Modeling and Analysis of Mechanical Properties For FDM Parts*

Lianhui Jiang and Guang Yang
School of Mechanical and Electronic Engineering, Wuhan University of Technology, Wuhan 430070, China

Abstract *Fused deposition manufacturing has the potential to fabricate parts with locally controlled properties. Modeling and analysis of the mechanical properties of the FDM parts are the foundation to produce this new class of products. A mechanics model is established for manufacturing FDM parts with required mechanical properties. A series of equations is proposed to determine the elastic constants that are used in the model. The results are evaluated by experiments.*

Keywords *Solid Freeform Fabrication Fused deposition manufacturing Locally controlled properties*

1.Introduction

Solid freeform fabrication (SFF), also referred to as rapid prototyping (RP) or layered manufacturing (LM), is a class of technologies that build objects in an additive fashion directly from a computer model. The ability of SFF technologies to fabricate parts with locally controlled properties creates opportunities for manufacturing a whole new class of parts with functionally gradient structures [1,2,3]. The functionally graded structures could be useful in controlling the mechanical properties of parts and tools on a local scale. Fused deposition manufacturing (FDM) has the potential to produce parts with locally controlled properties by changing deposition density and deposition orientation.

In the FDM process, the deposit head that extrudes a semi-molten filament through a heated nozzle in a prescribed pattern follows the trajectory defined by the chosen deposition strategy of the layer. After the semi-molten material is deposited onto the worktable, it begins to cool and bond to the neighboring layer. The bonding between the individual roads of the same layer and of neighboring layers is driven by the thermal energy of semi-molten material. FDM processes use several types of materials, such as acrylonitrile-butadiene-styrene (ABS), investment casting wax, ceramics and metals [5,6], to build conceptual or functional parts.

Most of the existing research efforts in FDM have been primarily directed to the development of new materials [2]. Research efforts were also made to characterize FDM part quality. Dutta and his associates investigated the relationship between deposition strategies and the resulting stiffness of FDM parts. Matas characterized the meso-structural features of unidirectional extruded material as a function of FDM process variables. Longmei Li researched the relationship between parts' properties and deposition parameters of FDM. [2,7]

Mechanical properties of FDM parts are governed by their meso-structures, which are determined by manufacturing parameters and bonding intension between filaments. Manufacturing parameters include width of filaments, layer thickness, deposition orientation and gap sizes between filaments. By modulating manufacturing parameters, FDM processes can produce parts with desired properties. Among the manufacturing parameters, deposition directions in layers and gap sizes between filaments are the most important parameters to control the mechanical properties. Thus, it is essential to establish mechanical models of FDM parts in

* This research has been supported by the National Natural Science Foundation of China (60904073)

relation to these two manufacturing parameters. The models can then be used to design parts with desired mechanical properties.

The remainder of this article is organized as follows. Section 2 is a study of the mechanical properties of FDM parts. Experimental analysis is given in section 3. Section 4 summarizes the conclusions of the paper.

2. Modeling and Analysis of Mechanical Properties of FDM Parts

2.1 Mechanical Models.

The FDM parts are composites of ABS filaments, bonding between filaments, and voids. As composite materials, they can be viewed and analyzed at different levels and on different scales, depending on the particular characteristics and behavior under consideration.

FDM parts are composed of layers, so lamination theory can be used to analyze the mechanical behaviors of FDM parts [8,9]. A unidirectional filament FDM part,

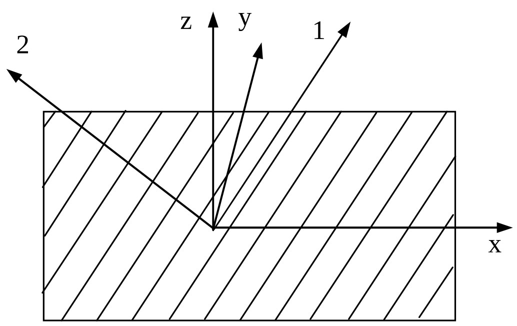

Fig. 1 Definition of principal material axes and loading axes for one layer

which can be regarded as a unidirectional fiber-reinforced composite, contains three orthogonal planes of material property symmetry, and is classified as an orthotropic material[8].

The coordinate system for analyzing mechanical behaviors of one layer of FDM parts was established as follows. Two right-handed coordinate systems are defined, namely, 1-2-z and x-y-z, as shown in Figure 1. Both 1-2 and x-y axes are in the plane of the lamina, and the z-axis is normal to this plane. In the 1-2-z systems, Axis 1, along the filament length direction, represents the longitudinal direction of the lamina. Axis 2 is normal to the fiber length and represents the transverse direction of the lamina. In the x-y-z system, Axes x and y represent the loading directions. The angle between positive Axis x and Axis 1 is called the fiber orientation angle, represented by θ. A laminate is constructed by stacking a number of laminas along the z direction. It should be pointed out that Axes 1 and 2 are constant from one lamina to the next in a unidirectional laminate.

For a general orthotropic ($\theta \neq 0$ or $90°$) lamina, the strain-stress relations can be expressed in matrix notation as (1):

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \left[\bar{S}\right] \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix}. \tag{1}$$

where $\left[\bar{S}\right]$ represents the compliance matrix for the lamina. The elements in $\left[\bar{S}\right]$ matrix can be expressed as (2):

$$\bar{S}_{11} = \frac{1}{E_{xx}} = S_{11}\cos^4\theta + (2S_{12} + S_{66})\sin^2\theta\cos^2\theta + S_{22}\sin^4\theta.$$

$$\bar{S}_{12} = \frac{\nu_{xy}}{E_{yy}} = S_{12}(\sin^4\theta + \cos^4\theta) + (S_{11} + S_{22} - S_{66})\sin^2\theta\cos^2\theta.$$

$$\bar{S}_{22} = \frac{1}{E_{yy}} = S_{11}\sin^4\theta + (2S_{11} + S_{66})\sin^2\theta\cos^2\theta + S_{22}\cos^4\theta. \tag{2}$$

$$\bar{S}_{16} = -m_x = (2S_{11} - 2S_{12} - S_{66})\sin\theta\cos^3\theta - (2S_{22} - S_{12} - S_{66})\sin^3\theta\cos\theta.$$

$$\bar{S}_{26} = -m_y = (2S_{11} - 2S_{12} - S_{66})\sin^3\theta\cos\theta - (2S_{22} - 2S_{12} - S_{66})\sin\theta\cos^3\theta.$$

$$\bar{S}_{66} = \frac{1}{G_{xy}} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})\sin^2\theta\cos^2\theta + S_{66}(\sin^4\theta + \cos^4\theta).$$

$$S_{11} = \frac{1}{E_{11}} \quad S_{12} = S_{21} = -\frac{\nu_{12}}{E_{11}} = -\frac{\nu_{21}}{E_{22}} \quad S_{22} = \frac{1}{E_{22}} \quad S_{66} = \frac{1}{G_{12}}.$$

where E_{11} is longitudinal Young's Modulus, E_{22} is transverse Young's Modulus, ν_{12} is major Poisson's ratio and G_{12} is shear modulus. From (1) it appears that there are six elastic constants that govern the stress-strain behavior of a lamina. However, a close examination of these equations would indicate that these six elements are functions of the four independent elements. Therefore, the four elastic constants can be used to determine the fully populated matrix of $[\bar{S}]$.

A laminate is constructed by stacking a number of laminas in the thickness direction. Therefore, in macro mechanical approaches, the stiffness matrix of a laminate is composed of properties of every lamina according to the Lamination Theory [8], while the in-plane elastic behavior of a unidirectional lamina may be fully described in terms of four elastic lamina properties. For the design purpose, it is desirable to have reliable predictions of lamina properties as a function of constituent properties and geometrical characteristics. One of the objectives of composite analysis is to obtain such relationships.

2.2 Modeling of Elastic Constants for FDM Parts.

When loaded in the longitudinal direction, a unidirectional FDM part is an aggregate of ABS filaments. Longitudinal Young's modulus E_{11} can be predicted by the rule of mixtures. Therefore, longitudinal Young's modulus should be as follows:

$$E_{11} = (1 - \rho_1)E. \tag{3}$$

where E is Young's modulus of ABS filament and ρ_1 is the area void density in the plane normal to filaments. The area void density can be calculated using the following equation:

$$\rho_1 = 1 - \frac{v_1 A_1}{v_2 dh}. \tag{4}$$

where v_1 is the velocity of the material supply system, A_1 is the section area of the supply filament,
v_2 is the velocity of the manufacturing head, d is the distance between two filaments of the same
layer and h is the thickness between two layers.

In the case of transverse normal loading, the bonding among the filaments is the load carrier. Because of imperfect bonding and the irregular interface between filaments, an accurate equation to calculate transverse modulus is impossible. An approximate equation is used to calculate transverse modulus:

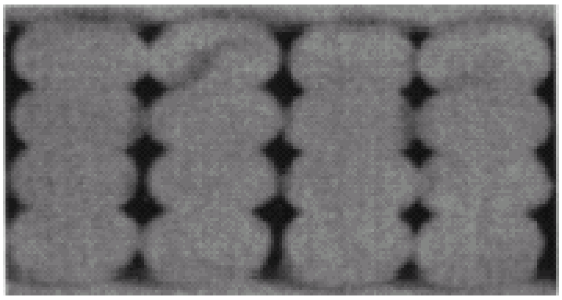

Fig. 2 Image of cross section of road geometry

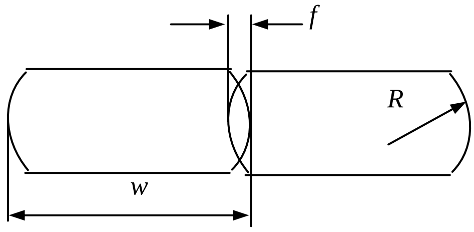

Fig. 3 Theoretical model between two adjacent roads

$$E_{22} = \zeta E. \tag{5}$$

where $\zeta = 0 \sim 1$ is the coefficient that is decided by distance between filaments, bonding status, etc. To calculate ζ, the geometry of the cross section should be studied. Fig. 2 shows the cross section image of road geometry [4]. Fig. 3 shows the theoretical model of two adjacent roads. From the cross section images of road geometry, the shape of the hole between filaments can be assumed as lozenge. The horizontal cater-corner a of the lozenge can be calculated as follow:

$$a = (2-\sqrt{3})R - (w-d). \tag{6}$$

where w is the deposition width as shown in Fig. 3.

The area of the lozenge is equal to the area of the cross section of the hole. So the perpendicular cater-corner of the lozenge b can be calculated as follow:

$$b = 2\rho_1 dh / a. \tag{7}$$

The coefficient ζ can be calculated using the following equation:

$$\zeta = \frac{d}{d - a + 2h\int_0^{a/2} a / \left[a(h-b) + 2bx\right] dx}. \tag{8}$$

Similarly, the prediction of the in-plane shear modulus takes the following form:

$$G_{12} = \zeta G. \tag{9}$$

Table 1
FDM Manufacturing Parameters Settings

Parameter	Value
Nozzle radius (R)	0.3[㎜]
Layer interval (h)	0.15[㎜]
Road width (w)	0.58[㎜]
Head temperature	220[℃]
Envelope temperature	60[℃]

The in-plane Poisson's ratio, ν_{12}, is the same as that of ABS filaments, ν, as follows:

$$\nu_{12} = \nu. \tag{10}$$

2.3Experimental Analysis

To evaluate the coefficient value in the equation, the experimental analysis was carried out. All the specimens were made using a FDM machine with the manufacturing parameters as shown in Table 1. The designs of unidirectional specimens are shown in Figure 4. The specimens with fiber orientation of $0°,45°$, and $90°$, were tested to determine the elastic constants for different deposition orientation. The tests followed the standard procedures of GB 1040-79.

In the 0^0 unidirectional specimen tests, the strain gages were mounted in the longitudinal and the transverse directions to determine the longitudinal Young's modulus E_{11} and Poison's ratio ν_{12}. Transverse modulus E_{22} can be derived from the results of the $90°$ unidirectional specimen tests. With the resultant Young's modulus

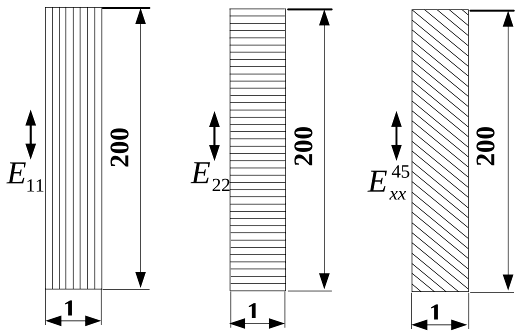

Fig. 4 Geometry of Unidirectional

E_{xx}^{45} in the x direction, and with the other constants of ν_{12}, E_{11}, and E_{22}, G_{12} can be determined as follows [8]:

$$G_{12} = \frac{1}{\dfrac{4}{E_{xx}^{45}} - \dfrac{1}{E_{11}} - \dfrac{1}{E_{22}} + \dfrac{2\nu_{12}}{E_{11}}}. \tag{11}$$

The theoretic and experimental results are listed in Table 2.

Table 2 Theoretical Results with Experimental Results

Constant, units	Experimental results	Theoretical Estimation	Difference, %
E_{11} [Mpa]	2238.7	2267.6	1.29%
E_{22} [Mpa]	2189.7	2156	1.54%
ν_{12}	0.4211	N/A	N/A
G_{12} [Mpa]	800	785.22	1.85%

3.Conclusion

SFF processes must become broadly used for manufacturing but not just prototyping. For fabrication of parts with locally controlled properties, a model of in-plane mechanical properties of FDM parts is established. A set of equations for calculating the elastic constants was found. The constants were used to determine the constitutive models of FDM parts. According to the model, different deposition densities, orientations and their combinations can be selected in producing the parts with required stiffness properties. This research is useful for exploring FDM technology fabricating parts with locally controlled properties.

4.ACKNOWLEDGEMENT

The authors would like to thank the National Nature Science Foundation of China (Grant No, 60904073) for its supporting for this research and to thank the people involved in the work. Without their efforts and input, the completion of the research would not have been possible.

References

[1] Cho.W, Sachs,E.M., Patrikalakis.N.M., Cima.M.J., Jackson.T.R, etc. “Methods for distributed design and fabrication of parts with local composition control.” Proceedings of the 2001 NSF Design and Manufacturing Grantees Conference. Jan. 2001, Tampa, Florida.

[2] Longmei Li, Qiao Sun, Celine Bellehumeur, Peihua Gu. “Modeling and Analysis for Fabrication of FDM Prototypes with Locally Controlled Properties.” Journal of Manufacturing Processes; 2002;vol4,No.2;pp129-141

[3] T.R. Jackson, H. Liua, N.M. Patrikalakisa,U, E.M. Sachsb, M.J. Cimac “Modeling and designing functionally graded material components for fabrication with local composition control” Materials and Design.2002, vol20, No.2-3,pp63-75

[4] Wenbiao Han, Mohsen A. Jafari, Stephen C. Danforth, Ahmad Safari, 2002, “Tool Path-Based Deposition Planning in Fused Deposition Processes”, Transactions of The ASME. Journal of Manufacturing Science and Engineering (v.124). pp462-472

[5] Agarwala, M.K., van weeren, R., Bandyopadhyay, a., Whalen, P.J., Safari, A., and Danforth, S.C., 1996, ”Fused Deposition of Ceramics and Metals: An Overview,” Proceedings of Solid Freeform Fabricaiton Sypposium, Austin, TX, pp.385-390.

[6] Jafari, M.A., Han, W., Mohammadi, F., Safari, A., Danforth, S.C., and Langrana, N.A., 2000, “A Novel System for Fused Deposition of Advanced Multiple Ceramics.” Rapid Prototyping Journal, 6, No.3, pp.161-174.

[7] Li, Longmei (2002), “Analysis and fabrication of FDM prototypes with locally controlled properties.” PhD dissertation, Calgary, Alberta, Canada: Dept. of Mecanical and Mfg. Engg, Univ. of Calgary.

[8] Mallick, P.K., 1988, Fiber-reinforced Composites: Materials, Manufacturing, and design, Marcel Dekker, Inc, Us.

[9] P.Gu, L.Li, 2002, “Fabrication of Biomedical Prototypes with Locally Controlled Properties Using FDM”, Annals of the CIRP, vol51(1), pp 181-184.