Characteristics of Item Replacement in Weibull Distribution

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Abstract *The aim of this paper is to study preventive replacement in order to increase system's MTBF by replacing item following the Weibull Distribution. Here, we discuss the periodic preventive replacement and random preventive replacement as preventive replacement. According to item preventive replacement following Weibull Distribution, based on the MTBF evaluation of item to study the characteristics of item replacement.*

Keywords *Weibull Distribution Preventive replacement MTBF*

1. Introduction

Outline and background of the preventive replacement theory
Notation used in this paper.
F(t): the failure distribution function of replacement item.

$$\overline{F}(t):1-F(t);f(t)=dF(t)/dt$$

G(t): the distribution function of the preventive replacement

$$\overline{G}(t):1-G(t);g(t)=dG(t)/dt$$

<t>: first moment or MTBF under the preventive replacement
$<t^n>$: n moment
μ : the replacement factor of preventive replacement
distribution function G(t)=1-exp(-μt)
T : time interval of preventive replacement
Γ(•): Gamma function;
Γ(•, •): in-complete Gamma function;
δ(•): Derta function
β : shape parameter of Weibull Distribution
η : scale parameter of Weibull Distribution
CV: coefficient of variation
τ : reliability improvement rate

Now let's gather up the result of fundamental and general theory about preventive replacement

(1) First moment or MTBF under preventive replacement

MTBF or the first moment<t> is given as:

$$<t>=\frac{\int_0^{\infty}\overline{F}(t)\overline{G}(t)dt}{1-\int_0^{\infty}\overline{F}(t)g(t)dt} \tag{1}$$

(2) Second moment and variance under preventive replacement

the Second moment$<t^2>$ is given as:

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$$<t^2>=\frac{2\{1-\int_0^\infty \overline{F}(t)g(t)dt\}\{\int_0^\infty t\overline{G}(t)\overline{F}(t)dt\}}{(1-\int_0^\infty \overline{F}(t)g(t)dt)^2}+\frac{2\{\int_0^\infty \overline{G}(t)\overline{F}(t)dt\}\{\int_0^\infty t\overline{F}(t)g(t)dt\}}{(1-\int_0^\infty \overline{F}(t)g(t)dt)^2} \quad (2)$$

thus the variance δ^2 is easily got from $\delta^2=<t^2>-<t>^2$

2. Periodic Preventive Replacement and Random

Preventive Replacement

The Periodic preventive replacement is the most general way in the preventive replacement, so that we study on this case, and then take up the random preventive replacement as extreme example

2.1 The Average $<t>_p$ and the 2nd Moment $<t^2>_p$ of the Periodic Preventive Replacement

If theb periodic preventive replacement is done at time T, non- preventive replacement $\overline{G}(t)$ is as Figure 1, then

$$\overline{G}(t)=\begin{cases}1 & 0\le t\le T\\ 0 & the\, other\end{cases} \quad (3)$$

$$g(t)=\delta(t-T) \quad (4)$$

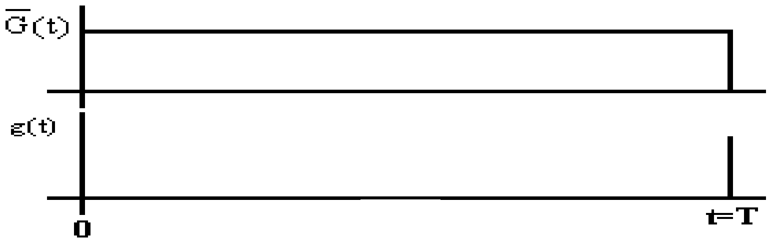

Figure 1

Consequently, the molecule and the denominator of (Eq.1) is described as following

$$\int_0^\infty \overline{G}(t)\overline{F}(t)dt=\int_0^T \overline{F}(t)dt$$

$$\int_0^\infty \overline{F}(t)g(t)dt=\int_0^\infty \overline{F}(t)\delta(t-T)dt=\overline{F}(T)$$

and

$$1-\int_0^\infty \overline{F}(t)g(t)dt=1-\overline{F}(T)=F(T)$$

Therefore the 1st moment $<t>_p$

$$<t>_p=\frac{\int_0^T \overline{F}(t)dt}{F(T)} \quad (5)$$

On the other hand, the 2nd moment $<t^2>_p$ can be also obtained in the same way.

$$<t^2>_p=\frac{2\int_0^T t\overline{F}(t)dt}{F(T)}+\frac{2T\overline{F}(T)\int_0^T \overline{F}(t)dt}{F(T)^2} \quad (6)$$

So the variance of preventive replacement can be obtained through formula (Eq.5) and (Eq.6).

2.2 The 1st Moment and the 2nd Moment of the Random Replacement

In the case of the random preventive replacement, as maintenance function we put G(t)=1-exp(-μ t) into formula (Eq.1) and (Eq.2), then 1st moment $<t>_R$ and 2nd moment $<t^2>_R$ are obtained by

$$< t >_R = \frac{\int_0^\infty \overline{F}(t)\exp(-\mu t)dt}{1-\mu\int_0^\infty \overline{F}(t)\exp(-\mu t)dt} \tag{7}$$

$$< t^2 >_R = \frac{2\int_0^\infty t\overline{F}(t)\exp(-\mu t)dt}{1-\mu\int_0^\infty \overline{F}(t)\exp(-\mu t)dt} + \frac{2(\int_0^\infty \overline{F}(t)\exp(-\mu t)dt)(\int_0^\infty t\overline{F}(t)\exp(-\mu t)dt)}{(1-\mu\int_0^\infty \overline{F}(t)\exp(-\mu t)dt)^2} \tag{8}$$

3. Applying to Weibull Distribution

When applying these theoretical, we consider preventive replacement characteristic from average value increase of the failure interval. We suppose the Weibull Distribution as

$$\overline{F}(t) = \exp\{-(t/\eta)^\beta\} \tag{9}$$

3.1 The Periodic Preventive Replacement

We insert it into (Eq.5), the average value is as follow:

$$< t >_p = \frac{\int_0^T \exp\{-(t/\eta)^\beta\}dt}{1-\exp\{-(T/\eta)^\beta\}} \tag{10}$$

Moreover, it can be rearrange as

$$< t >_p = \frac{\eta\Gamma[(1/\beta),(T/\eta)^\beta]}{\beta\Gamma[1,(T/\eta)^\beta]} \tag{11}$$

On the other hand, the 2nd moment can be got by (Eq.6)

$$< t^2 >_p = \frac{2\eta^2\Gamma[(2/\beta),(T/\eta)^\beta]}{\beta\Gamma[1,(t/\eta)^\beta]} + \frac{2T\eta\exp\{-(T/\eta)^\beta\}\Gamma[1/\beta,(T/\eta)^\beta]}{\beta\Gamma[1,(T/\eta)^\beta]^2} \tag{12}$$

3.2 The Random Preventive Replacement

We apply the Weibull Distribution into (Eq.7) and (Eq.8) yield the following:

Average value i.e.

$$< t >_R = \frac{\int_0^\infty \exp(-\lambda t)dt}{1-\mu\int_0^\infty \exp(-\lambda t)dt} \tag{13}$$

$$<t^2>_R = \frac{2\int_0^\infty t\cdot\exp(-\lambda t)dt}{1-\mu\int_0^\infty \exp(-\lambda t)dt} + \frac{2\mu(\int_0^\infty \exp(-\lambda t)dt)(\int_0^\infty t\cdot\exp(-\lambda t)dt)}{(1-\mu\int_0^\infty \exp(-\lambda t)dt)^2} \quad (14)$$

Simplify, here $(t/\eta)^\beta + \mu t = \lambda t$ is used

4. Consideration

4.1 Condition for Calculation

First, for convenience and simple expression, we suppose average of the Weibull Distribution E(t)= $\eta\Gamma(1/\beta+1)=1$ and normalize the real time.

If the periodic normalized time is T=0.1, the real time is $0.1\eta\Gamma(1/\beta+1)$. More once we define the replacement rate μ of the preventive replacement distribution function G(t)=1-exp(-μ t) in the random replacement

Because reverse of the μ is average replacement intervals. Suppose T is 0.1, μ is 10, it show random maintenance had 0.1 intervals in average, the real time is $0.1\eta\Gamma(1/\beta+1)$.Moreover in order to compare the preventive replacement characteristics, we use the evolution reliability improvement rate in MTBF and coefficient of variation in dispersion.

The reliability improvement rate is defined as following

$$\tau = \frac{<t>\eta\Gamma(1/\beta+1)}{\eta\Gamma(1/\beta+1)} = <t> - 1 \quad (15)$$

$$\tau = \frac{<t> - T_F}{T_F} = \frac{<t>}{T_F} - 1$$

And the coefficient of variation CV is defined as

$$CV = \frac{\sqrt{variance}}{average} = \sqrt{\frac{<t^2> - <t>^2}{<t>^2}} = \sqrt{\frac{<t^2>}{<t>^2} - 1} \quad (16)$$

4.2 Reliability improvement rate

In the case of the periodic replacement, according to formula (Eq.15) and $\eta\Gamma(1/\beta+1)=1$

Improvement rate of periodic replacement is

$$<\tau>_p = \frac{\eta\Gamma[(1/\beta),(T/\eta)^\beta]}{1-\exp(-(T/\eta)^\beta)} - 1 \quad (17)$$

About random replacement use the same way. Improvement rate of random replacement is

$$<\tau>_R = \frac{\int_0^\infty \exp(-\lambda t)dt}{1-\mu\int_0^\infty \exp(-\lambda t)dt} - 1 \quad (18)$$

4.3 Coefficient of Variation

The CV value of the periodic replacement is

$$cv_p = \sqrt{2\beta\{a+b\}-1}$$
$$a = \frac{\Gamma[2/\beta,(T\Gamma(\cdot))^\beta](1-\exp(-(T/\eta))^\beta)}{\Gamma[1/\beta,(T\Gamma(\cdot))^\beta]^2} \qquad (19)$$
$$b = \frac{\Gamma(\cdot)T\exp(-(T\Gamma(\cdot))^\beta)}{\Gamma[1/\beta,(T\Gamma(\cdot))^\beta]}$$

Here, $\Gamma(\bullet)$ is $\Gamma(1/\beta+1)$, $\eta=1/\Gamma(1/\beta+1)$

On the other hand, the CV value of the random replacement is

$$<cv>_R = \sqrt{\frac{2\int_0^\infty t\cdot\exp(-\lambda t)dt}{(\int_0^\infty \exp(-\lambda t)dt)^2}-1} \qquad (20)$$

4.4 Numerical Results

Table 1 Shows an example of calculated results in the periodic preventive replacement

T	β = 2.5		β = 3.0		β = 3.5		β = 4.0	
	τ_p	CV_p	τ_p	CV_p	τ_p	CV_p	τ_p	CV_p
0.1	479	0.664	1571	0.973	5085	1.234	16344	1.469
0.2	84	1.305	195	1.605	448	1.882	1020	2.141
0.3	30	1.688	57	2.000	107	2.287	200	2.554
0.4	14	1.986	23	2.305	38	2.595	62	2.862
0.5	7	2.250	11	2.575	17	2.865	25	3.128
0.6	4	2.510	6	2.844	8	3.136	11	3.397
0.7	2	2.791	3	3.147	4	3.451	5	3.716
0.8	1	3.126	2	3.526	2	3.866	2	4.158
0.9	1	3.557	1	4.052	1	4.481	1	4.857
1.0	0	4.144	0	4.837	0	5.486	0	6.099

Comparing to above table, we can get two picture as following:

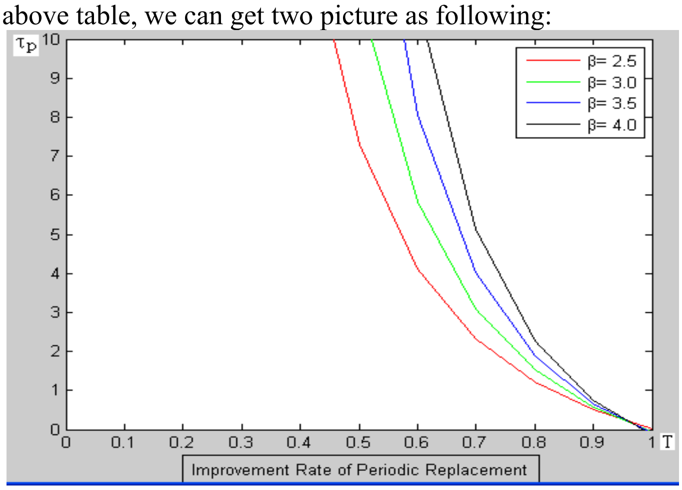

Figure 2 Improvement rate of periodic replacement

From the Figure 2 of above, we can see that: the interval T of exchange is smaller, reliability improvement rate τ_p is bigger. Turn over, the interval T of exchange is bigger, reliability improvement rate τ_p is smaller. This kind of trend has nothing to do with the size of β. The interval of exchange around one, the reliability improvement rate will become very small, the effect of preventive replacement will disappear. In a word, it is very important to exchange with the interval which is smaller to the average of item. The shape parameter β is bigger, the reliability improvement rate is bigger, the interval between breakdown and breakdown will become bigger.

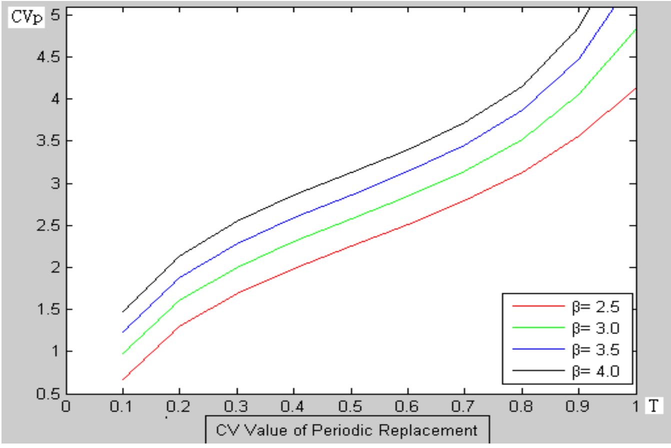

Figure 3 CV value of periodic replacement

From the Figure 3 of above, we can see that: the interval T of exchange is bigger, the coefficient of variation CV_P is bigger. Turn over, the interval T of exchange is smaller, the coefficient of variation CV_P is smaller. This kind of trend has nothing to do with the size of β.

The shape parameter β is bigger, the coefficient of variation is bigger. If we want to get the optimum value, we don't only consider the reliability improvement rate but also consider the coefficient of variation CV_P

Synthesize: It will obtain the good effect when short the interval time of exchange. This is in accordance with our experience.

5. Discussion and Conclusion

In this report, we obtain the results of theory about periodic preventive replacement and random preventive replacement and then introduced clearly the formula of average and variance in the both preventive replacement, more applied these result to the Weibull Distribution, and showed the example of replacement characteristics about preventive replacement.

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