# Research on Dynamic Pricing Model Based on e-Supply Chain Management*

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**Abstract** *With the influence of the uncertain factor in business trade, the research on dynamic pricing considering the random fluctuation of price has become an important project in managerial economics. In this paper, we introduce the uncertain factor, which produced by the random errors, into the pricing model of the dominant manufacturers. By introducing the expectation of retail price, variance and transfer price, we change the game of incomplete information into the game of complete information. Then we resolve the problem of optimal pricing about transfer price with extremum of function according to the optimal production and storage model.*

**Keywords** *Random fluctuation Dynamic pricing Incomplete information game Extremum of function*

## 1. Introduction

Based on the network and electronic commerce (EC), the internet can arrive any place of the world. It results in the substantial increase of demand or supply under non-prediction. At the same time, lower menu cost supports traders to change the price more frequently according to the condition of market. So it is less and less efficient to confirm the optimal price according to establish the static demand price model under the random fluctuation of demand and price.

Many scholars researched the dynamic pricing model. The article[4] discussed the dynamic pricing model of the dominant manufacturer after the time factor was added to the demand function. The article[5] suggested introducing uncertain factor into economic model and establishing the price model of rational expectation. The article[6] displayed the optimization of dynamic game and nonlinear pricing and the dominant firm price leadership model completely. Based on the above articles, this paper discuss the pricing model of the dominant manufacturer further with uncertain factor, which produced by the random errors in EC. By introducing uncertain expectation and variance, it changes the game of incomplete information into the game of complete information. Then it solves the problem of optimal pricing about transfer price with the extremum function method according to the optimal production and storage model.

## 2. Analyzing and Hypothesizing of the Model

Assumed that a new kind of production is in monopoly position, and there is only one manufacturer and one seller. Because of the new production, the manufacturer is in dominance and can predict the demand of market. The manufacturer confirms the transfer prices of its production according to the maximum profit. Then, the seller confirms the order quantity according to the maximum profit. This process can be considered as the incomplete information game with the decentralized decision. This paper considers uncertain factor as an endogenous variable and introduce it into the model. Then it assumes the demand in consumption market is

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related not only to the price, but also to the price fluctuation which is the uncertain factor of price, so it expresses price fluctuation with price variance.

## 3. Supply Chain Model

### 3.1 Optimal Order Strategy of Sellers

After manufacturer price the production $p_1$ with maximum profit according to some information, sellers choose the optimal ordering quantity. Supposed the lead time $T-K$ is fixed, and the period of order $T$ is constant. When sellers estimate the ordering quantity according to the profit maximization, they order $T-K$ days in advance. Then, the products will arrive after $K$ days.

After sellers introduce uncertain factor into economic model as inner variable, they can price according to the demand of the market in internet. The demand function of consumers in this period is:

$$q_s(t) = D(p_s(t), p_s^u, u_1) = a - bp_s(t) + \lambda p_s^u + u_1, \tag{1}$$

where $p_s(t)$ is the sell price at point t, and $q_s(t)$ is the demand amount at point t. Both of them are random PROCESS. $p_s^u$ is the variance of price (fixed), which reflects the fluctuation of price. If $\lambda$ is negative, consumers hate the price undulation. If $\lambda$ is positive, consumers like the price undulation. $u_1$ is a random disturbance. And the parameters $a, b > 0$. (And the parameters a and b are greater than zero.)

Because of the predomination of manufacturer, transferring price $p_1$ is assured. Seller assures sell price $p_s(t)$, and market gives the amount of demand $q_s(t)$. Assumed all the demands are satisfied under $p_s(t)$, the order quantity in this period is the accumulation of instantaneous demand $q_s(t)$, that is,

$$Q = \int_0^T q_s(t)dt, \tag{2}$$

The storage at point $x$ is

$$Q_S = Q - \int_0^x q_s(t)dt = \int_0^T q_s(t)dt - \int_0^x q_s(t)dt = \int_x^T q_s(t)dt, \tag{3}$$

If the storage cost of unit production is $c_2$ at unit time, the storage cost is

$$C_S = \int_0^T c_2 \cdot Q_S(x)dx = c_2\int_0^T (\int_x^T q_s(t)dt)dx = c_2 \iint_D q_s(t)dtdx,$$

$$= c_2\int_0^T (\int_0^t q_s(t)dx)dt = c_2\int_0^T t \cdot q_s(t)dt, \tag{4}$$

where $D$ is a triangle, surrounded by $x=0, x=T, t=x, t=T$. The purchase cost is

$$C_b = p_1 Q + C_1 = p_1\int_0^T q_s(t)dt + C_1, \tag{5}$$

where $C_1$ is fixed purchase cost, so total cost is

$$TC_s = C_b + C_S, \tag{6}$$

The revenue of the seller is

$$I_s = \int_0^T p_s(t)q_s(t)dt = \int_0^T [\frac{a}{b} - \frac{1}{b}q_s(t) + \frac{\lambda}{b}p_s^u + \frac{u_1}{b}]q_s(t)dt$$

$$= \int_0^T [\frac{a}{b} q_s(t) - \frac{1}{b} q_s^2(t) + \frac{\lambda}{b} p_s^u q_s(t) + \frac{u_1}{b} q_s(t)] dt \quad , \tag{7}$$

And the profit of the seller in order period is

$$R_s = I_s - TC_s = I - C_b - C_s$$

$$= \int_0^T [\frac{a}{b} q_s(t) - \frac{1}{b} q_s^2(t) + \frac{\lambda}{b} p_s^u q_s(t) + \frac{u_1}{b} q_s(t)] dt - p_1 \int_0^T q_s(t) dt - c_2 \int_0^T t \times q_s(t) dt - C_1$$

$$= \int_0^T [\frac{a}{b} q_s(t) - \frac{1}{b} q_s^2(t) + \frac{\lambda}{b} p_s^u q_s(t) + \frac{u_1}{b} q_s(t) - p_1 q_s(t) - c_2 t \times q_s(t)] dt - C_1 \quad . \tag{8}$$

Assumed supply quantity equal demand quantity in every period, the seller predicts and chooses supply quantity $q_s(t)$ and its sell price $p_s(t)$ to get maximize the profit $R_s$. If

$$F = \frac{a}{b} q_s(t) - \frac{1}{b} q_s^2(t) + \frac{\lambda}{b} p_s^u q_s(t) + \frac{u_1}{b} q_s(t) - p_1 q_s(t) - c_2 t \times q_s(t) .$$

We will get maximum profit $R_s$ with Euler Equation. Then

$$\frac{\partial F}{\partial q_s[t]} - \frac{d}{dt}(\frac{\partial F}{\partial q_s'[t]}) = 0 \tag{9}$$

Solving this differential equation, we can get the sell function at point *t* when the profit is maximum:

$$q_s(t) = \frac{1}{2}(a - bp_1 + \lambda p_s^u - bc_2 t + u_1) \tag{10}$$

$Q*$ is the optimal order quantity for sellers after the wholesale price is determined, that is,

$$Q* = \begin{cases} \int_0^T q_s(t) dt = -\frac{1}{4} bc_2 T^2 + (a - bp_1 + \lambda p_s^u + u_1) T & p_1 \le \frac{a + \lambda p_s^u - 0.5 bc_2 T + u_1}{b} \\ 0 & p_1 > \frac{a + \lambda p_s^u - 0.5 bc_2 T + u_1}{b}. \end{cases} \tag{11}$$

### 3.2 Rational Expectations of the Best Product Plan and Transferring Price That Manufacturer Determined

Assumed the demand function of the market and the order interval of the seller are open. Then, the order quantity of seller can be predicted, and the transferring price $p_1$ can be determined under the maximum profit. The manufacturer signs a contract for goods with the seller. According the contract, the seller purchase $Q*$ productions with $p_1$ and the manufacturer deliver at point $K$ .

For the manufacturers, they must consider sales revenue, production cost and storage cost. Sales revenue is multiplication of price and order quantity. Production cost depends on production rate, which is the quantity in unit time. And the production rate is higher, the production cost is more. Storage cost is determined by the finished productions and expiration time.

$x(t)$ is the cumulative production of product plan until point t without consideration of assets' time value. Because the product rate at point t (marginal product) is $x'(t)$, marginal product cost is $f(x'(t))$, marginal storage cost is $g(x(t))$ and marginal revenue is $\varphi(x'(t))$. Then, the total cost $C(x(t))$ is

$$C(x(t)) = \int_0^K [f(x'(t)) + g(x(t))]dt\,. \tag{12}$$

In order to confirm the actual expression of this function, we assume the following items:

(1) The cost of increasing one production in uniform time is in proportion to the product rate, where the coefficient is $2k_1$. So, we get $\frac{df(x'(t))}{dx'(t)} = 2k_1 x'(t)$, or,

$$f(x'(t)) = k_1[x'(t)]^2\,. \tag{13}$$

(2) The storage cost is in proportion to storage amount in unit time, where the coefficient is $k_2$, that is,

$$g(x(t)) = k_2 x(t) \tag{14}$$

The total revenue $I(x(t))$ from point 0 to point $K$ is:

$$I(x(t)) = \int_0^K \varphi(x'(t))dt = \int_0^K p_1 x'(t)dt \tag{15}$$

The profit $R$ from point 0 to point $K$ is:

$$R = R(x(t)) = I(x(t)) - C(x(t)) = \int_0^K \varphi(x(t))dt - \int_0^K [f(x'(t)) + g(x(t))]dt$$

$$= \int_0^K [p_1 x'(t) - k_1 x'^2(t) - k_2 x(t)]dt \tag{16}$$

$$x(0) = 0, \quad x(K) = Q^* \tag{17}$$

The problem may be attributed to find the maximum of functional $R(x(t))$, which can be solved by variation method. If $MR(t,x,x') = p_1 x' - k_1 x'^2 - k_2 x$, we will get the maximum profit $R$ according to Euler Equation, we have to let:

$$\frac{\partial MR}{\partial x} - \frac{d}{dt}\left(\frac{\partial MR}{\partial x'}\right) = 0$$

Then we can get the differential equation of second order:

$$-k_2 + 2k_1 x''(t) = 0$$

Under the constraint condition (17), its solution is:

$$x(t) = \frac{k_2}{4k_1}t^2 + \frac{4k_1 Q^* - k_2 K^2}{4k_1 K}t\quad. \tag{18}$$

It's the product plan of maximum profit. Obviously, with $x(t) \ge 0, 0 \le t \le K$, we can get

$$Q^* \ge \frac{k_2 K^2}{4k_1}\quad. \tag{19}$$

That is, when the order quantity of the sellers satisfies condition (19), the product plan confirmed by condition (18) is optimal. If $Q^* < \frac{k_2 K^2}{4k_1}$, we can delay the start time which make the time difference $\bar{K}$ satisfying equation $Q^* = \frac{k_2 \bar{K}^2}{4k_1}$ from $t_1$ to $K$, so the product plan is better.

After we substitute $x(t)$ in the equation (16), the maximum profit is solved according to $p_1$ and $x(t)$:

$$R_m = \frac{K^3k_2^2}{48k_1} - \frac{Kk_2Q^*}{2} + p_1Q^* - \frac{k_1(Q^*)^2}{K} \tag{20}$$

Take non-zero of $Q^*$ in equation (11) into equation (20), we get

$$R_m = \frac{K^3k_2^2}{48k_1} + \left(p_1 - \frac{Kk_2}{2}\right)\left[-\frac{1}{4}bc_2T^2 + (a - bp_1 + \lambda p_s^u + u_1)T\right]$$
$$-\frac{k_1}{K}\left[-\frac{1}{4}bc_2T^2 + (a - bp_1 + \lambda p_s^u + u_1)T\right]^2 \tag{21}$$

Obviously, R is a quadratic and convex function of price $p_1$. If we get maximum $R$ with appropriate $p_1$, that is $\frac{dR}{dp_1} = 0$, then

$$p_1 = \frac{-2b^2c_2k_1T^2 + 4a(K + 2bk_1T) + 4K(\lambda p_s^u + u_1) + 2bK^2k_2 - bc_2KT + 8bk_1T(\lambda p_s^u + u_1)}{8b(K + bk_1T)} \tag{22}$$

$p_1^e$ is the expectation of the best price $p_1$,

$$p_1^e = E(p_1) = \frac{4a(K + 2bk_1T) - 2b^2c_2k_1T^2 + 2bK^2k_2 - bc_2KT}{8b(K + bk_1T)} + \frac{K + 2bk_1T}{2b(K + bk_1T)}\lambda E(p_s^u) \ .$$

Assumed variance $p_s^u$ of transferring price $p_s$ is a constant. Then

$$p_1^e = \frac{4a(K + 2bk_1T) - 2b^2c_2k_1T^2 + 2bK^2k_2 - bc_2KT}{8b(K + bk_1T)} + \frac{K + 2bk_1T}{2b(K + bk_1T)}\lambda p_s^u \tag{23}$$

So, $p_1^e$ is made up of two terms. One is determined by the structure of model, the other involves retail price undulation. When $\lambda > 0$, $p_1^e$ is in proportion to $p_s^u$. $p_s^u$ is higher, $p_1^e$ is higher. It is coincident with the theory, which is the expectation revenue of asset increased with the risk. In a word, under the incomplete information, the expectation of economical people is scaled to uncertainty rather than related to it only.

## 4. Calculation Examples

$$q_s(t) = 970 - 3p_s(t) + u_1$$

Assumed $a = 1000, b = 3, \lambda = -2$ and $p_s^u = 15$ in equation (1), the demand function of the consumer in order cycle time is:

$$q_s(t) = 970 - 3p_s(t) + u_1$$

If $T = 7$, we can confirm the lead time is two days according to some conditions, such as transportation, so $K = T - 2 = 5$. The coefficient between rate of change and production rate $\frac{k_1}{2}$ is 1 when the product rate increase unit product cost, and the coefficient between storage cost and storage amount $k_2$ is 0.05. And the purchase cost $C_1$ is 100 Yuan, holding cost per unit production in unit time $c_2$ is 1 Yuan. Above data can be adjusted and modified according to the actual statistic and forecasting. Because zero is the average of $u_1$, we substitute mathematical expectation $q_s(t)$ for $q_s(t)$ when we calculated.

According to equation (23), we can get $p_1^e \approx 314.78 + 0.32\lambda p_s^u = 305.31$. And with $p_1^e \leq 321.58$ from equation (11), the optimal sale $Q*$ is 360 when the manufacturer determine price.

According to equation (21) and (8), the maximum profit of the manufacturers $R_m$ is 58028.6, and the maximum profit of the wholesalers $R_s$ is 1295.17.

From the data simulation, we can know that the manufacturers have the active power and control the profit mostly because of the open and dominant price under the monopoly and competition.

## 5. Conclusions

Comparing to definite situation，the hypothesis containing fluctuate price is more close to the fact. With the development of the electronic commerce, the forecast and decision should be more reliable and it is more important to study the random situations. We have made some effort on the related aspects in this paper and got the optimal strategy of pricing under the situation of the fluctuating price in random. (See Formula (23)) The application of this strategy is important to resolve the questions in commodity exchange with uncertainty factors

## References

[1] Luo Zhanghua, Liu Lu. Research on dynamic pricing mechanism in transactions[J]. *Beijing University of Aeronautics and Astronautics Journal* (Social Science Edition), 2005,18 (1).

[2] Kannan P K, Praveen K Kopalle. Dynamic Pricing on the Internet: Importance and Implications for Consumer Behaviour[J]. *International Journal of Electronic Commerce*, 2001, 5 (3): 63283.

[3] Gallego G, Van Ryzin G J. A Multiple Product Dynamic Pricing Problem with Applications to Network Yield Management [J]. *Operations Research*, 1997, (45): 24241.

[4] Liu Jian, Lai Ming-yong, Zhang Han-jiang. The optimization of supply chain ordering and pricing decisions based on the trend of demand [J]. *System Engineering*, 2003, 21 (5).

[5] Cao Caixia, Zhang Yan. Rational expectations model with uncertainties [J]. *Journal of the Beijing Union University* (Natural Science) 2004, 18 (1).

[6] Tang Xiaowo, Zeng Yong, Li Shiming, et al.. Management Economic Analysis -- Theory and Application [M]. Chengdu: University of Electronic Science and Technology of China Press, 2000.

[7] Jiang Qiyuan, Xie Jinxing, Ye jun. Mathematical model [M]. Beijing: Higher Education Press, 2003.