

Optimization-Based Comparison of Two Reliability Models in Repairable Systems

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Abstract: The reliability and performance of repairable systems have been of important concerns in industrial, manufacturing, and engineering environments. Modern production systems are expected to operate continuously with minimal interruptions, making reliability analysis a critical component of system design and maintenance planning. Frequent system failures can lead to production losses, increased maintenance costs, reduced customer satisfaction, and lower overall efficiency. Therefore, understanding the behavior of repairable systems and developing methods to improve their performance, remains an essential area of research. Motivated by these challenges, the present study focuses on the analysis of two different repairable system models. The models are evaluated using several widely accepted reliability performance measures, including Mean Time to System Failure (MTSF), Availability, Busy Period, and the Expected Number of Repairman's Visits. Such measures are particularly useful for maintenance managers and reliability engineers when making decisions regarding resource allocation and system improvements. To perform the analysis in our study, regenerative point graphical techniques are employed, offering a systematic and mathematically sound framework for studying stochastic repairable systems. These techniques enable the derivation of reliability characteristics and performance measures under varying operational conditions. In addition, the study incorporates a metaheuristic optimization approach to determine the optimal values of system parameters, particularly failure and repair rates. Among the available optimization methods, the Moth Flame Optimization (MFO) algorithm is selected due to its effectiveness in solving complex optimization problems and its ability to explore large search spaces efficiently. The optimized parameter values obtained through the MFO algorithm are used to evaluate and compare the performance of the two repairable system models. Numerical examples and computational illustrations are presented to demonstrate the applicability of the proposed methodology. Comparative analysis of the results highlights the strengths and limitations of each model under different operating conditions. The findings indicate that optimization-based reliability analysis can provide deeper insights into system performance, support maintenance planning, and contribute to improved decision-making processes in reliability engineering. Overall, the study emphasizes the importance of combining advanced reliability modeling techniques with modern optimization algorithms to enhance the efficiency, availability, and dependability of repairable systems.

Keywords: Reliability, Moth Flame Optimization, Comparative analysis, Metaheuristic optimization, System models.

1. INTRODUCTION

The reliability of repairable systems plays a vital role in ensuring the consistent performance of industrial and manufacturing processes. In this study, two different repairable system models are formulated and compared to evaluate their behavior under varying failure and repair conditions.

- Model-1 represents a repairable system characterized by operational, degraded, and repair states, with transitions governed by failure and repair rates. The system performance is evaluated in terms of its reliability measures such as Mean Time to

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System Failure (MTSF), Availability (A_0), Busy Period (B_0), and Expected Number of Repairman's Visits (V_0).

- Model-2 extends the complexity of the system by incorporating additional degraded and down states along with multiple repair loops. This model captures a wider range of operational scenarios, thereby offering a more detailed analysis of the system's performance indices.

To analyze and optimize both models, the Regenerative Point Technique (RPGT) is applied for deriving expressions of reliability indices, while Moth Flame Optimization (MFO), a nature-inspired metaheuristic algorithm, is employed to obtain optimal values of the system parameters. Numerical results and comparative analysis highlight the differences between the two models, providing insights into system efficiency, maintainability, and practical decision-making in reliability engineering. Singla et al. [1] studied SRGM parameter estimation using four metaheuristic methods—GSA, SCA, GWO, and RGA—on real loss datasets. Results show that metaheuristic estimates closely match LSE, with RGA and GWO and thus outperform others in accuracy, integration, and R^2 values. Thus, RGA and GWO emerge as reliable tools for complex optimization in software reliability analysis. Singla et al. [2] focused on optimizing reliability and cost in Distributed Redundancy Architectures (DRA) using advanced metaheuristic algorithms. Among MFO, WOA, DA, GOA, and COA, the Coati Optimization Algorithm (COA) outperforms others by providing faster, cost-efficient, and more reliable solutions. Numerical, graphical, and tabular results confirm COA's superiority, establishing it as a robust tool for enhancing system reliability and minimizing expenses. Singla et al. [3] analyzed a three-unit (A, B, and C) complex system using Markov processes and Laplace transformation to evaluate reliability indices such as MTSF, availability, reliability, and profit under deterioration and preventive maintenance rates. Sensitivity analysis reveals how varying failure and repair rates impact system behavior, showing that Unit C has the most significant influence on overall performance. Numerical results, presented graphically and tabularly, conclude that system reliability and profit decrease over time, offering practical insights for industries like dairy and fertilizer plants to achieve higher efficiency with lower maintenance costs. Singla et al. [4] then presented a study which discussed the reliability and profit of a two-out-of-three cold standby system with an external supporting system, preventive maintenance, and a single repairable server. Using RPGT, reliability measures such as MTSF, availability, and profit are derived through path probabilities, transition probabilities, and mean sojourn times under varying failure and repair rates. Sensitivity analysis with tables and graphs highlights system behavior and performance, offering insights into dependability optimization.

Gandhi et al. [5] worked on frog algorithm which is based on food search by an army of frogs. Inspired by frogs' food-search behavior, the Shuffled Frog Leaping Algorithm (SFLA) mimics how frogs leap between rocks and share information to improve search strategies. Mirjalili [6] introduced the Moth-Flame Optimization (MFO) algorithm, inspired by moths' transverse orientation navigation. MFO is tested against other metaheuristics on 29 real engineering problems, showing competitive and promising performance. Results highlight MFO's effectiveness in handling complex, constrained, and unknown search spaces. Khorshidi et al. [7] presented an optimization model for Reliability and Redundancy Allocation Problems (RRAP) in multi-state weighted k-out-of-n systems, aiming to maximize reliability and minimize cost. The model is solved using Genetic Algorithm (GA) and Imperialist Competitive Algorithm (ICA). Findings show that GA achieves better solutions, while ICA offers faster convergence, providing insights for complex reliability optimization. Nath & Muhuri [8] introduced a novel formulation namely Prioritized Many-objective Reliability Redundancy Allocation Problem which optimizes all objectives while preserving priority among them. A hybrid evolutionary method is proposed, dividing objectives into a leader (priority) module and a secondary optimization module. The approach is tested on series, series-parallel, bridge, and gas turbine systems, with

experiments showing superior performance compared to existing methods. Zhang et al [9] proposed a novel reliability optimization model for k-out-of-n:G subsystems using a mixed redundancy strategy, where both subsystem requirement k and redundancy type (active, cold-standby, or mixed) are treated as decision variables. A continuous-time Markov chain models system reliability, and a Pseudo-Parallel Genetic Algorithm (PPGA) is developed to solve RAP and RRAP efficiently. Numerical experiments demonstrate that the proposed model achieves higher reliability than existing methods, while PPGA ensures optimal solutions with fast convergence. Ravi [10] extended the Great Deluge Algorithm (GDA) into a Modified Great Deluge Algorithm (MGDA) for optimizing complex system reliability under cost and weight constraints. The algorithm is applied to reliability optimization and redundancy allocation problems, and compared with simulated annealing variants and other methods. Results show that MGDA achieves superior accuracy and speed, as compared to Ant Colony Optimization (ACO), making it an efficient alternative for reliability optimization. Singla et al. [11] studied a soap plant to optimize its cost for repairing under machine learning.

2. MODEL DESCRIPTION

2.1. Assumptions

- Failure and repair times of all units are exponentially distributed.
- Only one unit can fail or can be repaired at a time.
- Repair starts immediately after failure, subject to system condition.
- The repaired unit is as good as new.
- Switching between units is perfect and instantaneous.
- The system is initially in fully operational state S_0 .
- States S_1 to S_7 are regenerative states, while S_8 represents the absorbing (failed) state.
-  : Down State,  : Up state

2.2. Symbols and Notations

The various notations used for transition diagram i.e Figure 1 and Figure 2, and for mathematical expression is given in Table 1.

Table 1. Various Notations and symbols used in study.

$q_{ij}(t)$	Represents probability density function (p.d.f.) for changing states from one to another i.e. from 'i' to 'j' in $(0, t]$.
p_{ij}	$q_{ij}^*(0)$; State probability after taking Laplace transformation ('*') of q_{ij} .
$R_i(t)$	System reliability in state i
μ_i	MST in state i, $\mu_i = \int_0^\infty R_i(t)dt$
μ_i^1	Time gap for repairing in regenerative state 'i' at $t=0$.
pr	Probability
$\xi \xrightarrow{sf} i$	represents a directional failure-free path from the base state ξ to state i . It denotes a sequence of transitions that does not encounter any failures along the way, leading from ξ to i .
V_{kk}	represents the transitional probability density function from the terminal state k to state k within the k cycle.
$V_{\bar{k}\bar{k}}$	represents the Probability factor of reaching to the k^{th} terminal state from the $k - \text{cycle}$.
$i \xrightarrow{sr(sff)} j$	represents the r^{th} directional path from state i to j .
n_j	Total server rate or efficiency for the state
r_{ij}	Transition rate from state S_i to S_j

2.3. System Description

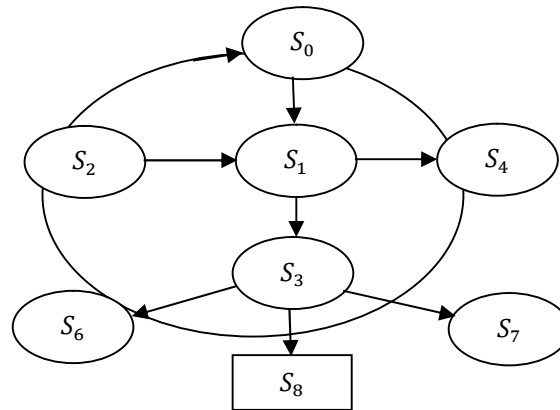


Fig. 1. Transition Diagram for repairable system for model 1

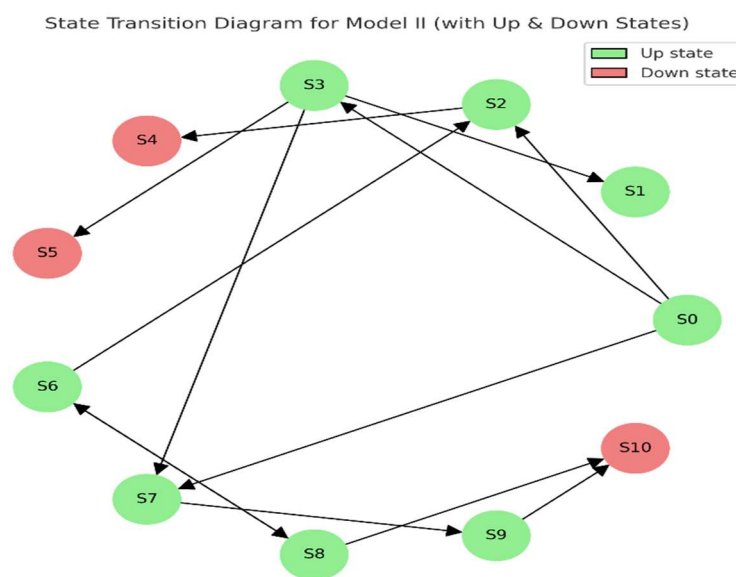


Fig. 2. Transition diagram for repairable system for model 2

3. MATHEMATICAL MODELLING

A regenerative state is a state from which the system starts afresh without any memory of previous transitions. At such a state, the future behavior of the system depends only on the present state and not on the past history. In the proposed RPGT models, state S_0 is considered the regenerative state because it represents the fully operational condition of the repairable system. Whenever the system returns to this state after repair, a new regenerative cycle begins, and the future evolution of the system becomes independent of the previous operating cycle.

3.1. Model 1 under RPGT

The steady-state probabilities of transitions from one regenerative state to another state for model 1 are given in Table 2.

Table 2. Transition probabilities from Various states to another one

From	To	$q_{ij}(t)$	p_{ij}
S_0	S_1	$r_{01}e^{-(r_{01}+r_{04})t}$	$\frac{r_{01}}{r_{01} + r_{04}}$
S_0	S_4	$r_{04}e^{-(r_{01}+r_{04})t}$	$\frac{r_{04}}{r_{01} + r_{04}}$
S_1	S_0	$r_{10}e^{-(r_{10}+r_{11}+r_{14})t}$	$\frac{r_{10}}{r_{10} + r_{11} + r_{14}}$
S_1	S_3	$r_{13}e^{-(r_{10}+r_{11}+r_{14})t}$	$\frac{r_{13}}{r_{10} + r_{11} + r_{14}}$
S_1	S_4	$r_{14}e^{-(r_{10}+r_{11}+r_{14})t}$	$\frac{r_{14}}{r_{10} + r_{11} + r_{14}}$
S_3	S_6	$r_{36}e^{-(r_{36}+r_{37}+r_{38})t}$	$\frac{r_{36}}{r_{36} + r_{37} + r_{38}}$
S_3	S_7	$r_{37}e^{-(r_{36}+r_{37}+r_{38})t}$	$\frac{r_{37}}{r_{36} + r_{37} + r_{38}}$
S_3	S_8	$r_{38}e^{-(r_{36}+r_{37}+r_{38})t}$	$\frac{r_{38}}{r_{36} + r_{37} + r_{38}}$
S_4	S_0	$r_{40}e^{-r_{40}t}$	1
S_6	S_0	$r_{60}e^{-r_{60}t}$	1
S_7	S_0	$r_{70}e^{-r_{70}t}$	1
S_8	S_0	Down state	1

3.2. Mean Sojourn Times (MST)

Mean sojourn time for Model 1 i.e. the time spent in one state before entering in another state is given in Table 3. The MST is helpful to understand the reliability for each state to perform evaluation for overall reliability.

Table 3. Mean Sojourn Times

State S_i	Mean Sojourn Time
S_0	$\frac{1}{r_{01} + r_{04}}$
S_1	$\frac{r_{01}}{r_{10} + r_{13} + r_{14}}$
S_3	$\frac{1}{r_{36} + r_{37} + r_{38}}$
S_4	$\frac{1}{r_{40}}$
S_6	$\frac{1}{r_{60}}$
S_7	$\frac{1}{r_{70}}$
S_8	$\frac{1}{r_{80}}$

3.3. Mean time to system failure (MTSF)(T_0)(For Model 1)

MTSF is defined as the expected time duration for which the system operates successfully before its first failure. It serves as a key reliability index to evaluate system performance under given failure and repair parameters.

$$MTSF(T_0) = \left[\sum_{i,sr} \left\{ \frac{\left\{ \text{pr} \left(\xi^{\text{sr} \rightarrow \text{sff}} \right) \right\} \mu_i}{\Pi_{m_1 \neq \xi} \{1 - V_{m_1 m_1}\}} \right\} \right] / \left[1 - \sum_{sr} \left\{ \frac{\left\{ \text{pr} \left(\xi^{\text{sr} \rightarrow \text{sff}} \right) \right\}}{\Pi_{m_2 \neq \xi} \{1 - V_{m_2 m_2}\}} \right\} \right] \quad (3.1)$$

$$T_0 = \frac{\frac{1}{r_{01}+r_{04}} + \frac{p_{01}}{r_{10}+r_{13}+r_{14}} + \frac{p_{01}p_{13}}{r_{36}+r_{37}+r_{38}} + \left(\frac{p_{04}+p_{01}p_{14}}{r_{40}} \right) + \left(\frac{p_{01}+p_{13}p_{36}}{r_{60}} \right) + \left(\frac{p_{01}+p_{13}p_{37}}{r_{70}} \right)}{1 - \left(\frac{r_{01}}{r_{01}+r_{04}} \right) \left(\frac{r_{13}}{r_{10}+r_{13}+r_{14}} \right) \left(\frac{r_{38}}{r_{36}+r_{37}+r_{38}} \right)} \quad (3.2)$$

3.4. Availability for Model 1

The states in which system is performing its function, are $j = 0, 1, 3, 4, 6$ and using fuzzy logic, the result obtained for the available time for working of system is:

$$A_0 = \left[\sum_{j,sr} \left\{ \frac{\left\{ \text{pr}(\xi^{\text{sr} \rightarrow j}) \right\} f_j \mu_j}{\Pi_{m_1 \neq \xi} \{1 - V_{m_1 m_1}\}} \right\} \right] \div \left[\sum_{i,sr} \left\{ \frac{\left\{ \text{pr}(\xi^{\text{sr} \rightarrow i}) \right\} \mu_i^1}{\Pi_{m_2 \neq \xi} \{1 - V_{m_2 m_2}\}} \right\} \right] \quad (3.3)$$

$(\sum V_{0 \rightarrow j} f_j \mu_j) \div (\sum V_{0 \rightarrow i} f_i \mu_i)$, Where $1 \leq j \leq 7, 1 \leq i \leq 8$, for $i \neq j$.

and $f_j = 1, \mu_j^1 = \mu_j$ for all j .

$$A_0 = \frac{\frac{1}{r_{01}+r_{04}} + p_{01} \left(\frac{1}{r_{13}+r_{14}} \right) + p_{01}p_{13} \left(\frac{1}{r_{36}+r_{37}+r_{38}} \right) + p_{04} \left(\frac{1}{r_{40}} \right) + p_{01}p_{14} \left(\frac{1}{r_{40}} \right) + p_{01}p_{13}p_{36} \left(\frac{1}{r_{60}} \right) + p_{01}p_{13}p_{37} \left(\frac{1}{r_{70}} \right)}{\frac{1}{r_{01}+r_{04}} + p_{01} \left(\frac{1}{r_{13}+r_{14}} \right) + p_{01}p_{13} \left(\frac{1}{r_{36}+r_{37}+r_{38}} \right) + p_{04} \left(\frac{1}{r_{40}} \right) + p_{01}p_{14} \left(\frac{1}{r_{40}} \right) + p_{01}p_{13}p_{36} \left(\frac{1}{r_{60}} \right) + p_{01}p_{13}p_{37} \left(\frac{1}{r_{70}} \right) + p_{01}p_{13}p_{38}} \quad (3.4)$$

3.5. Busy Period of the repairman for Model 1

The progression of maintenance, whether it be corrective or preventive, can be expressed as:

$$B_0 = \left[\sum_{j,sr} \left\{ \frac{\left\{ \text{pr}(\xi^{\text{sr} \rightarrow j}) \right\} n_j}{\Pi_{m_1 \neq \xi} \{1 - V_{m_1 m_1}\}} \right\} \right] \div \left[\sum_{i,sr} \left\{ \frac{\left\{ \text{pr}(\xi^{\text{sr} \rightarrow i}) \right\} \mu_i^1}{\Pi_{m_2 \neq \xi} \{1 - V_{m_2 m_2}\}} \right\} \right] \quad (3.5)$$

$$B_0 = \frac{p_{01} \left(\frac{1}{r_{10}+r_{13}+r_{14}} \right) p_{01}p_{13} \left(\frac{1}{r_{36}+r_{37}+r_{38}} \right) p_{04}+p_{01}p_{14} \left(\frac{1}{r_{40}} \right) p_{01}p_{13}p_{36} \left(\frac{1}{r_{60}} \right) + p_{01}p_{13}p_{37} \left(\frac{1}{r_{70}} \right)}{\frac{1}{r_{01}+r_{04}} + p_{01} \left(\frac{1}{r_{10}+r_{13}+r_{14}} \right) + p_{01}p_{13} \left(\frac{1}{r_{36}+r_{37}+r_{38}} \right) + p_{04}+p_{01}p_{14} \left(\frac{1}{r_{40}} \right) + p_{01}p_{13}p_{36} \left(\frac{1}{r_{60}} \right) + p_{01}p_{13}p_{37} \left(\frac{1}{r_{70}} \right) + p_{01}p_{13}p_{38}} \quad (3.6)$$

3.6 Expected Fractional Number of repairman's Visits for Model 1

The visit of a repairman for fixing the faults for continuity of the system is expressed as:

$$V_0 = \left[\sum_{j,sr} \left\{ \frac{\left\{ \text{pr}(\xi^{\text{sr} \rightarrow j}) \right\}}{\Pi_{k_1 \neq \xi} \{1 - V_{k_1 k_1}\}} \right\} \right] \div \left[\sum_{i,sr} \left\{ \frac{\left\{ \text{pr}(\xi^{\text{sr} \rightarrow i}) \right\} \mu_i^1}{\Pi_{k_2 \neq \xi} \{1 - V_{k_2 k_2}\}} \right\} \right] \quad (3.7)$$

$$V_0 = \frac{p_{01} + p_{01}p_{13} + p_{04} + p_{01}p_{14} + p_{01}p_{13}p_{36} + p_{01}p_{13}p_{37}}{\frac{1}{r_{01}+r_{04}} + p_{01} \left(\frac{1}{r_{10}+r_{13}+r_{14}} \right) + p_{01}p_{13} \left(\frac{1}{r_{36}+r_{37}+r_{38}} \right) + p_{04} + p_{01}p_{14} \left(\frac{1}{r_{40}} \right) + p_{01}p_{13}p_{36} \left(\frac{1}{r_{60}} \right) + p_{01}p_{13}p_{37} \left(\frac{1}{r_{70}} \right) + p_{01}p_{13}p_{38}} \quad (3.8)$$

3.7. Model 2 under RPGT

The steady-state probabilities of transitions and mean sojourn time from one state to another state for model 2 are given in Table 4 and Table 5 respectively.

Table 4. The transition probabilities from Various states to another one

From	To	$q_{ij}(t)$	p_{ij}
S ₀	S ₂	$r_{02}e^{-(r_{02}+r_{03}+r_{07})t}$	$\frac{r_{02}}{r_{02} + r_{03} + r_{07}}$
S ₀	S ₃	$r_{03}e^{-(r_{02}+r_{03}+r_{07})t}$	$\frac{r_{03}}{r_{02} + r_{03} + r_{07}}$
S ₀	S ₇	$r_{07}e^{-(r_{02}+r_{03}+r_{07})t}$	$\frac{r_{07}}{r_{02} + r_{03} + r_{07}}$
S ₁	S ₂	$r_{12}e^{-r_{12}t}$	1
S ₂	S ₀	$r_{20}e^{-(r_{20}+r_{24}+r_{26})t}$	$\frac{r_{20}}{r_{20} + r_{24} + r_{26}}$
S ₂	S ₄	$r_{24}e^{-(r_{20}+r_{24}+r_{26})t}$	$\frac{r_{24}}{r_{20} + r_{24} + r_{26}}$
S ₂	S ₆	$r_{26}e^{-(r_{20}+r_{24}+r_{26})t}$	$\frac{r_{26}}{r_{20} + r_{24} + r_{26}}$
S ₃	S ₁	$r_{31}e^{-(r_{31}+r_{35}+r_{37})t}$	$\frac{r_{31}}{r_{31} + r_{35} + r_{37}}$
S ₃	S ₅	$r_{35}e^{-(r_{31}+r_{35}+r_{37})t}$	$\frac{r_{35}}{r_{31} + r_{35} + r_{37}}$
S ₃	S ₇	$r_{37}e^{-(r_{31}+r_{35}+r_{37})t}$	$\frac{r_{37}}{r_{31} + r_{35} + r_{37}}$
S ₄	Down	$r_{46}e^{-r_{46}t}$	1
S ₅	Down	$r_{57}e^{-r_{57}t}$	1
S ₆	S ₂	$r_{62}e^{-(r_{62}+r_{68})t}$	$\frac{r_{62}}{r_{62} + r_{68}}$
S ₆	S ₈	$r_{68}e^{-(r_{62}+r_{68})t}$	$\frac{r_{68}}{r_{62} + r_{68}}$
S ₇	S ₀	$r_{70}e^{-(r_{70}+r_{79})t}$	$\frac{r_{70}}{r_{70} + r_{79}}$
S ₇	S ₉	$r_{79}e^{-(r_{70}+r_{79})t}$	$\frac{r_{79}}{r_{70} + r_{79}}$
S ₈	S ₆	$r_{86}e^{-(r_{86}+r_{8,10})t}$	$\frac{r_{86}}{r_{86} + r_{8,10}}$
S ₈	S ₁₀	$r_{8,10}e^{-(r_{86}+r_{8,10})t}$	$\frac{r_{8,10}}{r_{86} + r_{8,10}}$
S ₉	S ₇	$r_{97}e^{-(r_{86}+r_{9,10})t}$	$\frac{r_{97}}{r_{97} + r_{9,10}}$
S ₉	S ₁₀	$r_{9,10}e^{-(r_{86}+r_{9,10})t}$	$\frac{r_{9,10}}{r_{97} + r_{9,10}}$
S ₁₀	S ₆	$r_{10,6}e^{-(r_{10,6}+r_{10,7})t}$	$\frac{r_{10,6}}{r_{10,6} + r_{10,7}}$
S ₁₀	S ₇	$r_{10,7}e^{-(r_{10,6}+r_{10,7})t}$	$\frac{r_{10,7}}{r_{10,6} + r_{10,7}}$

Table 5. Mean Sojourn time for various states

State S_i	Mean Sojourn Time
S_0	$\frac{1}{r_{02} + r_{03} + r_{07}}$
S_2	$\frac{r_{01}}{r_{20} + r_{24} + r_{26}}$
S_3	$\frac{1}{r_{31} + r_{35} + r_{37}}$
S_4	$\frac{1}{r_{46}}$
S_5	$\frac{1}{r_{57}}$
S_6	$\frac{1}{r_{62} + r_{68}}$
S_7	$\frac{1}{r_{70} + r_{79}}$
S_8	$\frac{1}{r_{86} + r_{8,10}}$
S_9	$\frac{1}{r_{97} + r_{9,10}}$
S_{10}	$\frac{1}{r_{10,6} + r_{10,7}}$

3.8. Mean time to system failure (MTSF)(T_0) for Model 2

MTSF is defined as the expected time duration for which the system operates successfully before its first failure. It serves as a key reliability index to evaluate system performance under given failure and repair parameters.

$$\text{MTSF } (T_0) = \left[\sum_{i, sr} \left\{ \frac{\left\{ \text{pr} \left(\xi \xrightarrow{\text{sr}(\text{sff})} i \right) \right\} \mu_i}{\Pi_{m_1 \neq \xi} \{1 - V_{m_1 m_1}\}} \right\} \right] / \left[1 - \sum_{sr} \left\{ \frac{\left\{ \text{pr} \left(\xi \xrightarrow{\text{sr}(\text{sff})} \xi \right) \right\}}{\Pi_{m_2 \neq \xi} \{1 - V_{m_2 m_2}\}} \right\} \right] \quad (3.9)$$

$$T_0 = \frac{\mu_0 P_{04} + \mu_2 P_{24} + \mu_3 P_{35} + \mu_6 (P_{6,8} P_{8,10}) + \mu_7 (P_{7,9} P_{9,10})}{1 - (V_{0,0} + V_{2,2} + V_{3,3} + V_{6,6} + V_{7,7} + V_{8,8} + V_{9,9} + V_{10,10})} + \frac{\mu_8 P_{8,10} + \mu_9 P_{9,10} + \mu_{10} (P_{10,6} P_{6,8} P_{8,10}) + P_{10,7} P_{7,9} P_{9,10}}{1 - (V_{0,0} + V_{2,2} + V_{3,3} + V_{6,6} + V_{7,7} + V_{8,8} + V_{9,9} + V_{10,10})} \quad (3.10)$$

3.9. Availability for Model 2

$$A_0 = \left[\sum_{j, sr} \left\{ \frac{\left\{ \text{pr}(\xi^{sr \rightarrow j}) \right\} f_j \mu_j}{\Pi_{m_1 \neq \xi} \{1 - V_{m_1 m_1}\}} \right\} \right] \div \left[\sum_{i, sr} \left\{ \frac{\left\{ \text{pr}(\xi^{sr \rightarrow i}) \right\} \mu_i^1}{\Pi_{m_2 \neq \xi} \{1 - V_{m_2 m_2}\}} \right\} \right] \quad (3.11)$$

$$A_0 = \frac{\mu_0 P_{04} + \mu_2 P_{24} + \mu_3 P_{35} + \mu_6 (P_{6,8} P_{8,10}) + \mu_7 (P_{7,9} P_{9,10}) + \mu_8 P_{8,10} + \mu_9 P_{9,10}}{\mu_0 P_{04} + \mu_2 P_{24} + \mu_3 P_{35} + \mu_6 (P_{6,8} P_{8,10}) + \mu_7 (P_{7,9} P_{9,10}) + \mu_8 P_{8,10} + \mu_9 P_{9,10} + \mu_{10} (P_{10,6} P_{6,8} P_{8,10} + P_{10,7} P_{7,9} P_{9,10}) + \mu_4 + \mu_5} \quad (3.12)$$

3.10. Busy Period of the repairman for Model 2

$$B_0 = \left[\sum_{j, sr} \left\{ \frac{\left\{ \text{pr}(\xi^{sr \rightarrow j}) \right\} n_j}{\Pi_{m_1 \neq \xi} \{1 - V_{m_1 m_1}\}} \right\} \right] \div \left[\sum_{i, sr} \left\{ \frac{\left\{ \text{pr}(\xi^{sr \rightarrow i}) \right\} \mu_i^1}{\Pi_{m_2 \neq \xi} \{1 - V_{m_2 m_2}\}} \right\} \right] \quad (3.13)$$

$$B_0 = \frac{n_0 P_{04} + n_2 P_{24} + n_3 P_{35} + n_6 (P_{6,8} P_{8,10}) + n_7 (P_{7,9} P_{9,10}) + n_8 P_{8,10} + n_9 P_{9,10} + n_{10} (P_{10,6} P_{6,8} P_{8,10} + P_{10,7} P_{7,9} P_{9,10})}{\mu_0 P_{04} + \mu_2 P_{24} + \mu_3 P_{35} + \mu_6 (P_{6,8} P_{8,10}) + \mu_7 (P_{7,9} P_{9,10}) + \mu_8 P_{8,10} + \mu_9 P_{9,10} + \mu_{10} (P_{10,6} P_{6,8} P_{8,10}) + P_{10,7} P_{7,9} P_{9,10} + \mu_4 + \mu_5} \quad (3.14)$$

3.11. Expected Number of repairman's Visits for Model 2

$$V_0 = \left[\sum_{j, sr} \left\{ \frac{\{pr(\xi^{sr} \rightarrow j)\}}{\Pi_{k_1 \neq \xi} \{1 - V_{k_1 k_1}\}} \right\} \right] \div \left[\sum_{i, sr} \left\{ \frac{\{pr(\xi^{sr} \rightarrow i)\} \mu_i^1}{\Pi_{k_2 \neq \xi} \{1 - V_{k_2 k_2}\}} \right\} \right] \quad (3.15)$$

$$= [\sum_j V_{\xi, j}] \div [\sum_i V_{\xi, i}, \mu_i^1]$$

$$V_0 = \frac{P_{24} + P_{35} + P_{6,8} P_{8,10} + (P_{7,9} P_{9,10}) + P_{8,10} + P_{9,10} + (P_{10,6} P_{6,8} P_{8,10} + P_{10,7} P_{7,9} P_{9,10})}{\mu_0 P_{04} + \mu_2 P_{24} + \mu_3 P_{35} + \mu_6 (P_{6,8} P_{8,10}) + \mu_7 (P_{7,9} P_{9,10}) + \mu_8 P_{8,10} + \mu_9 P_{9,10} + \mu_{10} (P_{10,6} P_{6,8} P_{8,10}) + P_{10,7} P_{7,9} P_{9,10} + \mu_4 + \mu_5} \quad (3.16)$$

4. OPTIMIZATION BY MOTH FLAME OPTIMIZATION ALGORITHM (MFO)

4.1. Moth-Flame Optimization (MFO) Algorithm

The Moth-Flame Optimization (MFO) algorithm is a nature-inspired metaheuristic proposed by Mirjalili [6], motivated by the transverse orientation behavior of moths in nature. In this algorithm, moths represent candidate solutions, while flames denote the best solutions obtained during the optimization process. The movement of moths toward flames is modeled using a logarithmic spiral function, which effectively guides the search toward optimal regions of the solution space. MFO maintains a dynamic balance between exploration and exploitation by gradually reducing the number of flames as iterations progress, thereby enhancing convergence accuracy. Owing to its simple structure, few control parameters, and strong global search capability, MFO has been widely applied to solve complex, nonlinear, and multimodal optimization problems. The algorithm has demonstrated promising performance in various engineering applications, including reliability optimization, scheduling, power systems, and machine learning, making it a competitive alternative to other population-based optimization techniques.

- **Initialization**

Initialize a population of moths randomly within the defined search space. Set algorithm parameters such as population size and maximum number of iterations.

- **Fitness Evaluation**

Evaluate the fitness value of each moth using the objective function.

- **Flame Identification**

Sort the moths based on fitness values and identify the best solutions as flames.

- **Position Update**

Update the position of each moth with respect to its corresponding flame using a logarithmic spiral function.

- **Flame Reduction**

Gradually reduce the number of flames over iterations to shift the search from exploration to exploitation.

- **Boundary Handling**

Check and correct moth positions to ensure they remain within the permissible bounds.

- **Termination Check**

Repeat Steps 2–6 until the maximum number of iterations is reached or convergence criteria are satisfied.

- **Output**

Report the best flame as the optimal solution.

4.2. Result Analysis

By considering the optimization technique of Moth Flame with some initial parameters such as number of iterations are 100 with population size of 50. The various rates for both model moving from one state to another lies between 0 and 1. The following outcomes are obtained.

Table 6. (Model 1) By applying the Moth-Flame Optimization, get the following results

Various Parameters	Moth Flame Optimum Values
r_{01}	0.001
r_{04}	0.001
r_{10}	0.050
r_{13}	0.200
r_{14}	0.050
r_{36}	0.001
r_{37}	0.001
r_{38}	0.200
r_{40}	0.050
r_{60}	0.050
r_{70}	0.050
MTSF	0.9987
Availability	0.9993
B_0	0.0292
V_0	0.0027

Table 7. (Model 2) By applying the Moth-Flame Optimization, get the following results

Various Parameters	Moth Fly Optimum Values
r_{01}	0.001
r_{04}	0.001
r_{10}	0.050
r_{13}	0.200
r_{14}	0.050
r_{36}	0.001
r_{37}	0.001
r_{38}	0.200
r_{40}	0.050
r_{60}	0.050
r_{70}	0.050
MTSF	0.9999
Availability	0.9800
B_0	0.6371
V_0	0.0036

5. OUTCOMES AND DISCUSSION

The performance of the two repairable system models has been evaluated using key reliability indices, namely Mean Time to System Failure (MTSF), steady-state Availability (A_0), Busy Period of the repairman (B_0), and the Expected Number of Repairman's Visits (V_0). The analytical expressions of these indices are derived using the Regenerative Point Graphical Technique (RPGT), and optimal values of failure and repair parameters are obtained through the Moth Flame Optimization (MFO) algorithm.

5.1. Comparative Performance of Model 1 and Model 2

For Model 1, the optimized results obtained through MFO indicate a high availability (≈ 0.9993) and a large MTSF (≈ 0.9987), demonstrate that the system remains operational for a longer duration before failure. The busy period of the repairman (≈ 0.0292) and the expected number of repairman's visits (≈ 0.0028) are comparatively low, which implies reduced maintenance effort and lower operational burden. This suggests that Model 1 is efficient in terms of maintainability and resource utilization.

In contrast, Model 2, which incorporates additional degraded and down states along with multiple repair loops, exhibits a slightly higher MTSF (≈ 0.9999), indicating better long-term reliability. However, this improvement comes at the cost of lower availability (≈ 0.9800). Furthermore, Model 2 shows a significantly higher busy period of the repairman (≈ 0.6371) and a larger expected number of repairman's visits (≈ 0.0364). This highlights that while Model 2 can sustain operation for longer duration, it demands more frequent and prolonged maintenance activities.

5.2 Impact of System Complexity

The results clearly indicate that increased system complexity enhances reliability but adversely affects maintainability. Model 2 captures more realistic operational scenarios by including additional degraded and repair states, which improves MTSF but increases downtime due to frequent repairs. On the other hand, Model 1, with a simpler configuration, ensures higher availability and lower maintenance involvement, making it suitable for systems where uninterrupted operation is critical.

5.3. Effectiveness of Moth Flame Optimization (MFO)

The application of the MFO algorithm proves to be highly effective in optimizing system parameters. The algorithm successfully balances failure and repair rates to maximize reliability indices while minimizing maintenance-related measures. The obtained results confirm that MFO is a robust and efficient metaheuristic tool for solving nonlinear and multi-objective optimization problems in reliability engineering. Overall, the comparative analysis provides valuable insights into the trade-off between reliability and maintainability, enabling system designers to select an appropriate model based on operational priorities.

6. CONCLUSION

In this study, two repairable system models are formulated and analysed using Regenerative Point Graphical Technique to derive reliability measures such as MTSF, Availability, Busy Period, and Expected Number of Repairman's Visits. To enhance system performance, the Moth Flame Optimization algorithm is employed to obtain optimal values of failure and repair parameters. The results reveal that Model 1 offers higher availability and lower maintenance effort, making it more suitable for systems where continuous operation and reduced repair workload are essential. Conversely, Model 2 demonstrates superior long-term reliability (higher MTSF) but requires increased maintenance involvement due to its complex structure and multiple repair loops. The study highlights the effectiveness of metaheuristic optimization, particularly MFO, in reliability analysis and parameter tuning of repairable systems. The findings provide practical guidelines for system designers and reliability engineers to make informed decisions by considering the trade-offs between reliability, availability, and maintenance cost.

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