

# Thermal Error Modeling Analysis and Compensation Based on the Grey System Theory on Two Turntable 5-Axis NC Machine Tools

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**Abstract** *In this paper a novel concept of thermal error mode analysis is proposed in order to develop a better understanding of the thermal deformation on two turntable 5-Axis NC machine tools. The correlation analysis theory is used as finding the correlation between temperature data and thermal errors. Four key temperature points of two turntable 5-Axis NC machine tools were obtained based on the temperature field analysis. A novel thermal error model on the basis of grey system theory was proposed. The experiments proved that there was averagely a 40% increase in machine tool precision.*

**Keywords** *Thermal error, Modeling, Grey theory, Compensation*

## 1. Introduction

Because inaccuracy of machine tools is a major source of workpiece errors, control of machine error sources is critically important. Among the sources of machine error, thermally induced errors and geometric errors are known to be key contributors<sup>[1]</sup>. These errors could be reduced with structural improvement of the machine tool itself through design and manufacturing technology. However, there are, in many cases, physical limitations to accuracy improvement that cannot be overcome solely by production and design techniques. Therefore, error compensation is a cost effective way to solve the problem<sup>[2]</sup>. Accurate modeling of errors is a key part of error compensation. The thermal errors of a machine tool origin from the non-linear and time-varying thermal deformations caused by the non-uniform temperature variations in the machine structure. The temperature variations are related to the heat source location, heat source intensity, thermal resistance coefficient and the machine system configuration. Therefore, the thermal error model is usually achieved by non-linear empirical modeling approaches which correlate machine thermal errors to temperature measurements of a machine.

In this paper a novel concept of thermal error modeling analysis is proposed in order to develop a better understanding of the thermal deformation on two turntable 5-Axis NC machine tools. The thermal error prediction model, which based on grey system theory, was built on the x, y, and z directions. The prediction precision of model is very high. The robustness of model is very strong. Experiments proved that the model is more optimal model among available thermal error modeling of machine tools.

## 2. Temperature Field Identification and Thermal Error Measurement of Machine Tools

### 2.1 Sensor Location Distribution on Two Turntable 5-Axis NC Machine Tool

Two turntable 5-Axis NC machine tools is a precise machine tool, with high geometric and position accuracy. So, the thermal error is the important factor affecting the machine tool accuracy. To detect the temperature field of two turntable 5-Axis NC machine tools, a total of 24 temperature sensors were installed on the machine tools as shown in Fig. 1. The locations of the temperature sensors can be divided into 8 groups<sup>[1]</sup>, as shown in table 1. As shown in Fig. 1, the spindle house of the machine tools is installed on the Z direction slide, so the temperature field from the area is very complicated. In order to analyze the complex area, the more temperature

sensors are applied.

Table 1 Temperature sensor location & Function

Group number	Sensor serial	Installment place & Function
1	1,2,3,4,5,6,7,19,23	measuring the temperatures of the ball screw bearings and nuts of Z axis
2	8,12,15,18	measuring the temperatures of the machine tool bed
3	21,22,24	measuring the temperatures of the spindle housing
4	10,11	measuring the temperatures of the ball screw bearings and nuts of X axis
5	13,14	measuring the temperatures of the ball screw bearings and nuts of Y axis
6	9,17	measuring the temperatures of the ball screw bearings and nuts of A axis
7	16	measuring the temperatures of the ball screw bearings and nuts of C axis
8	20	measuring the ambient temperature

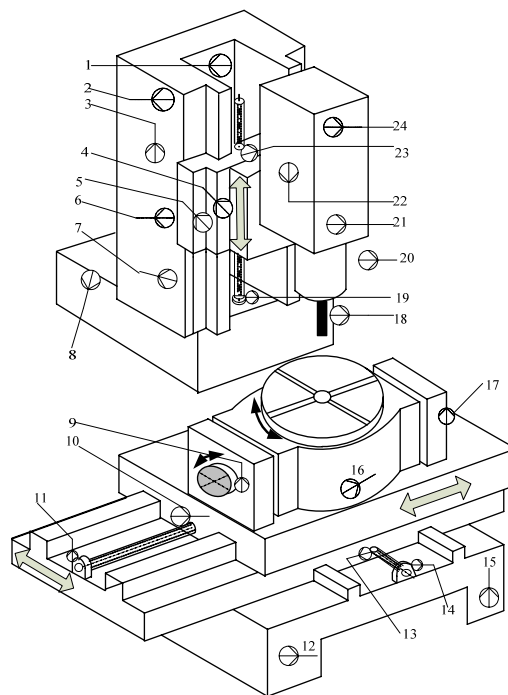


Fig.1. Sensor locations of the thermal sensing system

## 2.2 Temperature Field under a Cutting Cycle

In order to investigate the thermal behavior of the machine tool, simulation working conditions has been tested and the temperature field was recorded by the compensation control and temperature sensing systems.

Firstly, simultaneously moving the x, y, and z axes at low speed (800mm/min), with a spindle speed of 1500rpm for 2.5 hours. Secondly, stopping and cooling down the machine for 1.5 hours. Thirdly, moving the x, y, and z axes at high speed (1200mm/min) for another 2.5 hours, with a spindle speed of 3000rpm at the same time. In addition, during the testing moving A and C axes with 50rpm.

Fig. 3(a)–(d) show the temperature variations of the 8 groups of temperature sensors over time series respectively.

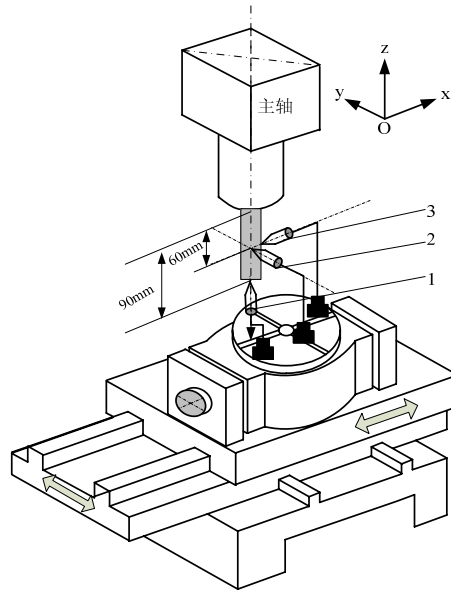


Fig.2. Measurement of thermal errors for machine tool

### 2.3 Thermal Error Measurement of Machine Tool

The experimental set-up for measuring thermal errors is shown in Fig. 2. Three displacement sensors mounted on the turntable were used for measuring the thermal drift errors of the spindle in the x, y, and z directions. The inclination error was neglected because the workpiece is short. Measurement results of thermal errors can be seen in Fig. 4.

### 2.4 Measurement Result Analysis of Temperature and Thermal Error

The conclusions are drawn from Figs. 3- Figs.4:

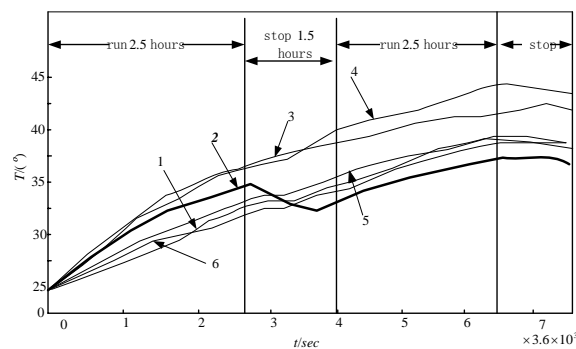
(1) Temperature change is slow with machining process. The highest measuring temperature is about  $45^{\circ}\text{C}$ .

(2) The temperatures of the spindle are about  $13^{\circ}\text{C}$  during the foregoing 2.5 hours and  $21^{\circ}\text{C}$  the latter 2.5 hours. The temperatures of the nut and screw bearing of the x, y, a, and c-axis rise about  $10^{\circ}\text{C}$  during the foregoing 2.5 hours and  $18^{\circ}\text{C}$  the latter 2.5 hours.

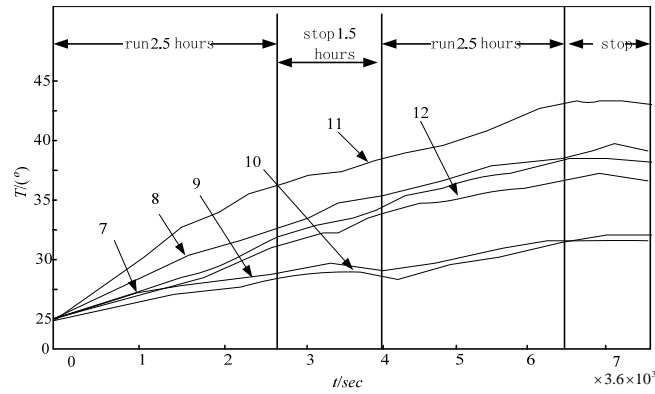
(3) The thermal error range is about  $28\ \mu\text{m}$ , which is larger than the expected value.

(4) The part error decreases with the machine temperature rise but having some time-lag.

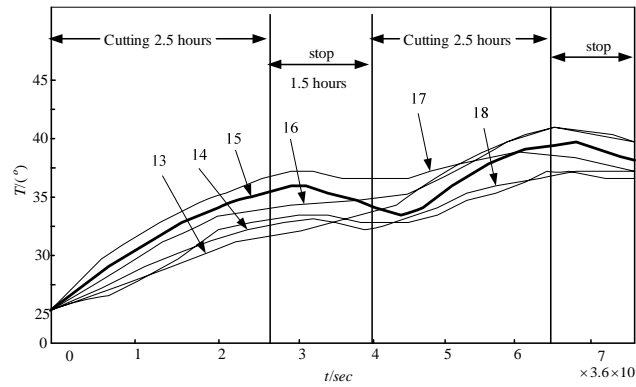
(5) When stopping cutting, the machine tool temperature does not drop immediately. Thermal error does not decrease with stopping machine tool.



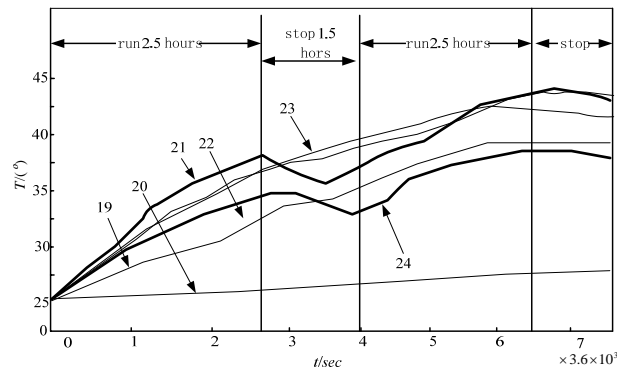
(a) 1-6



(b) 7-12



(c) 13-18



(d) 19-24 号

Fig.3. Temperature field and temperature collection systems

### 3. Choice of the Optimal Temperature Measuring Point

The amount of the temperature variable is a key factor to the accuracy of the thermal error model. Too fewer amount induces that the accuracy of the thermal error model is very low, which can't insure compensation precision. However, too much amount leads that the calculating time and additional cost are high. Therefore, the temperature variable amount should be determined before the thermal error modeling. In this study, the temperature variables are determined based on correlation analysis between the thermal errors and temperature variables<sup>[3,4]</sup>.

The optimal sensitive temperature measurement point should meet two factors (1) one is that the location should lie in the thermal sources (2) the other is that the correlation between the

thermal errors and temperature variables is highest. Correlation analysis is one of the most effective data processing methods on mathematic statistics, which describe correlative degree among multi-variables. The correlation among variables is often expressed by related coefficient  $\rho_{x,y}$  :

$$\rho_{XY} = \frac{Cov(X,Y)}{\sqrt{D(X)} \cdot \sqrt{D(Y)}} \tag{1}$$

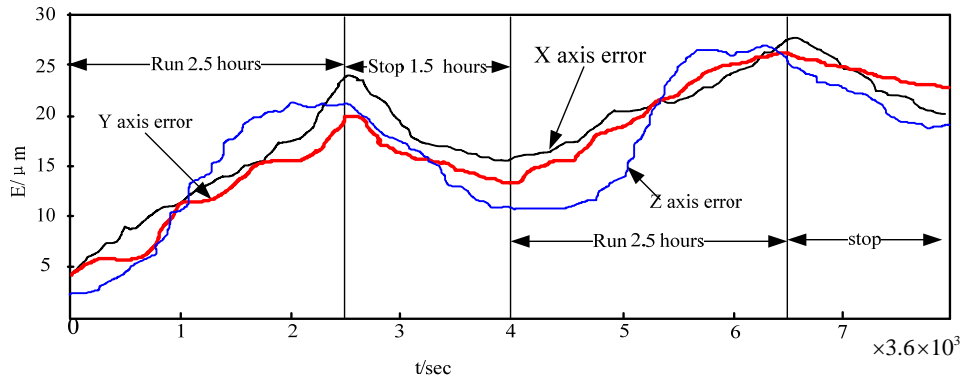


Fig.4. Output of displacement sensors from three directions

Table 2 The related coefficients among the temperature and error

Sensor Coefficient Error	T21	T2	T24	T15
X	0.693909	0.985567	0.704328	0.969573
Y	0.622667	0.710333	0.983336	0.958945
Z	0.987637	0.692565	0.732559	0.985667

In this article, the related coefficients between temperature and thermal error variables are acquired through data processing program and matlab software. Through calculating and analyzing the related coefficients between 24 temperature data series and x, y, and z direction thermal error series, the correlation coefficients  $\rho_{21,x}, \rho_{2,x}, \rho_{24,x}, \rho_{15,x}$ , as shown in Table 2, are more sensitive than other points. What's more, the thermal deformations of installment locations of four temperature measuring points seriously affect the machining precision of machine tool in the x, y, and z directions [1,3]. So the data series variable from 21, 2, 24, 15 temperature sensors are used for the thermal error modeling.

#### 4. Error Modeling Based on Grey System Theory

##### 4.1 Data Prediction and Process of Grey System Theory

If  $\{x^{(0)}(i)\}(i=1,2,\dots,n)$  is an original data series [5], new data series can be gotten by once accumulation calculation:

$$x^{(1)}(k) = \sum_{j=1}^k x^{(0)}(j) \quad k = 1, 2, \dots, n \tag{2}$$

Then the differential equation relative to GM(1,1) model is:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = u \tag{3}$$

Here,  $a$  is development grey parameter;  $u$  is the endogenous control grey parameter.

It is assumed that  $\hat{a}$  is the estimating parameter,  $\hat{a} = \begin{bmatrix} a \\ u \end{bmatrix}$  is acquired by the least square method.

$$\hat{a} = (B^T B)^{-1} B^T Y_n \tag{4}$$

Here

$$B = \begin{bmatrix} -\frac{1}{2}[x^{(1)}(1) + x^{(1)}(2)] & 1 \\ -\frac{1}{2}[x^{(1)}(2) + x^{(1)}(3)] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}[x^{(1)}(n-1) + x^{(1)}(n)] & 1 \end{bmatrix} \tag{5}$$

$$Y_n = (x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n))^T \tag{6}$$

Solving (3), grey model can be gotten:

$$\hat{x}(k+1) = \left[ X^{(0)}(1) - \frac{u}{a} \right] e^{-ak} + \frac{u}{a} \quad k = 0, 1, 2, \dots, n \tag{7}$$

#### 4.2 Improvement of Grey System Theory

During the thermal error modeling, the prediction model is acquired both by the collected all data and only a part of data. Generally, the model from different data series is different. So the parameter  $a$  is not same.

If  $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n))$ ,  $x^{(0)}(n)$  can be seen the origin of the time axis, and  $t \in i$ , then  $t < n$  is considered as past,  $t = n$  as present,  $t > n$  as future. So

(1) The grey model from  $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n))$  is called as all data model  $GM(1,1)$ .

(2) If  $\forall k_0 > 1$ , the grey model from  $X^{(0)} = (x^{(0)}(k_0), x^{(0)}(k_0+1), \dots, x^{(0)}(n))$  is called as part data model  $GM(1,1)$ .

(3) If  $x^{(0)}(n+1)$  is the new information, which is added to  $X^{(0)}$ , the grey model from  $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n), x^{(0)}(n+1))$  is called as new model  $GM(1,1)$ .

(4) Adding new information  $x^{(0)}(n+1)$  and eliminating the old information  $x^{(0)}(1)$ , the grey model from  $X^{(0)} = (x^{(0)}(2), \dots, x^{(0)}(n), x^{(0)}(n+1))$  is called as the metabolism  $GM(1,1)$ .

#### 4.3 Application of the Grey System Modeling

On the basis of grey system modeling analysis and selected temperature variables above sections, the metabolism  $GM(1,1)$  of grey model is individually used as building thermal error model in the x, y, and z directions. Detailed modeling process is neglected for the paper length limitation. This paper only offers built model based on experiment data, which include measurement values, forecast values, and residuals.

Experiment data graph, model prediction graph, and residuals graph of three directions have been drawn as shown in Fig.5. Fig. 5 shows that experiment data graph and model prediction graph are very coincident, the residuals of any data point is less  $5 \mu m$ . It can be seen that the prediction models is precise, which can be written in compensation control device.

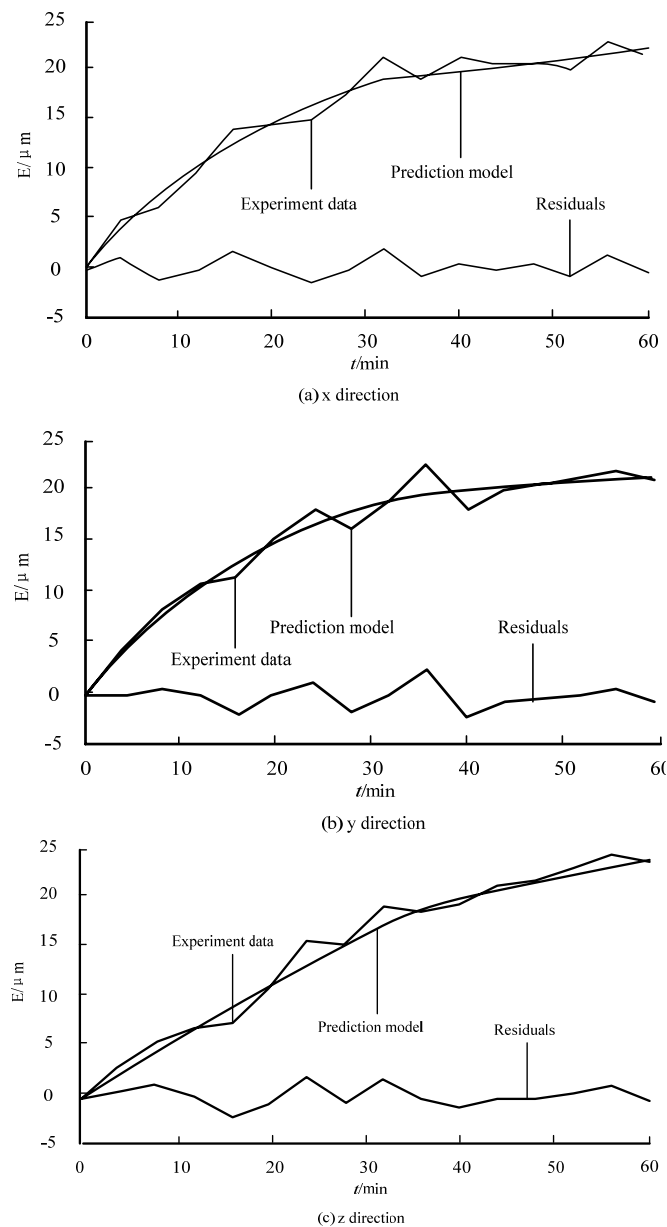


Fig.5. The output results of real measurement and model

### 5. Real Time Compensation of the Thermal Error

In order to evaluate the performance of the metabolism grey model  $GM(1,1)$ , other experiments are carried out according to the standard ISO.232. The experimental parameter setting is similar with the experimental setting described in Section 2. Four temperature sensors (No. 2, 15, 21 and 24) and the displacement sensors are only used in the experiments. Two group comparison experiments have been done, one group is 60 parts without compensation, and the other group is also 60 parts with compensation. The material of experiments parts is 45# Fe.

The Fig.6 is the trial result. The averagely thermal error of two turntable 5-Axis NC machine tools is  $20\ \mu\text{m}$  before compensation, and is  $7\ \mu\text{m}$  after compensation. There is averagely a 60%

increase on precision. Compensation experiments proved the metabolism grey model  $GM(1,1)$  is with higher precision, and the modeling method is accurate.

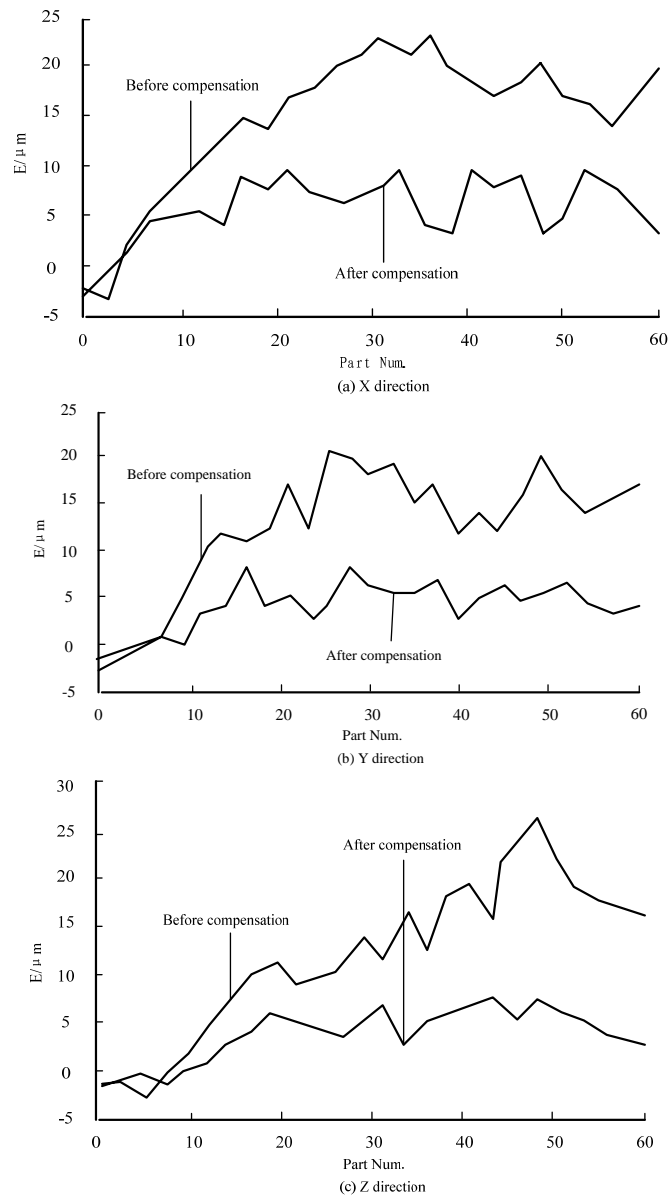


Fig. 6. X, Y, Z direction experiments before and after compensation

## 6. Conclusions

In this paper, the metabolism grey model  $GM(1,1)$  was proposed for on-line prediction of the dynamic and highly non-linear thermal errors on two turntable 5-Axis NC machine tools. The proposed model not only enhances the prediction accuracy of the thermal error but also reduces the computation cost. Relative analysis method is used as assuring the optimal temperature measurement point location of machine tool. The experimental results demonstrated that the machining accuracy was improved significantly after implementation of the compensation of the thermal errors.



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