Dynamic Stackelberg Games with Requirements to the Controlled System as a Model of Sustainable Environmental Management

Sergey A. Kornienko and Guennady A. Ougolnitsky
Southern Federal University, Russia

Abstract
A definition of sustainable environmental management based on the hierarchical game theoretic formalization is proposed. The definition includes requirements both to the state of the considered environmental system and to the control impact on it. A classification of leaders in the model context is given, a model example is described, and a qualitative assessment of the proposed game theoretic models is fulfilled.
Keywords dynamic Stackelberg games, sustainable environmental management

1 Introduction

The term "sustainable development" was introduced by the International Commission for Environment and Development in 1987. It determines a development that "meets the needs of the present without compromising the ability of future generations to meet their own needs" [1]. An apparently simple intergenerational rule is that development is sustainable "if it does not decrease the capacity to provide non-declining/capita utility for infinity" [2]. The weak sustainability rule requires that total net capital investment, or the rate of change of total net capital wealth, not allowed to be persistently negative. This definition entails the assumption that natural capital is similar to produced capital and can be substituted for it. Proponents of the strong sustainability concept argue that natural capital is to a greater or lesser extent non-substitutable [3].

The concept of sustainable development (sustainability) is very vague and fuzzy. Pezzey listed 60 published definitions of sustainable development[4]. A detailed analysis of the modern state of the art is made by Zaccai [5]. Glavic and Lukman have proposed a hierarchical classification of concepts and terms concerned with sustainability[6]. Some researchers argue for a special science about sustainable development (sustainability science)[7-9]. Necessity of sustainability economics as a complement to ecological economics is discussed by Baumgartner and Quaas [10]. There are many principles of sustainability assessment and measurement [11]. An example of the assessment is given in Literature[12]. Nooteboom puts the problem of environmental assessment procedures in the context of complexity theory[13].

Some authors tried to give the concept of sustainability a formal nature, either symbolic [14], discursive [15], or reflexive [16]. The axiomatic foundation of sustainable development based on the concept of sustainable preferences has been launched by Chichilnisky [17]. Asheim and Mitra have introduced sustainable discounted utilitarianism, allowing to resolve intergenerational conflicts while satisfying the two main Chichilnisky axioms[18]. D'Albis and Ambec addressed the question of fair intergenerational sharing of scarce natural resources[19]. A very interesting approach to the mathematical formalization of ecosystem sustainability based on optimal control theory is described in Literature[20-28]. In these papers, Fisher information as a sustainability measure for dynamic systems is proposed, and the sustainability hypotheses with particular focus on the natural ecosystems are formulated.

It was also marked that strong efforts on different levels are necessary to provide sustainability. The sustainable development of an environmental system could be defined as: 1) an integral and balanced development of all aspects of the system functioning; 2) concordance of interests of all agents associated with the system; 3) a trade-off between short-term and long-term criteria of the system efficiency. It is very important to notice that to ensure sustainability it is necessary to provide some requirements not only to the state of the environmental system, but also to the control impact on it. As far any environmental system is impacted by many associated agents, the environmental management is a conflict process that should be described by game theoretic models.

In many practical situations a traditional model of controlled dynamical system is not sufficient. For example, let the control subject be an industrial enterprise situated on the bank of a river which is the control object. The enterprise objective is to maximize its profit without consideration of the river water pollution by industrial sewage. So, in many cases the actions of control subject determined by his/her private interests and objectives are able to bring the control object in a state which is not acceptable from the point of view of sustainability. This suggests that an additional (higher) control level should be introduced to provide sustainability requirements to the state of control object. This new control subject is able to have his/her own interests too. To achieve his/her objective the new control subject can exert an impact to the initial control subject. For example, an environmental protection agency can control water quality by establishing sewage limits and charging penalties if an enterprise exceeds them. In this case a new subject of the hierarchical control arises which is a complexly structured system with its internal links and relations. Dynamic Stackelberg games [29] is a relevant model in this case.

From the other side, sustainable development requires strong collaborative efforts of states, corporations, social organizations and individuals. From this point of view, a cooperative game theoretic formalization is required and the main problem is time consistency of the trade-off solutions. Literature[30-35] presented classes of transferable-payoff cooperative games with solutions which satisfy group optimality and individual rationality. In literature[36-43] are presented solutions satisfying group optimality and individual rationality at the initial time in cooperative games with nontransferable payoffs. In literature[38-39] threats are used to ensure that no players will deviate from the initial cooperative strategies throughout the game horizon. The problem of time consistency in differential games has been intensively explored in the past decades [44]. Haurie [42] raised the problem of instability when the Nash bargaining solution is extended to differential games. Petrosyan [45] formalized the notion of time consistency in differential games. Kidland and Prescott [46] introduced the notion of time consistency related to economic problems. Petrosyan [47-48]and Petrosyan and Zenkevich [49-50] presented a detailed analysis of time consistency in cooperative differential games, in which a method of regularization was used to construct time consistent solutions. Yeung and Petrosyan [51] designed time consistent solutions in differential games and characterized the conditions that the allocation-distribution procedure must satisfy. Petrosyan and Zenkevich [52] proposed the conditions of sustainable cooperation.

Environmental game theoretic applications are considered in literature [32-35,39,44,53-63] and others.

In the author's publications [64-67] an approach to the mathematical formalization of sustainable development and sustainable management is developed. Our contribution in this paper is to describe a model of sustainable environmental management that includes requirements both to the state of the considered environmental system and to the hierarchical control impact on it. In the Section 2 of the paper a mathematical formalization of sustainable environmental management is proposed. In the Section 3 dynamic compulsion and impulsion Stackelberg games with requirements to the controlled system are introduced. In the Section 4 a classification of leaders in the games is given. Section 5 is dedicated to an illustrative model example. In Section 6 the principles of assessment of decision making models of sustainability [68] are used for the proposed model. Section 7 concludes.

2 Sustainable environmental management: mathematical formalization

Thus, to define sustainable environmental management it is necessary to formulate certain requirements both to the state of the environmental system and to the control impact on it. To some extent of conventionality the first group of requirements is called "homeostasis", and the second group "compromise" [65].

Requirements of homeostasis. Highly organized systems of the real world should resist to the external impacts or accommodate to them providing a conservation of the conditions of their existence and goal-oriented development. As a French physiologist Claude Bernard has said: "The constancy of the internal milieu is a condition of the free life of organism" (1878). The term "homeostasis" was introduced in 1932 by Walter Cannon who treated it as a relative dynamic constancy of the whole organism [69].

There is no escape from the conclusion about a similarity between the concepts of homeostasis as a dynamic constancy of the organism and sustainable development as a combination of the economic development (dynamics) and the environmental stability (constancy) on the biosphere level. We think that the concept of homeostasis could be considered not only on the level of a separate organism but also on the levels of populations, ecosystems, environmental-economic systems, and arbitrary dynamic systems including human beings.

Let's name the homeostasis of a system the domain of values of the essential parameters of the system in which its normal existence and development are possible. Functioning of any dynamic system is characterized by a set of parameters the values of which change in the time. The condition of homeostasis means that all parameters of the system functioning during a considered period of time (long enough or even infinite) take their values from a given range, and in the specific case take given point values. For example, the point requirements to the physiological parameters of a human organism are well known: temperature 36.6 degrees centigrade, blood pressure 120 on 80 millimeters of mercury column, and so on. In the same time certain deviations from the standard values are allowable: they form the admissible ranges of functioning of a healthy organism. The point and interval requirements of homeostasis of any dynamic system including human beings can be given similarly.

In the mathematical modeling a set of the essential parameters of functioning of a dynamic system is called its vector of state (phase vector) and is denoted $x(t) = (x_1(t), ..., x_n(t))$. Its components (state variables) $x_i(t)(i = 1, ..., n)$ are the values of parameters which characterize the system state in the moment of time t from the point of view and with the degree of detail which are determined by the objectives and resources of the research. For example, a population can be characterized by one parameter (its number or biomass) or, more precisely, by dozens of parameters considering its sex, age, genetic, and other structuring.

Then a requirement (condition) of homeostasis can be formulated in two forms: weak and strong. In the weak form the condition can be written as (Lagrange stability)

$$\forall t \in [0, T] : x(t) \in X^* \tag{1}$$

where X^* is the domain of homeostasis, T - the length of the considered time period. For example, $\forall t \in [0, T] : x(t) > 0$ (population does not extinct), or $\forall t \in [0, T] : x(t) > x_{cr}$ (number of population is not less than a dangerous threshold), or $\forall t \in [0, T] : x(t) \leq \bar{x}$ (concentration of a pollutant does not exceed the maximal allowable one).

Often it is possible to define the domain of homeostasis as $X^* = [x^* - \varepsilon, x^* + \varepsilon]$, where x_i^* is an "ideal" value of the i-th state variable, $\pm\varepsilon_i$ is an allowable deviation from the value $(i = 1, ..., n)$. In this case the condition (1) takes the form

$$\forall t \in [0, T] : x(t) \in [x^* - \varepsilon, x^* + \varepsilon] \tag{2}$$

and becomes in fact the condition of (neutral) Lyapunov stability of a stationary point x^* of the controlled environmental system.

The strong form of requirement of homeostasis means satisfaction of (2) together with the additional condition

$$\lim_{t \to \infty} x(t) = x^* \tag{3}$$

i.e. asymptotic Lyapunov stability of the stationary point x^* of the controlled environmental system [70]. Let's notice that it is possible to require not only stability of a stationary point, but also of another trajectory of a controlled dynamic system (periodical oscillations, linear or exponential growth, and so on). The choice of the specific form of the requirement of homeostasis (Lagrange stability, neutral or asymptotic Lyapunov stability of a stationary point or another trajectory) is determined by a specific environmental sustainable management problem under consideration.

Requirements of compromise. It is extremely important to notice again that the requirements of homeostasis are necessary but not sufficient to provide sustainable environmental management. Many agents are associated with any environmental system. From one side, their objectives and interests are determined to an extent by the state and dynamics of the system. From the other side, the agents impact the system and exert some effect on its functioning. Thus the sustainable development is possible only if a condition of the consideration and coordination of interests of the associated agents is satisfied. As the objectives and interests of agents do not coincide in the majority of cases then their interaction is conflict. But in the same time the objectives and interests are not antagonistic, therefore a compromise is possible. In the light of the requirements of sustainable development the compromise should consider the condition of homeostasis. It is this compromise between all agents associated with the system that forms another condition of its sustainable development. In the mathematical formalization of a conflict interaction by game theoretic models compromises are described by optimality principles for different classes of games. Thus from the mathematical point of view the condition of compromise means an existence of solution of the game describing a conflict interaction of the agents associated with the system. The condition of homeostasis reflects the requirement to the state of the system meanwhile the condition of compromise formalizes the demands to the impact on it. If a compromise is not achieved then the system will be permanently threatened by not appropriate impacts violating the homeostasis. A simple question should be answered: who will ensure the condition of homeostasis and why does (s)he need it?

Note that any solution of a game (optimality principle) has some properties that provide a stability of the compromise. So, Nash equilibrium is stable towards individual deviations, i.e. no player has incentives to violate an initial agreement. In Stackelberg equilibrium the follower chooses an optimal answer to the leaders strategy which in turn is chosen such as to provide the leader the maximal payoff on the set of the optimal answers. In the case of a cooperative solution the key property is Pareto optimality due to which a players payoff can be increased only at the cost of other players.

Another key problem of the sustainable management is a possible nonconformity of the short-term and long-term criteria of optimality. The condition of homeostasis is a long-term one because it should be satisfied along the whole period of existence and functioning of the environmental system. In the same time the agents associated with the system are often guided by short-term criteria with much smaller character times. As a result the compromise is under the threat of violation by those participants of the initial agreement for which it could be more advantageous in the current moment of time to take another strategy corresponding to their short-term interests.

In this connection a realization of the requirements of sustainable management is possible only for those compromises which keep their optimality for all associated agents along the whole period of existence of the environmental system. In the game theoretic formalization this principle was called a time consistency [51-56, 70]. The property of time consistency of the solution of a game means that a truncation of the solution is still optimal in all subgames arising along the optimal trajectory of the system development. This property provides a practical realization of the compromise solution from which it is not advantageous for any agent to deviate all along the time of system functioning.

Thus, the weak form of the requirement of compromise is existence of a solution of a game theoretic model of conflict interaction of the agents associated with the controlled environmental system. In the case of a hierarchically controlled environmental system that is under consideration in this paper the Stackelberg solution is used. The strong form of the requirement of compromise means additionally the time consistency of the solution.

It could be stated that separately the characterized requirements of homeostasis and compromise are necessary, and in their totality also sufficient conditions of the sustainable development of any environmental system. The condition of homeostasis expresses basic requirements to all aspects of the system functioning, the condition of compromise provides the adequacy of impacts by all agents associated with the system with acceptable consideration of their interests, including a coordination of short-term and long-term optimality criteria of the agents and the consequent non-advantageousness for them to deviate from the initially agreed compromise solution all along the time. If the homeostasis is provided and a dynamic consistent compromise between all the associated agents is achieved then a sustainable environmental management of the system takes place. The sustainable environmental management ensures both conditions of sustainable development and means of their realization.

The conditions of sustainable environmental management are characterized in Table 1.

Table 1 Conditions of sustainable environmental management.

	Homeostasis	Compromise
Weak form	Lagrange (1) or neutral Lyapunov stability (2) of a phase trajectory (for example, stationary point) of the controlled environmental system	Existence of a solution of the game theoretic model which has some strategic stability (for example, Nash of Stackelberg solution)
Strong form	Asymptotic Lyapunov stability (2)-(3) of a phase trajectory (for example, stationary point) of the controlled environmental system	Time consistency of the solution

3 Dynamic Stackelberg games with requirements to the controlled system.

In many practically important cases the stakeholders of an environmental system are organized hierarchically. Then three methods of hierarchical control [65] can be used (Table 2). Their game theoretic formalization is proposed in [64].

Consider the example from Introduction in more details. The higher level control subject (Leader) is an environmental protection agency, the lower level control subject (Follower) is an industrial enterprise, and the control object is river ecosystem. It is natural to treat the desirable strategy for Leader as such a strategy in which the industrial sewage doesnt exceed the maximum allowable concentration of pollutants in the river. In the case of compulsion the objective is reached by establishing some sewage limits and license recall from the enterprise if the limits are violated. In the case of impulsion if the enterprise exceeds the maximum allowable concentration of pollutants in the river then a penalty is charged. At last, in the case of conviction the enterprise administration is environmentally

Table 2 Characteristic of the methods of hierarchical control.

	Compulsion	Impulsion	Conviction
General description of the method	Leader provides choosing the desirable strategy of Follower by force	The strategy desirable for Leader is more profitable for Follower than undesirable ones	Follower chooses the strategy desirable for Leader voluntarily and consciously
Nature of impact	Administrative or legislative	Economic	Social-psychological
Type of relationships	Subject-to-object	Subject-to-object with partial consideration of Follower's interests	Subject-to-subject
Mathematical formalization	Leader's impact on the Follower's set of admissible strategies	Leader's impact on the Follower's payoff function	Transition of Leader and Follower to cooperation and maximization of the summarized coalitional payoff function

conscious and it provides the required sewage refinement voluntarily.

To find the strategies of sustainable environmental management it is necessary to build game theoretic models with requirements to the state of controlled system. We will consider dynamic Stackelberg games which formalize hierarchical relations between the stakeholders of an environmental system based on compulsion or impulsion. A possible transition to conviction (cooperation) is also considered.

Dynamic compulsion Stackelberg game with requirements to the controlled system. The game can be written as follows:

$$J_L = \int_0^T e^{-\alpha t}[g_{LI}(u(t), x(t)) - g_{LC}(q(t)) - M\rho(x(t), X^*)]dt \to \max \qquad (4)$$

$$q(t) \in Q \qquad (5)$$

$$J_F = \int_0^T e^{-\alpha t} g_F(u(t), x(t))dt \to \max \qquad (6)$$

$$u(t) \in U(q(t)) \qquad (7)$$

$$\dot{x} = f(x(t), u(t)), \quad x(0) = x_0 \qquad (8)$$

where J_L, J_F – payoff functionals of the leader and the follower respectively; T – period of consideration; α – discount factor; $x(t)$ – state vector of the hierarchically controlled environmental system; $u(t)$ – vector of controls of the follower (impact on the controlled environmental system); $q(t)$ – vector of compulsive controls of the leader; g_{LI} – function of the leader's personal interest (for example, income); g_{LC} – function of control cost of the leader ($g_{LC}(0) = 0$); M – penalty constant; $\rho(x(t), X^*) = \begin{cases} 0, \ x(t) \in X^*, \\ 1, \ \text{otherwise}; \end{cases}$ – indicator function; Q, U – sets of admissible controls of the leader and the follower respectively; g_F – instant payoff function of the follower; f – known function of controlled dynamics of the environmental system; x_0 – given initial condition.

Compulsion in the game (4) – (8) consists in that the leader by controls $q(t)$ exerts an impact to the set of the follower's admissible controls $U(q(t))$ (without control dependence). Requirements to the state of the controlled environmental system (without loss of generality in the form of Lagrange stability) are described by means of an indicator function ρ and a penalty constant M in the leader's payoff functional. Therefore, if the requirements to the controlled system (treated as its homeostasis conditions) are violated then the leader is charged a penalty M.

A Stackelberg equilibrium [29] is accepted as solution of the dynamic game (4) – (8). Its specific feature is presence of the indicator function ρ in the leader's payoff functional subject to which the functional can become discontinuous when $M \to \infty$. Therefore a key role belongs here to the solution of a parametrical inverse optimal control problem, i.e. the problem of building of the set of "homeostatic" controls of the follower

$$U^* = \{u(t) \in U(q(t)) : \ x(t) \in X^*\} \tag{9}$$

If $\exists q(t) : \ U^* \neq \emptyset$ then solution of the game (4) – (8) is reduced to the solution of an ordinary dynamic Stackelberg game without requirements to the controlled system, otherwise the game (4) – (8) has no solution.

Time consistency of a solution of the game (4) – (8) depends on its information structure. It is known [29] that in the class of open-loop strategies a Stackelberg solution is not time consistent, i.e. in this case only the weak form of the requirement of compromise can be discussed. To provide time consistency (the strong form of compromise) closed-loop strategies should be considered.

Dynamic impulsion Stackelberg game with requirements to the controlled system. The game can be written as follows:

$$J_L = \int_0^T e^{-\alpha t}[g_{LI}(p(t), u(t), x(t)) - g_{LC}(p(t)) - M\rho(x(t), X^*)]dt \to \max \tag{10}$$

$$p(t) \in P^U \tag{11}$$

$$J_F = \int_0^T e^{-\alpha t} g_F(p(t), u(t), x(t))dt \to \max \tag{12}$$

$$u(t) \in U \tag{13}$$

subject to (8), where in comparison with (4) – (7) $p(t)$ – vector of impulsive controls of the leader.

Impulsion in the game (8), (10) – (13) consists in that the leader by controls $p(t)$ exerts an impact to the follower's payoff function. A control dependence (feedback) also takes place, namely $\tilde{p}(t) = p(u(t))$. Therefore the set of the follower's admissible strategies is a set of maps $P^U = \{p : U \to P\}$. Considerations about the Stackelberg solution are the same as in the case of compulsion.

Transition to conviction. Both in the case of compulsion and in the case of impulsion a transition to conviction is possible. This transition means cooperation between the leader and the follower for joint solution of the problem of sustainable environmental management. From the mathematical point of view, conviction means a coalition of the leader and the follower and their joint maximization of the summarized payoff functional, or solution of the optimal control problem

$$J_{L+F} = \int_0^T e^{-\alpha t}[g_{LI}(u(t), x(t)) - g_{LC}(q(t)) - M\rho(x(t), X^*) + g_F(u(t), x(t))]dt \to \max \tag{14}$$

$q(t) \in Q; \quad u(t) \in U(q(t))$ subject to (8).

Suppose that $U^* \neq \emptyset$ (otherwise the sustainable environmental management problem has no solution). Then it is evident that the follower chooses $u(t) \in U^*$. Thus $q(t) \equiv 0$ (compulsion is not required), respectively $g_{LC}(0) = \rho(x(t), X^*) = 0$, and the problem (14) takes the form

$$J_{L+F} = \int_0^T e^{-\alpha t}[g_{LI}(u(t), x(t)) + g_F(u(t), x(t))]dt \to \max, \quad u(t) \in U \tag{15}$$

subject to (8), i.e. only the follower maximizes the coalitional payoff functional. After solution of the optimal control problem (15) the received payoff is shared between the leader and the follower accordingly to a cooperative optimality principle [36-37]. In the case of impulsion a transition to conviction is similar.

4 Classification of leaders

In the proposed conceptual framework the leader is treated as a regulator who is responsible for the requirements to the controlled environmental system, or a subject of the sustainable environmental management. The following classification attributes can be used accordingly to the form of the payoff functional (4) or (10).

Importance of the requirements to the controlled environmental system. From the point of view of the leader, the requirements can be: - mandatory ($M \to \infty$): in this case the leader can't solve her control problem without ensuring the requirements because she is charged an arbitrarily big penalty otherwise; - desirable ($M \approx g_{LI}, g_{LC}$): in this case the leader may compare what is more profitable: to ensure the requirements, to reduce cost or to increase her income; - neglectable ($M \approx 0$): in this case the leader is in fact not interested to ensure the requirements.

The first variant can be considered as an "ideal" setting of the problem of sustainable environmental management, the second one - as a more practical model of the real life. The third variant is a degenerated one because the requirements of homeostasis are neglectable (in fact, absent). Notice that the specific problem of sustainable environmental management arises only if the requirements are mandatory.

Presence of the personal interest. The leader can be disinterested ($g_{LI} = 0$) or interested ($g_{LI} > 0$). In the former case the only leaders objective is to ensure the requirements of homeostasis (considering her control cost) while in the latter case the leader has also her personal interest (for example, to get a share from the collected taxes or fines).

Control efficiency. A process of control generates cost for the leader. So, the efficiency of control can be high ($g_{LC} \approx 0$) or low ($g_{LC} >> 0$). In the former case the costs are very small, and the leader can concentrate on the requirements of homeostasis (probably considering her personal interest). In the latter case the control costs become an essential factor.

5 A model example

For the sake of parsimony, let's consider as an illustrative example the most simplistic Malthus model of a controlled environmental system

$$\dot{x} = (a - u(t))x(t), \quad x(0) = x_0 \tag{16}$$

where $x(t)$ – number (biomass) of a population; $u(t) \in [0, 1]$ – share of its exploitation (harvesting). Lets require that

$$\forall t \in [0, T]\ x_0 - \varepsilon \le x(t) \le x_0 + \varepsilon \tag{17}$$

be the weak condition of homeostasis (given that the initial population value is optimal for the habitat). Then the strong form of homeostasis is (17) plus

$$\lim_{t \to \infty} x(t) = x_0 \tag{18}$$

For each $u(t)$ the solution of Cauchy problem (16) takes the form

$$x(t) = x_0 e^{(a-u(t))t} \tag{19}$$

It is evident that both conditions (17) and (18) are satisfied for (19) when

$$u(t) \equiv a \tag{20}$$

(the solution of the inverse optimal control problem). Now lets investigate whether the leader can ensure (20). Compulsion Stackelberg game has the form

$$J_L = \int_0^T [kpu(t)x(t) - cq^2(t) - M\rho(x(t), X^*)]dt \to \max \tag{21}$$

$$0 \le q(t) \le 1 \tag{22}$$

$$J_F = \int_0^T (1-p)u(t)x(t)dt \to \max \tag{23}$$

$$0 \le u(t) \le 1 - q(t) \tag{24}$$

subject to (16). Here discounting on the finite time interval [0,T] is omitted for simplicity, p – constant tax rate, k – share of the collected taxes in the leader's income, c – factor of the leader's control efficiency, $M \to \infty$ – penalty constant. The leader can ensure the condition (20) by choosing

$$q(t) \equiv 1 - a \tag{25}$$

In this case is the Stackelberg solution for the game (16), (21) C (24), and the players' payoffs are

$$J_L^{comp} = (kpax_0 - c)T, \quad J_F^{comp} = (1-p)ax_0T \tag{26}$$

Notice that $J_L^{comp} \begin{cases} > 0, \ kpax_0 > c \text{ (the leader's control is efficient)}, \\ < 0, \ otherwise. \end{cases}$. In the case of transition to conviction the coalitional control problem is

$$J_{L+F} = \int_0^T [(1-(1-k)p)u(t)x(t) - cq^2(t) - M\rho(x(t), X^*)]dt \to \max$$

$$0 \le q(t) \le 1, \quad 0 \le u(t) \le 1 - q(t)$$

The team solution has the form ; in this case the condition (20) is satisfied and

$$J_{L+F}^{conv} = (1-(1-k)p)ax_0T > ((1-(1-k)p)ax_0 - c)T = J_L^{comp} + J_F^{comp}$$

Impulsion Stackelberg game has the form

$$J_L = \int_0^T [kp(t)u(t)x(t) - cp^2(t) - M\rho(x(t), X^*)]dt \to \max \tag{27}$$

$$0 \leq p(t) \leq 1 \tag{28}$$

$$J_F = \int_0^T (1 - p(t))u(t)x(t)dt \to \max \tag{29}$$

$$0 \leq u(t) \leq 1 \tag{30}$$

subject to (16). Here the leader can ensure the condition (20) by choosing the impulsive strategy

$$\tilde{p}(t) = \begin{cases} 1 - \delta, & u(t) = a, \\ 1, & \text{otherwise,} \end{cases} \tag{31}$$

and the follower's optimal reaction is also $u(t) \equiv a$. Thus, $(1 - \delta, a)$ is the Stackelberg solution in the game (16), (27)–(30), and the players' payoffs are

$$J_L^{imp} = (kax_0 - (1 - \delta)c)T, \quad J_F^{imp} = \delta a x_0 T \tag{32}$$

As in the case of compulsion,

$$J_L^{comp} \begin{cases} > 0, \ kax_0 > (1 - \delta)c \text{ (the leader's control is efficient)}, \\ < 0, \ otherwise. \end{cases}$$

Let's notice also that $J_L^{comp} < J_L^{imp}$, but in the same time $J_F^{comp} >> J_F^{imp}$.

Transition to conviction entails the following optimal control problem:

$$J_{L+F} = \int_0^T [(1 - (1 - k)p(t))u(t)x(t) - cp^2(t) - M\rho(x(t), X^*)]dt \to \max$$

$$0 \leq p(t) \leq 1, \quad 0 \leq u(t) \leq 1.$$

The team solution is also ; in this case the condition (20) is satisfied and

$$J_{L+F}^{conv} = ax_0T > ((k + \delta)ax_0 - (1 - \delta)c)T = J_L^{comp} + J_F^{comp}$$

i.e. cooperation is more advantageous for both players.

6 Qualitative assessment of the model

In the paper [68] five methodological criteria for sustainability models are proposed: an interdisciplinary approach, uncertainty management, a long-range or intergenerational point of view, "glocality" and participation. Let's use these criteria for the qualitative assessment of our model of sustainable environmental management.

Interdisciplinary approach. This approach is followed in two aspects. First, state variables of the modeled controlled environmental system characterize its ecological, economic, social and other components. Second, in the game theoretic models an equation of dynamics can describe environment, while payoff functions and sets of admissible strategies of the players determine socio-economic interests and possibility of their implementation.

Uncertainty. This requirement is naturally ensured by building and investigation of stochastic control (game theoretic) models. Note that random variables can enter both in the dynamics equation and in the payoff functions.

Long-term perspective. This criterion is considered by the very setting of a dynamic game theoretic model. The game can be defined both on finite (long enough for the character scale of functioning) and on infinite period of time. An additional comparison of the temporal effects is provided by discounting of the payoffs.

Global-local perspective. This requirement means a consideration of the hierarchical nature of modeled processes that is ensured by using Stackelberg games. In more complicated model settings the systems of equations describing the modeled system dynamics could include small parameters that reflect different scale of the modeled processes.

Participation. As in the case of “glocality”, this criterion is considered by the very game theoretic setting that means a compromise concordance of different interests of the stakeholders.

Thus, the qualitative analysis demonstrates that the proposed sustainable environmental management models satisfy the methodological criteria introduced by Boulanger and Brechet [68].

7 Conclusion

In spite of the long-term discussion, the concept of sustainability remains vague and fuzzy. In this paper a mathematical formalization of the concept based on requirements of neutral or asymptotic Lyapunov stability of an ideal trajectory in the state space of an environmental system is used. This approach can be considered as a formalization of the requirement of strong sustainability.

However, this is not sufficient. While speaking about sustainable environmental management, it is necessary to add some requirements concerning the control process. It is provided by the description of interests of all stakeholders in a game theoretic model. As far in many practical important situations relations between the stakeholders are asymmetric, dynamic Stackelberg games are used as the model. We consider compulsion Stackelberg games in which the leader exerts an administrative impact to the set of the followers admissible controls without control dependence (feedback), and impulsion Stackelberg games in which the leader exerts an economic impact to the followers payoff function and control dependence (feedback) takes place. In both cases a transition to conviction (cooperation) is possible that means a coalition of the leader and the follower and their team maximization of the joint payoff function. Conviction is the most perspective and advantageous for both players if they are able to overcome different obstacles on the way to cooperation. All proposed game theoretic models include requirements to the controlled environmental system that are treated as conditions of its homeostasis.

The proposed models are successfully tested by criteria described in [68]. It seems worthwhile to develop the proposed methodology for different classes of Stackelberg games and environmental systems.

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Corresponding author
Author can be contacted at: ougoln@gmail.com