Group Link Gear Locus Modeling Based on Motion Relativity

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Abstract *In order to solve the comprehensive analysis problem of the mechanism link locus and motion, this paper bring forward the parameter calculation of locus based on motion relativity and differential geometry theory. The problem of point-group calculation is thus solved, and the comprehensive analysis of motion and position is realized. Moreover, the method is simplified and the calculation accuracy is also improved. In the paper, a moving frame of axes is set up by using the lever axes to describe the complex structure of linkage. Motion relativity theory is adopted to study the overall motion rule of the structure. By using the calculation result of the linkage axes, the calculation model is set up to analyze its motion, which simplifies the point-group motion calculation. The point-groups form the same component have the same angle velocity, angle acceleration velocity, base point, and calculation model, which makes the calculation procedure modularized and the program shortened. Owning to complete digital analysis, the characteristic parameter calculations of locus are very accurate, the data-base is easy to establish, the transfer and analysis of data become convenient. This also makes it easy to establish the shape chart of the linkage locus and to analyze their resemblance. It's a useful method to analyze the locus and motion of the link gear comprehensively.*

Keywords *Link gear motion Motion analysing Group locus Multi-dimensional space*

1.Introduction

The basic task of computer-aided analysis of link gear is to calculate the motion parameter. But it is difficulty because of the diversification of complex trajectory and motion parameter on every point of the link gear. Especially the comprehensive analysis on motion and trajectory has been always important and difficult. So it's very necessary to find a easy theoretical method to calculate and improve efficiency.

Francois Isnard worked over the motion analysis of link gear, and has found out two arithmetic methods. John R M utilized graphics to analyze the characteristics of trajectory and motion with high speed. Zahn C T, Roskies R Z put forward a method to fit linkage curve by Fourier series. LuoKang proposed a easy method to process moving axes. By using two dimensional array to describe and store the coordinates of linkage curve, it has many merits in data processing and calculation, but too complicated. These methods ignored the physical characteristic linkage motion. Even the study on the internal rule of motion didn't utilize the integrated characteristic of the space configuration and the motion to simplify calculation. At present, there is no fundamental method to solve the unified problem on motion and geometry, also no method to establish and analyze linkage motion parameter accurately and swiftly.

2.Our Approach

According to motion relativity and differential geometry theory, this paper proposes a motion trajectory analysis method based on moving axes of link gear. A frame of axes is fixed on link lever which produces a moving frame of axes by using lever axes as coordinate. It can be very convenient to express the complex structure of linkage, and also can take full advantage of the linkage axes calculation to simplify the motion calculation on point groups, thus realize the comprehensive study on linkage motion and geometry trajectory. It's an efficient method to

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analyze the motion characteristic of link gear...

3.New Method For Design Of Link Mechanism

3.1.1 The basis problem for trajectory design

Generally, motion design on plane link gear can be arranged as three basis problems. First, component position, that is, the link gear can lead a certain component pass some preset positions accurately or approximately according to prescribed sequence. Second, preset motion rule, that is the driving part and the follower meet some required relationship of positions, including definite quick-return characteristic. Or, when the motion of driving part is defined, the follower can move accurately or approximately according to prescribed rule. Third, prescribed motion trajectory, some point on the linkage component in plane motion can move accurately or approximately according to prescribed trajectory. The problem can be summarized as: present digital design needs to analyze the motion of some point or a group points on the component, which requires record the data of its trajectory characteristic on different point.

The specific task of trajectory motion analysis is to analyze and calculate several adjacent points, define the most accordant articulated points which meet the preset trajectory motion requirements. On the other hand, the motion parameters on nonadjacent points are calculated, thus to establish the database on different trajectory motion characteristic, and prepare for the design.

The description of the integrated motion rule of link gear

It's simple and flexible to utilize lever group method to analyze link gear, moreover, easy programming. Paper [1] has worked out a general subprogram for second grade group of five basic type, and also several three and four grade lever group in common use. The kinematics analysis system of plane link gear based on fundamental lever group and single lever automatic identification is a general system for plane linkage motional performance analysis, which can analyze the motion of single and multi- freedom system in a wide range. As to its mathematic model and solving process, it's too complicated. Normally the results are angle velocity and angle acceleration velocity.

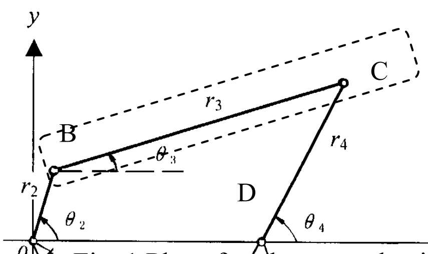

Fig. 1 Plane four lever mechanical linkage

This paper works on the mathematic modeling of motional link gear. To simplify the process, the model is a four lever linkage. The links on the articulated points are the object to study the overall motion rule of linkage. Fig. 1 shows a plane four lever link gear in XOY right-angle coordinate system. Supposing the length of the framework is r1 and the angle θ_1 ($\theta_1=0$) ; the length of the crank is r_2 and the angleθ_2; the length of the lever is r_3 and the angleθ_3; the length of the pendulum lever is r_4 and the angle θ_4. By using vector loop method to establish vector equation of linkage:

$$\vec{r_2}+\vec{r_3}=\vec{r_1}+\vec{r_4} \tag{1}$$

Supposing the angle between every lever and x axes is positive anticlockwise, and negative clockwise. The projections of every vector on coordinate system:

$$r_2\cos\theta_2 + r_3\cos\theta_3 - r_1\cos\theta_1 - r_4\cos\theta_4 = 0 \tag{2}$$

$$r_2\sin\theta_2 + r_3\sin\theta_3 - r_1\sin\theta_1 - r_4\sin\theta_4 = 0 \tag{3}$$

$$\theta_1 = 0 \quad \theta_2 = \omega t \quad \theta_3' = \omega_3 \quad \theta_4' = \omega_4$$

$$\theta_3'' = \varepsilon_3 \quad \theta_4'' = \varepsilon_4$$

$$\theta_3 = \arctan\frac{r_4\sin\theta_4 - r_2\sin\theta_2}{r_1 + r_4\cos\theta_4 - r_2\cos\theta_2} \tag{4}$$

$$\theta_4 = 2\arctan\frac{A \pm \sqrt{A^2 + B^2 - C^2}}{B - C} \tag{5}$$

$$A = -\sin\theta_2 \qquad B = r_1/r_2 - \cos\theta_2$$

$$C = \frac{r_1^2 + r_2^2 + r_4^2 - r_3^2}{2r_2r_4 - (r_1\cos\theta_2)/r_4}$$

$$\dot{\theta}_3 = \frac{-r_2\sin(\theta_2 - \theta_4)}{-r_3\sin(\theta_3 - \theta_4)}\dot{\theta}_2$$

$$\theta_4' = \frac{-r_2\sin(\theta_2 - \theta_3)}{-r_4\sin(\theta_4 - \theta_3)}\theta_2'$$

$$\theta_3'' = \frac{-r_2c\cos(\theta_2 - \theta_4) + r_3\theta_2'^2\cos(\theta_4 - \theta_3) - r_1\theta_2'^2}{-r_3\sin(\theta_4 - \theta_3)} \quad \theta_4'' = \frac{r_2\theta_2'^2c\cos(\theta_2 - \theta_3) + r_3\theta_3'^2 - r_4\theta_4'^2\cos(\theta_4 - \theta_3)}{r_1\sin(\theta_4 - \theta_3)}$$

For displacement solution of linkage, the key problems are to solve angle, angle velocity and angle acceleration velocity, on the base of which to solve those of one point. When crank angle changes between0~360°,the displacement of that point changes accordingly. If the length of every lever and crank angleθ2 are preset, θ_3 and θ_4 can be produced by the equation (2) (3). Because θ_2 is the function of t, so are θ_3 and θ4,supposing：

$$\theta_3 = f_1(t) \qquad \theta_4 = f_2(t)$$

Differentiate θ_3 and θ_4,we can get the angle velocity and angle acceleration velocity ω_3,ε_3,ω_4,ε_4 from lever and pendulum lever. Because the differentiation process is very complicated, so if the result cannot be in full use, it will be a big waste. We use these parameter in point group calculation.

4.Motion Analysis Of Moving Axes On Linkage

4.1.1 Trajectory and displacement solution of any point M on link gear

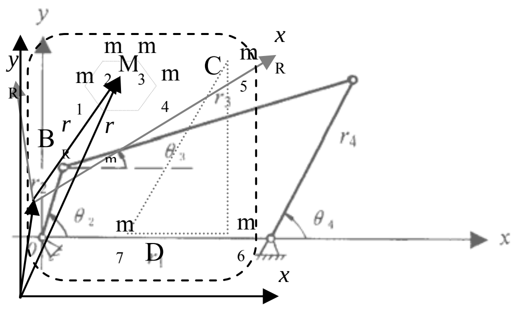

Fig. 2 Vector r_m point M on plane four lever linkage

When link gear with diversified shape moves in a plane, the description will be very complicated in fixed coordinates. Coordinates show in Fig. 2 are established on lever, so relative position of any point on lever can be expressed by one group of local coordinate, which won't change according to the moving of linkage. No matter how complex of linkage, the size of linkage can be expressed fully and normatively. The absolute position of any point can be expressed by relative coordinate and the origin of local coordinate. According to Fig. 2, position vector equation of any point M is:

$$\vec{r}_m = \vec{r}_2 + \vec{r}_{Rm} \tag{6}$$

Supposing the angle is positive anticlockwise, the coordinate of point M is:

$$x_m = r_2 \cos\theta_2 + x_{Rm} \cos\theta_3 \tag{7}$$

$$y_m = r_2 \sin\theta_2 + y_{Rm} \sin\theta_3 \tag{8}$$

Among which: xRm,yRm are the coordinates of point M in local axes, xm,ym are the absolute coordinates of point M. when preset a crank angle θ_2, the displacements in both x and y direction can be produced on point M with angleθ_2 by putting angle θ_3,θ_4 from equation(4), (5)into them. If preset:θ_2=0~360°, step 5°, and make 72 step loop computing, then we can get the trajectory point of point M in this range. These coordinates are more precise than those data produced by graphic chart. Differentiate equation(7), (8), the velocity and acceleration velocity on point M can be produced.

4.1.2 Trajectory solution of point group on mechanical linkage

By trajectory graphic chart, the size of lever and trajectory points can be defined approximately. Component size can be determined by using the range of trajectory size, and point position can be chosen by trajectory shape. Even lever size has been determined, but calculation and analysis are still necessary for determination whether point trajectory and motion parameter meet requirements. Generally speaking, tiny difference on point position can produce huge one on trajectory and motion characteristic. Trajectories form different points on lever are shown in Fig. 3.

The shape of trajectory on different point of lever differs. We can suppose that in $x_R B y_R$ coordinate system, there be at least one point can meet trajectory requirements. The ideal point may be near the one picked by graphic chart, so we need to choose one group of points nearby for analysis and calculation. Those points are all on one component, and share the same angle velocity and angle acceleration velocity ω_3,ε_3. As before, we take origin B as basing-point, the moving point for calculation. The motion of the moving point can be separated to translational moving of basing-point and rotary moving of moving point. As to moving points group, which have identical angle velocity, angle acceleration velocity and basing-point, the calculation load are no more than those of one point, but the results multiply.

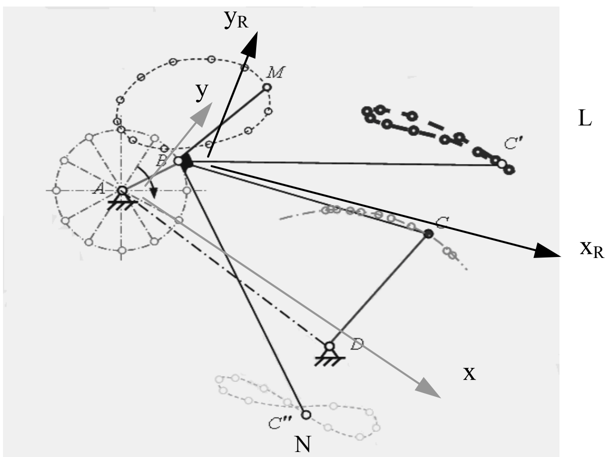

Fig. 3 Trajectory from different point on lever

5.Data Processing Of Lever'S Moting

5.1.1 The establishing of trajectory data group of point group on link gear

The calculation of adjacent point group solved the trajectory character problem of comparison and selection. Calculation on different point can solve the moving character problem of analysis and storage for many trajectories.

Nonadjacent points on lever also have the same angle velocity, angle acceleration velocity and basing-point, the calculation load of point group are no more than those of one point, but the analysis can be very accurate for many kinds of trajectory motion rule.

As to the plane link gear shown in Fig.1, we can set the number of the lever as 1, other moving levers are 2, 3, … randomly. Similarly, all the moving point for calculation can be set as 1,2,3,…,n2 randomly.

Form a three-dimensional data group grid1(n,q,l), which has n+l line, q+1 column and 2 page. Suppose the coordinates of point M on link gear to be (xnm,y_{nm}) , grid(n,m,0)= x_{nm} and grid(n,m,1)= y_{nm}, according to the simple rule above, we can put all the point coordinates into data group grid.

By accessing data group grid, we can get length between P and Q on any component L:

length=$[(x_{lp}-x_{lq})^2+(y_{lp}-y_{lq})^2]^{1/2}$

Among which, the local coordinates of point P on lever L are x_{lp}=grid (L,P,0),y_{lq} =grid (l,p,1), those of point Q are x_{lq}=grid (l,q,0),y_{lq}=grid (l,q,1).

5.1.2 Trajectory motion analysis based on local coordinate system

By accessing data group grid, we can also calculate motion parameter of point group. The absolute coordinate of any point L equals to its local coordinate plus the absolute coordinate of point B. The calculation shows in (7), (8).

As shown in Fig.3, the motion of point M is an overlap of two motions (translational moving of basing-point B and rotary moving of point M around B). The velocities of B and M are Vb and Vm, the rotary velocity of point M relative to B is V_{mb}:

$$\vec{v}_b = \vec{r}_2 \times \vec{\omega}_2 = (x_b i + y_b j)\omega_2 = (x_b \omega_2 i + y_b \omega_2 j)$$

$$\vec{v}_{mb} = \vec{r}_{Rm} \times \vec{\omega}_3 = (x_{mb} i + y_{mb} j)\omega_3 = x_{mb}\omega_3 i + y_{mb}\omega_3 j$$

$$\vec{v}_m = \vec{v}_{mb} + \vec{v}_b = (x_b\omega_1 + x_{mb}\omega_3)i + (y_b\omega_1 + y_{mb}\omega_3)j$$

The acceleration velocity of point M:

$$\vec{a}_m = \vec{a}_b + \vec{a}^n{}_{mb} + \vec{a}^\tau{}_{mb}$$
$$= \vec{r}_2 \times \omega^2{}_2 + \vec{r}_{Rm} \times \omega^2{}_3 + \vec{r}_{Rm} \times \vec{\varepsilon}_3$$

The equations above are combination of coordinates, angle velocity and angle acceleration velocity. After the size of linkage and the angle velocity of crank have been given, we can calculate the position, velocity and acceleration velocity of moving point. Equally, we can define two-dimensional data group, record position, velocity and acceleration velocity curve on trajectory.

5.1.3 Trajectory programming of point group

Trajectory programming is computer-aided computing. Because of the unification and standardization of equation, the programs for one motion parameter are the same model. Given crank's position (angle) and lever's length, you can calculate trajectory (position) , velocity and acceleration velocity curve by accessing program model. This method can analyze point motion on component precisely. On the other hand, it can process many points together, and store the calculation results as a database, so as to analyze the motion character on different point with digital method. With Fourier series proposed by Zahn C T, Roskies R Z to fit lever's curve, we can fit discrete point into mathematics equation. Or, we can pick out the shape character of graphs, and with computer-aided function to analyze trajectory character of plane linkage curve and its distribution rule.

The calculation process is the same for the same type of plane linkage, just to input different structure parameter and original condition, and you can get the trajectory and displacement. For other types of plane linkage such as crank block, we can deduce its overall motion equation with local axes, that is, the motion rule of moving axes, then the calculation equation of position, velocity and acceleration velocity on the component, then program according to process. The structure is different, but the method is the same.

6.Analysis Example For Linkage Motion

For four-lever linkage n, imaginary line shown in Fig.2 (rectangle with round angle), when move in a plane, we use local axes $x_R B y_R$, then the coordinate of any point m can be expressed as （x_{Rmi},y_{Rmi}). θ_3, ω_n ,ε_n are angle movement of local axes $x_R B y_R$. All the points' local coordinates are in data group grid. Basing-point is B, solve position and motion parameters on point m_1,m_2,……,m_n at any moment t: r_{mi},V_{miX},V_{miY},a_{mix},a_{miy}。

Solution: position vector equation at any moment

$$x_{nm_1} = r_2 \cos\theta_2 + x_{Rm_1} \cos\theta_3$$

$$y_{nm_1} = r_2 \sin\theta_2 + y_{Rm_1} \sin\theta_3$$

$$\cdots\cdots$$

$$x_{nm_i} = r_2 \cos\theta_2 + x_{Rm_i} \cos\theta_3$$

$$y_{nm_i} = r_2 \sin\theta_2 + y_{Rm_i} \sin\theta_3$$

Velocity vector equation at any moment t:

$$\vec{v}_{m_1} = \vec{v}_{m_1 b} + \vec{v}_b = (x_b\omega_1 + x_{m_1 b}\omega_3)i + (y_b\omega_1 + y_{m_1 b}\omega_3)j \quad \cdots\cdots$$

$$\vec{v}_{m_i} = \vec{v}_{m_i b} + \vec{v}_b = (x_b\omega_1 + x_{m_i b}\omega_3)i + (y_b\omega_1 + y_{m_i b}\omega_3)j$$

We can see from above, the equation for any point m_i is the same as for point m_1, so is the program. Similarly, the calculation of acceleration velocity also has common calculation equation and program.

The basic process to establish linkage curve database is: 1)set linkage basic size r_1,r_2,r_3,r_4, if only proportion factor matters for curve character calculation, then suppose r_2=1; 2)set position parameter r_{Rmi} of point M; 3)use plane linkage kinematics based on basic lever group and single automatic identification to analyze system, and calculate the overall motion rule of component; 4)calculate motion trajectory parameter of point M; 5)solve character parameter T for trajectory graph; 6)store basic parameter p and character parameter T according to certain data group grid1(n,q,l) format. Repeat the step above, we can get character parameter database of linkage curve for different linkage and point.

7.Conclusion

The trajectory parameter calculation method is based on motion relativity and differential geometry theory, which calculates points group motion parameter. It takes full advantage of the simple math relationship between local coordinates and the basing-point and overall motion rule, which makes the equation's deduction more simple, flexible and systematic, the programming and accessing of subprogram more handy, and accessing of data and analysis of graph more convenient. Local coordinate makes it more simple to construct storage model for linkage size information. It makes the best of the middle results for calculation, which reduced repetition in point group trajectory parameter calculation. It's a very effective resolution method.

Acknowledgements

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References

[1] Zahn C T,Roskies R Z. Fourier descriptors for plane closed curves. IEEE Transaction on Pattern Analysis and Machine Intelligence. C-21(3) (1992):269~281.

[2] Francois Isnard , Gordon Dodds. Dynamic Positioning ofClosed Chain Robot Mechanisms in Virtual Reality Environments. International Conference on Intelligent Robots and Systems , Victoria , B. C. , Canada ,1998.

[3] LuoKang,XiongNan The Structure Identification Of Link Gear Based On Connected Matrix Aggregtion Operation, Sichuan Industry Technology College Scientific Journal, 19 (2) (2000)：35～37.

[4] Wang Shuang. Functional analysis and optimization theory [M]. Beijing: Beijing Aerospace University Press, 2004.

[5] Isnard F , Dodds G ,Claude Vallée. Efficient Multi - Arm Closed Chain Dynamics Computation for Visualisation . International Conference on Intelligent Robots and Sys2 tems , Grenoble , France ,1997.