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Delay-dependent Stability Criteria of Stochastic Uncertain Hopfield Neural Networks with Unbounded Distributed Delays and Impulses

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Abstract *This paper is concerned with the stability analysis problem for a class of delayed stochastic uncertain Hopfield neural networks with unbounded distributed delays and impulses. A new Lyapunov-Krasovskii functional is constructed for the addressed system and several free-weighting matrices combined with the S-procedure are employed to derive the delay-dependent stability criterion. The criterion is derived and formulated in terms of linear matrix inequality (LMI). In addition to that, two illustrated examples with simulation results are given to show the effectiveness of the obtained theoretical results.*

Keywords *Delay-dependent Stochastic Hopfield neural networks Distributed delays Lyapunov Krasovskii functional Linear matrix inequality Impulses.*

1. Introduction

During the past several years, the stability of a unique equilibrium point of Hopfield neural networks [2] with delays have received especially considerable attention due to their extensive applications in solving optimization problem, traveling salesman problem and many other subjects in recent years [4, 5, 6, 14, 15, 16, 17, 18, 23, 24]. Basically, the stability results of delayed Hopfield neural networks can be classified into two categories: delay dependent stability and delay independent stability. Delay-dependent stability results are generally less conservative than delay-independent stability when the delays are small.

On the other hand, time-delays occurring in the interaction between neurons will affect the stability of a network by creating instability, oscillation and chaos phenomena. Recently, a number of global stability criteria of Hopfield networks with time-delays have been proposed (see [13, 15, 16, 17, 18]). The dynamical systems are often classified into two categories of either continuous-time or discrete-time systems. Apart from this two systems, yet there is a somewhat new category of dynamical systems, which is neither continuous-time nor purely discrete-time, these are called dynamical systems with impulses. A basic theory of impulsive differential equations has been developed in [11]. The stability conditions in [19, 20, 22] were established by using the impulsive condition. In the real world, there are two common disturbances that affects the

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network process, one is stochastic perturbations and the other one is uncertain parameters. Recently, there are some research papers about stochastic neural networks, has been investigated, see for example [3, 5, 6, 7, 9, 10, 16, 17, 18]. In [12] the stability problem for both discrete and distributed delays were discussed. Yang [21] has been investigated the stability of neural networks with distributed delays.

In practical, uncertainties often exist in most engineering and communication systems and may cause undesirable dynamic network behaviors such as oscillation, instability and chaos. More specifically, the connection weights of the neurons are inherent dependent on certain resistance and capacitance values that inevitably bring in uncertainties during the parameter identification process. In the literature, uncertainties can possibly be described by norm bounded, polytopic or linear fractional uncertainties characterizations and have been widely employed in the field of robust control and performance analysis.

Based on the above descriptions, this paper aims to develop the problem of asymptotic stability for delayed stochastic uncertain Hopfield neural networks with unbounded distributed delays and impulses. By constructing an appropriate Lyapunov-Krasovskii functional, employing several free-weighting matrices and S-procedure, we obtain a delay-dependent stability criterion in terms of LMIs. Finally, two numerical examples with simulation results are provided to demonstrate the usefulness of the main results in this paper.

2. Network Model and Preliminaries

The delayed stochastic Hopfield neural network model with unbounded distributed delays and impulses is defined by the following state equations:

$$\begin{aligned} dx_i(t) &= \Big[-a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau)) + \sum_{j=1}^{n} c_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(x_j(s))ds + J_i\Big]dt \\ &\quad + \sum_{j=1}^{n} \sigma_{ij}(t, x_j(t))dw_j(t), \quad t \neq t_k \qquad (1) \\ x_i(t_k) &= I_k x(t_k^-), \quad t = t_k, \quad k = 1, 2, ... \end{aligned}$$

where $x_i(t)$ is the state of the ith neuron at time t; $a_i > 0$ denotes the passive decay rate; b_{ij} and c_{ij} are the synaptic connection strengths; f_j denotes the neuron activation functions; J_i is the constant input from outside the system; τ represents the continuous delay and the delay kernel k_j is a real valued continuous function defined on $[0, +\infty]$ and satisfies, for each i, $\int_0^\infty k_j(s)ds = 1$. The stochastic disturbance $w(t) = (w_1(t), w_2(t), ..., w_m(t))^T$ is an m-dimensional Brownian motion; $\sigma_{ij}(\cdot, \cdot)$ is locally Lipschitz continuous and satisfies the linear growth condition as well; $x_i(t_k) = I_k x(t_k^-)$ is the impulse at moment t_k, the fixed moment of time t_k satisfy $t_1 < t_2 <, ..., \lim_{k\to+\infty} t_k = +\infty$ and $x(t^-) = \lim_{s\to t^-} x(s)$; I_k is a constant real matrix at the moments of time t_k.

Let $PC([-\tau, 0], \mathbb{R}^n)$ denotes the set of piecewise right continuous functions $\phi : [-\tau, 0] \to \mathbb{R}^n$ with the sup-norm $|\phi| = sup_{-\tau\leq s\leq 0}\|\phi(s)\|$. For given t_0, and $\phi \in (PC[-\tau, 0], \mathbb{R}^n)$, the initial condition of system (1) is described as $x(t_0 + t) = \phi(t), \ for \ t \in [-\tau, 0], \quad \phi \in$

$PC([-\tau, 0], \mathbb{R}^n)$.

The following assumptions is utilized throughout this paper:

(C1) The activation function $f(x)$ is boundless and satisfies

$$0 \leq \frac{f_i(\xi_1) - f_i(\xi_2)}{\xi_1 - \xi_2} \leq l_i, \quad \text{for any } \xi_1, \xi_2 \in R, \xi_1 \neq \xi_2,\ i = 1, 2, ..., n$$

(C2) The function $f_i(x_i(\cdot)) = 0$, $i = 1, 2, ..., n$ satisfy

$$0 \leq \frac{f_i(x_i(t))}{x_i(t)} \leq l_i, \quad f_i(0) = 0,\ \forall x_i(t) \neq 0,\ i = 1, 2, ..., n$$

where l_i, $i = 1, 2, ..., n$ are positive constants.

Remark 2.1 The above conditions ensures that the nonlinear resulting neuron activation functions should be non-monotonic and be more general than the usual sigmoid function as well as the commonly used Lipschitz condition.

The equilibrium point $y^* = [y_1^*, y_2^*, ..., y_n^*]$ of system (1) will be shifted to the origin by the transformation $y(\cdot) = x(\cdot) - x^*$, transforms system (1) into the following form

$$dy(t) = [-Ay(t) + Bg(y(t-\tau)) + C\int_{-\infty}^{t} K(t-s)g(y(s))ds]dt + \sigma(t, y(t))dw(t), \quad t \neq t_k \tag{2}$$

$$y(t_k) = I_k y(t_k^-), \quad t = t_k, \quad k = 1, 2, ...$$
$$y(t_0 + t) = \psi(t), \quad t \in [-\tau, 0]$$

where $y = [y_1, y_2, ..., y_n]^T$, $A = diag[a_1, a_2, ..., a_n]$, $B = [b_{ij}]$, $C = [c_{ij}]$, $K(t-s) = diag[k_1(t-s), k_2(t-s), ..., k_n(t-s)]$, $g(y) = [g_1(y_1), g_2(y_2), ..., g_n(y_n)]$ with $g_j(y_j(t)) = f_j(y_j(t) + x_j^*) - f_j(x_j^*)$. Note that since each function $f_j(\cdot)$ satisfies the assumptions (C_1) and (C_2), hence each $g_j(\cdot)$ satisfies

$$g_j^2(\xi_j) \leq L_j^2 \xi^2, \quad \xi_j g_j(\xi_j) \geq \frac{g_j^2(\xi_j)}{L_j}\ \forall \xi_j \in \mathbb{R}, \quad g_j(0) = 0$$

(C3) There exist a constant matrix D_0 such that

$$trace[\sigma^T(t, y(t))\sigma(t, y(t))] \leq y^T(t)\ D_0\ y(t)$$

Lemma 2.2 [1] (S-Procedure) Let $T_i \in \mathbb{R}^{n\times n}$ $(i = 0, 1, ..., p)$ be symmetric matrices. The conditions on T_i, $(i = 0, 1, ..., p)$

$$\alpha^T T_0 \alpha > 0, \quad \forall \alpha \neq 0\ s.t.\ \alpha^T T_i \alpha \geq 0\ (i = 0, 1, ..., p)$$

hold, if there exist $\tau_i \geq 0$ $(i = 0, 1, ..., p)$ such that

$$T_0 - \sum_{i=1}^{p} \tau_i T_i > 0$$

Lemma 2.3 Let U, V, W and M be real matrices of appropriate dimensions with M satisfying $M = M^T$, then

$$M + UVW + W^T V^T U^T < 0, \quad \forall \quad V^T V \leq I$$

if and only if there exist a scalar $\epsilon > 0$ such that

$$M + \epsilon^{-1} UU^T + \epsilon W^T W < 0$$

Definition 2.4 [25] The function $V : [t_0, \infty) \times \mathbb{R}^n \to \mathbb{R}^+$ belongs to class v_0 if

(1) the function V is continuous on each of the sets $[t_{k-1}, t_k) \times \mathbb{R}^n$ and for all $t \geq t_0$, $V(0, t) \equiv 0$;

(2) $V(x, t)$ is locally Lipschitzian in $x \in \mathbb{R}^n$;

(3) for each $k = 1, 2, ...$, there exist finite limits

$$\lim_{(q,t)\to(x,t_k^-)} V(q,t) = V(x, t_k^-)$$
$$\lim_{(q,t)\to(x,t_k^+)} V(q,t) = V(x, t_k^+)$$

with $V(x, t_k^+) = V(x, t_k)$ satisfied.

In the following section, we will develop delay-dependent condition for the given system such that the origin of the delayed stochastic Hopfield neural network (2) is asymptotically stable.

3. Asymptotic Stability Criterion

Before discussing the stability analysis of the problem, we firstly introduce the Ito's formula for a general stochastic system.

Let $V(y(t), t) : \mathcal{C}([-\tau, 0], \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}_+)$ be a positive function which is continuously twice differentiable in y and once differentiable in t. Thus, an operator $\mathcal{L}$ acting on $V(y(t), t)$, is defined by

$$\mathcal{L}V(y(t), t) = V_t(y(t), t) + V_y(y(t), t)[-Ay(t) + Bg(y(t-\tau)) + C\int_{-\infty}^{t} K(t-s)g(y(s))ds] + \frac{1}{2}trace[\sigma^T(t, y(t))V_{yy}(y(t), t)\sigma(t, y(t))] \quad (3)$$

where

$$V_t(y(t), t) = \frac{\partial V(y(t), t)}{\partial t}, \quad V_y(y(t), t) = \Big(\frac{\partial V(y(t), t)}{\partial y_1}, \frac{\partial V(y(t), t)}{\partial y_2}, ..., \frac{\partial V(y(t), t)}{\partial y_n}\Big)$$
$$V_{yy}(y(t), t) = \Big(\frac{\partial^2 V(y(t), t)}{\partial y_i \partial y_j}\Big)_{n\times n}$$

Now, the following theorem gives a new stability criterion for system (2) without uncertain parameters.

Theorem 3.1 Suppose that the assumption $(C1) - (C3)$ is satisfied. If there exist matrices

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \geq 0 \text{ with } P_{11} > 0, \quad R = \begin{bmatrix} R_{11} & R_{12} \\ R_{12}^T & R_{22} \end{bmatrix} \geq 0$$

$Q_1 > 0, Q_2 > 0$, H_1, H_2, H_3, H_4, diagonal matrices $E > 0$, $S > 0$ and a positive scalar $\rho > 0$ such that the following inequalities hold:

$$P < \rho I \tag{4}$$

$$I_k^T P_{11} I_k + 2 I_k^T P_{12} I_k + I_k^T P_{22} I_k - P_{11} - 2P_{12} - P_{22} < 0 \tag{5}$$

$$\Omega = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & -\tau A^T P_{12}^T + \tau P_{22}^T & -\tau H_1 & -\tau A^T R_{22} \\ * & \Xi_{22} & \Xi_{23} & -H_4^T & -\tau P_{22}^T & -\tau H_2 & 0 \\ * & * & \Xi_{33} & 0 & \tau B^T P_{12} & -\tau H_3 & -\tau B^T R_{22} \\ * & * & * & -E & \tau C^T P_{12} & -\tau H_4 & -\tau C^T R_{22} \\ * & * & * & * & -\tau R_{11} & -\tau R_{12} & 0 \\ * & * & * & * & * & -\tau R_{22} & 0 \\ * & * & * & * & * & * & -\tau R_{22} \end{bmatrix} < 0 \tag{6}$$

where

$\Xi_{11} = -P_{11}A - A^T P_{11}^T + P_{12} + P_{12}^T + \rho D_0 + Q_1 + LEL + \tau R_{11} + H_1 + H_1^T - \tau R_{12} A$

$\Xi_{12} = P_{12} - H_1 + H_2^T, \quad \Xi_{13} = P_{11}B + H_3^T + \tau R_{12} B$

$\Xi_{14} = P_{11}C + H_4^T + \tau R_{12} C, \quad \Xi_{22} = -Q_1 - H_2 - H_2^T, \quad \Xi_{23} = LS - H_3^T, \quad \Xi_{33} = -Q_2 - 2S,$

$L = \{l_1, l_2, ..., l_n\}$. Then the origin of system (2) is the unique equilibrium point and it is globally asymptotically stable.

Proof. Define new state variables

$$\begin{aligned} g_1(t) &= -Ay(t) + Bg(y(t-\tau)) + C \int_{-\infty}^{t} K(t-s) g(y(s)) ds \qquad (7) \\ g_2(t) &= \sigma(t, y(t)) \qquad (8) \end{aligned}$$

To prove the asymptotic stability result, let us consider the following Lyapunov functional candidate for system (2) as

$$V_1 = \delta_1^T(t) P \delta_1(t), \quad V_2 = \int_{t-\tau}^{t} y^T(s) Q_1 y(s) ds, \quad V_3 = \int_{t-\tau}^{t} g^T(y(s)) Q_2 g(y(s)) ds$$

$$V_4 = \int_{-\tau}^{0} \int_{t-\tau}^{t} \delta_2^T(s) R \delta_2(s) ds d\sigma, \quad V_5 = \sum_{j=1}^{n} e_j \int_{0}^{\infty} k_j(\xi) \int_{t-\xi}^{t} g_j^2(y_j(\gamma)) d\gamma d\xi \tag{9}$$

$$\text{where} \quad P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \geq 0 \text{ with } P_{11} > 0, \quad R = \begin{bmatrix} R_{11} & R_{12} \\ R_{12}^T & R_{22} \end{bmatrix} \geq 0$$

$$\text{and} \quad \delta_1(t) = \begin{bmatrix} y(t) \\ \int_{t-\tau}^t y(s)ds \end{bmatrix}, \quad \delta_2(t) = \begin{bmatrix} y(t) \\ g_1(t) \end{bmatrix},$$

By Newton-Leibnitz formula, the following equation is true for any matrices H_i $(i = 1, 2, 3, 4)$ with appropriate dimensions:

$$\begin{aligned} 2\Big[y^T(t)H_1 + y^T(t-\tau)H_2 + g^T(y(t-\tau))H_3 + \Big(\int_{-\infty}^t K(t-s)g(y(s))ds\Big)H_4\Big] \\ \times \Big[y(t) - y(t-\tau) - \int_{t-\tau}^t g_1(s)ds\Big] = 0 \end{aligned} \tag{10}$$

when $t \neq t_k$ the derivative of V can be calculated by using Ito's differential formula. Then the trajectories of the system (2) is given as:

$$\begin{aligned} \mathcal{L}V_1 &= 2\delta_1^T(t)P\dot{\delta_1}(t) \\ &= 2\begin{bmatrix} y(t) \\ \\ \int_{t-\tau}^t y(s)ds \end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} \\ \\ P_{12}^T & P_{22} \end{bmatrix} \begin{bmatrix} -Ay(t) + Bg(y(t-\tau)) + C\int_{-\infty}^t K(t-s)g(y(s))ds \\ \\ y(t) - y(t-\tau) \end{bmatrix} \\ &= -2y^T(t)P_{11}Ay(t) + 2y^T(t)P_{11}Bg(y(t-\tau)) + 2y^T(t)P_{11}C\Big(\int_{-\infty}^t K(t-s)g(y(s))ds\Big) \\ &+ 2y^T(t)P_{12}y(t) - 2y^T(t)P_{12}y(t-\tau) - 2\int_{t-\tau}^t y^T(s)P_{12}^T Ay(t)ds + \int_{t-\tau}^t y^T(s)P_{12}^T B \\ &\times g(y(t-\tau))ds + 2\int_{t-\tau}^t y^T(s)P_{22}y(t)ds - 2\int_{t-\tau}^t y^T(s)P_{22}y(t-\tau)ds + 2\Big(\int_{t-\tau}^t y^T(s)ds\Big) \\ &\times P_{12}^T C\Big(\int_{-\infty}^t K(t-s)g(y(s))ds\Big) + trace[\sigma^T(t, y(t))P\sigma(t, y(t))] \end{aligned} \tag{11}$$

$$\mathcal{L}V_2 = y^T(t)Q_1y(t) - y^T(t-\tau)Q_1y(t-\tau) \tag{12}$$

$$\mathcal{L}V_3 = g^T(y(t))Q_2g(y(t)) - g^T(y(t-\tau))Q_2g(y(t-\tau)) \tag{13}$$

$$\mathcal{L}V_4 = \tau\delta_2^T(t)R\delta_2(t) - \int_{t-\tau}^t \delta_2^T(s)R\delta_2(s)ds \tag{14}$$

$$\begin{aligned} \mathcal{L}V_5 &= \sum_{j=1}^n e_j \int_0^\infty k_j(\xi)g_j^2(y_j(t))d\xi - \sum_{j=1}^n e_j \int_0^\infty k_j(\xi)g_j^2(y_j(t-\xi))d\xi \\ &= g^T(y(t))Eg(y(t)) - \sum_{j=1}^n e_j \int_0^\infty k_j(\xi)d\xi \int_0^\infty k_j(\xi)g_j^2(y_j(t-\xi))d\xi \\ &\leq y^T(t)LELy(t) - \sum_{j=1}^n e_j\Big(\int_0^\infty k_j(\xi)g_j^2(y_j(t-\xi))d\xi\Big)^2 \\ &= y^T(t)LELy(t) - \Big(\int_{-\infty}^t K(t-s)g(y(s))ds\Big)E\Big(\int_{-\infty}^t K(t-s)g(y(s))ds\Big) \end{aligned} \tag{15}$$

It is noted from $(C2)$ that,

$$g_i(y_i(t-\tau))[g_i(y_i(t-\tau)) - l_i y_i(t-\tau)] \leq 0, \quad i = 1, 2, ..., n \tag{16}$$

Now, by applying the S-procedure, we find that system (2) is asymptotically stable, if there exist $S = diag\{s_1, s_2, ..., s_n\}$ such that

$$\begin{aligned}
\mathcal{L}V &= \mathcal{L}V_1 + \mathcal{L}V_2 + \mathcal{L}V_3 + \mathcal{L}V_4 + \mathcal{L}V_5 + 2\Big[y^T(t)H_1 + y^T(t-\tau)H_2 + g^T(y(t-\tau))H_3 \\
&\quad + \Big(\int_{-\infty}^{t} K(t-s)g(y(s))ds\Big)H_4\Big]\Big[y(t) - y(t-\tau) - \int_{t-\tau}^{t} g_1(s)ds\Big] \\
&\leq \mathcal{L}V_1 + \mathcal{L}V_2 + \mathcal{L}V_3 + \mathcal{L}V_4 + \mathcal{L}V_5 + 2\Big[y^T(t)H_1 + y^T(t-\tau)H_2 + g^T(y(t-\tau))H_3 \\
&\quad + \Big(\int_{-\infty}^{t} K(t-s)g(y(s))ds\Big)H_4\Big]\Big[y(t) - y(t-\tau) - \int_{t-\tau}^{t} g_1(s)ds\Big] \\
&\quad -2\sum_{i=1}^{n} s_i g_i(y_i(t-\tau))(g_i(y_i(t-\tau)) - l_i y_i(t-\tau)) \\
&\leq \mathcal{L}V_1 + \mathcal{L}V_2 + \mathcal{L}V_3 + \mathcal{L}V_4 + \mathcal{L}V_5 + 2\Big[y^T(t)H_1 + y^T(t-\tau)H_2 + g^T(y(t-\tau))H_3 \\
&\quad + \Big(\int_{-\infty}^{t} K(t-s)g(y(s))ds\Big)H_4\Big]\Big[y(t) - y(t-\tau) - \int_{t-\tau}^{t} g_1(s)ds\Big] \\
&\quad -2g^T(y(t-\tau(t)))Sg(y(t-\tau(t))) + 2g^T(y(t-\tau(t)))Sg(y(t-\tau(t)))
\end{aligned}$$

$$\begin{aligned}
&\leq y^T(t)[-P_{11}A - A^T P_{11}^T + P_{12} + P_{12}^T + \rho D_0 + Q_1 + LEL + \tau R_{11} + H_1 + H_1^T - \tau R_{12}A] \\
&\quad \times y(t) + y^T(t)[P_{12} - H_1 + H_2^T]y(t-\tau(t)) + y^T(t)[P_{11}B + H_3^T + \tau R_{12}B]g(y(t-\tau)) \\
&\quad + y^T(t)[P_{11}C + H_4^T + \tau R_{12}C]\Big(\int_{-\infty}^{t} K(t-s)g(y(s))ds\Big) + y^T(t)[-\tau A^T P_{12}^T B + \tau P_{22}^T \\
&\quad \times \Big(\int_{t-\tau}^{t} y(s)ds\Big) + y^T(t)[-\tau H_1]\Big(\int_{t-\tau}^{t} g_1(s)ds\Big) + y^T(t-\tau)[-Q_1 - H_2 - H_2^T] \\
&\quad \times y(t-\tau) + y^T(t-\tau)[-H_3^T + LS]g(y(t-\tau)) + y^T(t-\tau)[-\tau P_{22}^T]\Big(\int_{t-\tau}^{t} y(s)ds\Big) \\
&\quad + y^T(t-\tau)[-\tau H_2]\Big(\int_{t-\tau}^{t} g_1(s)ds\Big) + g^T(y(t-\tau))[-Q_2 - 2S]g(y(t-\tau)) + y^T(t-\tau)) \\
&\quad \times [-H_4^T]\Big(\int_{-\infty}^{t} K(t-s)g(y(s))ds\Big) + g^T(y(t-\tau)[\tau B^T P_{12}]\Big(\int_{t-\tau}^{t} y(s)ds\Big) \\
&\quad + g^T(y(t-\tau)[-\tau H_3]\Big(\int_{t-\tau}^{t} g_1(s)ds\Big) + \Big(\int_{-\infty}^{t} K(t-s)g(y(s))ds\Big)^T[-E]
\end{aligned}$$

$$
\begin{aligned}
&\times\Big(\int_{-\infty}^{t} K(t-s)g(y(s))ds\Big) + \Big(\int_{-\infty}^{t} K(t-s)g(y(s))ds\Big)^T[\tau C^T P_{12}]\Big(\int_{t-\tau}^{t} y(s)ds\Big)\\
&+\Big(\int_{-\infty}^{t} K(t-s)g(y(s))ds\Big)^T[-\tau H_4]\Big(\int_{t-\tau}^{t} g_1(s)ds\Big) + \Big(\int_{t-\tau}^{t} y(s)ds\Big)^T[-\tau R_{11}]\\
&\times\Big(\int_{t-\tau}^{t} y(s)ds\Big) + \Big(\int_{t-\tau}^{t} y(s)ds\Big)^T[-\tau R_{12}]\Big(\int_{t-\tau}^{t} g_1(s)ds\Big) + \Big(\int_{t-\tau}^{t} g_1(s)ds\Big)^T[-\tau R_{22}]\\
&\times\Big(\int_{t-\tau}^{t} g_1(s)ds\Big) \qquad (17)
\end{aligned}
$$

It is easy to see

$$
y(t) - y(t-\tau) - \int_{t-\tau}^{t} g_1(s)ds \;=\; \int_{t-\tau(t)}^{t} g_1(s)ds = \frac{1}{\tau}\int_{t-\tau(t)}^{t} \tau(\tau^{-1}\tau(t)g_1(s))ds \quad (18)
$$

This together with (16), implies

$$
\mathcal{L}V \;=\; \frac{1}{\tau}\int_{t-\tau(t)}^{t} \beta^T(t,s)\Omega\beta(t,s)ds
$$

where $\beta^T(t,s) \;=\; [y^T(t)\;\; y^T(t-\tau)\;\; g^T(y(t-\tau))\;\; \Big(\int_{-\infty}^{t} K(t-s)g(y(s))ds\Big)^T\;\; y^T(s)\;\; g_1^T(s)]$

This implies that $\mathcal{L}V(y(t),t) < 0$. When $t = t_k$, we obtain the following result:

$$
V(y(t_k),(t_k)) - V(y(t_k),t_k^-) = \delta_1^T(t_k)\begin{bmatrix} P_{11} & P_{12}\\ P_{12}^T & P_{22}\end{bmatrix}\delta_1(t_k) - \delta_1^T(t_k^-)\begin{bmatrix} P_{11} & P_{12}\\ P_{12}^T & P_{22}\end{bmatrix}\delta_1(t_k^-)
$$

$$
\begin{aligned}
&= y^T(t_k^-)\Big\{I_k^T\begin{bmatrix} P_{11} & P_{12}\\ P_{12}^T & P_{22}\end{bmatrix} I_k - \begin{bmatrix} P_{11} & P_{12}\\ P_{12}^T & P_{22}\end{bmatrix}\Big\}y(t_k^-)\\
&= y^T(t_k^-)I_k^T P_{11} I_k y(t_k^-) + y^T(t_k^-)I_k^T P_{12} I_k y(t_k^-) + y^T(t_k^-)I_k^T P_{12}^T I_k y(t_k^-)\\
&+ y^T(t_k^-)I_k^T P_{22} I_k y(t_k^-) - y^T(t_k^-)P_{11}y(t_k^-) - y^T(t_k^-)P_{12}y(t_k^-)\\
&- y^T(t_k^-)P_{12}^T y(t_k^-) - y^T(t_k^-)P_{22}y(t_k^-)\\
&= y^T(t_k^-)\Big[I_k^T P_{11} I_k + 2I_k^T P_{12} I_k + I_k^T P_{22} I_k - P_{11} - 2P_{12} - P_{22}\Big]y(t_k^-)
\end{aligned}
$$

Based on the Lyapunov stability theorem, it follows that the delayed stochastic Hopfield neural network (2) is globally asymptotically stable in the mean square. The proof of the theorem is completed. □

4. Robust Asymptotic Stability Criterion

Consider the system (2) with norm bounded parameter uncertainties that is

$$\begin{aligned} dy(t) &= [-(A+\Delta A(t))y(t) + (B+\Delta B(t))g(y(t-\tau)) + (C+\Delta C(t))\int_{-\infty}^{t} K(t-s) \\ &\qquad g(y(s))ds]dt + \sigma(t, y(t))dw(t), \quad t \neq t_k \qquad (19) \\ y(t_k) &= I_k y(t_k^-), \quad t = t_k, \quad k = 1, 2, ... \end{aligned}$$

where $A+\Delta A(t)$, $B+\Delta B(t)$ and $C+\Delta C(t)$ are of the following structure:

$$[\Delta A(t) \ \ \Delta B(t) \ \ \Delta C(t)] = MF(t)[N_1 \ \ N_2 \ \ N_3]$$

where M, N_1, N_2, N_3 are known constant matrices with appropriate dimensions and bounded which satisfies

$$F^T(t)F(t) \leq I, \quad t \geq 0$$

Theorem 3.2 Suppose that the assumption $(C1)-(C3)$ is satisfied. If there exist matrices

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \geq 0 \text{ with } P_{11} > 0, \quad R = \begin{bmatrix} R_{11} & R_{12} \\ R_{12}^T & R_{22} \end{bmatrix} \geq 0$$

$Q_1 > 0, Q_2 > 0$, H_1, H_2, H_3, H_4, diagonal matrices $E > 0$, $S > 0$ and four positive scalars $\rho > 0$, $\epsilon_1 > 0$, $\epsilon_2 > 0$, $\epsilon_3 > 0$ such that the following inequalities hold:

$$P < \rho I \qquad (20)$$

$$I_k^T P_{11} I_k + 2I_k^T P_{12} I_k + I_k^T P_{22} I_k - P_{11} - 2P_{12} - P_{22} < 0 \qquad (21)$$

$$\Omega_1 =$$

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} & -\tau A^T P_{12}^T + \tau P_{22}^T & -\tau H_1 & P_{11}M & 0 & -\tau A^T R_{22} & 0 \\ * & \Xi_{22} & \Xi_{23} & -H_4^T & -\tau P_{22}^T & -\tau H_2 & 0 & 0 & 0 & 0 \\ * & * & \Theta_{33} & 0 & \tau B^T P_{12} & -\tau H_3 & 0 & 0 & -\tau B^T R_{22} & 0 \\ * & * & * & \Theta_{44} & \tau C^T P_{12} & -\tau H_4 & 0 & 0 & -\tau C^T R_{22} & 0 \\ * & * & * & * & -\tau R_{11} & -\tau R_{12} & 0 & \tau R_{12}M & 0 & 0 \\ * & * & * & * & * & -\tau R_{22} & P_{22}M & 0 & 0 & 0 \\ * & * & * & * & * & * & -\epsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\epsilon_2 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\tau R_{22} & \tau R_{22}M \\ * & * & * & * & * & * & * & * & * & -\epsilon_3 I \end{bmatrix}$$

$$< 0 \tag{22}$$

where

$$\begin{aligned} \Theta_{11} &= -P_{11}A - A^T P_{11}^T + P_{12} + P_{12}^T + \rho D_0 + Q_1 + LEL + \tau R_{11} + H_1 + H_1^T - \tau R_{12}A \\ &\quad + \epsilon_1 N_1^T N_1 + \tau\epsilon_2 N_1^T N_1 + \tau\epsilon_3 N_1^T N_1 \\ \Theta_{13} &= P_{11}B + H_3^T + \tau R_{12}B - \epsilon_1 N_1^T N_2 - \tau\epsilon_2 N_1^T N_2 - \tau\epsilon_3 N_1^T N_2, \\ \Theta_{14} &= P_{11}C + H_4^T + \tau R_{12}C - \epsilon_1 N_1^T N_3 - \tau\epsilon_2 N_1^T N_3 - \tau\epsilon_3 N_1^T N_3, \\ \Theta_{33} &= -Q_2 - 2S + \epsilon_1 N_2^T N_2 + \tau\epsilon_2 N_2^T N_2 + \tau\epsilon_3 N_2^T N_2, \\ \Theta_{44} &= -E + \epsilon_1 N_3^T N_3 + \tau\epsilon_2 N_3^T N_3 + \tau\epsilon_3 N_3^T N_3 \end{aligned}$$

and Ξ_{22}, Ξ_{23}, L are stated as in Theorem 3.1. Then the origin of system (2) is the unique equilibrium point and it is globally asymptotically stable.

Proof. In order to prove the robust asymptotic stability, we use the same Lyapunov-Krasovskii functional as defined in (9). By replacing A, B and C in (2) with $A + \Delta A(t), B + \Delta B(t)$ and $C + \Delta C(t)$, respectively and by Ito's differential formula, we can calculate the trajectories of the system (19), then we have

$$\Omega_1 = \Omega + \epsilon_1^{-1}\begin{bmatrix} P_{11}M \\ 0 \\ 0 \\ 0 \\ 0 \\ P_{22}M \end{bmatrix} \times \begin{bmatrix} M^TP_{11} & 0 & 0 & 0 & 0 & M^TP_{22} \end{bmatrix} + \epsilon_1 \begin{bmatrix} -N_1 \\ 0 \\ N_2 \\ N_3 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} -N_1^T & 0 & N_2^T & N_3^T & 0 & 0 \end{bmatrix}$$

$$+\tau\epsilon_2^{-1}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ R_{12}M \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 & 0 & M^TR_{12} & 0 \end{bmatrix} + \tau\epsilon_2 \begin{bmatrix} -N_1 \\ 0 \\ N_2 \\ N_3 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} -N_1^T & 0 & N_2^T & N_3^T & 0 & 0 \end{bmatrix}$$

$$+\tau\epsilon_3^{-1}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ R_{22}M \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & M^TR_{22} \end{bmatrix} + \tau\epsilon_3 \begin{bmatrix} -N_1 \\ 0 \\ N_2 \\ N_3 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} -N_1^T & 0 & N_2^T & N_3^T & 0 & 0 \end{bmatrix}$$

Therefore, under condition (20)-(22), system (19) is robustly globally asymptotically stable with respect to the uncertain parameters $\Delta A(t)$, $\Delta B(t)$ and $\Delta C(t)$. This completes the proof of the theorem. $\square$

If we neglect the impulsive term and stochastic perturbations in (2), then it reduces to

$$\dot{y}(t) = -Ay(t) + Bg(y(t-\tau)) + C\int_{-\infty}^{t} K(t-s)g(y(s))ds \tag{23}$$

Corollary 3.3 Suppose that the assumption $(C1)-(C3)$ is satisfied. If there exist matrices

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \geq 0 \text{ with } L_{11} > 0, \quad R = \begin{bmatrix} R_{11} & R_{12} \\ R_{12}^T & R_{22} \end{bmatrix} \geq 0$$

$Q_1 > 0, Q_2 > 0$, H_1, H_2, H_3, H_4 and diagonal matrices $E > 0$, $S > 0$ such that the following inequalities hold:

$$\Omega = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & -\tau A^T P_{12}^T + \tau P_{22}^T & -\tau H_1 & -\tau A^T R_{22} \\ * & \Xi_{22} & \Xi_{23} & -H_4^T & -\tau P_{22}^T & -\tau H_2 & 0 \\ * & * & \Xi_{33} & 0 & \tau B^T P_{12} & -\tau H_3 & -\tau B^T R_{22} \\ * & * & * & -E & \tau C^T P_{12} & -\tau H_4 & -\tau C^T R_{22} \\ * & * & * & * & -\tau R_{11} & -\tau R_{12} & 0 \\ * & * & * & * & * & -\tau R_{22} & 0 \\ * & * & * & * & * & * & -\tau R_{22} \end{bmatrix} < 0 \tag{24}$$

where

$$\Xi_{11} = -P_{11}A - A^T P_{11}^T + P_{12} + P_{12}^T + Q_1 + LEL + \tau R_{11} + H_1 + H_1^T - \tau R_{12}A$$

$$\Xi_{12} = P_{12} - H_1 + H_2^T, \quad \Xi_{13} = P_{11}B + H_3^T + \tau R_{12}B$$

$$\Xi_{14} = P_{11}C + H_4^T + \tau R_{12}C, \quad \Xi_{22} = -Q_1 - H_2 - H_2^T, \quad \Xi_{23} = LS - H_3^T, \quad \Xi_{33} = -Q_2 - 2S,$$

$L = diag\{l_1, l_2, ..., l_n\}$. Then the origin of system (2) is the unique equilibrium point and it is globally asymptotically stable.

Proof. By arguing similar to the proof of Theorem 1, we can show that the equilibrium point of system (2) is globally asymptotically stable in the mean square. This completes the proof of the theorem. □

Remark 3.4 The authors Chen and Cao, [2] discussed the global asymptotic stability of delayed Hopfield neural networks. Wan et al. investigated the mean square exponential stability of stochastic delayed Hopfield neural networks. In [18], Wang et al. proposed the robust stability for stochastic Hopfield neural networks with time delays and Zhang et al., [24, 26] obtained the global stability results for delayed Hopfield neural network. As a result, in all the above mentioned references, impulsive effect has not been taken into account. However, in our paper, we derived the delay-dependent stability results for stochastic Hopfield neural networks with impulsive effects. Therefore, our main result is new, quite effective and leads to less conservative results when compared with some existing works [2, 8, 19, 20].

Remark 3.5 It is noteworthy that in our paper, we employed several free weighting matrices and S-procedure to derive the delay-dependent stability criterion. The derived criterion is obtained in LMI forms whose feasibility can be readily checked by using the Matlab LMI toolbox. Different from the conventional stability criteria that depend on the M-matrix computation, no tuning of parameters will be needed when employing our LMI-based stability criteria. Moreover, two numerical examples with simulation results will show the effectiveness of the stability conditions in this paper.

5. Illustrated Examples

In this section, we provide two numerical examples to demonstrate the effectiveness of the main results presented in this paper.

Example 4.1 Consider a delayed stochastic Hopfield neural network (2) with parameters as:

$$A = \begin{bmatrix} 1.7679 & 0 \\ 0 & 1.8860 \end{bmatrix}, \quad B = \begin{bmatrix} -0.2376 & -0.4769 \\ -0.6707 & -0.7654 \end{bmatrix}, \quad C = \begin{bmatrix} -0.1052 & -0.5069 \\ -0.0257 & -0.2808 \end{bmatrix},$$

$$L = \begin{bmatrix} 0.5219 & 0 \\ 0 & 1.8993 \end{bmatrix}, \quad D_0 = \begin{bmatrix} 0.33 & 0 \\ 0 & 0.25 \end{bmatrix}, \quad I_k = I = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$$

It can be checked that system (2) satisfies the assumptions $(C1) - (C3)$. For the delay bound $\tau = 0.957$, we have obtained the following feasible solutions to the LMIs (4) - (6) in Theorem 1

$$P_{11} = \begin{bmatrix} 25.7039 & -1.6632 \\ -1.6632 & 39.0949 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} 4.3286 & -0.9483 \\ -0.9483 & 3.8977 \end{bmatrix}, \quad P_{22} = \begin{bmatrix} 8.8576 & -1.6157 \\ -1.6157 & 7.3501 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 18.9787 & -6.6658 \\ -6.6658 & 16.4166 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 29.9330 & 20.6811 \\ 20.6811 & 51.6907 \end{bmatrix}, \quad R_{11} = \begin{bmatrix} 21.3444 & -6.6192 \\ -6.6192 & 19.6772 \end{bmatrix}$$

$$R_{12} = \begin{bmatrix} 6.4801 & -1.9881 \\ -1.9881 & 4.5859 \end{bmatrix}, \quad R_{22} = \begin{bmatrix} 7.2495 & -2.6422 \\ -2.6422 & 3.5791 \end{bmatrix}, \quad H_1 = \begin{bmatrix} -1.5463 & 0.9452 \\ -1.2486 & 1.3717 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 1.6552 & 0.8485 \\ -0.6090 & 1.9241 \end{bmatrix}, \quad H_3 = \begin{bmatrix} 1.8527 & 1.2408 \\ 2.4549 & 1.7661 \end{bmatrix}, \quad H_4 = \begin{bmatrix} -1.0116 & 2.4402 \\ -1.5775 & 1.8661 \end{bmatrix}$$

$$S = \begin{bmatrix} 18.3098 & 0 \\ 0 & 8.9915 \end{bmatrix}, \quad E = \begin{bmatrix} 22.2513 & 0 \\ 0 & 16.6959 \end{bmatrix}, \quad \rho = 1.8663 \times 10^3$$

In order to show the significant improvement of our results, we summerize the comparisons between the previous works and the obtained result. For this example, the delay-dependent stability analysis in [3, 26, 27], cannot be satisfied for any $\tau > 0$. Table 1 shows the maximum upper bound of the previous works [8, 19, 20] as $0.4121, 1.7484$ and 1.7644, respectively. However, by theorem 1, we have that the origin of delayed stochastic Hopfield neural networks with impulsive effect is globally asymptotically stable for any constant allowable upper bound $\tau > 0$. Hence, it is clear that the proposed method shows the less conservativeness than the existing works [2, 8, 19, 20].

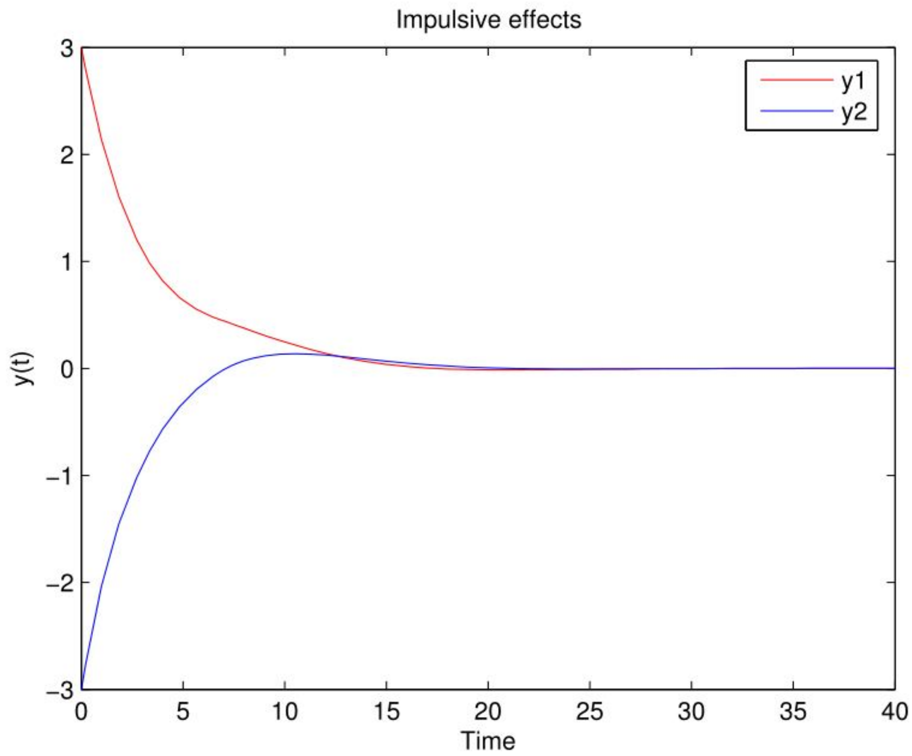

Fig. 1 State trajectories of y_1, y_2 for Example 1

Example 4.2 Consider a delayed stochastic uncertain Hopfield neural network (2) with the following parameters:

$$A = \begin{bmatrix} 0.7679 & 0 \\ 0 & 0.8860 \end{bmatrix}, \quad B = \begin{bmatrix} -0.1746 & -0.8642 \\ -0.2892 & -0.7300 \end{bmatrix}, \quad C = \begin{bmatrix} -0.8252 & -0.4912 \\ -0.4732 & -0.8858 \end{bmatrix},$$

$$M = \begin{bmatrix} 0.07051 & 0 \\ 0 & 0.0342 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.3526 & -0.1904 \\ 0.3322 & -0.1564 \end{bmatrix}, \quad N_2 = \begin{bmatrix} 0.2446 & 0.3674 \\ -0.1753 & 0.2956 \end{bmatrix},$$

Table 1: Maximum allowable bound of the delay

Method	Maximum upper bound of τ
In Ref [2, 26, 27]	-
In Ref [8]	0.4121
In Ref [19]	1.7484
In Ref [20]	1.7644
In this paper	for any large finite $\tau > 0$

$$N_3 = \begin{bmatrix} 0.1981 & -0.1313 \\ 0.1185 & 0.1645 \end{bmatrix}, \quad L = \begin{bmatrix} 0.07051 & 0 \\ 0 & 0.0342 \end{bmatrix}, \quad D_0 = \begin{bmatrix} 0.85 & 0 \\ 0 & 0.85 \end{bmatrix},$$

For $\tau = 0.957$ and by solving the LMIs (20)-(22) in Theorem 3.2, we get the following feasible solution as follows:

$$P_{11} = \begin{bmatrix} 133.1390 & -39.2261 \\ -39.2261 & 103.1396 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} 5.2825 & -5.7032 \\ -5.7032 & 10.4098 \end{bmatrix}, \quad P_{22} = \begin{bmatrix} 18.1026 & -10.6945 \\ -10.6945 & 25.6053 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 30.8319 & -27.8887 \\ -27.8887 & 47.6789 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 90.1163 & 28.7709 \\ 28.7709 & 147.2412 \end{bmatrix}, \quad R_{11} = \begin{bmatrix} 39.5332 & -37.1370 \\ -37.1370 & 66.5761 \end{bmatrix}$$

$$R_{12} = \begin{bmatrix} 21.2050 & -14.1850 \\ -14.1850 & 21.1791 \end{bmatrix}, \quad R_{22} = \begin{bmatrix} 31.5030 & -17.8344 \\ -17.8344 & 21.7477 \end{bmatrix}, \quad H_1 = \begin{bmatrix} -13.2142 & 4.3068 \\ -3.1365 & -1.7307 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 13.9993 & -1.7353 \\ -3.6838 & 8.4480 \end{bmatrix}, \quad H_3 = \begin{bmatrix} -9.5150 & 11.0562 \\ -15.8051 & 17.3187 \end{bmatrix}, \quad H_4 = \begin{bmatrix} 12.1911 & -1.3902 \\ -7.3453 & 12.0990 \end{bmatrix}$$

$$S = \begin{bmatrix} 72.1101 & 0 \\ 0 & 263.5610 \end{bmatrix}, \quad E = \begin{bmatrix} 491.5915 & 0 \\ 0 & 339.6219 \end{bmatrix}, \quad \rho = 7.1793 \times 10^3$$

$$\epsilon_1 = 26.1631, \quad \epsilon_2 = 15.5658, \quad \epsilon_3 = 16.3759.$$

Thus, all the conditions in Theorem 3.2 are satisfied. Therefore, the model (19) with above given parameters is globally asymptotically stable in the mean square.

6. Conclusion

This paper has studied the problem of stability analysis for a class of delayed stochastic uncertain Hopfield neural networks with distributed time-varying delays and impulses. A delay-dependent asymptotic stability condition is developed in terms of an LMI, which can be easily checked by using recently developed algorithms in solving LMIs. Finally, two numerical examples has been provided to demonstrate the usefulness and the reduced conservatism of the proposed results.

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