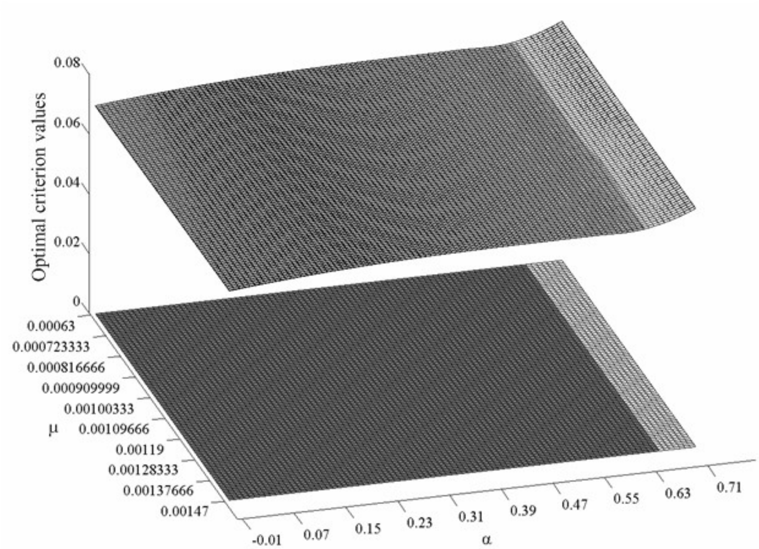

Fig.5 Plots of the dependencies of the optimal value of criterion K on exogenous parameters μ, a

in absence of "extra charge" shocks (those are in the model of actual economy).

The nonlinear DSGE model in question has the following vector form :

$$E_t F^{\theta}(X_{t-1}, X_t, X_{t+1}, H_t^{\Sigma_H}) = 0 \tag{12}$$

Here E_t is sign of conditional mathematical expectation given information available at the point of time t $(t = 1, 2, ...)$; F^{θ} is known vector function ; θ is parameters set, consisting of structural parameters of the model and auto regression parameters of shocks ; X_t is vector, consisting of endogenous variables and shocks, determined by first-order auto regressions ; X_0 is given ; $H_t^{\Sigma_H}$ is vector, consisting of Gaussian white noises, Σ_H is corresponding diagonal covariance matrix.

According to the technique taken for DSGE model, linear approximation of nonlinear DSGE model (12) was built in neighborhood of its stationary point X [13]. Mentioned point is found by solving vector equation (15) obtained from (14) by dropping time subscripts and nulling white noises :

$$F^{\theta}(X, X, X, 0) = 0 \tag{13}$$

Log-linearization of DSGE model of F. S mets and R. Wouters in neighborhood of its estimated stationary point gives linear DSGE model of the following form :

$$A^{\theta}\hat{X}_{t-1} + B^{\theta}\hat{X}_t + C^{\theta}E_t\hat{X}_{t+1} + D^{\theta}H_t^{\Sigma_H} = 0, t = 1, 2, 3... \tag{14}$$

Here the sign ≪^≫ corresponds to linearized variable, A^θ, B^θ, C^θ, D^θ are matrices of corresponding dimensions.

In this example we consider the case of implementing state economic policy by the Taylor rule [15], describing behavior of National bank insetting interest rate, and rules for determining the size of government spending as well.

In the framework of linear model (14) the Taylor rule for determining governmental bonds yield is presented in the following form :

$$\begin{aligned}\hat{R}_t = \rho\hat{R}_{t-1} + (1-\rho)\left(\bar{\pi}_t + r_\pi(\hat{\pi}_{t-1} - \bar{\pi}_{t-1}) + r_Y(\hat{Y}_t - \hat{Y}_t^p\right) \\ + r_{\Delta\pi}(\hat{\pi}_{t-1} - \hat{\pi}_{t-1}) + r_{\Delta Y}\left(\hat{Y}_t - \hat{Y}_t^P - \left(\hat{Y}_{t-1} - \hat{Y}_{t-1}^p\right)\right) + \eta_t^R\end{aligned} \tag{15}$$

and the rule of government spending in the following form :

$$\hat{G}_t = \rho_G\hat{G}_{t-1} + \eta_t^G \tag{16}$$

Here $\hat{R}_t$, $\hat{Y}_t$, $\hat{Y}_t^p$, $\hat{G}_t$, $\hat{\pi}_t$ are variables, corresponding to : governmental bondsyield (1 + interest rate), output, potential output, government spending and inflation. η_t^R is interest rate shock, given in terms of Gaussian white noise ; $\bar{\pi}_t$ is inflation shock ; η_t^G is government spending shock, ρ, r_π, $r_{\Delta\pi}$, $r_{\Delta Y}$, ρ_G are the parameters of the equations (15), (16).

Estimating parameters of linear DSGE model on the basis of statistical data of the economy of the Republic of Kazakhstan

Model (16) solution was obtained by the Blanchard-Kahn algorithm [16-17]. This solutionis presented in the form of first-order vector auto regression :

$$\begin{bmatrix} \hat{X}_t \\ \hat{R}_t \\ \hat{G}_t \end{bmatrix} = Q^\theta \begin{bmatrix} \hat{X}_{t-1} \\ \hat{R}_{t-1} \\ \hat{G}_{t-1} \end{bmatrix} + F^\theta H_t^{\Sigma_H}, t = 1, 2, 3... \tag{17}$$

Hereinafter $\hat{X}_t$ is vector-column consisting of all endogenous variables in the model (including shocks determined in terms of auto regression), excluding state policy tools of governmental bonds yield $\hat{R}_t$, and the size of government spending $\hat{G}_t$. Vectors $\hat{X}_0, \hat{R}_0, \hat{G}_0$ are given ; Q^θ, F^θ are matrices of corresponding dimensions.

Estimating parameters of the model in question (14) (using (17)) was made by the Bayesian estimation method (the Metropolis-Hastings algorithm with the number of simulations of 4 000 000) using the Kalman filter [18]. As observations were used quarterly data for seven macroeconomic indicators of Kazakhstan (GDP, Investments, Consumption, Employment, Average wage, Refinancing rate, and Inflation) from 2002I till 2010III. We found the logarithm of mentioned statistical indicators and linearly detrended them. For using the Kalman filter within the Bayesian approach the model (17) was supplemented with vector equation of the dimension :

$$\hat{S}_t = M \begin{bmatrix} \hat{X}_t \\ \hat{R}_t \\ \hat{G}_t \end{bmatrix} \tag{18}$$

Here M is matrix, each row of which contains one unity, all of the rest its elements are equal to 0. As the results of measuring of observed variables were taken log-deviations from its linear trends of macroeconomic indicators values (Consumption, Investments, GDP, Inflation, Average wage, Employment, Refinancing rate), corresponding to observed variables. Statistical data for the Republic of Kazakhstan from 2002I till 2011III was used in this study.

For using the Bayesian approach there were given a priori density distribution $p = p_0(\theta', \Sigma_H)$ of parameters θ', Σ_H. The form and probabilistic characteristics of this distribution from were used in the research [13], with the exception of mathematical expectations of a priori distribution of parameters Σ_H. Mentioned mathematical expectations were increased 2.5 times relative to corresponding values from S mets F. and Wouters R. in connection with large sampled standard deviations of economic indicators of Kazakhstanin comparison with Eurozone.

According to the Bayesian approach method [18], using like lihood function, obtained on the basis of the model(17), (18) using the Kalman filter, and a priori distribution of parameters $p_0(\theta', \Sigma_H)$ as well,was found posterior joint density distribution of initial estimates of parameters : $p = p_1(\theta', \Sigma_H)$. Then using the Metropolis-Hastings algorithm with density was $p_1(\theta', \Sigma_H)$ generated a sample, consisting of 4 000 000 sets of parameters θ', Σ_H. Finally, as required estimates of parameters were taken corresponding sampled averages.

Quality of applied method for finding the parameters estimates was tested by retro prognosis. For this purpose there were made predictions for mentioned observed economic indicators for four periods from 2010 IV till 2011 III. Root mean square deviations of obtained expected predicted values of economic indicators from corresponding statistical data were about 3%.

Analysis of shocks effectson GDP and inflation using estimating of impulse responses on disturbances within the framework of internal shocks of the economy of the Republic of Kazakhstan

Fig.6 and Fig.7 present relatively impulse responses of real GDP and inflation on (unit positive) shocks. Each diagram presented in figures is obtained by calculation of linear model(17) for initial zero values of all endogenous variables of the model and the value of chosen shock equal to its standard deviation for zero period. Under this all of the values of this shock for non-zero time values, and also the values of all other shocks of the model for all of time values were taken as the null.

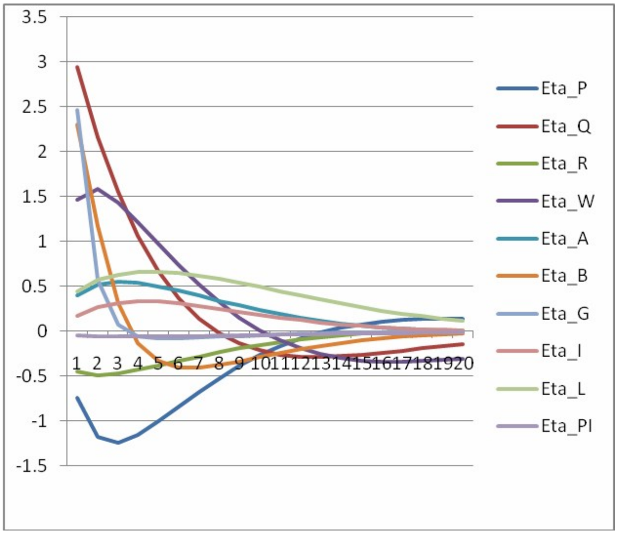

Fig.6 Impulse responses of real GDP on singular positive shocks

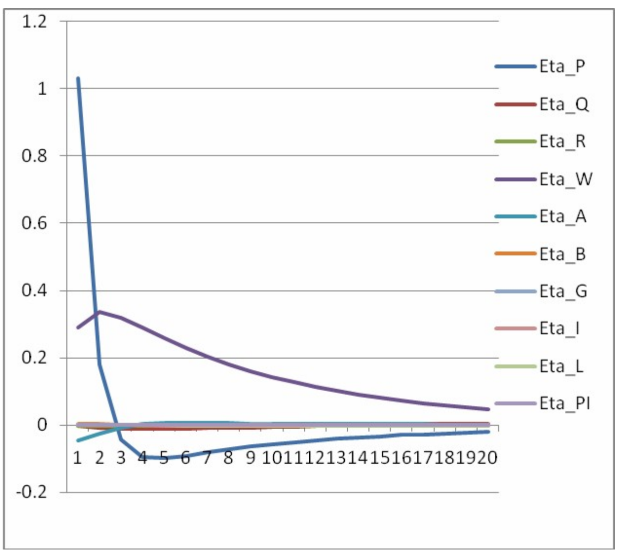

Fig.7 Impulse responses of inflation on singular positive shocks

Analysis of the diagrams of impulse response of real GDP presented in Fig.6 shows the following :

• Given positive shocks of productivity (ε_t^A), labor supply (ε_t^L) and investments (ε_t^I) GDP increases.

• Positive shocks of preferences (ε_t^B) and government spending (η_t^G) also increase GDP (since these shocks increase,respectively, consumption and government spending)

• Positive shock of extra charge for goods (η_t^P) decreases GDP, and shock of extra charge for wage (η_t^W) increases GDP.

• Positive monetary shock (η_t^R) results in production decline (because of interest rate growth).

Analysis of the diagrams of impulse response of inflation presented in the Fig.7 shows the following :

• Given positive shocks of extra charge for goods and extra charge for wage inflation increases

• Given positive shock of productivity inflation negligible decreases.

The rest of shocks do not practically have an effect on inflation.

These responses of the model to shocks correspond with theoretical propositions.

Decomposing indicators evolution to shocks effect parts in retrospective period

Indicators decomposition in retrospective period, presented in Fig.8 and 9,illustrates the contribution (in percentage) of each shock effect of the estimated model (17) to deviations of actual values of GDP and inflation indicators from corresponding equilibrium values for the period 2002I-2012I.

Presented diagrams show how deviations of GDP and inflation from their corresponding trends in retrospective period (from 2002 I till 2012 III) emerged according to positive and negative effects of the shocks in question.

For instance, deviation of GDP from trend in 2009IIIequal to -4.93% is the sum of positive summands :

1. Shock effects of labor supply (ε_t^L) equal to 3.02% deviation of GDP from trend,

2. Shock effects of interest rates (η_t^R) equal to 1.51% deviation of GDP from trend,

3. Shock effects of extra charge for goods (η_t^P) equal to 0.85% deviation of GDP from trend and negative summands :

4. Shock effects of preferences (ε_t^B) equal to -3.88% deviation of GDP from trend,

5. Shock effects of extra charge for wage (η_t^W) -3.63% deviation of GDP from trend,

6. Shock effects of extra charge for capital (η_t^Q) -1.16% deviation of GDP from trend,

7. Shock effects of government spending (η_t^G) -1.12% deviation of GDP from trend,

8. Shock effects of productivity (ε_t^A) -0.69% deviation of GDP from trend (rest

shock effects of investments (ε_t^I) and shock effects of inflation ($\bar{\pi}_t$) are negligible).

That is deviation of GDP from trend equal to -4.93% is the sum of all shock effects of the model for mentioned period.

For instance, deviation of inflation from trend in 2007IV equal to 6.32% is the sum of only positive summands (all of the rest effects are negligible) :

1. Shock effects of extra charge for goods equal to 4.87% deviation of inflation from trend ;

2. Shock effects of extra charge for wage 1.43% deviation of inflation from trend.

Analysis of the diagram in Fig.8 shows also that break-neck growth of GDP during the period 2004-2007 was mainly because of extra charge shocks (for wage, good, capital), lessening of GDP growth in 2008-2011 was mainly because of negative shock effects of preferences, extra charge for capital and wage.

Analysis of the diagram in Fig.9 shows that inflation deviation from the equilibrium level in the period 2002-2011 was almost fully because of extra charge shock for good and wage. In other words, all of the rest shocks of the model do not practically have an effect on inflation values in mentioned period.

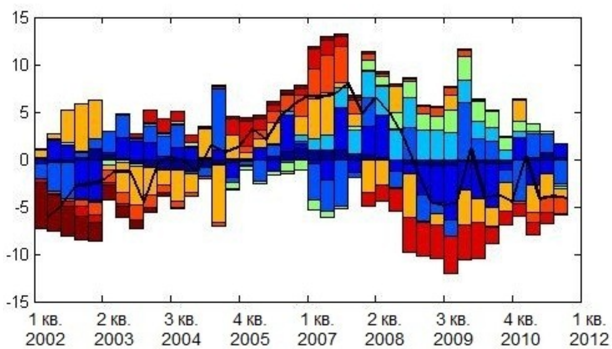

Fig.8 Shocks effects on GDP rate

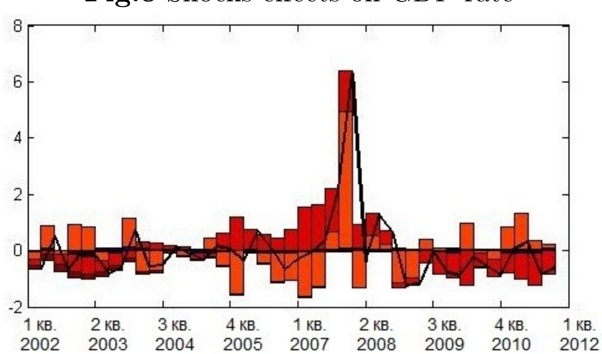

Fig.9 Shocks effects on the quarterly inflation

Prediction of shocks effects on economic indicators and suppression of their effects based on DSGE model of Smets-Wouters

In the paper by estimated model (17) were obtained predicted values of macroeconomic indicators (GDP and inflation) for 1, 4, 10, 20, 30 and 40 quarters (i.e. correspondingly for 2011IV, 2012III, 2014I, 2016III, 2019I, 2021III). For estimating shock effects on error variances of economic indicators predictions were used the standard technique for defining decomposition of error variance of predictions for the models of vector auto regressions [19]. The results obtained by the software Dynare Matlab Toolbox [http ://www.dynare.org] are presented in Tables 1 and 2. There are no shocks, in these tables, which effects on variance less than by 0.01%.

Analysis of the Tables 3 and 4 shows the following. Error variances of prognosis of GDP generally are determined by preference shocks, government spending shock, and extra charge shocks on the cost of capital. Error variances of prognosis of inflation generally are determined by extra charge shocks on goods and extra charge shocks on wages.

Table 3 Prognosis (billion tenge in average prices of 1994) and decomposition of variance of the quarterly GDP prognosis

Time hori -zon	Prognosis		Decomposition of variance (in %)								
	Mathematical expectation	Standard deviation	η_t^A	η_t^B	η_t^G	η_t^L	η_t^I	η_t^R	η_t^Q	η_t^P	η_t^W
1	271.413	8.17	2.25	22.90	17.57	1.47	0.09	1.07	45.32	6.13	3.19
4	292.166	10.960	4.04	21.73	15.30	3.25	0.15	1.56	41.99	8.27	3.71
10	327.034	13.229	4.04	21.73	15.30	3.25	0.15	1.56	41.99	8.27	3.71
20	385.810	16.067	4.09	20.69	14.20	3.70	0.14	1.51	39.84	7.90	7.92
30	456.450	19.136	4.03	20.39	13.98	3.67	0.14	1.49	39.47	8.01	8.82
40	542.395	22.751	4.02	20.37	13.97	3.67	0.14	1.49	39.45	8.03	8.86

Table 4 Prognosis (in %) and decomposition of variance of the quarterly inflation prognosis

Time horizon	Prognosis		Decomposition of variance (in %)			
	Mathematical expectation	Standard deviation	η_t^A	η_t^Q	η_t^P	η_t^W
1	1.22%	0.75%	0.29	0.01	92.24	7.47
4	1.36%	0.78%	0.30	0.06	72.31	27.33
10	1.62%	0.80%	0.28	0.15	62.65	36.92
20	1.78%	0.81%	0.27	0.16	60.71	38.85
30	1.83%	0.81%	0.28	0.17	60.55	39.00
40	1.85%	0.81%	0.28	0.17	60.53	39.01

In realization of the state policy for minimizing shock effects on economic indicators in the capacity of its tools we choose additive summands η_t^R, η_t^G in the expressions (14), (15), desired values of which are searched in terms of deterministic values instead of respective shocks.

The parametric control approach for minimizing shock effects consists in the following. Let T be number of quarter, starting from which the state realizes parametric control policy for minimizing shock effects. At each time point $t = T, T+1, T+2, ...$ on the basis of estimated model (17), written in the form

$$\begin{bmatrix} \hat{X}_{t+i} \\ \hat{R}_{t+i} \\ \hat{G}_{t+i} \end{bmatrix} = Q^{\theta} \begin{bmatrix} \hat{X}_{t+i-1} \\ \hat{R}_{t+i-1} \\ \hat{G}_{t+i-1} \end{bmatrix} + F^{\theta} H_{t+i}^{\Sigma_H}, i = 1, 2, 3, ..., 40 \tag{19}$$

such deterministic values of tools η_{t+1}^R, ..., η_{t+40}^R, η_{t+1}^G,..., η_{t+40}^G are defined, those give the minimum for criterion

$$L_t = E_t \sum_{i=1}^{40} \beta^i (\hat{\pi}_{t+i}^2 + \lambda_Y \hat{Y}_{t+i}^2), \min(\eta_{t+1}^R, ..., \eta_{t+40}^R, \eta_{t+1}^G, ..., \eta_{t+40}^G) L_t \tag{20}$$

(characterizing expected discounted total deviation of GDP and inflation values from respective equilibrium values (trend)) under the following constraints on endogenous variables of the model. Mathematical expectations for inflationand bond yield in this time horizon should not deviate from respective equilibrium values more than by 0.5% :

$$|E_t \hat{\pi}_{t+i}| \leq 0.005, i = 1, 2, 3, ..., 40 \tag{21}$$

$$|E_t \hat{R}_{t+i}| \leq 0.005, i = 1, 2, 3, ..., 40 \tag{22}$$

and mathematical expectation for the size of government spending by 5.0% from their trend values :

$$|E_t \hat{G}_{t+i}| \leq 0.05, i = 1, 2, 3, ..., 40 \tag{23}$$

Moreover, at each specified time point t, the value of current state of economy for the time point $t - (\hat{X}_t, \hat{R}_t, \hat{G}_t)$ is known. After receiving a new information (the values of variables $(\hat{X}_{t+1}, \hat{R}_{t+1}, \hat{G}_{t+1})$ at the next time point $(t+1)$), the values of tools η_{t+2}^R, ..., η_{t+41}^R, η_{t+2}^G,..., η_{t+41}^G are calculated again by solving the Problem (19)-(23) for respective period. Here discount factor, is some weight coefficient.

Introduce a new minimization criterion :

$$\tilde{L}_t = \sum_{i=0}^{40} \beta^i \left((E_t \hat{\pi}_{t+i})^2 + \lambda_Y (E_t \hat{Y}_{t+i})^2 \right), \min(\eta_{t+1}^R, ..., \eta_{t+40}^R, \eta_{t+1}^G, ..., \eta_{t+40}^G) \hat{L}_t \tag{24}$$

It is not difficult to check, that this criterion differs from by the value, independent from variables $\eta^R_{t+1},...,\eta^R_{t+40},\eta^G_{t+1},...,\eta^G_{t+40}$.

From the relation (19), by taking mathematical expectations for both of its parts, we get

$$E_t\begin{bmatrix}\hat{X}_{t+i}\\ \hat{R}_{t+i}\\ \hat{G}_{t+i}\end{bmatrix}=Q^\theta E_t\begin{bmatrix}\hat{X}_{t+i-1}\\ \hat{R}_{t+i-1}\\ \hat{G}_{t+i-1}\end{bmatrix}+F^{\theta''}\begin{bmatrix}\eta^R_{t+i}\\ \eta^G_{t+i}\end{bmatrix}, i=1,2,3,...,40 \qquad (25)$$

Here $F^{\theta''}$ is matrix, comprised by two corresponding columns of the matrix F^θ Consequently, optimal values of variables of the problems (19)-(23)and (21)-(25) coincide between each other. Derived optimization problem (21)-(25) applies to classical (deterministic) type of variational calculus problems, which for each time point t is solved by the linear and quadratic programming method (with Matlab application).

Below in the paper it is estimated the effectiveness of application of formulated above parametric control in assumption that the state will implement this policy during 30 years (120 periods). That is, assume that at each time point t ($t = T, T+1, T+2, ..., T+119$, T is number of quarter, corresponding to 2011III) the stated etermines the values of parameters $\eta^R_{t+1},\eta^G_{t+1}$ by solving above mentioned optimization problem(19)-(23) (or, that is the same, (21)-(25)). In computing experiment it is assumed that the economy is precisely described by estimated model :

$$\begin{bmatrix}\hat{X}_{T+i}\\ \hat{R}_{T+i}\\ \hat{G}_{T+i}\end{bmatrix}=Q^\theta\begin{bmatrix}\hat{X}_{T+i-1}\\ \hat{R}_{T+i-1}\\ \hat{G}_{T+i-1}\end{bmatrix}+F^\theta H^{\Sigma_H}_{t+i}, i=1,2,3,...,160$$

where $\hat{X}_T$ is known.

In the paper, for estimating the effectiveness of application of formulated above parametric control approach the Monte-Carlo method was used with estimate of 100 development scenarios of economy. Let us present aggregative algorithm for estimating application of the parametric control approach.

1. Generation of the sample, consisting of 100 elements-sets of values of vector Gaussian random values (white noises) $\{H^{'\Sigma_H}_{T+1},H^{'\Sigma_H}_{T+2},...,H^{'\Sigma_H}_{T+120}\}_j$ where $j = 1,...,100$, $H^{\Sigma_H}_{T+i}=[H^{'\Sigma_H}_{T+i},\eta^R_{T+i},\eta^G_{T+i}]^T$ with known probabilistic characteristics of noises Σ_H.

2.For each element of the sample $\{H^{'\Sigma_H}_{T+1},H^{'\Sigma_H}_{T+2},...,H^{'\Sigma_H}_{T+120}\}_1$, $\{H^{'\Sigma_H}_{T+1},H^{'\Sigma_H}_{T+2},...,H^{'\Sigma_H}_{T+120}\}_2,...,$ $\{H^{'\Sigma_H}_{T+1},H^{'\Sigma_H}_{T+2},...,H^{'\Sigma_H}_{T+120}\}_{120}$:

2.1 The calculation of the model with parametric control :

We solve optimization problem (21)-(25) for period $t = T$ i.e. we find respective

$\eta^R_{T+1}, ..., \eta^R_{T+40}, \eta^G_{T+1}, ..., \eta^G_{T+40}$; From obtained set of valueswe take $\eta^R_{T+1}, \eta^G_{T+1}$ (we drop rest values $\eta^R_{T+2}, ..., \eta^R_{T+40}, \eta^G_{T+2}, ..., \eta^G_{T+40}$.

2.2. The model is calculated for 1 step with shocks values $H^{\Sigma_H}_{T+1}$, which consist of tools values $\eta^R_{T+1}, \eta^G_{T+1}$ determined by the state and shocks values $H'^{\Sigma_H}_{T+1}$,which were realized by economy independently (exogenously) from the state policy.

$$\begin{bmatrix} \hat{X}_{T+1} \\ \hat{R}_{T+1} \\ \hat{G}_{T+1} \end{bmatrix} = Q^\theta \begin{bmatrix} \hat{X}_T \\ \hat{R}_T \\ \hat{G}_T \end{bmatrix} + F^\theta H^{\Sigma_H}_{T+1}$$

2.3 The steps 2.1 and 2.2 are iteratedfor values $t = T+1, T+2, T+3, ..., T+120$.

3. On the bases of obtained 100 trajectories of GDP and inflation is built average trajectory (expected prognosis value) and standard deviations of prognosis for the period $t = T, T+1, T+2, T+3, ..., T+120$. Obtained values are compared with basic prognosis.

Realization results of formulated algorithm show that for used sample of shocks the parametric control of suppressing shocks effects provides diminishing predicted standard deviations of GDP by 58.3% at the average in prognosis horizon from 2011III till 2021III (see Fig.10).

The parametric control of suppressing shocks effects provides diminishing predicted standard deviations of inflation by 32.0% at the average in prognosis horizon from 2011III till 2021III and diminishing samples tan dard deviation of inflation by 47.8% in comparison with actual data during the period2002Itill 2011III (see Fig.11)

3.3 Macroeconomic Analysis and Parametric Control of the Economic Growth Based on Computable General Equilibrium Model for the Economic Sectors

Presentation of computable general equilibrium model

Non-autonomous computable general equilibrium model (CGE model) in general form is presented by the following system of relations [2], [20].

1) Subsystem of differential equations, connecting endogenous variables values for two successive years :

$$x_1(t+1) = f_1\left(x_1(t), x_2(t), x_3(t), \mu(t), a(t)\right) \tag{26}$$

Here $t = 0, 1, ..., n-1$ is number of year, discrete time ; $x(t) = (x_1(t), x_2(t), x_3(t)) \in R^m$ is vector of endogenous variables of the system ;

$$x_i(t) \in X_i(t) \subset R^{m_i}, i = 1, 2, 3 \tag{27}$$

Here the variables

$x_1(t)$ involve the values of capital assets of the sectors-producers, budgets of

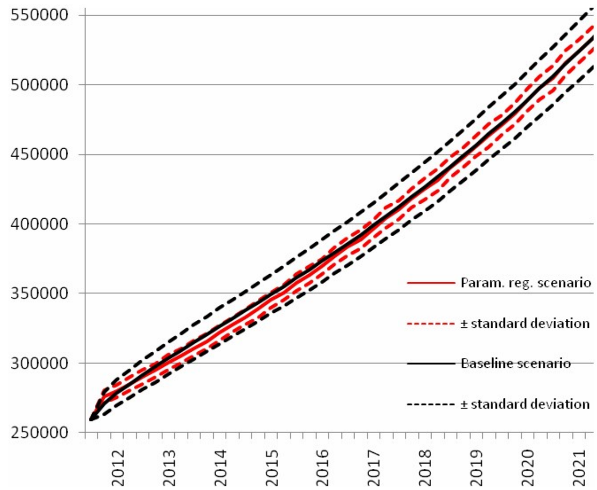

Fig.10 Prognostic values of real GDP for the basic scenario and the parametric control approach

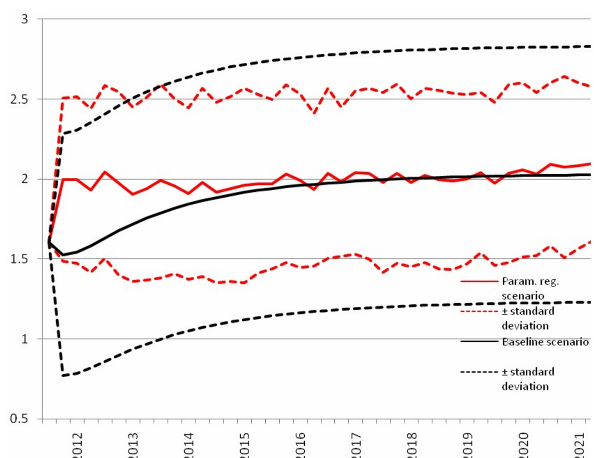

Fig.11 Prognostic values of inflation for the basic scenario and the parametric control approach (in %)

economic agents and so on ;

$x_2(t)$ involve the values of demand and supply of agents in different markets and so on ;

$x_3(t)$ are different kinds of market prices and budget parts in markets with state-set prices for various economic agents ; $m_1 + m_2 + m_3 = m$;

$u(t) \in U(t) \subset R^q$ is vector function of controllable (adjustable) parameters. Coordinate values of this vector correspond to various state economic policy tools, for instance, such as state budget parts and budget parts of economic agents, various tax rates, governmental bonds yield and so on ;

$a(t) \in A \subset R^s$ is vector function of uncontrollable parameters (factors). Coordinate values of this vector characterize various external and internal social and economic factors depending on time : export and import goods prices, population size of the country, production functions parameters and so on ;

$X_1(t),\ X_2(t),\ X_3(t),\ U(t)$ are compact sets with non-empty interiors ; $X_i = \bigcup_{t=1}^{n} X_i(t),\ i = 1, 2, 3$; $X = \bigcup_{t=1}^{3} X_i$; $U = \bigcup_{t=0}^{n-1} U(t)$, is open connected set ;

$f_1 : X \times U \times A \to R^{m_1}$, is continuous mapping.

2) Subsystem of algebraic equations, describing behavior and interaction of agents in various markets within sampled year, these equations allow expressing the variables by exogenous parameters and rest endogenous parameters :

$$x_2(t+1) = f_2\left(x_1(t), x_3(t), x_3(t), u(t), a(t)\right) \tag{28}$$

Here $f_2 : X_1 \times X_3 \times U \times A \to R^{m_2}$ is continuous mapping.

3) Subsystem of recurrence relations for iterative calculations of equilibrium values of market prices in various markets and budget parts in markets with state-set prices for various economic agents :

$$x_3[Q+1] = f_3\left(x_2(t)[Q], x_3(t)[Q], L, u(t), a(t)\right) \tag{29}$$

Here $Q = 0, 1, ...$ is number of iteration ; L is the set of positive numbers (adjustable constants of iterations, when their values decrease,economic system comes faster to its equilibrium condition, however,at the same time the risk of the case when prices go to negative range increases ; $f_3 : X_2 \times X_3 \times (0, +\infty)^{m_s} \times U \times A \to R^{m_2}$ is continuous mapping (that is compressing at fixed t ; $x_1(t) \in X_1(t)$; $u(t) \in U(t)$; $a(t) \in A$ and some fixed L. In this case the mapping f_3 has the only fixed point, to which converges the iterative process (28), (29).

Computable model (26), (28), (29) under fixed values of functions $u(t)$ and $a(t)$ for each time point t determine the value of exogenous variables $x(t)$, corresponding to price equilibrium of demand and supply in the markets of goods and services of agents in the framework of the following algorithm.

1) In the first step it is assumed that t=0 and it is determined the initial values of variables $x_1(0)$.

2) In the second step for current the initial values of variables $x_3(0)[0]$ are determined in various markets and for various agents ; the values
$x_2(t)[0] = f_2\left(x_1(t), x_3(t)[0], x_3(t), \mu(t), a(t)\right)$ (the initial values of demand and supply of agents in the markets of goods and services) are calculated by (28).

3) In the third step for current t it is run iterative process (28), (29). In this,

for each value Q current values of demand and supply are found from (29) : $x_2(t) = f_2\left(x_1(t), x_3(t)[Q], x_3(t), u(t), a(t)\right)$ by improvement of market prices and budget parts of economic agents.

Condition for stopping iterative process is equality of demand and supply values in various markets accurate within 0.01%. Consequently,there are determined equilibrium values of market prices in each market and budget parts in markets with state-set prices for various economic agents. We omit the index Q for such equilibrium values of endogenous variables.

4) In the following step on the basis of obtained equilibrium solution for the time point using differential equations (26) we define the values of variables $x_1(t+1)$. The value increases by unity. Transition to the step 2.

Quantity of iterations of steps 2, 3, and 4 are determined in accordance with the parametric identification problems, prognosis and control in chosen in advance periods.

Considered CGE model can be presented in the form of continuous mapping $f : X \times U \times A \to R^m$, determining transformation of the values of endogenous variables of the system for zero year to respective values of the next year according to presented above algorithm. Here the compacts $X(t) = X_1(t) \times X_2(t) \times X_3(t)$, determining the compact X in the space of endogenous variables are defined by the set of possible values of variables x_1 and respective equilibrium values of variables x_2 and x_3 calculated by relations (30)-(31).

We will assume that for chosen point $x_1(0) \in \mathrm{Int}(X_1)$ and corresponding, calculated by (28), (29) points $x(0) = (x_1(0), x_2(0), x_3(0))$, the inclusion $x(t) = f^t(x(0)) \in \mathrm{Int}(X(t))$ is true under some fixed $u(t) \in \mathrm{Int}(U(t))$, $a(t) \in A$ for $t = 0, ..., n$. (n-fixed positive integer). This mapping f defines discrete dynamical system in the set X, on the trajectory of which imposed appropriate initial condition :

$$\{f^t, t = 0, 1, ...\}, x|_{t=0} = x_0 \tag{30}$$

Based on this conception specific CGE model of economic sectors is considered below.

Parametric identification of CGE model of economic sectors

The model in question by statistical data of the republic of Kazakhstan is presented by the following 19 economic agents.

Economic agent No.1. Agriculture, hunt and forestry ;

Economic agent No.2. Fishery, fish breeding ;

Economic agent No.3. Mining ;

Economic agent No.4. Manufacturing ;

Economic agent No.5. Production and distribution of electricity, gas and water ;

Economic agent No.6. Construction ;

Economic agent No.7. Trade ; automobile and house articles maintenance ;

Economic agent No.8. Hotels and restaurants;

Economic agent No.9. Transportation and communication;

Economic agent No.10. Financial activities;

Economic agent No.11. Transactions with real estates, lease and services to enterprises;

Economic agent No.12. Public administration;

Economic agent No.13. Education;

Economic agent No.14. Public health and social services;

Economic agent No.15. Other municipal, social and personal services;

Economic agent No.16. Housekeeping services;

Economic agent No.17. Aggregate consumer, combining households;

Economic agent No.18. Government, presented by the sum of central, regional and local governments, and non-budget funds as well. Government determines tax rates and amount of subsidies for agents-producers and the size of social transfers for households. Moreover, this sector includes non-commercial organizations, serving households (political parties, labor unions, social associations, etc.);

Economic agent No.19. Bank sector, involving National bank and commercial banks.

Economic agent No.20. Outside world.

Here,economic sectors No.1-16 are agents-producers.

The considered model is presented as general expressions of relations of (26), (28), (29) respectively $m_1 = 67$, $m_2 = 597$, $m_3 = 34$ by expressions, which help to calculate values of its 698 endogenous variables. This model contains also 2045 estimated exogenous parameters.

In the result of combined solution of the problems A and B as per the constructed algorithm of parametric identification (Section 1.2) using statistical data on evolution of the economy of the Republic of Kazakhstan. The relative value of deviations of estimated values of variables used mostly as criteria of corresponding observed values was equal to less than 0.63%.

Further the calculation of the model outside the period of parametric identification (forecast calculation) using extrapolated for the forecast period values of functions $u(t), a(t)$ will be called as basic calculation.

The results of calculation and retrospective basic calculation of the model for 2008 partially presented in the Table 5 show estimated values, observed values and deviations of estimated values of main output variables of the model from corresponding observed values. Here, the period 2000-2007corresponds to the period of parametric identification of the model; 2008 is the retroprognosis period; Y is gross output (in prices of 2000); Y_g is GDP (in prices of 2000); P is consumer price index in percentages relative to the previous year; sign $\ll * \gg$ corresponds to observed values, sign $\ll\Delta\gg$ corresponds to deviations (in percentages)of esti-

Table 5 Observed, calculated values of output variables of the model and corresponding deviations

Indicator	Year								
	2000	2001	2002	2003	2004	2005	2006	2007	2008
$Y^*(t)$	5.44	6.32	6.47	6.86	7.72	8.52	9.25	9.69	9.84
$Y(t)$	5.38	6.32	6.47	6.86	7.72	8.52	9.27	9.64	9.82
$\Delta Y(t)$	-1.22	-0.02	0.00	0.00	0.05	0.08	0.21	-0.51	-0.26
$Y_g^*(t)$	2.45	2.78	3.05	3.36	3.72	4.09	4.55	5.01	5.18
$Y_g(t)$	2.47	2.78	3.05	3.35	3.72	4.09	4.55	5.01	5.20
$\Delta Y_g(t)$	0.88	0.07	-0.04	-0.02	-0.02	-0.02	-0.04	-0.15	0.38
$P^*(t)$		106.4	106.6	106.8	106.7	107.5	108.4	118.8	109.5
$P(t)$		107.6	106.8	106.9	106.7	107.3	108.2	118.6	109.4
$\Delta P(t)$		1.13	0.18	0.08	-0.05	-0.23	-0.22	-0.24	-0.05

mated values from respective observed values.

Analysis of sources for economic growth based on computable general equilibrium model of economic sectors

Now, let us analyze sources for economic growth of economic sectors based on CGE model of economic sectors and based on retrospective data for 2000-2009. For this, using expressions of production functions of economic agents of the model, we estimate the effect of change of arguments of these functions on the rates of growth of GAV of sectors $VY\_i[t+1]$ in assumption about constancy of coefficients $CA\_z\_ji[t]$ under consumed by sector intermediate products $VD\_pj\_iz[t]$, coefficients $CA\_k\_i[t]$ under capital $(VK\_i[t]+VK\_i[t+1])/2$ and coefficients $CA\_l\_i[t]$ under labor $VD\_pi\_il[t]$. Here $i,j=1,...,16$ are numbers of economic agents; t is number of year; Power (X,Y) corresponds to X^Y, Exp(X) corresponds to e^X; CA_r_i is coefficient, characterizing technical progress in i-th sector.

$$
\begin{aligned}
&VY\_i[t+1] = CA\_r\_i \times Exp(VD\_p1\_iz[t] \times CA\_z\_1i) \times Exp(VD\_p2\_iz[t]\\
&\times CA\_z\_2i]) \times Exp(VD\_p3\_iz[t] \times CA\_z\_3i) \times Exp(VD\_p4\_iz[t] \times CA\_z\_4i)\\
&\times Exp(VD\_p5\_iz[t] \times CA\_z\_5i) \times Exp(VD\_p6\_iz[t] \times CA\_z\_6i)\\
&\times Exp(VD\_p7\_iz[t] \times CA\_z\_7i) \times Exp(VD\_p8\_iz[t] \times CA\_z\_8i)\\
&\times Exp(VD\_p9\_iz[t] \times CA\_z\_9i) \times Exp(VD\_p10\_iz[t] \times CA\_z\_10i)\\
&\times Exp(VD\_p11\_iz[t] \times CA\_z\_11i) \times Exp(VD\_p12\_iz[t] \times CA\_z\_12i)\\
&\times Exp(VD\_p13\_iz[t] \times CA\_z\_13i) \times Exp(VD\_p14\_iz[t] \times CA\_z\_14i)\\
&\times Exp(VD\_p15\_iz[t] \times CA\_z\_15i) \times Exp(VD\_p16\_iz[t] \times CA\_z\_16i)\\
&\times Power(((VK\_i[t]+VK\_i[t+1])/2), CA\_k\_i)\times\\
&\times Power(VD\_pi\_il[t], CA\_l\_i)
\end{aligned}
\tag{31}
$$

After finding the logarithms of both parts (31) and total increment of the function ln (VY_i) and dropping members of the highest order infinitesimal, we get the following estimate of the growth rate y_i of real GAV of i-th sector depending on growth of arguments of production function : CA_r_i, VD_pj_iz, $Ki_m = (VK\_i[t] + VK\_i[t+1])/2$, and $VD\_pi\_il[t]$.

$$\begin{aligned} y_i = \frac{\Delta VY\_i}{VY\_i} = \frac{\Delta CA\_r\_i}{CA\_r\_i} + \sum_{j=1}^{16}(CA\_z\_ji \times VD\_pj\_iz)\frac{\Delta VD\_pj\_iz}{VD\_pj\_iz} \\ + CA\_k\_i\frac{\Delta Ki_m}{Ki_m} + CA\_l\_i\frac{\Delta VD\_pi\_il}{VD\_pi\_il} \end{aligned} \tag{32}$$

Denote by $a_i = \frac{\Delta CA\_r\_i}{CA\_r\_i}$ the rate of technical progress in i-th sector ; $z_{ij} = \frac{\Delta VD\_pj\_iz}{VD\_pj\_iz}$ is the rate of intermediate products consumed by i-th sector, and produced by j-th sector ; $k_i = \frac{\Delta Ki_m}{Ki_m}$ is the rate of capital accumulation in i-th sector ; $l_i = \frac{\Delta VD\_pi\_il}{VD\_pi\_il}$ is the growth rate of labor inputs in $i-th$ sector, where the sign "Δ" means change of variable ; time values in (32) were dropped for short.

Coefficients on the right-hand-side of formula (32) at stated above rates and characterize degree of effect of the factors in question on economic growth and allow comparing their effect with effect of technical progress, coefficient at which is equal to 1. By denoting these coefficients by $a_{ij} = CA\_z\_ji \times VD\_pj\_iz$, $\beta_i = CA\_k\_i$, $\gamma_i = CA\_l\_i$, from (32) we get its reduced writing :

$$y_i = a_i + \sum_{j=1}^{16} \alpha_{ij} z_{ij} + \beta_i k_i + \gamma_i l_i \tag{33}$$

Let us present the values of coefficients, defining contributions of sources of economic growth of sectors based on the model in question for 2008. (see Table 6). Coefficients in the Table show how much the growth rate of GAV would increase given the one percent increase of growth factors (fixed assets, labor or demand for intermediate goods of economic agents).

Analysis of coefficients table β_i, γ_i, $\alpha_i^s = \sum_{j=1}^{16} \alpha_{ij}$ of the Table 4 shows that if we exclude the rate of technical progress, effect of which on the rate of growth of all sectors in given model is the same, then from rest three factor rates of economic growth, the largest effect on the rate of real output of sectors 1, 5, 7, 12, 13, 16 of the economy has the rate of labor inputs ; for sectors 4, 6, 8, 10, 15-the rate of capital accumulation ; and for other sectors 2, 3, 9, 11, 14 -the rate of consumed by the sector intermediate products, produced by all of the sectors.

Also note that for sectors 5, 7 12, 16 the rates of capital accumulation practically do not have effect on corresponding rate of output growth ; the rates of labor inputs haven on-zero effect on the rates of output growth of all sectors ; for sectors

6, 16 the rates of consumed intermediate good shave no effect on corresponding rate of output growth.

The results of analysis allow choosing the following budget parts of 16 economic sectors in the capacity of tools for solving the problems of economic growth.

O_{ij}-budget part of i-th sector, which is for payment for goods and services, bought from j-th sector ;

O_i^l-budget part of i-th sector, which is for payment for labor ;

O_i^n-budget part of i-th sector, which is for payment for investment goods.

Table 6 Coefficients, characterizing an impact of economic growth factors

Number of sector i	β_i	γ_i	α_{i1}	α_{i2}	α_{i3}	α_{i4}	α_{i5}	α_{i6}	α_{i7}
1	0.3089	0.9051	$1.345\cdot 10^{-18}$	$9.480\cdot 10^{-02}$	$2.897\cdot 10^{-14}$	$2.171\cdot 10^{-13}$	$1.602\cdot 10^{-14}$	$2.028\cdot 10^{-14}$	$1.345\cdot 10^{-12}$
2	0.2426	2.4964	$1.590\cdot 10^{-16}$	$7.308\cdot 10^{-01}$	$7.087\cdot 10^{-16}$	$9.884\cdot 10^{-15}$	$1.390\cdot 10^{-15}$	$1.120\cdot 10^{-15}$	$1.590\cdot 10^{-16}$
3	0.9650	0.6886	$2.886\cdot 10^{-15}$	0.000	$2.970\cdot 10^{-12}$	$8.269\cdot 10^{-13}$	$9.894\cdot 10^{-14}$	1.478	$2.886\cdot 10^{-15}$
4	1.2900	0.0805	$2.227\cdot 10^{-13}$	$1.343\cdot 10^{-1}$	$8.634\cdot 10^{-13}$	$9.989\cdot 10^{-13}$	$9.029\cdot 10^{-03}$	$2.641\cdot 10^{-16}$	$2.227\cdot 10^{-13}$
5	$1.0\cdot 10^{-10}$	2.4083	$3.199\cdot 10^{-17}$	$5.537\cdot 10^{-17}$	$8.913\cdot 10^{-14}$	$6.720\cdot 10^{-4}$	$5.406\cdot 10^{-3}$	$5.300\cdot 10^{-4}$	$3.199\cdot 10^{-17}$
6	0.9343	0.7721	$2.078\cdot 10^{-16}$	$9.186\cdot 10^{-18}$	$3.313\cdot 10^{-14}$	$3.940\cdot 10^{-13}$	$2.467\cdot 10^{-15}$	$1.324\cdot 10^{-14}$	$2.078\cdot 10^{-16}$
7	$1.0\cdot 10^{-10}$	1.8792	$8.133\cdot 10^{-16}$	$1.061\cdot 10^{-15}$	$3.115\cdot 10^{-14}$	$2.660\cdot 10^{-13}$	$4.124\cdot 10^{-14}$	$2.508\cdot 10^{-13}$	$8.133\cdot 10^{-16}$
8	1.0691	0.4706	$2.003\cdot 10^{-02}$	$1.498\cdot 10^{-15}$	$4.343\cdot 10^{-17}$	$6.171\cdot 10^{-14}$	$3.375\cdot 10^{-15}$	$7.288\cdot 10^{-15}$	$2.003\cdot 10^{-02}$
9	0.8660	0.2153	$4.289\cdot 10^{-16}$	$2.265\cdot$	$1.666\cdot 10^{-13}$	$8.753\cdot 10^{-15}$	$3.057\cdot 10^{-14}$	$4.005\cdot 10^{-14}$	$4.289\cdot 10^{-16}$
10	0.6702	0.5492	0.000	0.000	0.000	$4.397\cdot 10^{-15}$	$5.602\cdot 10^{-15}$	$8.811\cdot 10^{-15}$	0.000
11	1.2022	0.1006	$2.929\cdot 10^{-05}$	2.267	$5.064\cdot 10^{-14}$	$1.030\cdot 10^{-12}$	$1.664\cdot 10^{-04}$	$5.308\cdot 10^{-13}$	$2.929\cdot 10^{-05}$
12	$1.0\cdot 10^{-10}$	2.5822	$5.415\cdot 10^{-14}$	0.000	0.000	$4.255\cdot 10^{-13}$	$2.573\cdot 10^{-14}$	0.000	$5.415\cdot 10^{-14}$
13	0.2635	1.7177	$2.370\cdot 10^{-14}$	0.000	0.000	$8.847\cdot 10^{-13}$	$1.142\cdot 10^{-13}$	$5.739\cdot 10^{-15}$	$2.370\cdot 10^{-14}$
14	0.0227	1.7814	$6.736\cdot 10^{-14}$	$2.018\cdot 10^{-1}$	$6.280\cdot 10^{-15}$	$1.288\cdot 10^{-12}$	$1.919\cdot 10^{-13}$	$4.809\cdot 10^{-13}$	$6.736\cdot 10-14$
15	0.9304	0.2173	$1.041\cdot 10^{-15}$	$1.408\cdot 10^{-3}$	$7.681\cdot 10^{-15}$	$3.712\cdot 10^{-13}$	$3.926\cdot 10^{-14}$	$1.579\cdot 10^{-13}$	$1.041\cdot 10^{-15}$
16	0	1.9372	0	0	0	0	0	0	0

Continuation of Table 6.

Number of sector i	α_{i8}	α_{i9}	α_{i10}	α_{i11}	α_{i12}	α_{i13}	α_{i14}	α_{i15}	α_{i16}
1	$4.325 \cdot 10^{-15}$	$4.595 \cdot 10^{-2}$	$1.807 \cdot 10^{-14}$	$1.177 \cdot 10^{-14}$	0	$1.681 \cdot 10^{-16}$	$1.178 \cdot 10^{-15}$	$9.304 \cdot 10^{-18}$	0
2	$8.054 \cdot 10^{-17}$	$1.087 \cdot 10^{-14}$	$2.616 \cdot 10^{-17}$	2.280	0	0·	0	0	0
3	$8.054\ 10^{-14}$	$2.221 \cdot 10^{-2}$	$1.436 \cdot 10^{-13}$	$4.626 \cdot 10^{-12}$	0	$1.581 \cdot 10^{-15}$	$4.991 \cdot 10^{-4}$	$1.571 \cdot 10^{-14}$	0
4	$8.038 \cdot 10^{-15}$	$1.444 \cdot 10^{-13}$	$4.353 \cdot 10^{-14}$	$6.338 \cdot 10^{-16}$	0	$1.247 \cdot 10^{-16}$	$3.379 \cdot 10^{-03}$	$5.445 \cdot 10^{-16}$	0
5	$1.642 \cdot 10^{-15}$	$2.760 \cdot 10^{-1}$	$6.132 \cdot 10^{-3}$	$2.512 \cdot 10^{-14}$	0	$3.273 \cdot 10^{-5}$	$2.026 \cdot 10^{-17}$	$5.494 \cdot 10^{-4}$	0
6	$7.977 \cdot 10^{-15}$	$1.117 \cdot 10^{-2}$	$6.525 \cdot 10^{-16}$	$2.014 \cdot 10^{-14}$	0	$8.436 \cdot 10^{-17}$	$1.554 \cdot 10^{-17}$	$1.089 \cdot 10^{-16}$	0
7	$6.067 \cdot 10^{-14}$	$4.548 \cdot 10^{-1}$	$3.407 \cdot 10^{-13}$	$1.505 \cdot 10^{-12}$	0	$5.394 \cdot 10^{-16}$	$8.576 \cdot 10^{-5}$	$4.404 \cdot 10^{-16}$	0
8	$1.528 \cdot 10^{-15}$	$8.267 \cdot 10^{-15}$	$3.053 \cdot 10^{-14}$	$1.507 \cdot 10^{-14}$	0	$2.339 \cdot 10^{-17}$	$1.338 \cdot 10^{-17}$	$2.244 \cdot 10^{-16}$	0
9	$3.346 \cdot 10^{-14}$	$1.002 \cdot 10^{-12}$	$3.203 \cdot 10^{-13}$	$4.339 \cdot 10^{-13}$	0	$3.657 \cdot 10^{-15}$	$4.803 \cdot 10^{-16}$	$1.091 \cdot 10^{-15}$	0
10	$1.991 \cdot 10^{-14}$	$3.270 \cdot 10^{-01}$	$2.862 \cdot 10^{-13}$	$5.408 \cdot 10^{-15}$	0	$1.347 \cdot 10^{-14}$	0	$8.805 \cdot 10^{-18}$	0
11	$6.664 \cdot 10^{-14}$	$6.596 \cdot 10^{-13}$	$2.965 \cdot 10^{-13}$	$2.646 \cdot 10^{-12}$	0	$9.260 \cdot 10^{-14}$	$1.661 \cdot 10^{-03}$	$9.599 \cdot 10^{-06}$	0
12	$4.185 \cdot 10^{-1}$	$1.548 \cdot 10^{-1}$	$9.506 \cdot 10^{-13}$	$7.964 \cdot 10^{-13}$	0	$2.126 \cdot 10^{-14}$	$1.408 \cdot 10^{-15}$	$2.886 \cdot 10^{-15}$	0 0
13	$4.820 \cdot 10^{-14}$	$5.807 \cdot 10^{-1}$	$1.794 \cdot 10^{-14}$	$1.331 \cdot 10^{-14}$	0	$3.939 \cdot 10^{-15}$	$7.738 \cdot 10^{-17}$	$1.836 \cdot 10^{-14}$	0
14	1.891	$2.118 \cdot 10^{-13}$	$4.749 \cdot 10^{-17}$	$2.743 \cdot 10^{-13}$	0	$2.219 \cdot 10^{-16}$	$1.782 \cdot 10^{-14}$	$6.646 \cdot 10^{-14}$	0
15	$6.285 \cdot 10^{-14}$	$2.139 \cdot 10^{-1}$	$9.739 \cdot 10^{-14}$	$8.436 \cdot 10^{-14}$	0	$9.538 \cdot 10^{-17}$	$1.416 \cdot 10^{-17}$	$1.222 \cdot 10^{-13}$	0
16	0	0	0	0	0	0	0	0	0

Finding the optimal parametric control laws on the basis of CGE model of economic sectors

In computational experiments with CGE model of economic sectors,the criterion below was used as maximization criterion

$$K = \frac{1}{6} \sum_{t=2010}^{2015} VY[t] \tag{34}$$

Here K is average value of country's gross output for 2010-2015 in prices of 2000.

In experiments with optimization criterion (34), the constraints on growth of consumer price index of the following form were used :

$$VPr[t] \leq 1.0$$

Here is calculated consumer price index of the model without parametric control, is consumer price index with parametric control.

In computational experiments control was performed for 1536 exogenous parameters - j-th agent-producer's budget parts for purchasing goods and services, produced by i-th agent-producer for 2000-2015 : $O_i^j[t]$; t=2010,...,2015 ; i,j=1,...,16. Here $\sum_{i=1}^{16} O_i^j(t) \leq 1$ for mentioned values of t. Basic values of mentioned parts, obtained by solving the parametric identification problem of the model on data of 2000-2008, we will denote by $\bar{O}_i^j$; i,j=1,...,16.

The following problem of finding optimal values of adjustable parameters vectors was considered. On the basis of CGE model of economic sectors to find mentioned values of budget parts of agents-producers $O_i^j[t]$, which provide the maximum of criterion K under additional constraints on these parts of the following form :

$$0.5 \leq O_i^j[t]/\bar{O}_i^j \leq 2; i, j = 1, ..., 16; t = 2010, ..., 2015.$$

Solutions of these optimization problems were made using the Nedler-Mead algorithm. After using parametric control of budget parts of the model, criterion value turned out $K = 1.6283 \cdot 10^{13}$, criterion value increased by 33.14% in relation to the basic variant.

References

[1] Tarasevich L.S., GrebennikovP.I., Leusskiy A.I. (2006), "Macroeconomics", *Moscow: Vysshee obrazovanie*, in Russian.

[2] Ashimov A.A., Sultanov B.T., Adilov Zh.M., Borovskiy Yu.V., Novikov D.A., Alshanov R.A. Ashimov As.A. (2013), " Macroeconomic analysis and parametrical control of a national economy", *New York: Springer.*

[3] E.S. Said, D.A. Dickey. (1984), "Testing for unit roots in autoregressive-moving average models of unknown order", *Biometrika*, Vol.71, No.3, pp.599-607.

[4] R.C. Fair. (2004), "Estimating how the macroeconomy works", *Cambridge, Mass.: Harvard University Press.*

[5] R.F. Engle, C.W.J. Granger. (1987) "Co-integration and error correction: Representation, estimation, and testing", *Econometric*, Vol.55, No.2, pp.251-276.

[6] Turnovsky S.J. (2000), "Methods of macroeconomic dynamic", *Cambridge: MIT Press.*

[7] Kolemaev V.A.Matematnqeskaya. (2002), "konomnka", *Moskva: Yuniti.*

[8] Kydland F.E., Prescott E.C. (1977), *Rules rather than Discretion: the Inconsistency of Optimal Planes*, Springer, *Journal of Political Economy*, Vol.87.

[9] Dornbusch R., Fischer S. (1990), "Macroeconomics", *New York: McGraw-Hill.*

[10] Tumanova E.S., Shagas N.A. (2004), "Macroeconomics", *Moscow: Infra-M.*

[11] Dubovskiy S.V. (1995), "The Kondratiev cycle as the simulation object", *Mathematical modelling*, Vol.7, No.6, pp.65-74, Russian.

[12] K.S. Abdiev. (2001-2009), "Statistical yearbook of Kazakhstan", Astana: Agency on Statistics of the Republic of Kazakhstan.

[13] Smets F., Wouters R (2003), "An estimated dynamic stochastic general equilibrium model of the Euro Area", *Journal of the European Economic Association*, Vol.1, No.5, pp.1123-1175.

[14] Sims C. (2002), "Random lagrange multipliers and transversality", ECO 504, Spring, pp.1-7.

[15] Orphanides A. (2007) , *Taylor Rules, Federal Eeserve Board, Working Paper*, No.18, Washington, D.C..

[16] Fernández-de-Córdoba G., Torres J.L. (2009), "Forecasting the Spanish economy with an Augmented VAR-DSGE model", *Malaga Economic Theory Research Center Working Papers*, No.1.

[17] Blanchard O.J., Kahn C.M. (1980), "The Solution of Linear Difference Models under Rational Expectation, Econometrica", *Comput. Math. Math. Phys*, Vol.24, pp.1812.

[18] Fernández-Villaverde J. (2010), "The econometrics of DSGE models, SERIEs", Vol.48, No.5, pp.1305-1312.

[19] Hamilton J.D. (1994), "Time series analysis", *New Jersey: Princeton.*

[20] Makarov V.L., Bahtizin A.R., and Sulakshin S.S. (2007), "Application of computable models in state administration", *Moscow: Nauchniy expert.*

Corresponding author

A. Ashimov can be contacted at: ashimov37@mail.ru