A new decision method for multi-criteria decision making with numerical values based on criteria reduction

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Annotation
The work is contribution a new decision method to address the challenge (large number of criteria) in multi-criteria decision making (MCDM) problems with numerical values. This new method involves criteria reduction based on the rough set theory and the relation of criteria values (tolerance and advantage relations). Using this method and building a discrenibility matrix for numerical value MCDM problems, find useful criteria and avoid useless criteria. Then, we find a new way to obtain the weights based on the discernibility matrix when criteria weights of alternatives are completely unknown. Later, we also propose a new method to rank the alternatives according to weighted combinatorial advantage values (WCAV). Finally, we use a realistic voting example to demonstrate the proposed method.

Key words: Multi-criteria decision making, Criteria reduction, Discernibility matrix, Obtain weight, Relation.

1 Introduction

With the development of information technology, most decision makers (DMs) face the problem of how to make a wise decision when there are massive data in the decision table. How can we filter information is a potential application and development area for MCDM/MAUT in an internet or mobile environment that was proposed by Wallenius et al. on *Management Science*, 2008 [1]. Undoubtedly, we need to find out the useful data that really affect the decision making also take to human subjective. The related problems have been one of the most popular research topics in decision making science since 2008 [2-4]. Massive data of MCDM problems contain three situations that are large number of criteria, large 4umber of alternatives or both. In this paper, we focus on the problems which are large number of criteria in the decision table.

Extracting useful information from large quantity of uncertain problems has become an important research field in computer science-attribute reduction [5-6]. Rough set theory has been recognized as one of the most powerful techniques to deal with uncertainty problems since its appearance in 1982 [7-8]. The original rough set approach validated to be very useful in dealing with discrete problems. Rough set theory [9-10] is based on equivalence relation and captures useful information from a great deal of information through attribute reduction is a basic research method.

Attribute (Criteria) reduction find useful information by using smaller criteria set $B \subseteq A$ which is to make the criteria set *A* describes replaced and described by B. In this paper, we want to find out the useful criteria set B from A, and use B to make a decision will address the problem proposed by Wallenius et al. on *Management Science*, 2008.

Obtaining criteria weights is also an important research topic of MCDM problems. For uncertain of criteria weights problems, there are several methods to obtain them, such as obtaining criteria weights approach of subject [11-12], objective obtaining criteria weights approach[13-14], subjective and objective obtaining criteria weights approach [15], the OWA operator weights [16] and feedback model [17], etc. As for the uncertain criteria weights problems, most approaches obtain the criteria weights according to their deviation of criteria values to facilitate the ranking of alternatives. Generally speaking, the bigger the deviation of criteria values are, the larger the weight will be [18]. In reality, we find that some criteria values change lager than others but these criteria only have a little influence on the result, while some tiny changes of few criteria would lead to different consequences for some MCDM problems.

According to the traditional methods of MCDM problems, we need several procedures, such as unifying criteria and obtaining criteria weights and information fusion as well as ranking and selecting the most desirable alternative(s) [18-19]. In order to rank alternatives, we need to compare expectations of combinatorial criteria values [20-21]. But we can not filter off the absolute disparity through unifying criteria, so different criteria are incomparable even if we unify them into the same meaning. In this paper, we think the criteria can be compared only on the same criteria. Some errors would be produce in the processing of uniform criteria. In order to filter off these errors, we propose through comparing the weighted combinatorial advantage values (WCAV) [22] to rank alternatives in this paper.

This paper is organized as follows: In section 2, for numerical value MCDM problems, there are a large number of criteria in the decision table. We propose a criteria reduction technique based on tolerance and advantage relation and Rough set theory to find out useful criteria, respectively. In section 3, we propose a new way of obtaining criteria weights according to the discernibility matrix and using weighted combinatorial advantage value (WCAV) instead of traditional methods to rank those alternatives. In section 4, we validate the method a useful and effective tool for MCDM problems. Finally, section 5 discusses the conclusion and future work.

2 a Large Number Criteria MCDM Problems and Criteria Reduction

In this section, we will give one type of numerical value MCDM problems that involve a large number of criteria in decision tables. The primary goal of MCDM problems is to rank the alternatives and select the most desirable one(s). To achieve this goal, several processing steps are needed to compare the alternatives based on multiple criteria. In each processing step, specific algorithms and operations are involved. The problems concerned in this paper are how to find out the useful criteria and obtain weights of useful criteria and to rank alternatives or select the most desirable alternative(s).

2.1 Addressing Numerical Value MCDM Problems

Example MCDM problem (Supplier choice): The Commercial Aircraft Corporation of China, Ltd. (CACC) builds huge commercial aircrafts to serve commercial airlines in China. To build aircrafts the company needs to buy and use some key parts from international or domestic suppliers. Therefore, the CACC must make a scientific decision to choose the most desirable supplier that relate to the success of commercial aircraft program. There are lots of complicated factors that affect decision makers (DMs) to make decision and they need to combine all information for every supplier and analyze them as well as select the most desirable supplier(s) [22].

Suppose that there are five international suppliers in the first round competing for the CACC demand of some key parts of the huge commercial aircrafts, and these five suppliers are represented by $\boldsymbol{A}=\{A_1,A_2,A_3,A_4,A_5\}$. Suppose we invite 100 experts to make judgments for the sake of obtaining the degrees to which alternative Ai satisfies and does not satisfy criteria C_j (i=1, 2, 3, 4, 5; j=1, 2, ……, m). There are two kinds of poll results “yes” or “no” to the question whether alternative Ai satisfies criteria C_j. Then we should choose the most desirable choice based on the results in Table 1.

Table 1 The result of “yes” answers from 100 experts

U	C_1	C_2	……	C_m	……
A_1	55	50	……	58	……
A_2	61	62	……	60	……
A_3	55	55	……	70	……
A_4	65	65	……	65	……
A_5	89	85	……	55	……

2.2.1 the Challenge of Numerical Value MCDM Problems

Obviously, there are a large number of criteria in decision Table 1. The first problem is how to make a wise decision within limited time when the DMs face a large number of criteria? What kind of decision support do DMs want under large number of criteria environment? To address the problem, we need to find out useful criteria that really affect the decision results. This is a significant potential research area for MCDM problems.

2.2.2 the Principles and Methods of Criteria Reduction for Numerical Value MCDM Problems

We often face a question whether we can remove some criteria from an information table while preserving its basic properties, that is, whether a table contains some superfluous criteria. Through criteria reduction we want to find out useful information that really affects the processing of making decision. This work is to our best knowledge the first one that applies Rough set theory [7-10] to do criteria reduction. In addition, the Rough set theory contains many attributes reduction [5-6] techniques, such as the reduction of attributes based on similarity relation [23], advantage relation or disadvantage relation [22] and automatic threshold estimation [24] etc. In this paper, we apply Rough set theory to find out useful information from the decision table by using criteria reduction based on tolerance relation and advantage relation of criteria value.

Now, let’s use the concrete example shown in Table 1 to illustrate our idea in using tolerance relation or advantage relation and Rough set theory to find out useful information. Generally speaking, there are three different type criteria

- The criteria of cost type (the smaller of criteria values, the better)
- The criteria of benefit type (the larger of criteria values, the better)
- The criteria of middle type (at a special point of criteria values, the better)

Obviously, all the criteria belong to benefit type in Table 1.

There are two different criteria values of alternatives A_1 and A_2 on criterion C_1 are $f(A_1, C_1)$ and $f(A_2, C_1)$, such as $f(A_1, C_1)=55$, $f(A_2, C_1)=61$. Obviously, both of them have different criteria values on C_1, i.e., $f(A_1, C_1)< f(A_2, C_1)$. For benefit criteria, if $f(A_1, C_1)$is smaller than $f(A_2, C_1)$, it means that the alternative A_2 locates at a more advantage position than A_1 on criterion C_1 when we want to compare them. In other words, alternative A_2 is better than A_1 on criteria C_1. In this paper, we use $A_2 \succ A_1/C_1$ to indicate this advantage relation of them. At this situation, C_1 is a useful criterion when we compare these two alternatives A_1 and A_2.

Two criteria values of alternatives A_2 and A_4 on criterion C_1 are $f(A_2, C_1)$ and $f(A_4, C_1)$, i.e., $f(A_2, C_1)=61$ and $f(A_4, C_1)=65$. Obviously, both of them have different criteria values on C_1, i.e., $f(A_2, C_1)< f(A_4, C_1)$. For benefit criteria, if $f(A_2, C_1)$is smaller than $f(A_4, C_1)$, it means that the alternative A_2 locates at a more disadvantage position than A_4 on criterion C_1 when we compare them. At the same time, the alternative A_4 locates at a more advantage position than A_2 on criterion C_1. In this paper, we use $A_2 \prec A_4/C_1$ to indicate this disadvantage relation of them. Thus, C_1 is still a useful criterion when we compare these two alternatives A_2 and A_4.

Two criteria values of alternatives A_1 and A_3 on criterion C_1 are $f(A_1, C_1)$ and $f(A_3, C_1)$, obvious $f(A_1, C_1)$=55 and $f(A_3, C_1)$=55. Both of them have the same criteria value on C_1, i.e., $f(A_1, C_1)= f(A_3, C_1)$. For benefit or cost criteria, if two alternatives have the same criteria value on a criterion, it means that these two alternatives locate at the same position on this criterion when we compare them. That means the alternative A_1 and A_3 locate at the same position on criterion C_1. According to attribute reduction based on tolerance relation and Rough set theory, if two alternatives have the same values on the same criteria, it means that these two criteria values have a tolerance relation for numerical value MCDM problems. When two alternatives have different values on the same criterion, it means that a tolerance relation does not exist. In this paper, we use $A_1 \cong A_3/C_1$ to indicate this tolerance relation of them. Thus, C_1 is a useless criterion when we compare these two alternatives A_1 and A_3, so we can remove it.

As the previous discussion, there are three different relations of criteria values for numerical value MCDM problems. These three relations are advantage relation, disadvantage relation and equivalence relation.

Definition 1. Suppose that $\{A_1, A_2, \dots, A_n\}$ indicates a set of n alternatives, and $\{C_1, C_2, \dots, C_m\}$ is a set of m criteria for numerical value MCDM problems. $f(A_i, C_j)$ and $f(A_k, C_j)$ indicate the possible outcome of alternatives A_i and A_k on criterion C_j, *three relations of the criteria values are* defined as follows:

$$\begin{cases} A_i \succ A_k/C_j & f(A_i,C_j) > f(A_k,C_j) \\ A_i \cong A_k/C_j & f(A_i,C_j) = f(A_k,C_j) \\ A_i \prec A_k/C_j & f(A_i,C_j) < f(A_k,C_j) \end{cases} \quad (1)$$

Our idea regarding how to find out useful information: As the previous discussion, when we want to compare two alternatives, we do not need to consider those criteria with the same value on the same criteria. That means we do not need to consider those criteria that two alternatives locate at the same position on these criteria. However, we need to consider those criteria that two alternative locate at different positions on these criteria. From the previous discussion, criterion C_1 is useless to compare alternatives A_1 and A_3, but it is needed when we want to know which is better between alternatives A_1 and A_2. That means the same criteria may play different roles for different alternatives in the decision table. Thus, if we want to find out the useful criteria that we need, we need to make a comparison on *every* pair of alternatives in the decision table. To check *every* pair of alternatives separately, we need to construct a discernibility matrix [25] and find out useful criteria for all the alternatives in the decision table. The data in the discernibility matrix indicate a set of criteria that must be considered when we want to compare two corresponding alternatives.

We think this criteria reduction method reflect and convey the below information.

- From the perspective of the alternative: every alternative is chosen as the most desirable alternative(s) in decision making. Thus, it is necessary to find out these kinds of desirable criteria that locate in a comparative advantage position as the useful criteria in decision making for every alternative and the larger these criteria's weights are the better. Thus, the alternative could have its weighted combinatorial advantage in decision making.
- From the perspective of the competitors: Those criteria that locate at a less advantage position could be chosen as the useful criteria in decision making for their opponents and the heavier of those kinds of criteria are, the better. Thus, the competitors could have their own weighted combinatorial advantage in decision making and they will have more chance to be selected as the best choice in decision making.

From the perspective of the decision makers: they want to make a scientific and reasonable decision within limited time. So they need to remove those useless criteria that locate in the same position when they compare two different alternatives.

2.2 Building the discernibility matrix

As we proposed in section 2.1.2, need to construct three discernibility matrixes as criteria reduction based on tolerance relation, advantage relation and disadvantage relation and Rough set theory in order to find out useful criteria from the decision table. The discernibility matrix for numerical values MCDM problems as follows:

Definition 2. Suppose that $\{A_1, A_2, \dots, A_n\}$ indicates a set of n alternatives, and$\{C_1, C_2, \dots, C_m\}$ is a set of m criteria for numerical value MCDM problems. $f(A_i, C_j)$ and $f(A_k, C_j)$ indicate the possible outcome of alternatives A_i and A_k on criterion C_j, M is a *discernibility matrix for numerical value MCDM problems based on tolerance relation* defined as follows:

$$M = \begin{array}{c} \\ A_1 \\ \vdots \\ A_n \end{array} \begin{array}{c} A_1 \quad \cdots \quad A_n \\ \begin{bmatrix} m_{11} & \cdots & m_{1n} \\ \vdots & \ddots & \vdots \\ m_{n1} & \cdots & m_{nn} \end{bmatrix} \end{array}, \text{ where } m_{ik} = \begin{cases} \{C_j \in \boldsymbol{C} : f(A_i, C_j) \neq f(A_k, C_j)\} \\ \phi \qquad else \end{cases} \tag{2}$$

And m_{ik} denotes those criteria of two alternatives A_i and A_k have the different criteria values on the same criteria in set $\boldsymbol{C}$. In other words, m_{ik} is a set of criteria that contains all the criteria of two alternatives A_i and A_k have different criteria values in set $\boldsymbol{C}$. Obviously, if there is $A_i \cong A_k / C_j$, there will be $A_k \cong A_i / C_j$ when $f(A_i, C_j) = f(A_k, C_j)$. So there is $m_{ik}=m_{ki}$ in the discernibility matrix. Thus, the discernibility matrix is a symmetric matrix.

Definition 3. Suppose that $\{A_1, A_2, \dots, A_n\}$ indicates a set of n alternatives for numerical value or interval number MCDM problems, $\{C_1, C_2, \dots, C_m\}$ is a set of m criteria. For the numerical value MCDM problems, $M^{\succ}$ is a *discernibility matrix for numerical value MCDM problems based on advantage relation* defined as follows:

$$M^{\succ} = \begin{array}{c} \\ A_1 \\ \vdots \\ A_n \end{array} \begin{array}{c} A_1 \quad \cdots \quad A_n \\ \begin{bmatrix} m_{11}^{\succ} & \cdots & m_{1n}^{\succ} \\ \vdots & \ddots & \vdots \\ m_{n1}^{\succ} & \cdots & m_{nn}^{\succ} \end{bmatrix} \end{array}, \text{ where, } m_{ik}^{\succ} = \begin{cases} \{C_j \in \boldsymbol{C} : A_i \succ A_k / C_j \\ \phi \qquad else \end{cases} \tag{3}$$

And $m_{ik}^{\succ}$ is a set of criteria which contains those criteria that the alternative A_i locates at a more advantage position than A_k on those criteria.

Definition 4. Suppose that $\{A_1, A_2, \dots, A_n\}$ indicates a set of n alternatives for numerical value or interval number MCDM problems, $\{C_1, C_2, \dots, C_m\}$ is a set of m criteria. For the numerical value MCDM problems, $M^{\prec}$ is a *discernibility matrix for numerical value MCDM problems based on disadvantage relation* defined as follows:

$$M^{\prec} = \begin{array}{c} \\ A_1 \\ \vdots \\ A_n \end{array} \begin{array}{c} A_1 \quad \cdots \quad A_n \\ \begin{bmatrix} m_{11}^{\prec} & \cdots & m_{1n}^{\prec} \\ \vdots & \ddots & \vdots \\ m_{n1}^{\prec} & \cdots & m_{nn}^{\prec} \end{bmatrix} \end{array}, \text{ where, } m_{ik}^{\prec} = \begin{cases} \{C_j \in \boldsymbol{C} : A_i \prec A_k / C_j \\ \phi \qquad else \end{cases} \tag{4}$$

And $m_{ik}^{\prec}$ is a set of criteria which contains those criteria that the alternative A_i locates at a more disadvantage position than A_k on those criteria.

There is $M^{\prec} = M^{\succ T}$ between these two discernibility matrixes. So we will get the same useful criteria by using the discernibility matrix based on advantage relation, the discernibility matrix based on disadvantage relation or both. Thus, we will just use the discernibility matrixes based on advantage and tolerance relations to find out the useful criteria avoid the useless criteria in this paper.

We use the relative discernibility function of discernibility matrix by using Boolean reasoning techniques [6-7, 25-26]. We can get the useful criteria for all the alternatives.

3 The Method for a Large Number of Criteria MCDM Problems

According to the traditional methods of MCDM problems, we need several procedures, such as unifying criteria and obtaining criteria weights and information fusion as well as ranking and selecting the most desirable alternative(s)[18]. In this paper, we apply the method of criteria reduction based on tolerance and advantage relations and Rough set theory to a large number criteria MCDM problems. So, we will change the traditional procedure of MCDM problems.

3.1 Resolution Procedure for the a Large Number of Criteria MCDM Problems

To solve the above problems, a new resolution procedure is proposed, as shown in Fig. 1. A brief description of the resolution procedure is given below.

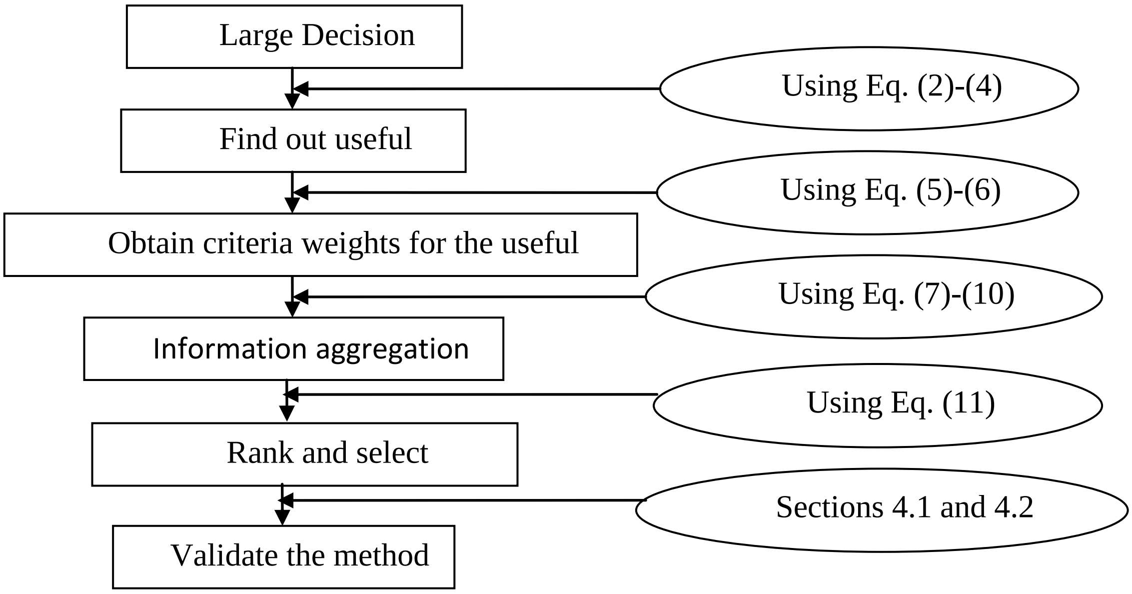

Fig.1 The resolution procedure for a large number of criteria MCDM problems

First, by using Eq. (2)-(4), we construct a discernibility matrix to find out the useful criteria for numerical value MCDM problems.

Second, by using Eq. (5) and (6), we find a way to obtain criteria weights for useful criteria and all criteria in the decision table, respectively.

Third, by using Eq. (7)-(11), we find a new algorithm to rank alternatives of numerical value of MCDM problems.

Finally, we validate this new resolution procedure a feasibility and validity method for MCDM problems.

All equations (Eq. (5)-(11)) and details (sections 4.1 and 4.2) will be provided in the next sections.

3.2 Obtaining Criteria Weights of the Useful Criteria

The challenge of obtaining criteria weights: There are several methods to obtain the criteria weights. According to the traditional opinion, the most popular one in the existing methods is to obtain the criteria weights based on the deviation of criteria values [18]. But we think this method has two places that deserve further consideration.

- First, for some MCDM problems, some criteria values change lager than others but these criteria only have a little influence on the result, while some tiny changes of few criteria would lead to different consequences.
- Second, building the discernibility matrix and finding out useful criteria depend on whether the criteria values are the same or not, but do not depend on whether the deviation of criteria values larger or smaller.

How to find a scientific and reasonable method and obtain the criteria weights is becoming a very important research area in MCMD fields. Based on the previous representing, we propose a new way to obtain the criteria weights of MCMD problems.

Our idea about how to obtain criteria weights: In the discernibility matrix, $m_{ik}(i,k \in 1,2,\cdots,n)$ indicates a useful set which contains those criteria that two alternatives A_i and A_k have different criteria values. For the DMs, $m_{ik}(i,k \in 1,2,\cdots,n)$ contains all the criteria that we must compare if we want to know which is the better between the alternatives A_i and A_k in Table 2. The times of the criteria appear in the discernibility matrix mean how many times we need to consider it. The times of criteria appear in the advantage discernibility matrix means how many alternatives locate at a more advantage position than others on these criteria. Every alternative wants to become the best one in decision making. Thus, these alternatives want those criteria with big weights in advantage matrix. At the same time, the times of these criteria appear in the disadvantage discernibility matrix means how many alternatives locate at a disadvantage position on these criteria. Every alternative hopes its competitors having big weights in disadvantage matrix. Thus, the criteria weights should have a proportional relation with the times it appears in the discernibility matrix.

Definition 5. Suppose that $\{A_1,A_2,\cdots,A_n\}$ is a set of n alternatives of MCDM problems, $\{C_1,C_2,\cdots,C_{m'}\}$ indicates a set of m' useful criteria, and $\{\omega_1,\omega_2,\cdots,\omega_{m'}\}$ is a set corresponding to all useful criteria weights. ω_j $(j=1,2,\cdots m')$ for useful criteria are listed as follows:

$$\omega_j = \frac{|C_j|}{|\sum_{j=1}^{m'} C_j|} \tag{5}$$

Where, $|C_j|$ indicates the times of criteria C_j appearing in the discernibility matrix, $|\sum_{j=1}^{m'} C_j|$ indicates the total times of all useful criteria appearing in the discernibility matrix, and m' indicates how many useful criteria in Table 2.

Definition 6. Suppose that $\{A_1,A_2,\cdots,A_n\}$ is a set of *n* alternatives of MCDM problems, $\{C_1,C_2,\cdots,C_m\}$ indicates a set of *m* criteria o, and $\{\omega_1,\omega_2,\cdots,\omega_m\}$ is a set corresponding to all criteria weights. ω_j $(j=1,2,\cdots,m)$ for all criteria are listed as follows:

$$\omega_j = \frac{|C_j|}{|\sum_{j=1}^{m} C_j|} \tag{6}$$

Where, $|C_j|$ indicates the times of criteria C_j appearing in the discernibility matrix, $|\sum_{j=1}^{m} C_j|$ indicates the total times of all criteria appearing in the discernibility matrix, and *m* indicates how many criteria in the decision table.

As previous representing in section 2.2.2, we also use the same reasons to explain through Eq. (5) and (6) to obtain the criteria weights is reasonable and scientific. As the previous discussion of the risk preference assumptions, we get different advantage orders for the same two interval numbers under a special situation. Different DMs will get different discernibility matrixes for the interval number MCDM problems. Thus, we will get different criteria weights for different types of DMs in the same decision table.

3.3 Ranking and Selecting the Most Desirable Alternative(S) for Numerical Value MCDM Problems

The challenge of ranking alternatives: There are several existing techniques to rank alternatives in MCDM, such as Gower Plots and Decision Balls method [21], THESEUS method [20], information fusion [18], weight restrictions (Reza Farzipoor Saen 2009) and rational research method [27] etc. We must make all criteria the same meaning if we want to compare them. But it would produce some errors in this step. Furthermore, uniting criteria and comparing the weighted combinatorial expectation of alternatives still has another problem. We can not filter off the absolute disparity through unifying criteria, so different criteria are incomparable even if we unify them into the same meaning.

Our idea of how to rank and select alternatives: We think the criteria only on the same criteria can be compared. Through criteria reduction and obtaining criteria weights we don't need to consider the deviation between two criteria values, because we only consider whether they are the same or not and find out the useful criteria as well as obtain the criteria weights. For the DMs, they just need to know the criteria that locate at an advantage or a disadvantage in decision making. They do not need to consider these useless criteria. In this paper, we propose through comparing the weighted combinatorial advantage values (WCAV) of alternatives to rank alternatives and select the most desirable alternative(s) [22].

For benefit type criteria, criteria values of two alternatives A_i and A_k on criterion C_j are $f(A_i, C_j)$ and $f(A_k, C_j)$. If $f(A_i, C_j)$is larger than $f(A_k, C_j)$ it means that the alternative A_i locates at a more advantage position than A_2 on criterion C_j. In other words, alternative A_i is better than A_k on criteria C_j. Undoubtedly, if alternative A_i is better than A_k on criteria C_j, it also means that alternative A_k locates at a more disadvantage position than A_i on criterion C_j. In this paper, we use $A_i \succ A_k / C_j$ to indicate the relation. If there is $A_i \prec A_k / C_j$, it means that alternative A_k locates at a more advantage position than A_i on criterion C_j, respectively, there has $f(A_i, C_j) < f(A_k, C_j)$. Two criteria values of two alternatives on the same criterion. If two alternatives have the same criteria values, it means that these locate at the same position on this criterion in decision making.

We use $AV_{A_i \succ A_k / C_j}$ to express an advantage value (AV) between decision alternative A_i and A_k on criterion C_j. So we get the advantage value as follows:

$$AV_{A_i \succ A_k / C_j} = \begin{cases} 1 & A_i \succ A_k / C_j \\ 0 & A_i \cong A_k / C_j \\ -1 & A_i \prec A_k / C_j \end{cases} \tag{7}$$

$WAV_{A_1 \succ A_2}$ represents a weighted advantage value (WAV) of criteria between A_i and A_k for useful criteria as follows:

$$WAV_{A_i \succ A_k} = AV_{A_i \succ A_k / C_1} \cdot \omega_1 + AV_{A_i \succ A_k / C_2} \cdot \omega_2 + \cdots + AV_{A_i \succ A_k / C_{m'}} \cdot \omega_{m'} \tag{8}$$

$WAV_{A_1 \succ A_2}$ represents a weighted advantage value (WAV) criteria between A_i and A_k for all criteria as follows:

$$WAV_{A_i \succ A_k} = AV_{A_i \succ A_k / C_1} \cdot \omega_1 + AV_{A_i \succ A_k / C_2} \cdot \omega_2 + \cdots + AV_{A_i \succ A_k / C_m} \cdot \omega_m \tag{9}$$

Comparing every pair of all alternatives, we construct the weighted advantage relation matrix (WARM), so we get the WADM for all the alternatives of MCDM problems as follows:

$$WARM = \begin{bmatrix} WAV_{A_1 \succ A_1} & WAV_{A_1 \succ A_2} & \cdots & WAV_{A_1 \succ A_n} \\ WAV_{A_2 \succ A_1} & WAV_{A_2 \succ A_2} & \cdots & WAV_{A_2 \succ A_n} \\ \vdots & \vdots & \ddots & \vdots \\ WAV_{A_n \succ A_1} & WAV_{A_n \succ A_2} & \cdots & WAV_{A_n \succ A_n} \end{bmatrix} \tag{10}$$

For the WAV there are the following characteristics:

1. $WAV_{A_i \succ A_k} + WAV_{A_k \succ A_i} = 0$
2. $WAV_{A_i \succ A_k} = 0 \Leftrightarrow A_i \cong A_k$
3. $WAV_{A_i \succ A_k} > 0 \Leftrightarrow A_i \succ A_k$
4. $WAV_{A_i \succ A_k} = -WAV_{A_k \succ A_i}$

$WCAV_{A_k}^{\succ}$ represents a weighted combinatorial advantage value (WCAV) of alternatives for alternative A_k in decision tables, we can get the WCAV as follows:

$$WCAV_{A_k}^{\succ} = \frac{1}{n-1} \sum_{i \neq k} WAV_{A_k \succ A_i} \tag{11}$$

The larger the WCAV of A_k, the better alternative A_k. Therefore, all the alternatives can be ranked according to the WCAV, respectively. Thus, the best alternative can be selected. As the previous discussion of the risk preference assumptions, we get different advantage values for the same two intuitionistic fuzzy sets under a special situation. Thus, we will get different weighted combinatorial advantage values for different types of DMs in the same decision table.

4 Applying the method to numerical value MCDM problems

In this section, an example for numerical value MCDM problem is used to illustrate the feasibility and validity of the proposed method. In order to demonstrate the method of criteria reduction based on tolerance relation and Rough set theory, we proposed an effective tool for MCDM problems.

Suppose that the DMs have seven criteria of $\boldsymbol{C}=\{C_1,C_2,C_3,C_4,C_5,C_6,C_7\}$ to make a decision, such as quality C_1, competitive C_2, price C_3, design plan C_4, delivery time C_5, safety index C_6, and sale service C_7. Suppose we invite 100 experts to make their judgement and voting for all the competitors based on seven criteria, there are two kinds of poll results “yes” or “no”. Which is the best one in Table 2?

Table 2 The result of “yes” from 100 experts

U	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	45	50	75	20	50	40	48
A_2	61	62	65	54	45	50	50
A_3	45	55	30	54	45	45	70
A_4	65	65	30	65	70	45	65
A_5	89	85	65	65	65	50	50

As denoted in Fig. 1, we need several steps (find useful criteria, obtain criteria weights, rank alternatives, etc.) to select the most desirable alternative(s). We can build two different discernibility matrixes based on tolerance and advantage relations to find out the useful criteria, respectively. First, we will build the discernibility matrix based on tolerance relation and Rough set and validate the method in section 4.1. Second, we will build the discernibility matrix based on advantage relation and Rough set and validate it in section 4.3. As we have denoted in Fig. 1 that we need.

4.1 The Procedures for Numerical Value MCDM Problems Based on Tolerance Relation

Step 1 *Find out the useful criteria.* According to criteria of Table 2 by using Eq. (2), we construct the discernibility matrix based on tolerance relation and find out the useful criteria as follows:

$$M=\begin{bmatrix} \phi & \boldsymbol{C} & C_2C_3C_4C_5C_6C_7 & \boldsymbol{C} & \boldsymbol{C} \\ & \phi & C_1C_2C_3C_6C_7 & \boldsymbol{C} & C_1C_2C_4C_5 \\ & & \phi & C_1C_2C_3C_5C_7 & \boldsymbol{C} \\ & & & \phi & C_1C_2C_3C_5C_6C_7 \\ & & & & \phi \end{bmatrix}$$

As the previous representing, we know discernibility matrix is a symmetric matrix. In this paper we just give the upper triangular matrix of it.

If we want to compare alternatives A_1 and A_2, we must compare those criteria that m_{12} contains. As we proposed in section 2, $\boldsymbol{C}$ is a criterion set which contains all the criteria in Table 2. Thus, we need to compare all criteria in Table 2 if we want to know which is better between alternatives A_1 and A_2.

We use a relative discernibility function of discernibility matrix by using Boolean reasoning techniques[27-30]. We get $\{C_1,C_2,C_4,C_5\}$ a group of useful criteria. If we want to compare alternatives these four criteria are needed.

Step 2 *Obtain criteria weights of the useful criteria.* By using Eq. (5) and the useful criteria of $\{C_1,C_2,C_4,C_5\}$, we can get the criteria weights as follows:

$$\omega_1=\frac{1}{4},\ \omega_2=\frac{5}{18},\ \omega_4=\frac{2}{9},\ \omega_5=\frac{1}{4}.$$

Step 3 *Rank and select the most desirable alternative(s).* By using Eq. (7), (8) and (10) as well as the useful criteria of $\{C_1,C_2,C_4,C_5\}$, we construct the WARM as follows:

$$WARM=\begin{bmatrix} 0 & -0.5 & -0.25 & -1 & -1 \\ 0.5 & 0 & 19/36 & -1 & -1 \\ 0.75 & -19/36 & 0 & -1 & -1 \\ 1 & 1 & 1 & 0 & -5/18 \\ 1 & 1 & 1 & 5/18 & 0 \end{bmatrix}$$

By using Eq. (11), we get the WCAV for all alternatives as follows:

$$WCAV_{A_1}^{\succ}=-0.6875, WCAV_{A_2}^{\succ}=-0.2431, WCAV_{A_3}^{\succ}=-0.4444, WCAV_{A_4}^{\succ}=0.6806, WCAV_{A_5}^{\succ}=0.8194.$$

Therefore, the ranking order of all the alternatives is $A_5 \underset{0.2778}{\succ} A_4 \underset{1}{\succ} A_2 \underset{0.5278}{\succ} A_3 \underset{0.75}{\succ} A_1$.

Thus, the alternative A_5 is the best choice in Table 2.

4.1.2 Validating the method for numerical value MCDM problems based on tolerance relation

In section 4.1.1, we got the alternatives order for numerical value MCDM problems of CACC by using the new method based on the Rough set and tolerance relation. Several algorithms are needed in it, such as finding out useful criteria through criteria reduction and making a decision using useful criteria. In this section, we will validate this method that we proposed an effective and useful tool of MCDM problems. To address this problem, we use all the criteria in Table 2 to make a decision. If we get the same ranking order and the most desirable alternative as we have got in section 4.1 by using the useful criteria, it means that our method is correct.

Using all criteria in Table 2 of MCDM problems to make a decision, by using the Eq. (6) to obtain the criterion weights as follows:

$$\omega_{C_1}=9/61,\ \omega_{C_2}=10/61,\ \omega_{C_3}=8/61,\ \omega_{C_4}=8/61,\ \omega_{C_5}=9/61,\ \omega_{C_6}=8/61,\ \omega_{C_7}=9/61.$$

By using Eq. (7), (9), (10) and all the criteria，we get the WARM as follows:

$$WARM=\begin{bmatrix} 0 & -27/61 & -18/61 & -45/61 & -45/61 \\ 27/61 & 0 & 26/61 & -29/61 & -36/61 \\ 18/61 & -26/61 & 0 & -36/61 & -43/61 \\ 45/61 & 29/61 & 36/61 & 0 & -17/61 \\ 45/61 & 36/61 & 43/61 & 17/61 & 0 \end{bmatrix}$$

By using Eq. (11), we get the WCAV for all alternatives as follows:

$$WCAV_{A_1}^{\succ}=-0.5533, WCAV_{A_2}^{\succ}=-0.0492, WCAV_{A_3}^{\succ}=-0.3566, WCAV_{A_4}^{\succ}=0.3811, WCAV_{A_5}^{\succ}=0.5779.$$

Therefore, the ranking order of all the alternatives is $A_5 \underset{0.2787}{\succ} A_4 \underset{0.4754}{\succ} A_2 \underset{0.4262}{\succ} A_3 \underset{0.2951}{\succ} A_1$.

Obviously, we get the same ranking order and the most desirable choice in two different situations. So, finding out and using the useful criteria to make decision is an effective and useful tool for numerical value MCDM problems, especially, when there are large number of criteria in decision tables. In section 4.1, we validate the new method for criteria reduction based on Rough set and tolerance relation. Through this method, we can find out the useful criteria and avoid the useless criteria. Just use these useful criteria to make decision. Thus, we find a useful method for criteria reduction based on tolerance relation to make decision when there have large number of criteria of numerical value MCDM problems.

4.2 The Procedures for Numerical Value MCDM Problems Based on Advantage Relation

Step 1 *Find out the useful criteria*. According to criteria of Table 2 by using Eq.(3), we construct the discernibility matrix based on advantage relation and find out the useful criteria as follows:

$$M^{\succ}=\begin{pmatrix} \phi & C_3C_5 & C_3C_5 & C_3 & C_3 \\ C_1C_2C_4C_6C_7 & \phi & C_1C_2C_3C_6 & C_3C_6 & \phi \\ C_2C_4C_6C_7 & C_7 & \phi & C_7 & C_7 \\ C_1C_2C_4C_5C_6C_7 & C_1C_2C_4C_5C_7 & C_1C_2C_4C_5 & \phi & C_5C_7 \\ C_1C_2C_4C_5C_6C_7 & C_1C_2C_4C_5 & C_1C_2C_3C_4C_5C_6 & C_1C_2C_3C_6 & \phi \end{pmatrix}$$

As the previous representing, we know that the discernibility matrixes based on advantage and disadvantage relations have a relation of transposed, it means that they are transposed matrix each other. In this paper we just use the advantage discernibilty matrix to find out the useful criteria and make a decision.

As the discussion in section 2.2, we know that if we want to compare alternatives A_1 and A_2, alternatives A_1 locates at a more advantage position than A_2 on criteria C_3 and C_5 in Table 2. At the same time, alternatives A_2 locates at a more advantage position than A_1 on criteria C_1, C_2, C_4, C_6, and C_7 in Table 2.

We use a relative discernibility function of discernibility matrix by using Boolean reasoning techniques [24-26, 27-30]. We get $\{C_1,C_2,C_3,C_4,C_5,C_7\}$ a group of useful criteria. If we want to compare alternatives these four criteria are needed.

Step 2 *Obtain criteria weights of the useful criteria*. By using Eq. (5) and the useful criteria of $\{C_1,C_2,C_3,C_4,C_5,C_7\}$, we can get the criteria weights as follows:

$$\omega_1=\frac{9}{53},\ \omega_2=\frac{10}{53},\ \omega_3=\frac{8}{53},\omega_4=\frac{8}{53},\ \omega_5=\frac{9}{53},\omega_7=\frac{9}{53}.$$

Step 3 *Rank and select the most desirable alternative(s)*. By using (7), (8) and (10) as well as the useful criteria of $\{C_1,C_2,C_3,C_4,C_5,C_7\}$, we construct the WARM as follows:

$$WARM=\begin{pmatrix} 0 & -19/53 & -10/53 & -37/53 & -37/53 \\ 19/53 & 0 & 18/53 & -37/53 & -36/53 \\ 10/53 & -18/53 & 0 & -27/53 & -35/53 \\ 37/53 & 37/53 & 27/53 & 0 & -9/53 \\ 37/53 & 36/53 & 35/53 & 9/53 & 0 \end{pmatrix}$$

By using Eq. (11), we get the WCAV for all alternatives as follows:

$$WCAV_{A_1}^{\succ}=-0.4858,\ WCAV_{A_2}^{\succ}=-0.1698,\ WCAV_{A_3}^{\succ}=-0.3302,\ WCAV_{A_4}^{\succ}=0.2480,\ WCAV_{A_5}^{\succ}=0.5519.$$

Therefore, the ranking order of all the alternatives is $A_5 \underset{0.1698}{\succ} A_4 \underset{0.6981}{\succ} A_2 \underset{0.3396}{\succ} A_3 \underset{0.1887}{\succ} A_1$.

Thus, the alternative A_5 is the best choice.

4.2.2 Validating the model for numerical value MCDM problems

In this section, we will validate this method based on the Rough set and advantage relation an effective and useful tool of MCDM problems. To address this problem, we still use all the criteria in Table 2 to make a decision. If we get the same ranking order and the most desirable alternative as we have got in section 4.1 by using the useful criteria, it means that our model is effectively.

Using all criteria in Table 2 of MCDM problems to make a decision, by using the Eq. (6) to obtain the criterion weights as follows:

$$\omega_1=\frac{9}{61},\ \omega_2=\frac{10}{61},\ \omega_3=\frac{8}{61},\ \omega_4=\frac{8}{61},\ \omega_5=\frac{9}{61}\ \ \omega_6=\frac{8}{61},\ \omega_7=\frac{9}{61}.$$

By using Eq. (7), (9), (10) and all criteria，we get the WARM as follows:

$$WARM=\begin{pmatrix} 0 & -27/61 & -18/61 & -45/61 & -45/61 \\ 27/61 & 0 & 26/61 & -29/61 & -36/61 \\ 18/61 & -26/61 & 0 & -27/61 & -43/61 \\ 45/61 & 29/61 & 27/61 & 0 & -17/61 \\ 45/61 & 36/61 & 43/61 & 17/61 & 0 \end{pmatrix}$$

By using Eq. (11), we get the WCAV for all alternatives as follows:

$$WCAV_{A_1}^{\succ}=-0.5533,\ WCAV_{A_2}^{\succ}=-0.0492,\ WCAV_{A_3}^{\succ}=-0.3197,\ WCAV_{A_4}^{\succ}=0.3343,\ WCAV_{A_5}^{\succ}=0.5779.$$

Therefore, the ranking order of all the alternatives is $A_5 \underset{0.2787}{\succ} A_4 \underset{0.4754}{\succ} A_2 \underset{0.4262}{\succ} A_3 \underset{0.2951}{\succ} A_1$.

Obviously, we get the same ranking order and the most desirable choice in two different situations. So, finding out and using the useful criteria to make a decision is an effective and useful tool for numerical value MCDM problems, especially, when there are large number of criteria in decision tables. Thus, we find a useful model to make a decision when DMs face large number of criteria of numerical value MCDM problems. If we can get the same ranking order, it means that this model is a useful tool. In fact, if we get the same the most desirable alternative(s), we can say this model is still a useful tool for MCDM problems. Because the decision makers are always concerned with finding the best alternative(s), so getting the most desirable alternative(s) seems to be more important than the other ranks.

In this section, we use two different methods to address the “large decision table” (e.g. a large number of criteria) challenge in multiple criteria decision making. This new method involves criteria reduction based on Rough set and criteria value relation (tolerance and advantage relations). We also validate the new method. Thus, we can say that we find an effective method to address the problem “There may be not one of having insufficient information, but rather one of having too much or an unknown quality of information how we filter information for MCDM in an internet or mobile environment.”

4.3 Validating the model using maximizing deviation method

In this paragraph, we will compare the results from our method with the one based on the *maximum deviation method* to obtain the criteria weights. By using the *maximizing deviation method* for interval numbers proposed in [18], we need to take all the criteria into consideration in Table 2. If we use all the criteria in table 4 to make a decision, the ranking order of all the alternatives is also $A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$.

Alternative A_5 is the best choice.

We get the same best choice and ranking order of all alternatives.

5 Conclusion

This paper mainly focuses upon how to make a wise and reasonable decision within limited time when DMs face a large number of criteria in MCDM problems. In this paper, our solution mainly focuses on four aspects. First, according to numerical value MCDM problems, we proposed a method of criteria reduction based on tolerance relation and then build a discernibility matrix to find out useful criteria. Just use useful criteria to make a decision. Second, through the idea of building the discernibility matrix, we proposed a new method to obtain criteria weights. Third, a new method based on weighted combinatorial advantage criteria value and criteria reduction, we give the model of advantage matrix of criteria. Finally, we compare the ranking result by using useful criteria and all criteria in the Table to make a decision. We validated the method of finding out the useful criteria through criteria reduction a useful and scientific method for MCDM problems. Our work is an underline research field of MCDM problems with large number of criteria.

In future work, we will study the criteria reduction algorithms to find the useful criteria when the criteria are dependent.

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References

[1] J. Wallenius, J. S. Dyer, P. C. Fishburn, R. E. Steuer, S. Zionts and K. Deb(2008), "Multiple Criteria Decision Making, Multicriteria Utility Theory: Recent Accomplishments and What Lies Ahead", *Management Science*, Vol.54, No. 7, pp. 1336-1349.

[2] E. M. Feit, M. A. Beltramo, F. M. Feinberg and Reality Check(2010), "Combining Choice Experiments with Market Criteria to Estimate the Importance of Product Criteria", *Management Science*, Vol.56, No. 2, pp. 785-800.

[3] P. Ghemawat and D. Levinthal(2008), "Choice Interactions and Business Strategy", *Management Science*, Vol. 54, No. 9, pp. 1638-1651.

[4] V. V. Podinovski (2010), "Set choice problems with incomplete information about the preferences of the decision maker", *European Journal of Operational Research*, Vol. 207, No. 1, pp. 371-379.

[5] W. Ziarko(1993), "Variable Precision Rough Set Method", *Journal of Computer and System Sciences*, Vol.46, No. 1, pp. 39-59.

[6] Z. Plawlak(1997), "Rough set approach to knowledge-based decision support", *European Journal of Operational Research*, Vol. 99, No. 1, pp. 48-57.

[7] Z. Plawlak(1982), "Rough Sets", *International Journal of Computer and Information Sciences*, Vol. 11, No. 5, pp. 341-356.

[8] W. C. Lee and C. C. Wu(2009), "Some single-machine and m-machine flowshop scheduling problems with learning considerations", *Information Sciences*, Vol. 179, No. 22, pp. 3885-3892.

[9] Z. Pawlak and A. Skowron(2007), "Rough sets: Some extensions", *Information Sciences*, Vol. 177, No. 1, pp. 28-40.

[10] D.Q. Miao, Y. Zhao, Y.Y. Yao, H.X. Li and F.F. Xu(2009), "Relative reducts in consistent and inconsistent decision Tables of the Pawlak rough set method", *Information Sciences*, Vol. 179, No. 24, pp. 4140-4150.

[11] T. L. Saaty and L. Vargas (1987), "Uncertain and rank order in the analytic hierarchy process", *European Journal of Operational Research*, Vol. 32, No. 1, pp. 107-117.

[12] R. F. Saen(2009), "A decision method for ranking suppliers in the presence cardinal and ordinal criteria, weight restrictions, and nondiscretionary factors", *Annals of Operations*, Vol. 172, No. 1, pp. 177-192.

[13] G. R. Jahanshahloo, L. F Hosseinzadeh and M Izadikha(2006), "An algorithmic method to extend TOPSIS for decision-making problems with interval criteria", *Applied Mathematics and Computation*, Vol. 175, No. 2, pp. 1375-1384.

[14] J. J. Zhang, D. S. Wu and D. L. Olson(2005), "The method of grey related analysis to multiple criteria decision making problems with intuitionistic fuzzy set", *Mathematical and Compute Modeling* , Vol. 42, No. 9-10, pp. 991-998.

[15] J. Ma, Z. P. Fan and L. H. Huang(1999), "A alternative and objective integrated approach to

Obtain criteria weights", *European Journal of Operational Research*, Vol. 112, No. 2, pp. 397-404.

[16] Y. M. Wang and K. S. Chin(2011), "The use of OWA operator weights for cross-efficiency aggregation", *Omega*, Vol. 39, No. 5, pp. 493-503.

[17] C. Fu and S. L. Yang(2011), "An attribute weight based feedback method for multiple Attributive group decision analysis problems with group consensus requirements in evidential reasoning context", *European Journal of Operational Research*, Vol. 212, No. 1, pp. 179-189.

[18] Z. S. Xu(2004), *Uncertain Multiple Criteria Decision Making Methods and Applications*, Tsinghua University Press, Beijing, Vol. 11, No. 5, pp. 341-356.

[19] Y. H. Qian, J. Y. Liang and C. Y. Dang(2008), "Interval ordered information systems", *Computers and Mathematics with Applications*, Vol. 58, No. 8, pp. 1994-2009.

[20] E. Fernazdez and J. Navarro(2011), "A new approach to multi-criteria sorting based on fuzzy outranking relations: The THESEUS method", *European Journal of Operational Research*, Vol. 213, No. 2, pp. 405-413.

[21] L. C. Ma and H.L. Li(2011), "Using Gower Plots and Decision Balls to rank alternatives involving inconsistent preferences", *Decision Support Systems*, Vol. 51, No. 3, pp. 712-719.

[22] Liu J., Liu P., Liu S. F., Zhou X. Z. and Zhang T(2015). "A study of decision process in MCDM problems with large numbers of criteria", *International Transactions in Operational Research*, Vol. 22, No. 2, pp. 237-264.

[23] E.A. Abo-Tabl(2011), " A comparison of two kinds of definitions of rough approximations based on a similarity relation", *Information Sciences*, Vol. 181, No. 12, pp. 2587-2596.

[24] J. B. dos Santos, C. A. Heuser, V. P. Moreira and L.K. Wives(2011), "Automatic threshold estimation for data matching applications", *Information Sciences,*Vol.181, No. 13, pp. 2685-2699.

[25] Y.Y. Guan and H.K. Wang(2006), "Set-valued information systems", *Information Sciences*, Vol. 176, No. 17, pp. 2507-2525.

[26] S. Greco, B. Matarazzo and R. Slowinski(2001), "Rough sets theory for multi-criteria decision analysis", *European Journal of Operational Research*, Vol. 129, No. 1, pp. 1-47.

[27] W. Wang, B. Payam and B. Andrzej (2011), "Rational Research method for ranking semantic entities", *Information Sciences*, Vol. 181, No. 13, pp. 2823-2840.

[28] A. Skowron and C. Rauser(1992), *The discernibility matrices and functions in information system*, in: R. Slowinski (Eds.), Intelligent Decision Support: Handbook of Application and Advances of Rough Sets Theory, Kluwer Academic Publisher, Dprdrecht, pp. 331-362.

[29] A. Skowron(1995), "Extracting laws from decision tables: a rough set", *Computational Intelligence*, Vol. 11, No. 2, pp. 371-388.

[30] R. W. Swiniarskia and A. Skowronb(2006), "Rough set methods in feature selection and recognition", *Pattern Recognition Letters*, Vol. 24, No. 6, pp. 833-849.

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