A Novel Multi-attribute Decision Making Methodology and Application

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Abstract
In the multi-attribute decision making problems, how to effectively extract the decision making rules and rank the schemes are much more important research contents. However, the acquisitions of the decision making rules often are ignored. In view of this, with respect to the problems that there exist a lot of preference information and fuzzy information in the real decision making information system, a novel decision making methodology based on dominance intuitionistic fuzzy rough set is constructed in the paper, and then it is applied to audit risk assessment and risk judgment. Based on the analysis of model and example, the result shows that the proposed model can well realize the extraction of the decision making rules and the ranking of the schemes, and effectively deal with the intuitionistic fuzzy information system with preference information.
Keywords Preference information; Decision making rule; Dominance distance index

1 Introduction

As a useful mathematical tool to deal with knowledge with inaccuracy, uncertainty and fuzziness, rough set theory was initially proposed by Pawlak [1]. It has been widely applied in many fields, such as knowledge discovery, data mining, decision analysis, and pattern recognition[2–4]. The classical rough set theory conducts data reasoning on the basis of equivalence relationship, while it is difficult to satisfy the harsh conditions of equivalence relationship in the practical applications. At the same time, the binary relationship existing on the field of discourse is often a fuzzy relationship and a similarity relationship instead of an equivalence relationship. In view of this, based on the idea and method of the fuzzy set theory[5] put forward the fuzzy rough sets theory. Because this new theory can well describe the uncertainty of various types of knowledge and more objectively reflect the physical world, it has been rapidly becoming a research focus of rough set theory, leading to its rapid development.

As a result of simultaneously considering the positive, negative and hesitancy degrees for an object to belong to a set, intuitionistic fuzzy sets possess stronger ability of information expression and well describe and portray delicate ambiguities of the nature of the objective world when compared with the traditional fuzzy sets[6, 7]. Therefore, intuitionistic fuzzy sets and rough sets are first proposed to hybrid, leading to the construction of the intuitionistic fuzzy rough set model [8]. Due to the important theoretical value and application implications, intuitionistic fuzzy rough set theory has soon become a hot academic research area. Currently, most of the related research on intuitionistic fuzzy rough set lies in the aspects of constructing different models and exploring their relevant properties.

For the related researches on constructing different models and exploring their properties, the relationship between intuitionistic fuzzy set theory and rough set theory firstly is revealed, and then they employ intuitionistic fuzzy set to define approximation operators in the intuitionistic fuzzy approximation space. By making use of the cut set of intuitionistic fuzzy sets, the upper and lower approximation operators of intuitionistic fuzzy rough set and the axiomatic method of the approximation operators based on general binary intuitionistic fuzzy relationship are respectively constructed [8–10], and then it is well known that the upper and lower approximation sets of intuitionistic fuzzy rough sets are intuitionistic fuzzy sets by making the proof [11]. Based on intuitionistic fuzzy residual implication and intuitionistic fuzzy relationship, an intuitionistic fuzzy rough set model is established[12]. However, it is difficult to apply this model to deal with an information system with noise data. With the intuitionistic fuzzy triangle model $T = min$, intuitionistic fuzzy t-conorms S= max, and intuitionistic fuzzy inverse operator N, the approximation operators of the intuitionistic fuzzy rough sets is defined, and the intuitionistic fuzzy rough set models based on the general intuitionistic fuzzy logic operators are developed[13–15]. By using the thought of intuitionistic fuzzy set and rough set, an improved intuitionistic fuzzy rough set model based on Hamming distance and establish the models such properties as interval, symmetry, complete similarity and complete dissimilarity are proposed[16], while the novel intuitionistic fuzzy rough set based on general intuitionistic fuzzy information systems is constructed in order to expand the model and its application[17]. the interval-valued intuitionistic fuzzy rough set based on the thought of implication is established, and then the related properties of the models are developed[18, 19]. By combining interval intuitionistic fuzzy set and rough set, the interval intuitionistic fuzzy rough set models based on interval intuitionistic fuzzy relationship are constructed[19–21]. By using interval-valued intuitionistic fuzzy compatibility relationship, the interval-valued intuitionistic fuzzy rough set model based on the concept of double universes and relevant properties is constructed, and then it is applied into the decision making[22, 23].

For the related literature on attribute reduction, a genetic algorithm is proposed to reduce attributes by making use of the characteristics of intuitionistic fuzzy information systems[24], while the kind of attribute reduction algorithm of intuitionistic fuzzy rough set based on mutual information by combining information entropy theory and intuitionistic fuzzy rough set is constructed and designed[25]. By using the intuitionistic fuzzy distance formula, an intuitionistic fuzzy rough model and an attribute reduction algorithm is constructed[26–28].

As discussed above, although there exist many related literatures on the intuitionistic fuzzy rough set model to construct models and discuss their theories from different levels, there are few intuitionistic fuzzy rough set models in the existing literatures those can be applied into the multi-attribute decision making and effectively deal with the real problems. In the multi-attribute group decision making problems, the dominated and dominating relationship for the attributes should be considered, and the subjective preferences of attribute value from decision-makers are based on some methods to aggregate, so that the acquisition of the decision rules and the ranking of the schemes can be made. However, it is difficult for the group decision making methods based on rough set[29–38] to deal with multi-attribute decision making problems with noise data, preference information and fuzzy information. In view of this, with respect to the shortcoming of the size comparison of the intuitionistic fuzzy numbers, the concept of the intuitionistic fuzzy dominance distance index is defined, and then it is used to construct a novel intuitionistic fuzzy rough set model, finally an example illustrates the effectiveness and applicability of the proposed model.

2 Dominance Distance Index

Intuitionistic fuzzy sets proposed by Atanassov[6, 7] are an expansion and further development of the traditional fuzzy sets. As a result of taking into account to the membership and non-membership information by adding a new attribute parameter: non-membership function in intuitionistic fuzzy sets, it provides additional options for describing the properties of things and possesses a stronger capability of dealing with uncertainty information. That is, intuitionistic fuzzy sets can well describe and portray delicate ambiguities of the nature of the objective world.

2.1 Intuitionistic Fuzzy Dominance Distance Index

Definition 1. (Intuitionistic fuzzy sets[6, 7]. Suppose that $X = \{x_1, x_2, ..., x_n\}$ is a nonempty, finite set of objects with $x_i (i = 1, 2, ..., n)$ being the ith object. Then the set $A = \{< x, \mu_A(x), \upsilon_A(x) > |x \in X\}$ of triplets is called an intuitionistic fuzzy set, where $\mu_A(x)$ and $\upsilon_A(x)$ are respectively known as the membership and non-membership for the object to belong to , that is,

$$\mu_A(x) : X \to [0, 1], x \in X \to \mu_A(x) \in [0, 1] \tag{1}$$

$$\upsilon_A(x) : X \to [0, 1], x \in X \to \upsilon_A(x) \in [0, 1] \tag{2}$$

satisfying $0 \leq \mu_A(x) + \upsilon_A(x) \leq 1$, for any $x \in X$. And $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x), x \in X$ stands for the degree of hesitation or uncertainty for the object to belong to . So the intuitionistic fuzzy number is denoted as $\alpha =< \mu_A(x), \upsilon_A(x) >$.

Definition 2. (Intuitionistic fuzzy sets[6, 7]. For any intuitionistic fuzzy number $\alpha =< \mu, \nu >$, the score function $S(\alpha)$ of this number is defined as follows:

$$S(\alpha) = \mu - \nu, \;\; S(\alpha) \in [-1, 1] \tag{3}$$

The larger $S(\alpha)$ is, the greater the intuitionistic fuzzy number $\alpha =< \mu, \nu >$ is. For example, assume that both intuitionistic fuzzy numbers are respectively $\alpha_1 =< 0.8, 0.1 >$ and $\alpha_1 =< 0.9, 0.1 >$, because $S(\alpha_1) = 0.7$ and $S(\alpha_2) = 0.8$, then $S(\alpha_1) < S(\alpha_2)$. So we can regard the intuitionistic fuzzy number α_2 as being greater than α_2.

Definition 3. (Intuitionistic fuzzy sets[6, 7]. For any intuitionistic fuzzy number $\alpha =< \mu, \nu >$, the accuracy function $H(\alpha)$ of this number is defined as follows:

$$H(\alpha) = \frac{\mu + \nu}{2} \tag{4}$$

The larger $H(\alpha)$ is, the greater the intuitionistic fuzzy number $\alpha =< \mu, \nu >$ is. For example, for intuitionistic fuzzy numbers $\alpha_3 =< 0.7, 0.2 >$ and $\alpha_4 =< 0.4, 0.2 >$, according to Definition 4, the accuracy functions of these intuitionistic fuzzy numbers are respectively $H(\alpha_3) = 0.45$ and $H(\alpha_3) = 0.3$. Therefore,$\alpha_3 > \alpha_4$.

According to the definition 2 and 3, based on the score function $S(\alpha)$ and precision function $H(\alpha)$, the intuitionistic fuzzy numbers are compared. For any both intuitionistic fuzzy numbers $\alpha_i =< \mu_i, \upsilon_i >$ and $\alpha_k =< \mu_k, \upsilon_k >$, if $S(\alpha_i) \geq S(\alpha_k)$, then $\alpha_i \geq \alpha_k$; if $S(\alpha_i) = S(\alpha_k)$ and $H(\alpha_i) \geq H(\alpha_k)$, then $\alpha_i \geq \alpha_k$. For example, for intuitionistic fuzzy numbers $\alpha_5 =< 0.8, 0.1 >$ and $\alpha_6 =< 0.6, 0.3 >$, due to $S(\alpha_5) = 0.7$, $S(\alpha_6) = 0.3$, therefore $\alpha_5 > \alpha_6$; while for the intuitionistic fuzzy numbers $\alpha_7 =< 0.7, 0.3 >$ and $\alpha_8 =< 0.5, 0.1 >$, due to $S(\alpha_7) = 0.4$, $S(\alpha_8) = 0.4$, $H(\alpha_7) = 0.5$, $H(\alpha_8) = 0.3$, therefore $\alpha_7 > \alpha_8$. However, there exist the shortcomings that how much their uncertainty allotted to the membership and non-membership for two intuitionistic fuzzy numbers, so that the size of the intuitionistic fuzzy numbers cannot be exactly determined, for example, the intuitionistic fuzzy numbers α_7, α_8. In order to determine the size relationship of the intuitionistic fuzzy numbers, a novel method should be proposed.

Definition 4. For any given two intuitionistic fuzzy numbers $\alpha_i =< \mu_i, \upsilon_i >$ and $\alpha_k =< \mu_k, \upsilon_k >$, then the dominance distance index $IFDD(x_i, x_k)$ of the intuitionistic fuzzy numbers α_i and α_k can be defined as follows:

$$IFDD(x_i, x_k) = \begin{cases} 1 & \mu_i \geq \mu_k, \upsilon_i \leq \upsilon_k \\ 0 & \mu_i < \mu_k, \upsilon_i > \upsilon_k \\ \frac{1}{2} + \frac{1}{4}\frac{\mu_i - \upsilon_i - \mu_k + \upsilon_k}{\mu(\Omega)} & other \end{cases} \tag{5}$$

Where, $\mu(\Omega)$ stands for the background measure, and it is generally taken as $\mu(\Omega) = max\{1-\upsilon_1, 1-\upsilon_2, ..., 1-\upsilon_n\} - min\{\mu_1, \mu_2, ..., \mu_n\}$, while $IFDD(x_i, x_k)$ expresses the degree of intuitionistic fuzzy numbers υ_i better than the intuitionistic fuzzy number υ_k.

Based on the definition 4, intuitionistic fuzzy numbers can be compared. For example, suppose that the background measure $\mu(\Omega)$ is 1, for the intuitionistic fuzzy numbers $\alpha_9 =< 0.8, 0.1 >$ and $\alpha_{10} =< 0.5, 0.3 >$, such that $IFDD(x_i, x_k) = 0.55$, therefore, $\alpha_{11}$is greater than α_{12} by the possibility of 55%.

2.2 Dominance Relationship Based on Dominance Distance Index

Definition 5. Assume that an ordered 4-tuple $S = (U, A, V, f)$ is known as an preference information system, if $U = \{U_1, U_2, ..., U_n\}$ is a finite and nonempty set, known as the universe; $A = C \cup D$ is a finite, nonempty attribute set, while $C = \{a_1, a_2, ..., a_m\}$ and D are respectively the condition attribute set and the decision attribute set; $V = \cup V_a$ stands for the value domain of the information system S, where V_a is the value of U with respect to the attribute $a \in A$; $f : U \times A \to V$ is an information function. If V_a is an intuitionistic fuzzy number $\alpha_a =< \mu_a, \upsilon_a >$, then the information system S is called as the intuitionistic fuzzy preference information system, and denoted as IFS.

Definition 6. Suppose that there exist an intuitionistic fuzzy preference information system $IFS = (U, A, V, f)$, for $\forall a \in P \subseteq A$, $x_i, x_k \in U$, $f(x_i, a) = \alpha_{ia} =< \mu_{ia}, \upsilon_{ia} >\in V$, $f(x_k, a) = \alpha_{ka} =< \mu_{ka}, \upsilon_{ka} >\in V$, $\lambda \in (0.5, 1]$, if $IFDD(x_i, x_k) \geq \lambda$, then the dominance relationship between the objects x_i and x_k with respect to the α attribute or the attribute set P can be called as a dominance relationship based on the dominance distance index with threshold value λ, written as $(x_i, x_k) \in R_a^{\geq\lambda}$, $(x_i, x_k) \in R_P^{\geq\lambda}$.

Property 1. Suppose that there exist an intuitionistic fuzzy preference information system $IFS = (U, A, V, f)$, $\forall a \in P \subseteq A$, $\forall x_i, x_k \in U$, $f(x_i, a) = \alpha_{ia} =< \mu_{ia}, \upsilon_{ia} >\in V$, $f(x_k, a) = \alpha_{ka} =< \mu_{ka}, \upsilon_{ka} >\in V$, it holds true:

$$0 \leq IFDD_a(x_i, x_k) \leq 1 \tag{6}$$

$$0 \leq IFDD_P(x_i, x_k) \leq 1 \tag{7}$$

Proof. According to the definition 4,

(1)if $\mu_{ia} \geq \mu_{ka}, \upsilon_{ia} \leq \upsilon_{ka}$, then $IFDD_a(x_i, x_k) = 1$;

(2)if $\mu_{ia} < \mu_{ka}, \upsilon_{ia} > \upsilon_{ka}$, then $IFDD_a(x_i, x_k) = 0$;

(3)if $\mu_{ia} \geq \mu_{ka}, \upsilon_{ia} \geq \upsilon_{ka}$ or $\mu_{ia} \leq \mu_{ka}, \upsilon_{ia} \leq \upsilon_{ka}$, because of the background value measure $\mu(\Omega) = max\{1-\upsilon_1, 1-\upsilon_2, ..., 1-\upsilon_n\} - min\{\mu_1, \mu_2, ..., \mu_n\}$, then $2(\min_i(\mu_{ia}) - \max_i(1-\upsilon_{ia})) \leq \mu_{ia} - \upsilon_{ia} - \mu_{ka} + \upsilon_{ka} \leq 2(\max_i(1-\upsilon_{ia}) - \min_i(\mu_{ia}))$, so that $-2 \leq \frac{\mu_{ia} - \upsilon_{ia} - \mu_{ka} + \upsilon_{ka}}{\max_i(1-\upsilon_{ia}) - \min_i(\mu_{ia})} \leq 2$, and then there exists

$0 \leq \frac{1}{2} + \frac{1}{4} \frac{\mu_{ia} - \upsilon_{ia} - \mu_{ka} + \upsilon_{ka}}{\max\limits_i(1-\upsilon_{ia}) - \min\limits_i(\mu_{ia})} \leq 1$, therefore, $0 \leq IFDD_a(x_i, x_k) \leq 1$.

In conclusion, $0 \leq IFDD_a(x_i, x_k) \leq 1$ can be obtained.

Let the weight vector of attributes is $w = (w_1, w_2, ..., w_p)$, and it satisfies $w_t > 0$ and $\sum\limits_{t=1}^{p} w_t = 1$, and then there exists $IFDD_P(x_i, x_k) = \sum\limits_{t=1}^{p} w_t IFDD_{a_t}(x_i, x_k)$, therefore, $0 \leq IFDD_P(x_i, x_k) \leq 1$.

Property 2. For $\forall a \in P \subseteq A$, $x_i, x_k \in U$, $f(x_i, a) = \alpha_{ia} =< \mu_{ia}, \upsilon_{ia} >\in V$ and $f(x_k, a) = \alpha_{ka} =< \mu_{ka}, \upsilon_{ka} >\in V$, $\lambda \in (0.5, 1]$, if $(x_k, x_s) \in R_{\overline{a}}^{\geq\lambda}$, the following holds true: $IFDD_a(x_i, x_k) \leq IFDD_a(x_i, x_s)$.

Proof. It suffices to show $IFDD_a(x_i, x_k) - IFDD_a(x_i, x_s) \leq 0$. For $IFDD_a(x_i, x_k)$, there exist three cases:$IFDD_a(x_i, x_k) = 1$. $IFDD_a(x_i, x_k) = \frac{1}{2} + \frac{1}{4} \frac{\mu_{ia} - \upsilon_{ia} - \mu_{ka} + \upsilon_{ka}}{\max\limits_i(1-\upsilon_{ia}) - \min\limits_i(\mu_{ia})}$ and $IFDD_a(x_i, x_k) = 0$.

For $IFDD_a(x_i, x_s)$, there also are three cases. For $(x_k, x_s) \in R_{\overline{a}}^{\geq}$, according to the dominance relationship $R_{\overline{a}}^{\geq\lambda}$, that is, $IFDD_a(x_k, x_s) \geq \lambda$ By analyzing the situation, there exit the following 6 cases.

(1) When the object x_i is definitely better than the objects x_k and x_s, that is $\alpha_i > \alpha_k$ and $\alpha_i > \alpha_s$, thus $IFDD_a(x_i, x_s) = 1$, $IFDD_a(x_i, x_k) = 1$, therefore, $IFDD_a(x_i, x_s) - IFDD_a(x_i, x_k) = 0$.

(2)When the objectx_i is definitely better than the object x_s, and not necessarily better than the object x_k, $IFDD_a(x_i, x_s) = 1$ and $IFDD_a(x_i, x_k) = \frac{1}{2} + \frac{1}{4} \frac{\mu_{ia} - \upsilon_{ia} - \mu_{ka} + \upsilon_{ka}}{\max\limits_i(1-\upsilon_{ia}) - \min\limits_i(\mu_{ia})}$ can be acquired. Therefore, the following holds true: $IFDD_a(x_i, x_s) - IFDD_a(x_i, x_k) = \frac{1}{2} - \frac{1}{4} \frac{\mu_{ia} - \upsilon_{ia} - \mu_{ka} + \upsilon_{ka}}{\max\limits_i(1-\upsilon_{ia}) - \min\limits_i(\mu_{ia})}$. According to Definition 4, there exits $-2 \leq \frac{\mu_{ia} - \upsilon_{ia} - \mu_{ka} + \upsilon_{ka}}{\max\limits_i(1-\upsilon_{ia}) - \min\limits_i(\mu_{ia})} \leq 2$, That is, $IFDD_a(x_i, x_k) - IFDD_a(x_i, x_s) \leq 0$.

(3) When the object x_i is definitely better than the objects x_s and must be inferior to the object x_k, $IFDD_a(x_i, x_s) = 1$ and $IFDD_a(x_i, x_k) = 0$ can be obtained. Thus, there exists $IFDD_a(x_i, x_s) - IFDD_a(x_i, x_k) = 1$ that is, $IFDD_a(x_i, x_k) - IFDD_a(x_i, x_s) \leq 0$.

(4) When the object x_i may be superior to the object x_s and may be superior to the object x_k, $IFDD_a(x_i, x_s) = \frac{1}{2} + \frac{1}{4} \frac{\mu_{ia} - \upsilon_{ia} - \mu_{sa} + \upsilon_{sa}}{\max\limits_i(1-\upsilon_{ia}) - \min\limits_i(\mu_{ia})}$ and $IFDD_a(x_i, x_k) = \frac{1}{2} + \frac{1}{4} \frac{\mu_{ia} - \upsilon_{ia} - \mu_{ka} + \upsilon_{ka}}{\max\limits_i(1-\upsilon_{ia}) - \min\limits_i(\mu_{ia})}$ can be acquired. Thus, $IFDD_a(x_i, x_s) - IFDD_a(x_i, x_k) = \frac{1}{4} \frac{\mu_{ka} - \upsilon_{ka} - \mu_{sa} + \upsilon_{sa}}{\max\limits_i(1-\upsilon_{ia}) - \min\limits_i(\mu_{ia})}$ Due to $IFDD_a(x_k, x_s) \geq \frac{1}{2}$, thus $\mu_{ka} - \upsilon_{ka} - \mu_{sa} + \upsilon_{sa} \geq 0$, therefore, it then follows that $IFDD_a(x_i, x_s) - IFDD_a(x_i, x_k) \geq 0$.

When the object x_i may be superior to the object x_s and must be inferior to the object x_k, $IFDD_a(x_i, x_s) = \frac{1}{2} + \frac{1}{4} \frac{\mu_{ia} - \upsilon_{ia} - \mu_{sa} + \upsilon_{sa}}{\max\limits_i(1-\upsilon_{ia}) - \min\limits_i(\mu_{ia})}$ and $IFDD_a(x_i, x_k) = 0$ can be obtained. Thus $IFDD_a(x_i, x_s) - IFDD_a(x_i, x_k) = \frac{1}{2} + \frac{1}{4} \frac{\mu_{ia} - \upsilon_{ia} - \mu_{sa} + \upsilon_{sa}}{\max\limits_i(1-\upsilon_{ia}) - \min\limits_i(\mu_{ia})}$.

From the fact that $-2 \leq \frac{\mu_{ia}-\upsilon_{ia}-\mu_{sa}+\upsilon_{sa}}{\max\limits_i(1-\upsilon_{ia})-\min\limits_i(\mu_{ia})} \leq 2$, it follows that $IFDD_a(x_i,x_s)-IFDD_a(x_i,x_k) \geq 0$.

When the object x_i must be inferior to the objects x_s and x_k, there exist $IFDD_a(x_i,x_s) = 0$ and $IFDD_a(x_i,x_k) = 0$, and then $IFDD_a(x_i,x_s) - IFDD_a(x_i,x_k) = 0$ can be concluded.

In conclusion, $IFDD_a(x_i,x_s)-IFDD_a(x_i,x_k) \geq 0$ therefore, $IFDD_a(x_i,x_k) \leq IFDD_a(x_i,x_s)$. QED

Property 3. The dominance relationship based on dominance distance index satisfies the condition of transitivity.

Proof. It suffices to show that for $\forall a \in P \subseteq A,\ x_i, x_k \in U$, if $IFDD_a(x_i,x_k) \geq \frac{1}{2}$ and $IFDD_a(x_k,x_s) \geq \frac{1}{2}$, then $IFDD_a(x_i,x_s) \geq \frac{1}{2}$.

When $IFDD_a(x_i,x_k) \geq \frac{1}{2}$, there exists $\mu_{ia} - \upsilon_{ia} - \mu_{ka} + \upsilon_{ka} \geq 0$. Accordingly there are two cases which are respectively certainly better and perhaps superior for the object x_i than the object x_k. That is, if $\mu_i \geq \mu_k, \upsilon_i \leq \upsilon_k$, then $IFDD_a(x_i,x_k) = 1$. If $IFDD_a(x_i,x_k) = \frac{1}{2} + \frac{1}{4}\frac{\mu_{ia}-\upsilon_{ia}-\mu_{ka}+\upsilon_{ka}}{\max\limits_i(1-\upsilon_{ia})-\min\limits_i(\mu_{ia})}$ and $\mu_{ia} - \upsilon_{ia} - \mu_{ka} + \upsilon_{ka} \geq 0$ then $IFDD_a(x_i,x_k) \geq \frac{1}{2}$.

when $IFDD_a(x_k,x_s) \geq \frac{1}{2}$ similarly there are also two cases which are respectively and certainly better and perhaps superior for the object x_k than the object x_s. Thus there exist the following 4 cases where the object x_i is better than the object x_s.

(1)If $IFDD_a(x_i,x_k) = 1$ and $IFDD_a(x_k,x_s) = 1$, then $\mu_i \geq \mu_k, \upsilon_i \leq \upsilon_k$ and $\mu_k \geq \mu_s, \upsilon_k \leq \upsilon_s$, and then $\mu_i \geq \mu_s, \upsilon_i \leq \upsilon_s$. Therefore $IFDD_a(x_i,x_s) = 1$.

(2)If $IFDD_a(x_i,x_k) = 1$, $IFDD_a(x_k,x_s) = \frac{1}{2} + \frac{1}{4}\frac{\mu_{ka}-\upsilon_{ka}-\mu_{sa}+\upsilon_{sa}}{\max\limits_i(1-\upsilon_{ia})-\min\limits_i(\mu_{ia})}$ and $\mu_{ka} - \upsilon_{ka} - \mu_{sa} + \upsilon_{sa} \geq 0$, then $\mu_{ia} - \upsilon_{ia} - \mu_{sa} + \upsilon_{sa} \geq 0$. According to Definition 4, $IFDD_a(x_k,x_s) \geq \frac{1}{2}$ can be obtained.

(3)$IFDD_a(x_k,x_s) = \frac{1}{2} + \frac{1}{4}\frac{\mu_{ka}-\upsilon_{ka}-\mu_{sa}+\upsilon_{sa}}{\max\limits_i(1-\upsilon_{ia})-\min\limits_i(\mu_{ia})}$ and $\mu_{ia} - \upsilon_{ia} - \mu_{ka} + \upsilon_{ka} \geq 0$ and $IFDD_a(x_k,x_s) = \frac{1}{2} + \frac{1}{4}\frac{\mu_{ka}-\upsilon_{ka}-\mu_{sa}+\upsilon_{sa}}{\max\limits_i(1-\upsilon_{ia})-\min\limits_i(\mu_{ia})}$ and $\mu_{ka} - \upsilon_{ka} - \mu_{sa} + \upsilon_{sa} \geq 0$, such that $\mu_{ia} - \upsilon_{ia} - \mu_{sa} + \upsilon_{sa} \geq 0$ And according to the definition of the dominance distance index, $IFDD_a(x_k,x_s) \geq \frac{1}{2}$ can be obtained.

(4) If $IFDD_a(x_i,x_k) = \frac{1}{2} + \frac{1}{4}\frac{\mu_{ia}-\upsilon_{ia}-\mu_{ka}+\upsilon_{ka}}{\max\limits_i(1-\upsilon_{ia})-\min\limits_i(\mu_{ia})}$, $\mu_{ia} - \upsilon_{ia} - \mu_{ka} + \upsilon_{ka} \geq 0$ and $IFDD_a(x_k,x_s) = 1$, then $\mu_{ia} - \upsilon_{ia} - \mu_{sa} + \upsilon_{sa} \geq 0$. According to the definition of the dominance distance index, therefore, $IFDD_a(x_k,x_s) \geq \frac{1}{2}$.

In conclusion, $IFDD_a(x_k,x_s) \geq \frac{1}{2}$ can be always obtained. QED.

Property 4. For $\forall a \in P \subseteq A$, $x_i, x_k \in U$, $\alpha_{ia} =< \mu_{ia}, \upsilon_{ia} > \in V$, $\alpha_{ka} =< \mu_{ka}, \upsilon_{ka} > \in V$,then the following hold true: $IFDD_a(x_i,x_k)+IFDD_a(x_k,x_i) = 1$.

Proof. According to equation (5),

(1)If $\mu_{ia} \geq \mu_{ka}, \upsilon_{ia} \leq \upsilon_{ka}$, then $IFDD_a(x_i,x_k) = 1$ and $IFDD_a(x_k,x_i) = 0$, therefore $IFDD_a(x_i,x_k) + IFDD_a(x_k,x_i) = 1$ can be obtained.

(2) When $\mu_{ia} < \mu_{ka}, \upsilon_{ia} > \upsilon_{ka}$, it follows that $IFDD_a(x_i, x_k) = 0$ and $GDD_a(x_k, x_i) = 1$. Thus, we have $GDD_a(x_i, x_k) + GDD_a(x_k, x_i) = 1$.

(3) When the other conditions, $IFDD_a(x_i, x_k) = \frac{1}{2} + \frac{1}{4}\frac{\mu_{ia}-\upsilon_{ia}-\mu_{ka}+\upsilon_{ka}}{\mu_a(\Omega)}$ and $IFDD_a(x_k, x_i) = \frac{1}{2} + \frac{1}{4}\frac{\mu_{ka}-\upsilon_{ka}-\mu_{ia}+\upsilon_{ia}}{\mu_a(\Omega)}$ can be acquired.

Thus, $IFDD_a(x_i, x_k) + IFDD_a(x_k, x_i) = 1$ can be obtained. QED.

3 Dominance Intuitionistic Fuzzy Variable Precision Rough Set Model

In order to deal with preference attributes, with respect to the information system with preference value[39**?** –41], by introducing dominance relationship into the rough set mode and taking advantage of it to substitute for the indistinguishable relationship constructed the dominance rough set model, and then acquired the decision making rules. Based on the thought of dominance rough set model, the dominance relationship based on intuitionistic fuzzy dominance distance index can be used to substitute the equivalence relationship of rough set, and then the dominance intuitionistic fuzzy rough set model is constructed.

3.1 The Construction of Model

Definition 7. Suppose that there exist an intuitionistic fuzzy preference information system $IFS = (U, A, V, f)$, for $P \subseteq A$, $Cl_t^{\geq} \subseteq D$, a threshold value $\lambda \in (0.5, 1]$ and $\beta \in (0.5, 1]$, if $D_P^+(x) = \{y \in U\,|IFDD_a(x) \geq \lambda, \forall a \in P\}$ and $D_P^-(x) = \{y \in U\,|IFDD_a(x) < \lambda, \forall a \in P\}$ respectively stand for the $\lambda - P$ dominating set and dominated set with respect to x, and then the lower approximation and upper approximation of the decision making class $Cl_t^{\geq}$ are respectively defined as follows:

$$\underline{apr}_P^{\lambda}(Cl_t^{\geq}) = \cup\{x \in U : D_P^+(x) \subseteq Cl_t^{\geq}\} \tag{8}$$

$$\overline{apr}_P^{\lambda}(Cl_t^{\geq}) = \cup\{x \in U : D_P^-(x) \cap Cl_t^{\geq} \neq \emptyset\} \tag{9}$$

And thus, $\left[\underline{apr}_P^{\lambda}(Cl_t^{\geq}), \overline{apr}_P^{\lambda}(Cl_t^{\geq})\right]$ is called as the dominance intuitionistic fuzzy rough set.

The $\lambda - P$ -lower approximation $\underline{apr}_P^{\lambda}(Cl_t^{\geq})$ of $(Cl_t^{\geq})$ can be called as the positive domain of the dominance intuitionistic fuzzy rough set and interpreted as the set of the union of all condition classes with confidence threshold value , where the classified objects definitely belong to the upward union $(Cl_t^{\geq})$. Accordingly, the $\lambda - P$ -upper approximation $\underline{apr}_P^{\lambda}(Cl_t^{\geq})$ of $(Cl_t^{\geq})$ can be interpreted as the union of all classes with confidence threshold value λ, where the classified objects possibly belong to the upward union $(Cl_t^{\geq})$.

Accordingly, based on the lower approximation and upper approximation of the intuitionistic fuzzy rough set, the λ boundary domain and classification quality of the set $(Cl_t^{\geq})$ can be defined as follows:

$$bnd_P^{\lambda}(Cl_t^{\geq}) = \overline{apr}_P^{\lambda}(Cl_t^{\geq}) - \underline{apr}_P^{\lambda}(Cl_t^{\geq}) \tag{10}$$

$$\alpha_P^\lambda(Cl_t^{\geq}) = \frac{|\underline{apr}_P^\lambda(Cl_t^{\geq})|}{|\overline{apr}_P^\lambda(Cl_t^{\geq})|} \tag{11}$$

Analogously, the λ -lower and λ -upper approximations of can be respectively defined as follows:

$$\underline{apr}_P^\lambda(Cl_t^{\leq}) = \cup\{x \in U : D_P^-(x) \subseteq Cl_t^{\leq}\} \tag{12}$$

$$\overline{apr}_P^\lambda(Cl_t^{\leq}) = \cup\{x \in U : D_P^+(x) \cap Cl_t^{\leq} \neq \emptyset\} \tag{13}$$

And thus, $\left[\underline{apr}_P^\lambda(Cl_t^{\leq}), \overline{apr}_P^\lambda(Cl_t^{\leq})\right]$ stands for the dominance intuitionistic fuzzy rough set.

Accordingly, based on the lower approximation and upper approximation of the intuitionistic fuzzy rough set, the λ boundary domain and classification quality of the set $(Cl_t^{\geq})$ can be defined as follows:

$$bnd_P^\lambda(Cl_t^{\leq}) = \overline{apr}_P^\lambda(Cl_t^{\leq}) - \underline{apr}_P^\lambda(Cl_t^{\leq}) \tag{14}$$

$$\alpha_P^\lambda(Cl_t^{\leq}) = \frac{|\underline{apr}_P^\lambda(Cl_t^{\leq})|}{|\overline{apr}_P^\lambda(Cl_t^{\leq})|} \tag{15}$$

3.2 Attribute Reduction and Preference Rules

Definition 8. Suppose that that there exist an intuitionistic fuzzy preference information system $IFS = (U, A, V, f)$, for $P \subseteq A$ and the given threshold value $\lambda \in (0.5, 1]$, the classification quality of Cl can be defined as follows:

$$\gamma_P^\lambda(Cl) = \frac{|U - ((\cup bnd(Cl_t^{\geq})) \cup (\cup bnd(Cl_t^{\leq})))|}{|U|} \tag{16}$$

The classification quality $\gamma_P^\lambda(Cl)$ of Cl stands for the ratio of the relation between all the correctly classified objects and all the objects with respect to the attribute set P in the information system.

For every minimal subset $P \subseteq C$, the attribute set P satisfying $\gamma_P^\lambda(Cl) = \gamma_C^\lambda(Cl)$ s called as the reduction of C with respect to Cl and denoted by $RED_{Cl}(P)$.

The preferential decision rule is one kind of dependence form between condition preference attribute and decision preference attribute. Based on dominance relationship with the dominance distance index, the rough approximation is acquired, and then the preferential decision rule can be induced and shown as follows:

For the given threshold value λ, $D_{\geq}$ -decision rules can take on the following form:

If $f(x, q_1) \geq r_{q1} \wedge f(x, q_2) \geq r_{q2} ... \wedge f(x, q_p) \geq r_{qp} then x \in Cl_t^{\geq}$;

For the given threshold value λ, $D_{\leq}$ -decision rules can take on the following form:

If $f(x,q_1) \le r_{q1} \wedge f(x,q_2) \le r_{q2} ... \wedge f(x,q_p) \le r_{qp} then x \in Cl_t^{\ge}$;

where $\{q_1, q_2, ...q_p\} \subseteq C$, $iff f(x,q_1) \le r_{q1} \wedge f(x,q_2) \le r_{q2} ... \wedge f(x,q_p) \le r_{qp} then x \in Cl_t^{\ge}$, $t \in \{1,2,...,l\}$.

3.3 Comprehensive Dominance Degree

Definition 9. Suppose that the intuitionistic fuzzy preference information system $IFS = (U,A,V,f)$, for $\forall a_j \in P \subseteq A$, $x_i, x_k \in U$, $f(x_i,a_j) = \alpha_{ij} =< \mu_{ij}, \upsilon_{ij} > \in V$, $f(x_k,a_j) = \alpha_{kj} =< \mu_{kj}, \upsilon_{kj} > \in V$ the following is called as the dominance degree with equal weight with respect to the attribute set P for the object x_i over the object x_k based on the dominance distance index:

$$IFDD_P(x_i,x_k) = \frac{1}{|P|} \sum_{\forall a \in P} IFDD_a(x_i,x_k) \tag{17}$$

where $|P|$ is the cardinality of the attribute set P.

In the situation of real-life decision making, due to the fact that different decision makers may very well weigh the attributes differently, the results, obtained on the basis of this uniform treatment of equal weights, can be obviously expected to be inconsistent with the reality. Therefore, there is a need to study the situation that the attributes are given different weights.

Definition 10. Suppose that the intuitionistic fuzzy preference information system $IFS = (U,A,V,f)$, for $\forall a_j \in P \subseteq A$, $x_i, x_k \in U$, $f(x_i,a_j) = \alpha_{ij} =< \mu_{ij}, \upsilon_{ij} > \in V$, $f(x_k,a_j) = \alpha_{kj} =< \mu_{kj}, \upsilon_{kj} > \in V$, let the weight vector of the attribute be $w = (w_1, ..., w_t, ..., w_{|P|})$, satisfying $w_t > 0$ and $\sum\limits_{t=1}^{|P|} w_t = 1$. Then

$$IFDD_P(x_i,x_k) = \sum_{t=1}^{|P|} w_t IFDD_a(x_i,x_k) \tag{18}$$

is referred to as the different weight dominance degree with respect to the attribute set P for the object x_i over the object x_k based on the dominance distance index.

Definition 11. Suppose that the intuitionistic fuzzy preference information system is $IFS = (U,A,V,f)$, for $\forall x_i \in U$, $P \subseteq C$, the comprehensive dominance degree of the object x_i in all the objects based on the dominance distance index is defined as follows:

$$IFDD_P(x_i) = \frac{1}{|U|-1} \sum_{i \ne k} IFDD_P(x_i,x_k) = \frac{1}{|U|-1} \sum_{i \ne k} \sum_{t=1}^{|P|} w_t IFDD_a(x_i,x_k) \tag{19}$$

3.4 The Decision Steps Based on Dominance Intuitionistic Fuzzy Rough Set

To sum up, the decision steps are following:

Step1: by adjusting the given threshold value , based on the condition attribute set, the different classifications can be determined.

Step2: based on the decision attribute set, all classifications can be determined.

Step3: according to all classifications based on the condition attribute set and the decision attribute set, the lower approximation and the upper approximation of the decision classifications can be acquired, and then the attribute reduction can be obtained, so that t the decision rules can be extracted.

Step4: by the classification results based on the condition attribute set or the decision attribute set, based on the expressions (18) and (19), the comprehensive dominance degrees of the objects are calculated, and then the ranking of the objects can be determine.

4 Case Analysis

Information system security design is that by testing the legal compliance, the confidentiality, the usability, and the reliability of the audiles, the security risk of information system is assessed, and then some audit opinions and suggestions of information system security can be given. While the risk guidance information system security audit based on the risk identification and evaluation is that through the comprehensive analysis of the audiles operation environment and information system running environment, the factors affected the system security are extracted, and then according to the risk assessment, the implementation audit scope and key target are determined, so that the substantive tests are implemented. Therefore, the evaluation and judgment of audit risk possess are much more important in the information system security audit process. Currently, few scholars take advantage of rough set to make the audit risk judgment of the information system security, and the evaluation is mainly based on the knowledge and experience of audit personnel in the practice process, however, it lacks the scientific and rationality, which has effect on the implementation of information system security audit, and ultimately affects the audit results. In recent years, the audit risk assessment and risk judgment expert system is established to effectively decrease audit judgment deviation and reduce the audit risk. however, due to the complexity and uncertainty of the objective world, as well as the limitation of human ability to understand, there always exist a variety of preference information and fuzzy information in the audit risk assessment and risk judgment expert system, while it is difficult for the existing representation and processing methods of the information system security audit based on the expert experience to acquire exactly knowledge. In view of this, based on the proposed model in this paper, the attribute reduction is used to extract the decision rule of the audit risk assessment of the information system security and acquire the key factors and bottleneck factors of affecting the audit risk assessment of the information system security, and then the ranking of the different information system security can be determined, so that the proposed model can provide for audit personals a much reasonable and effective audit risk assessment method and tool.

According to the actual audit cases, the related data can be collected and shown in the table 1. In the table 1, there exist 10 audited objects denoted as $U = \{x_1, x_2, ..., x_{10}\}$, and the five attributes $C = \{a_1, a_2, a_3, a_4, a_5\}$ which respectively good system environment, good system control, reliable financial data, reliable audit software and standard operation. For each condition attribute, based on the audit results and their own professional quality, its values can be obtained by the comprehensive judgment from the information system audit experts, for example, $f(x_2, a_1) =< 0.5, 0.4 >$ stands for the fact that 50% experts think that the system environment of the audited object x_2 is good, while 40% experts think that it is bad and 10% experts hesitate. The decision attribute set is denoted as $D = \{d\}$, and it indicates whether the audit risk of information system security is acceptable. For example, $f(x_2, d) = 1$ expresses that experts regard that the audit risk of the audited object x_2 is acceptable.

Table 1 the audit risk assessment intuitionistic fuzzy information system of the information system security

U	a_1	a_2	a_3	a_4	a_5	d
U1	$< 0.2, 0.6 >$	$< 0.1, 0.7 >$	$< 0.4, 0.4 >$	$< 0.5, 0.4 >$	$< 0.4, 0.4 >$	0
U2	$< 0.5, 0.4 >$	$< 0.3, 0.6 >$	$< 0.3, 0.6 >$	$< 0.5, 0.2 >$	$< 0.5, 0.4 >$	0
U3	$< 0.2, 0.6 >$	$< 0.1, 0.8 >$	$< 0.2, 0.7 >$	$< 0.3, 0.6 >$	$< 0.3, 0.6 >$	0
U4	$< 0.7, 0.1 >$	$< 0.6, 0.4 >$	$< 0.7, 0.2 >$	$< 0.8, 0.2 >$	$< 0.7, 0.2 >$	1
U5	$< 0.3, 0.6 >$	$< 0.2, 0.7 >$	$< 0.2, 0.7 >$	$< 0.2, 0.6 >$	$< 0.3, 0.7 >$	0
U6	$< 0.6, 0.3 >$	$< 0.6, 0.4 >$	$< 0.6, 0.3 >$	$< 0.7, 0.2 >$	$< 0.5, 0.4 >$	1
U7	$< 0.2, 0.6 >$	$< 0.2, 0.6 >$	$< 0.5, 0.4 >$	$< 0.5, 0.4 >$	$< 0.2, 0.6 >$	1
U8	$< 0.1, 0.6 >$	$< 0.2, 0.6 >$	$< 0.4, 0.5 >$	$< 0.3, 0.6 >$	$< 0.2, 0.7 >$	0
U9	$< 0.7, 0.2 >$	$< 0.6, 0.4 >$	$< 0.8, 0.1 >$	$< 0.6, 0.3 >$	$< 0.8, 0.2 >$	1
U10	$< 0.6, 0.2 >$	$< 0.6, 0.2 >$	$< 0.8, 0.2 >$	$< 0.4, 0.5 >$	$< 0.4, 0.5 >$	1

Step1: according to the condition attribute set C, when $\lambda = 0.55$, based on the dominance distance index, the universe can be divided into the following classifications:

$$U/C = \{X_1, X_2, X_3, X_4\}$$

where $X_1 = \{x_1, x_3, x_5\}$, $X_2 = \{x_7, x_8\}$, $X_3 = \{x_2, x_6, x_9, x_{10}\}$, $X_4 = \{x_4\}$ and

$X_1 < X_2 < X_3 < X_4$.

Step2:according to the decision attribute set D, the universe can be divided into the following classifications: $U/D = \{Cl_1, Cl_2\}$.
where $Cl_1 = \{x_1, x_2, x_3, x_5, x_7, x_8\}$, $U/D = \{Cl_1, Cl_2\}$.

Step3: according to the classifications based on the condition attribute set and the decision attribute set, the lower approximation $\underline{apr}_P^\lambda(Cl_1^{\geqslant})$ of $(Cl_1^{\geqslant})$ and the upper approximation $\underline{apr}_P^\lambda(Cl_1^{\geqslant})$.the lower approximation $\underline{apr}_P^\lambda(Cl_2^{\leqslant})$ of $(Cl_2^{\leqslant})$ and the upper approximation $\underline{apr}_P^\lambda(Cl_2^{\leqslant})$ respectively are:

$$\underline{apr}_P^\lambda(Cl_1^{\leqslant}) = \{x_1, x_3, x_5, x_7, x_8\}\,,\ \overline{apr}_P^\lambda(Cl_1^{\leqslant}) = \{x_1, x_2, x_3, x_5, x_6, x_7, x_8, x_9, x_{10}\}$$

$$\underline{apr}_P^\lambda(Cl_2^{\geqslant}) = \{x_4\}\,,\ \overline{apr}_P^\lambda(Cl_2^{\geqslant}) = \{x_2, x_4, x_6, x_9, x_{10}\}$$

$$\gamma_P^{\lambda,\beta}(Cl) = \frac{|\,\{x_1, x_3, x_5, x_7, x_8\} \cup \{x_4\}\,|}{|\,\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}\,|} = \frac{6}{10} = 0.6$$

When $\lambda = 0.55$, the reduction $\{a_1, a_2\}$ based on the genetic algorithm can be acquired, therefore, it is well known that the factors of good system environment and good system control should be considered in the audit risk assessment of information system. According to the reduction $\{a_1, a_2\}$, the probabilistic decision rules can be generated and shown in the table 2.

Table 2 The probabilistic decision rules based on the reduction $\{a_1, a_2\}$ with $\lambda = 0.55$.

rules	support number	confidence
$a_1 \leqslant< 0.3, 0.6 > and\ a_2 \leqslant< 0.2, 0.7 > \xrightarrow{100\%} d = 0$	2	100%
$a_1 \leqslant< 0.2, 0.6 > and\ a_2 \leqslant< 0.2, 0.6 > \xrightarrow{100\%} d = 0$	3	100%
$a_1 \geqslant< 0.6, 0.3 > and\ a_2 \geqslant< 0.6, 0.4 > \xrightarrow{100\%} d = 1$	4	100%

Step4: Because there are four classifications based on the condition attribute set, and it satisfies $X_1 < X_2 < X_3 < X_4$, that is, $\{x_1, x_3, x_5\} < \{x_7, x_8\} < \{x_2, x_6, x_9, x_{10}\} < x_4$. For the object x_4, due to the fact that there only exists the object x_4 in the X_4, so its comprehensive dominance degree no longer needs to calculate; for the classes $\{x_1, x_3, x_5\}$, $\{x_7, x_8\}$ and $\{x_2, x_6, x_9, x_{10}\}$, their comprehensive dominance degrees should be computed. According to the attribute dependency based on the proposed the model, the attribute dependency degree of each attribute can be acquired and then they are standardized, thus the weight vector is obtained as follows $w = (w_1, w_2, w_3, w_4, w_5) = (0.2125, 0.2024, 0.1964, 0.1998, 0.1889)$.

For $\{x_1, x_3, x_5\}$, based on the formula (18) and (19), $IFDD_A(x_1) = 0.5204$,

$IFDD_A(x_3) = 0.1745$ and $IFDD_A(x_5) = 0.3035$ can be calculated, therefore, $x_1 > x_5 > x_3$. For $\{x_7, x_8\}$, because the both objects are in the classification, it is necessary to only calculate the dominance degree with weights, and then it will exist $IFDD_A(x_7) = IFDD_A(x_7, x_8) = 0.8167$ and $IFDD_A(x_8) = IFDD_A(x_8, x_7) = 0.1833$, therefore, $x_7 > x_8$. For $\{x_2, x_6, x_9, x_{10}\}$, based on the formula (18) and (19), $IFDD_A(x_2) = 0.1469$, $IFDD_A(x_6) = 0.4039$, $IFDD_A(x_9) = 0.5743$ and $IFDD_A(x_{10}) = 0.3563$ can be calculated, therefore, $x_2 < x_{10} < x_6 < x_9$. To sum up, the ranking of the audited objects is $x_3 < x_5 < x_1 < x_7 < x_8 < x_2 < x_{10} < x_6 < x_9 < x_4$.

Based on the above calculation and analysis, the proposed model possesses certain fault-tolerant ability by adjusting parameter λ, and it can well do with the intuitionistic fuzzy information system with preference information and realize the extraction of group decision rules and the ranking of schemes, so that it can well deal with the real multi-attribute decision making problems with preference information and fuzzy information.

5 Conclusion

In order to achieve the law of mining the real decision information system and extract decision rules, with respect to the intuitionistic fuzzy preference information system, we construct the dominance intuitionistic fuzzy rough set based on dominance distance index, and then exploit it to extract decision-making rules and determine the ranking of decision schemes. The results show that the hybrid model can well treat the decision making problems with fuzzy information and preference information and realize the mining for the law of the decision information system, the extraction of group decision rules and the ranking of schemes, meanwhile, the proposed model can be applied into the project evaluation, military system decision and other fields. However, the proposed model cannot well deal with the intuitionistic fuzzy preference information system consisting of noise data. For how to solve the multi-attribute decision making problems with noise data, the further study will involve in it.

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