Computer Investigation of a Game Theoretic Model of Social Partnership in the System of Continuing Education

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Abstract
A game-theoretic model of social partnership in the system of continuing education is proposed and investigated in the simulation mode. Results of the model identification and investigation based on simulation modeling are considered. A comparative analysis of egoistic and cooperative approaches to the social partnership is conducted.

Keywords Social partnership; Continuing education; Simulation modeling; Identification; Difference games.

1 Introduction
A system of higher education should be flexible enough to prepare qualified and competitive specialists capable to improve their knowledge and skills in the changing environment. One of the important mechanisms providing the solution of this problem is social partnership that allows for the cooperation of employers, universities, and students. Social partnership relations are very important in the system of higher education [1,2]. An overview of the papers concerned with mathematical modeling of the problems of social partnership is proposed in some previous studies of the authors [3].

Social partnership in continuing education is a specific system of joint activities of the education system agents characterized by trust, common objectives and values, and providing highly qualified, competitive, and mobile specialists for the labor market. The main research hypothesis is that social partnership permits to increase a level of professional competence of the specialists. It seems natural to use the formalism of differential games [4] for description of social partnership relationships. Due to the high complexity of the differential game model the techniques of simulation modeling [5] are applied for its investigation in the difference form.

The paper develops authors approach exposed in [3]. In the section 2 a general description of the model is given and the new moments in comparison with the previous studies [3] are shown. In the section 3 the model identification is described. In the section 4 planning and implementation of the computer simulation experiments are discussed. Section 5 deals with processing and analysis of the modeling results. Section 6 concludes.
2 A general Description of the Model

In the proposed model social partnership relations among employers, students, and university are considered. The payoff functions are as follows:

\[
J_P = \sum_{t=0}^{T} g_P(u_P(t), u_B(t), u_C(t), x(t)) \to \max, u_P(t) \in U_P;
\]

\[
J_B = \sum_{t=0}^{T} g_B(u_P(t), u_B(t), u_C(t), x(t)) \to \max, u_B(t) \in U_B;
\]

\[
J_C = \sum_{t=0}^{T} g_C(u_P(t), u_B(t), u_C(t), x(t)) \to \max, u_C(t) \in U_C;
\]

where \( N = \{P, B, C\} \) is a set of players, namely: \( P \) – employer; \( B \) – university; \( C \) – student.

The systems dynamics are given by the equation

\[
x(t + 1) = x(t) + f(x(t), u_P(t), u_B(t), u_C(t)), x(0) = x_0;
\]

Here \( u_P(t), u_B(t), u_C(t) \) are strategies of the players describing their efforts directed to the development of social partnership relations; \( U_P, U_B, U_C \) – domains of feasible strategies; \( J_P, J_B, J_C \) – payoff functionals of the players; \( g_P, g_B, g_C \) – instantaneous payoff functions; \( P = \{P_1, ..., P_r\} \) – a finite set of employers; \( B = \{B_1, ..., B_m\} \) – a finite set of universities; \( C = \{C_1, ..., C_S\} \) – a finite set of students; \( T = 4 \) (period = 5 years).

The strategies determine a share of the annual budget assigned by a player to the needs of continuing professional education (CPE): \( u_P(t) \) – a share of the annual budget assigned to CPE programs by an employer, \( U_P = [0, r_P] \); \( u_C(t) \) – a share of the annual budget assigned to CPE programs by a student, \( U_C = [0, r_C] \); \( u_B(t) \) – a share of the annual budget assigned to CPE programs by the university, \( U_B = [0, r_B] \); \( r_P, r_C, r_B \) – the annual budgets of the players.

The main contribution of this paper in comparison with [3] is that the payoff functionals are taken in the form

\[
J_P = \sum_{t=0}^{T} e^{-\xi t}[g_P(r_P - u_P(t)) + G_P(x(t))] \to \max, u_P(t) \in U_P;
\]

\[
J_B = \sum_{t=0}^{T} e^{-\xi t}[g_B(r_B - u_B(t)) + G_B(x(t))] \to \max, u_B(t) \in U_B;
\]

\[
J_C = \sum_{t=0}^{T} e^{-\xi t}[g_C(r_C - u_C(t)) + G_C(x(t))] \to \max, u_C(t) \in U_C;
\]
This idea was firstly proposed in [6] and means that each player shares her resources between common \((u_i)\) and private \((r_i - u_i)\) interests, \(i=P, B, C\). The first summand describes the private interests, and the second one describes the common interest. In this model it is supposed that the common interest consists in the development of the CPE system and each player gains her own gain \(G_i\) of it. The private interests of the players are different and describe their payoffs from investments beyond the CPE system. Discounting is also used, and the random variable \(\xi\) reflects a state of the social environment.

In the current investigation ten university departments, five employers and ten students are considered. The strategies of generalized players in any moment of time are calculated as an arithmetic mean of the strategies of those agents, namely:

Strategy of \(P\) (Employer):

\[
u_P(t) = \frac{1}{r} \sum_{i=1}^{r} u_{P_i}(t); u_P(t) \in U_P, r = 5;
\]

Strategy of \(C\) (Student):

\[
u_C(t) = \frac{1}{s} \sum_{j=1}^{s} u_{C_j}(t); u_C(t) \in U_C, s = 10;
\]

Strategy of \(B\) (University):

\[
u_B(t) = \frac{1}{z} \sum_{k=1}^{z} u_{B_k}(t); u_B(t) \in U_B, z = 10;
\]

The state variable of the model considered as a time function \(x(t)\) is also specified in comparison with [3] and now characterizes a level of professional training of the student; \(f\) – a function of the system dynamics depending on the players’ strategies.

It is assumed that the function of system dynamics increases in respect of all arguments (the efforts of players positively influence to the results of social partnership). To give the system dynamics the modified Verhulst-Pearl model is used, i.e. \(f\) is taken in the form

\[
f(x(t), u_P(t), u_B(t), u_C(t)) = h(u_P(t), u_B(t), u_C(t))x(t)(1 - \frac{x(t)}{K});\]

where \(K\) – maximal feasible value of the state variable in the given conditions; \(h\) – function of impact of the players’ strategies, namely:

\[
h(u_P(t), u_B(t), u_C(t)) = \sum_{i=1}^{3} a_i u_i(t); a_i \geq 0; \sum_{i=1}^{3} a_i = 1; i = P, B, C;\]
\(a_i\) – relative weights of the strategies.

For the estimation of the relative weights the following considerations are used. The sum of the weights is equal to 1, and all weights are positive. The most important influence is made by students as key agents of the CPE system. The two other weights are approximately equal to each other and don’t differ from the former weight (students) too significantly. So, the weights are chosen as follows (Table 1).

**Table 1 Relative weights of the strategies \(a_i\)**

<table>
<thead>
<tr>
<th>Relative Weight (i)</th>
<th>Employer</th>
<th>University</th>
<th>Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_i)</td>
<td>0.3</td>
<td>0.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>

### 3 Identification of the Model

Suppose that the state variable \(x(t)\) characterizes a level of professional training of the students. For the estimation of the initial training level a known approach proposed by Donald Kirkpatrick [7] is used. D. Kirkpatrick has developed a model of evaluation of the training effectiveness which considers four levels:

1) Reaction: to what degree participants react favorably to the training. For the estimation the results of sociological polls conducted among PCE students of the Southern Federal University are used. The sample consisted of 2,110 respondents, including 827 female (39\%) and 1283 male (61\%).

2) Learning: to what degree participants acquire the intended knowledge, skills, attitudes, confidence and commitment based on their participation in a training event. Here the data about the students’ progress together with data characterizing the material base of education are considered.

3) Behavior: to what degree participants apply what they learned during training in their professional practice. The results of sociological polls conducted among employers in the Rostov region are used. The sample consisted of 2,580 respondents, including 785 female (30\%) and 1795 male (69\%).

4) Results: to what degree targeted outcomes occur as a result of the training event and subsequent reinforcement. The data of polls among employers on the topic “A level of professional knowledge and skills of the graduates” are used.

As a result, an initial value of the professional training level is evaluated as \(x_0\) in the scale \([0, 10]\). Also, the maximal feasible value of the state variable in the given conditions is taken equal to \(K = 10\).

The identification of payoff functions was conducted as follows.

Let \(g_i\) be the i’s player payoff from the investments which are not concerned with CPE. The most important type are bank deposits; suppose that \(Vklad_i\) is a
dividend function:

\[ V_{klat_i} = \alpha_i \cdot (r_i - u_i(t)) \cdot st; \quad i = P, B, C; \]

\( \alpha_i \) – a share of the investments;
\( st \) – an average annual percent rate.

Use also a function \( in_i, \ i=P,B,C \), that determines some fixed payoffs or income of a university professor from other sources:

\[ g_B(r_B - u_B(t)) = V_{klad_B} + Rpt + in_B; \]

**Rpt** – a gain from additional training services:

\[ Rpt = time \cdot (stavka - \beta \cdot (r_B - u_B(t))); \]

\( time \) – a total amount of lectures;
\( stavka \) – an average price of a lecture;
\( \beta \) – a share of expenditures required for a lecture;

\[ g_P(r_P - u_P(t)) = V_{klad_P} + Ent + in_P; \]

**Ent** – a function of income business (for an Employer):

\[ Ent = \sum_j product_j \cdot (price_j - \gamma_j \cdot (r_P - u_P(t))); \]

\( product_j \) – an amount of the good \( j \);
\( price_j \) – a price of the good \( j \);
\( \gamma_j \) – expense for purchasing and storage of the good \( j \);

\[ g_C(r_C - u_C(t)) = V_{klad_C} + Earn + in_C; \]

**Earn** – an income function for Student:

\[ Earn = \sum_k (serv_k \cdot value_k) - \varepsilon \cdot (r_C - u_C(t)); \]

\( serv_k \) – a volume of services;
\( value_k \) – an average price of a service;
\( \varepsilon \) – a share of the expenditures.

Farther, \( G_i \) – a payoff of each player from increase of the Students level of professional training:

\[ G_i(x(t)) = (x(t) - x(t - 1)) \cdot (\sum_{k=1}^{lengthmat_i} mat_k^i \cdot pricemat_k^i + \sum_{j=1}^{lengthmat_i} nem_j^i \cdot pricenem_j^i), \quad i = P, B, C; \]
mat\textsubscript{k}^i – a set of evaluations of the material payoffs of the player \( i \) from CPE;

nem\textsubscript{j}^i – a set of evaluations of the non-material payoffs of the player \( i \) from CPE;

The evaluations are made in the segment \([0, 10]\).

pricemat\textsubscript{k}^i – a price of one point of the material payoff of the player \( i \);

pricenem\textsubscript{j}^i – a price of one point of the non-material payoff of the player \( i \);

lengthmat\textsubscript{i} – a number of evaluations of the material payoff of the player \( i \);

lengthnem\textsubscript{i} – a number of evaluations of the non-material payoff of the player \( i \);

Material and non-material payoffs for different players are represented by different factors and determine the characteristics and types of the payoffs. A preliminary expert estimation of the points is made. A maximal possible value of \( G_i \) for each player is found. A logically substantiated relation between material and non-material payoffs is deduced.

4 Planning and Implementation of the Computer Simulation Experiments

The model investigation was conducted by computer simulation on the base of scenario method [5]. The scenarios are formed according to plausible behavior patterns of the players. For simplicity the arithmetic mean strategies are described. It is supposed that for all scenarios for any moment of time the values of strategies are equal: \( u_P(t) = u_C(t) = u_B(t) = u_O(t), \ t = 0, \ldots, 4 \). Six scenarios are considered:

1) Maximal (max) one corresponds to the maximal possible financing when the whole budget of a player is assigned for training: \( u_O(t) = r_i \);

2) Medium (med) one assigns a half of the budget for training: \( u_O(t) = 0.5r_i \);

3) Minimal (min) allows for training only a small part of the budget: \( u_O(t) = 0.2r_i \);

4) Absence of financing (abs) is clear: \( u_O(t) = 0 \);

5) Decreasing of financing (dec) means that initially an eminent part of the players budget is assigned for training but then the share decreases to a small value, namely: \( u_O(t) = (0.8 - 0.15t)r_i \);

6) Increasing of financing (inc) describes the opposite strategies: \( u_O(t) = (0.2 + 0.15t)r_i \).

Thus, in the former four scenarios the strategies are constant in time, while in the fifth and sixth scenarios the shares of financing are time-dependent.

5 Processing and analysis of the modeling results

The processing of modeling data includes a comparative analysis of the graphs of state variable and payoff functionals for different scenarios. For example, in Fig.1 the graphs of all payoff functions for the scenario of maximal financing are shown. In contrary, in Fig.2, the graphs for Students payoffs for all scenarios are
given.

**Fig. 1** A comparison of the graphs of payoff functions of Employer ($J_p(t)$), Student ($J_c(t)$), and University ($J_b(t)$) for the scenario of maximal financing.

**Fig. 2** A comparison of the graphs of the Students payoff function for different scenarios.
The graphs of payoff functions show a moderate growth considering discounting. Instead of abstract utilities in [3], in this paper financial estimations of the payoffs were made. The comparative analysis has shown that the biggest payoff goes to Employer, a less to Student and the smallest to University. This result is new and was absent in [3]. Preliminary analysis of compensation of the investments has shown that in the first period only 71% of the expenses of Student, 63% of the expenses of Employer and 18% of the expenses of University are returned. Considering the future investments, only the expenses of Student and Employer for two scenarios: minimal one and increasing of financing – are compensated at all. Probably, a more precise identification of the model parameters is needed.

The received values of payoffs are very sensitive to the amounts of annual budgets of the players. As test data were used for calculations, the results are still qualitative. Nevertheless, the investigation is very important for debugging of the new model and preparation of the special sociological research. More precise values are expected to be received after this research.

The results about the impact of strategies on the payoff functions are similar to the previous ones [3]: the best results are provided by the maximal financing scenario, and the worst ones – for the case of no financing. A comparison of the level of professional training for different scenario is made in Fig.3 (for periods).

![Fig. 3 Comparison of the level of professional training for different scenarios](image)

This graph is qualitatively similar to its counterpart in [3]. Accordingly to the new questionnaires, the players will be able to estimate the changes in the level of professional training themselves.
6 Conclusion

A new version of the model of social partnership in continuing education system is considered. The payoff functions were specified as sums of private and common payoffs, where common payoff depends on the level of professional training of students.

The transitive model has determined a set of questions to be answered by a special sociological research among students, professors, and employers. The next model version will be implemented after the research. The current version of the model shows that even a small investment to the development of continuing education system gives a certain gain to its participants. A detailed analysis should be made for the problem of compensation: not all proposed scenario are substantiated from the point of view of investments. For the University the compensation is problematic.

In the model the players receive the maximal payoffs if the financing is maximal. It is evident that to increase the level of professional training is possible only if investments are made. In general, the model emphasizes the necessity of development of the continuing education system by all its participants.

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References


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