The Application of Norm Functional Multi-dimensional Space Theory in Dynamic Design

Y. G. Jiang
Machine-electrical Engineering Department, Chengdu Electromechanical College, Chengdu, 610031, China

Abstract *The dynamic design for space over three-dimension is a tough problem in mechanical design, because there are no proper cognition and analysis methods for multi-dimensional space. So, more abstract and comprehensive methods are in need. Normed function space theory is a fully-fledged mathematics theory for multi-dimensional space, which is a powerful aid in analysis and representation. By using arbitrary function, dynamic design problem can be solved, such as the description of multi-variable system, relation modeling and optimization. Variable systems with totally different physical meanings may have an identical norm function structure, which makes the abstraction and analysis of complex problem easier. The design is the modeling based on physical essence and the analysis and calculation of model, instead of traditional geometry analysis. The method is not analogy but precise calculation, which can produce an optimum solution in norm function. Centrifugal governing system is a typical mechanical power system. According to dynamic requirement, the energy arbitrary function is set up and solved. That completes the mechanical structure design. The example shows, the arbitrary function design is workable and digitized.*

Keywords *Multi-dimensional Space Route planning Dynamic design Norm function space*

1.Introduction

Along with the development of science and technology, the major task in mechanical design is the dynamic design of multi-variable system. The dimensions of multi-variable dynamic space are over three, at least four including time, without visual geometry. The major task of dynamic design is the representation and analysis of ultra three-dimension space. Nowadays, the mechanical design is basically in analogy and static design stage [1-2]. It's difficult to represent and analyze dynamic space without scientific method to express multi-dimension space. In mechanical structure design, the following problems cannot be solved at the same time: the representation of motion position and performance in mechanical drive system, the elastodynamic characteristic of mechanism motion, the time-sharing occupation of space, the spatial encounter and capture of moving object, the influence of different motion parameter on mechanical and motion characteristics, the structure of high-speed mechanism and dynamic strength and so on. These involve position, tangent line, normal line, time, velocity, accelerated velocity, which can be concluded to study function and its derivative.

Norm function space theory is a mathematic theory to describe and analyze multi-dimension space, which establish the relation between three-dimension and infinite space. The normal three-dimension space is a fully-fledged mathematic method, with direct geometry view, which helps the comprehension of the concepts in high-dimension, even infinite dimension space [3-4].

Dynamic design problem, such as the description, the representation and the overall optimization of multi-dimension space in mechanical design, can be analyzed with Norm function space theory. So, mechanical design can be transferred from geometry analogy to precise calculation, from three-dimension to multi-dimension. The content is expanded by

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changing the method of design, thus complete the fullness and scope in mechanical design.

2.The arbitrary function problem of N-dimension design space

2.1 The expansion and composition of design space

Mechanical design has developed from static to dynamic design, from three-dimension to multi- dimension, from mechanism to inter-discipline of mechanism, electricity, magnetism, liquid and light, from geometry analogy to precise calculation, from sequential to parallel design. The emphasis is transferred from structure design to life process of product. The continual development of mechanical design will expand dimension and content in design space further.

The structure parameter and the function or differential equations set which expresses physical process are foundation to form overall performance. The multi-dimension space of mechanical design consists of structure parameter or function sequential set. Normally, the spatial variable is represented by vector:

$$\boldsymbol{X} = \left[x_1, x_2, \cdots, x_n\right]^T$$

The sub-spaces are collections of parameters with identical characteristics. The design space can be divided into sub-space according to following functions: geometry shape, shape restrict, geometry factor; requirements on tolerance and surface technology; statics performance; motion performance; dynamics performance; process performance; and other performances concerned with geometry topology. All these performances are represented with functions and derivatives, such as the design of contour curve which need to consider its function and derivative, with nth continual derivatives to ensure smoothness of contour. The mechanical motion design hopes for motion equation with continual derivatives over 3 to ensure steady movement.

2.1.2 The arbitrary function problem of design space

Arbitrary function is more abstract and comprehensive than higher mathematics and linear algebra. To study space composed of different functions, the property and trend can be analyzed with abstract operator. Normally, the problems of arbitrary function are the representation and analysis of multi-dimension function space and to choose one set optimized function which meets the requirement from functions group.

The design process is to choose representation parameter according to preset dynamic characteristics from the system, to derive the solution which meets the design requirement, to choose one set optimized function, such as contour, motion, strength function and so on, in order to meet the standard. For example, a design with, in certain range, least deflection in vibrating frequency and lightest mass is a reverse problem of dynamics. That is, to preset requirements of the system on frequency, stress, strain and displacement, which determine the material and geometry characteristics in mechanical system. The problem is to solve the extremum of design arbitrary function. So, dynamic design problem is also arbitrary function problem: to represent and analyze multi-dimension space in mechanical design and to solve the extremum of arbitrary function.

The first arbitrary function problem of dynamic design is to study N-dimensional variable and establish a space which can express N-dimensional variable.

The second arbitrary function problem of dynamic design is to study the variable and its change rate at the same time. The key problem is to establish a norm function space which can express the function and its N th derivative at the same time.

The third arbitrary function problem of dynamic design is to choose optimized solution which meets dynamic requirements and proper operator to solve and design the extremum of arbitrary function.

3.The representation and simplification of N-dimension design space with arbitrary function

3.1.1 The representation of design space

Traditional mechanical design space is three-dimensional Euclid space, which is represented by distance and angle, and can be analyzed continuously and with extremes. For multi-dimension, such as linear space, normally there is no need to represent but only introduce norm into it, and make it a normed space. With norm, the concepts of open set, closed set, convergence and continuation can be introduced correspondingly. In order to solve the first arbitrary function problem of dynamic design, a normed space should be defined.

Definition: supposing X is the vector space in number field K (real number field R or complex number field C), if for every $x \in \boldsymbol{X}$, appointed one real number $\|x\|$ which called norm of x, it meets following norm axioms.

(1) homogeneity： $\|\alpha x\| = |\alpha|\|x\|$;

(2) triangle inequality： $\|x + y\| \leqslant \|x\| + \|y\|$;

(3) positive definiteness： $\|x\| \geqslant 0 \quad \|x\| = 0 \Leftrightarrow x = 0$

As above, x， $y \in X$, if $\alpha \in K$, X is called a nomed vector space in K， normed space for short. When norm must be expressed definitely, it is marked as (X， $\|\bullet\|$) . if K＝R (or C)， the normed space in K is real (or complex) .

The way to define norm diversifies. Norm can be defined according to its physical meaning. Furthermore, the distance of three-dimensional space, the displacement, velocity and accelerated velocity of motion can also be defined into norm separately, which can also be combined together for norm. The same function can be bestowed different norm. For example, C[a， b] is defined all continuous derivable functions in [a， b], which forms linear space X. For every $x \in$ X , we define norm:

$$\|x\| = \max_{a \le t \le b} |x(t)|$$

or

$$\|x\| = \max_{a \leqslant t \leqslant b} |x(t)| + \max_{a \leqslant t \leqslant b} |\dot{x}(t)|$$

or

$$\|x\| = \int_a^b |x(t)| dt$$

They can all be normed space. N-dimensional Euclid space set distance as norm:

$$\|x\| = \left(\sum_{i=1}^{n} |\xi_i|^2 \right)^{\frac{1}{2}}$$

So it is also a normed space.

Norm is a representation of multi-dimension. The definition of norm is artificial, which makes its physical meaning diversified. Some norm has such a complicated physical meaning that it is unfathomed by using three-dimension. Multi-dimension can express much richer physical meaning than three. Norm can express N variables at the same time, which can represent and analyze variables from several design spaces systematically and comprehensively at the same time.

3.1.2 The simplification of N-dimension design space with norm

Norm analysis is highly comprehensive and abstract. It is not focused on specific structure and physical meaning of function, but on the function system and its commonality. Mechanical design space analyzed with arbitrary function can unify geometry, statics, kinematics and dynamics design into one norm structure.

The diversification of norm definitions makes the study comprehensive and flexible. The normed space with distance as its norm is a traditional geometry space. And the one with the linear combination of displacement, velocity and accelerated velocity can show the difference among different moving modes easily and clearly. Normed space with linear combination of contour function and N th derivative as norm can also express the smoothness of curve and the continuity of processing easily.

Define for any $u \in C^m(t)$, set norm:

$$\|u\|_{m,p} = \left(\sum_{|a| \le m} \|\partial^a u\|_p^p \right)^{1/p}$$

$S = \{u \in C^m(t) : \|u\|_{m,p} < \infty\}$ is a normed space. Its completion is a Banach space, called Sobolev space.

The second arbitrary function problem of dynamic design can be solved by Sobolev space definition, that is, to study variable and its change rate at the same time. Though two functions with equal norm $\|u\|$m,p may not be the same, the norm of identical functions must be equal. So, accompanied by other conditions, norm can be used to study function. Distance, motion and energy space all involves function and its derivatives. Sobolev space is concerned with motion: suppose the motion space is composed of displacement, velocity, acceleration velocity and accelerating acceleration velocity, represented with u , u' , u'' , u''' respectively, $u \in C^m(t)$, set norm:

$$\|u\|_{m,p} = \left(\sum_{|a| \le m} \|\partial^a u\|_p^p \right)^{1/p} = \left(\int_{\xi_1}^{\xi_2} |u|^2 \, dt + \int_{\xi_1}^{\xi_2} |u'|^2 \, dt + \int_{\xi_1}^{\xi_2} |u''|^2 \, dt + \int_{\xi_1}^{\xi_2} |u'''|^2 \, dt \right)^{1/2}$$

$\|u\|$m,p norm can express for variables u , u' , u'' , u''' at the same time. Space variables in dynamics are: u , u' , u'' , mu'' , in which m is mass. Space variables of contour curve are $C^0(t)$, $C^1(t)$, $C^2(t)$, …, $C^n(t)$, which can also be defined into norm with similar method. They share the same form, all belonging to Sobolev space.

Sobolev space, which combines parameter and its derivative to define norm, includes the main content in dynamic design by unified structure of norm function. This highly abstract and comprehensive structure makes the analysis of mechanical design space more simple and comprehensive.

4.Example for dynamic design analyzed with arbitrary function

4.1.1 Flexible and digital characteristics

To define norm with several parameters or only one, no matter which way, the analysis of arbitrary function directly with norm is discrete and fussy. If to analyze directly with calculus, it's also discrete and fussy for function space because calculus can only study one function. Operators in arbitrary function are all effective methods to analyze arbitrary function.

The optimization of arbitrary function, effective theoretical method, is an expansion of differential calculus. Gateaux differential is equivalent to vatiation in variational calculus, or direction derivative in differential calculus. Fréchet differential is equivalent to gradient in differential calculus or complete differential or gradient. By using Gateaux and Fréchet differential, the extreme value of arbitrary function in linear space can be solved easily, and so can the necessary condition for local extreme. If X is a linear space and f is real value arbitrary function in X, with Gateaux or Fréchet differentiable, the necessary condition for extreme in $x_0 \in \mathbf{X}$ is, for every $h \in \mathbf{X}$, $\delta f(x;h) = 0$. That is, the extremum of arbitrary function can be solved by vatiation, which is the commonest and simplest method in analysis and calculation.

At present, the main task for dynamic design is to solve kinematics and dynamics problems, such as vibration frequency, periodic and inertial force, which involves mainly in analytical mechanics. The major theory of analytical mechanics is deduced according to the necessary condition for arbitrary function extreme, such as Hamilton's principle and Lagrange equation.

Hamilton's principle focuses on energy arbitrary function composed of general displacement and its derivative, time and so on. If the constraint is ideal and complete, and the conservative force is drive force, the real motion is the motion which makes get exrtreme value.

$$S(q)=\int_{t_1}^{t_2} L(q,q',t)\mathrm{d}t=\int_{t_1}^{t_2}[T(q,q',t)-V(q,t)]\mathrm{d}t$$

If the function which makes arbitrary function S(q) get extremum is existing and exclusive, according to arbitrary function differential theory, the necessary condition equivalent to arbitrary function extremum is:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial q_a'}\right)-\frac{\partial L}{\partial q_a}=0$$

This is called Lagrange equations set [5-6]. The solution can be very easy and simple only by partial differential or differential.

For example: to design a centrifugal governor shown in Fig.1, a slide C with mass M can slide along a vertical axis, and the whole system can also rotate on this axis. Point A is fixed, point B and D hinge two particles with mass m.

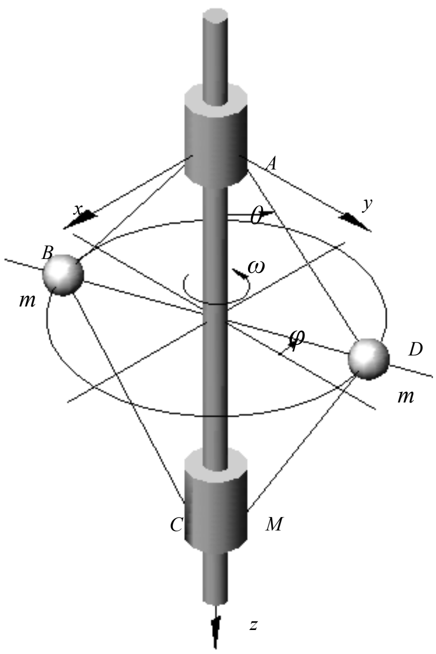

Fig.1 The vibration periodic design of centrifugal

The requirement: when the system acquires stable angular velocity ω, the vibration periodic is T, and determine the length of lever a.

Solution: (1) list Laplace equation: the angle between plane ABCD and axis x is ϕ, $\theta=\angle DAC$, degree of freedom F=2. to choose θ, ϕ is generalized coordinates, constraint is ideal and stable. The Cartesian coordinate of particle m:

$$x=a\sin\theta\cos\varphi \quad y=a\sin\theta\cos\theta \quad z=a\cos\theta$$

Lagrange function:

$$L(\theta,\varphi,\theta',\varphi')=2\frac{m}{2}\left[(a\theta')^2+(a\varphi'^2\sin\theta)\right]+\frac{M}{2}\left[4a^2(\theta'\sin\theta)^2\right]+2mga\cos\theta+2Mga\cos\theta$$

To solve partial derivative:

To solve partial derivative and simplification:

$$\frac{\partial}{\partial\theta'}L(\theta,\varphi,\theta'\varphi') = 2ma^2\theta' + 4Ma^2\theta'\sin^2\theta$$

Lagrange equation:

$$\frac{d}{dt}(2ma^2\theta' + 4Ma^2\theta'\sin^2\theta) = \left[(2m\varphi'^2 + 4M\theta'^2)a^2\cos\theta - (2m+2M)ga\right]\sin\theta$$

$$\left(m + 2M\sin^2\theta\right)\theta'' = \left[\left(m\varphi'^2 - 2M\theta'^2\right)\cos\theta - \left(m+M\right)\frac{g}{a}\right]\sin\theta$$

(2) To solve relatively balanced condition and micro-vibration periodic, the system rotates on vertical axis at even angular velocity ω, $\phi' = \omega$, on balanced position:

$$\left[m\omega^2\cos\theta - (m+M)\frac{g}{a}\right] = 0$$

to simplify, then:

$$m + 2M\theta'\sin^2\theta = \left[m\omega^2\cos\theta - (m+M)\frac{g}{a}\right]\sin\theta$$

At balanced position:

$$m\omega^2\cos\theta_s = (m+M)\frac{g}{a}$$

Solve angle θ_s, then put it into Lagrange equation:

$$(m + 2M\sin^2\theta)\theta'' = \left[m\omega^2(\cos\theta - \cos\theta_s)\right]\sin\theta$$

By using formula:

$$\cos\theta - \cos\theta_s = -2\sin\left(\frac{\theta+\theta_s}{2}\right)\sin\left(\frac{\theta-\theta_s}{2}\right)$$

Define $\varphi = \theta - \theta_s$, take micro-vibration approximation: $\frac{\theta+\theta_s}{2} = \theta_s$

$$\cos\theta - \cos\theta_s = -2\sin\theta_s\sin\left(\frac{\varphi}{2}\right) = -\varphi\sin\theta_s$$

Define and set the micro-vibration approximation:

$$(m + 2M\sin^2\theta_s)\varphi'' = \left[m\omega^2(-\sin\theta_s\varphi)\right]\sin\theta_s$$

$$\varphi'' = \frac{-m\omega^2\sin^2\theta_s}{m + 2M\sin^2\theta_s}\varphi$$

$$\Omega^2 = \frac{m\omega^2\sin^2\theta_s}{m + 2M\sin^2\theta_s}$$

Micro-vibration frequency:

$$\Omega = \omega\sqrt{\frac{\sin^2\theta_s}{1 + 2\frac{M}{m}\sin^2\theta_s}}$$

The micro-vibration periodic near the balanced position is:

$$T = \frac{2\pi}{\Omega} = \frac{2\pi}{\omega}\sqrt{\frac{1+2\frac{M}{m}\sin^2\theta_s}{\sin^2\theta_s}}$$

Solve:

$$\sin\theta_s = \left[\left(\frac{\omega T}{2\pi}\right)^2 - \frac{M}{m}\right]^{-\frac{1}{2}}$$

Set: $T = 0.6\,\mathrm{s}$, $\frac{M}{m} = 3$, $\omega = 10\pi/\mathrm{s}$

Solve: $\sin\theta_s = 0.4082$, $\theta_s = 24.0949^\circ$

By the angle θ_s at balanced position:

$$m\omega^2\cos(\theta_s) = (m+M)\frac{g}{a}$$

$$a = \frac{\left(1+\frac{M}{m}\right)g}{\omega^2\cos\theta_s} = 0.0435\ (\mathrm{m})$$

By simple dynamic design example above, we know that according to the expression of micro-vibration periodic, the structure of a centrifugal governor which meets dynamics requirements can be produced. The step is that based on the physical characteristics of system, to describe and establish arbitrary function rule by choosing motion parameters, and to deduce function solution that is in accordance with the design condition. It is different from tradition. This design is not confined to size or three-dimension any more, but dealing with the physical essence of system directly, choosing parameters flexibly, establishing arbitrary function rule and solving it.

Compared with traditional die forming and hard jig assemblies, MPF/MPP can perform in dieless and jigless manufacturing. The flexibility characteristic can be completely shown by using active elements in the EG of the MPF/MPP tooling. For example, using MPPF tooling, the deformation path can be changed to gain the best loading for raising the formability limit through increases in the contact area between the punches and the sheet, which changes the loading condition.

Various outputs from a 3D surface model can be digitally used to adjust the EG using a computer control system to configure the surfaces of tools. By means of the flexible and digital characteristics of MPF/MPP, larger and more complex panel parts can be formed and assembled incrementally and continuously with the DT/JT system.

5.Conclusion

To describe multi-dimension in mechanical design with arbitrary function theory, the selection of parameter is not geometry size, the object is not geometrical body, the representation is not limited to three-dimension figure any more, but to build model just according to the system's physical essence, and with diverse expression, to express different kinds of parameter at the same time. The design is not limited to the arrangement of three-dimensional figure, but to analyze and compare parameters form different design field with space theory. By using normed function, the convergence and extreme problems of function in Banach space can be discussed and designed. By utilizing Hilbert space, the optimization problem can be discussed. And by using differential operator, dynamic space function, derived function and optimized solution of parameter can be solved. To sum up, arbitrary function theory can solve the difficulty in dynamic design, and facilitate dynamic design.

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