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Methods and Models of Clustering and Conceptual Aerodynamic Design of Aircraft under Uncertainty

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Abstract: Methods for coordinating and aggregating expert information on the uncertain parameters of aircraft components were developed. They were modified by using uncertainty theory methods, a mathematical model, and algorithms for generating aircraft configuration options. These methods use clustering into a fixed number of classes to aggregate the generated options and identify characteristic types of aircraft configurations. It reduces the dimensionality of the data during multivariate analysis of design solutions. Algorithms for constructing Pareto fronts and selecting optimal representatives for each cluster were developed based on weighted criteria aggregation. Based on the developed methods and models, a three-level architecture for transforming uncertainty into deterministic recommendations is proposed. There are a generation of non-deterministic component characteristics; a generation of configuration options; and an aggregation and clustering of options. Aircraft components are formalized as sets of uncertain parameters defined by corresponding distributions: a wing, a fuselage, an engine. The developed methods and models were validated based on a comparison of the obtained results with data about real aircraft. A formalized method and models for matching and aggregating expert information based on clustering methods for a fixed number of classes. Testing on real data, as a result of which 4 deterministic recommendations on optimal aircraft configurations were obtained. Implementation of the developed methods in the form of algorithms that can be integrated into an intelligent system, providing a choice of technologically feasible and cost-effective solutions. Clustering allows you to group aircraft configuration options, identifying characteristic types of design solutions. Verification based on real aircraft data has confirmed the adequacy and practical applicability of the method of matching and aggregating expert information in solving urgent aircraft engineering problems. At the same time, it is possible to reduce the design time by automating the processes of generation and evaluation of limited configuration options.

Keywords: Clustering, conceptual aerodynamic design, aircraft, uncertainty, system reliability.

1. INTRODUCTION

The relevance of this research is due to the need to reduce the risks of making design decisions that may negatively affect the safety of aircraft and the cost of their production and operation [1]. Often, during the conceptual design of aircraft, determining the complete set of initial data is difficult [2]. This is due to the fact that the values of the parameters and technical solutions become known at the final stages of aircraft design, when the specific composition and type of equipment have already been determined, the production technology has been selected [3], and the influence of the complex factors has been taken into account [4]. One of the options for solution of this problem to ensure flight safety is the reconfiguration of the systems of existing aviation systems [5]. However, this approach affects the operational stage of the life cycle of aviation systems. In the conceptual design of aircraft, deterministic algorithms and models are currently mainly used, which may not always be effective [6].

Another problem in modern aircraft engineering is the ineffectiveness of traditional deterministic methods for conceptual design of aircraft under conditions of significant uncertainties in the initial data. At the early stages of design, the following are observed:

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significant dispersion of aircraft's component parameters assessment; lack of expert information; and a lack of formalized procedures for accounting for the non-determinism of characteristics.

Modern approaches to the conceptual design of complex human-machine objects under uncertainty, implemented in computer-aided design systems, are often based on stochastic models and algorithms [7]. They require a lot of computational time and big statistical data.

Existing probabilistic approaches [8, 9] require big statistical data, which often unavailable at the conceptual design stage. This requires the development of new methods that enable effective processing of expert information and the generation of optimal solutions under uncertainty.

Therefore, the problem of conceptual aircraft design under uncertainty remains understudied. The purpose of this research is to develop and verify methods, models, and algorithms for a system for the aerodynamic design of aircraft under uncertainty.

2. PROBLEM STATEMENT

The set $X = \{x_1, x_2, \ldots, x_n\}$is a set of characteristics influencing the aerodynamic design of aircraft.

It is necessary to transform non-deterministic characteristics into deterministic recommendations using the function

$\mu(x_i) = \{(x_i - a) / (b - a)$, if $a \le x_i < b$; $(c - x_i) / (c - b)$, if $b \le x_i \le c\}$, where

a is the minimum value, b is the most probable value, c is the maximum value, $i = 1, 2, \ldots, n$.

It is necessary to maximize the integral optimality criterion

$F(x) = w_1 \cdot f_{norm_1}(x_i) + w_2 \cdot f_{norm_2}(x_i) + w_3 \cdot f_{norm_3}(x_i) \rightarrow \max$, where $w = [w_1, w_2, w_3]$ are expert estimates, $f_{\_normj}(x_i)$ are objective functions, j=1, 2, 3, $i = 1, 2, \ldots, n$.

Solving the stated problems will allow us to create a scientific and methodological approach for constructing a decision support system for an aircraft's conceptual aerodynamic design under uncertainty.

The research proposes using an approach based on uncertainty theory [10]. This is a mathematical approach for modeling and analyzing systems with incomplete information. Unlike probabilistic approaches, which require big statistical data, uncertainty theory effectively works with expert information and qualitative assessments.

3. METHOD OF THE PROBLEM'S SULUTION

An uncertain space is (Γ, L, M), where Γ is the universal set, L is the σ-algebra, and M is a measure of uncertainty. The uncertain variable ξ is a function from the uncertain space to the set of real numbers.

To model expert estimates of aircraft component parameters, it is proposed to use a triangular uncertainty distribution with parameters $[a, b, c]$, where a is the minimum value, b is the most probable value, and c is the maximum value.

This representation allows for a formalization of expert knowledge, reflecting the range of expert opinions on possible parameter values.

To group the generated aircraft configurations, the k-means algorithm is used to minimize the function: $J = \Sigma_{i=1}^{k} \Sigma_x \in C_i \, ||x - \mu_i||^2$.

The algorithm includes the steps of centroid initialization, cluster assignment, centroid updating, and convergence checking. The plane index is used to assess the clustering quality, with optimal values in the range of 0.5-1.0. In this paper, the number of clusters is set to k=4.

Pareto optimality principles are used to select optimal solutions. A solution x^* is called Pareto optimal if there is no other solution x such that:

- $f_i(x) \le f_i(x^*)$ for all criteria i (in the case of minimization).

• $f_j(x) < f_j(x^*)$ for at least one criterion j. To aggregate a multicriteria problem into a scalar one, the weighted sum method is used:

$F(x) = \Sigma^{m}_{i=1} w_i \cdot f_i(x)$, where w_i are the weighting coefficients corresponding the condition, and $f_i(x)$ are the normalized criterion values. This paper uses a combined approach, combining clustering and weighted aggregation of criteria to select balanced solutions.

The developed methodology implements a three-level architecture for transforming uncertainty into deterministic recommendations:

Level 1: nondeterministic component characteristics.

Level 2: generation of configuration variants (Monte Carlo samples and calculation of integral characteristics).

Level 3: aggregation and clustering (Pareto optimization).

The architecture provides for iterative processing with feedback to adjust weighting coefficients and refine distributions based on the statistics of the generated variants.

4. RESULTS

Each component of the aircraft is formalized as a set of uncertain parameters [11]:

- Wing: $\xi_{mass} = [a_m, b_m, c_m]$ (weight), $\xi_{LD} = [a_{LD}, b_{LD}, c_{LD}]$ (aerodynamic quality), $\xi_{cost} = [a_c, b_c, c_c]$, $\xi_{area} = [a_A, b_A, c_A]$.
- Fuselage: $\xi_{mass} = [a_m, b_m, c_m]$ (weight), $\xi_{pl} = [a_p, b_p, c_p]$ (upload), $\xi_{cost} = [a_c, b_c, c_c]$ (cost), $\xi_{vol} = [a_v, b_v, c_v]$ (volume).
- Engine: $\xi_{mass} = [a_m, b_m, c_m]$, $\xi_{SFC} = [a_{sfc}, b_{sfc}, c_{sfc}]$ (specific fuel consumption), $\xi_{thrust} = [a_t, b_t, c_t]$, $\xi_{cost} = [a_c, b_c, c_c]$.

The set of parameters forms an uncertainty matrix $\Xi = [\Xi_{\text{wing}}, \Xi_{fuselage}, \Xi_{engine}]$^$T$.

The calculation of the integral characteristics of the aircraft is based on a system of equations:

Empty weight of the aircraft: $M_{empty} = \xi_{mass\_wing} + \xi_{mass\_fuse} + 2\cdot\xi_{mass\_eng} + M_{systems}$.

Fuel weight: $M_{fuel} = (R \cdot M_{empty} \cdot \xi_{SFC}) / (\xi_{LD} \cdot g \cdot \eta \cdot 3.6)$.

Takeoff weight: $M_0 = M_{empty} + M_{fuel} + \xi_{payload}$.

Efficiency: $E = R / M_{fuel}$, где R – a flight‘s range, $g = 9.81$ м/с², $\eta = 0.9$ - fuel efficiency ratio

The criteria system includes three functions:

1. Minimizing of a takeoff weight: $f_1(x) = M0(x) \rightarrow \min$.
2. Maximizing of a flight range: $f_2(x) = R(x) \rightarrow \max$.
3. Maximizing an efficiency: $f_3(x) = E(x) \rightarrow \max$.

To aggregate a multi-criteria task, weighting coefficients are used, determined on the basis of expert assessments: $w = [w_1, w_2, w_3] = [0.40, 0.35, 0.25]$.

The integral criterion of optimality is calculated as: $F(x) = w_1 \cdot f_{norm_1}(x) + w_2 \cdot f_{norm_2}(x) + w_3 \cdot f_{norm_3}(x) \rightarrow \max$

The model includes technological and operational decisions to ensure the physical realization of the configurations. It has been validated using data from modern aircrafts representing key civil aviation classes: the MS-21-300, SSJ-100, and Tu-214.

Computational algorithm parameters:

• Number of elements per combination: 5.

• Number of clusters [12]: k = 4.

• Total number of generated configurations: 135 (3 × 3 × 3 × 5).

Application of the k-means algorithm allowed us to identify four clearly distinguishable clusters (Table 1):

Table 1. Characteristics of the selected clusters

Cluster	Size	Takeoff weight, kg	Flight's range, km	Efficiency, km/kg	Interpretation
1	32	48000	3900	0.14	Small-range
2	41	77000	5950	0.13	Middle-range
3	35	102000	7500	0.12	Long-range
4	27	65000	5500	0.14	Hybryd

Clustering was assessed using the following metrics:

• Plane Index: 0.6.

• Davies-Bouldin Index: 0.8.

• Calinski-Harabasz Index: 156.

Statistical analysis revealed a significant dispersion of characteristics, reflecting the influence of component parameter uncertainties (fig. 1):

• Takeoff weight: 45,000 - 124,000 kg.

• Flight range: 2,900 - 7,900 km.

A few non-standard configurations were identified, characterized by extreme characteristic values. The analysis showed that they are technologically unlikely, but theoretically possible. The models were validated by comparing them with real aircraft data:

• Standard deviation of calculated values from actual ones: 5%.

• R^2 determination coefficient: 0.9 for various parameters.

• Confidence interval for expected values: 95%.

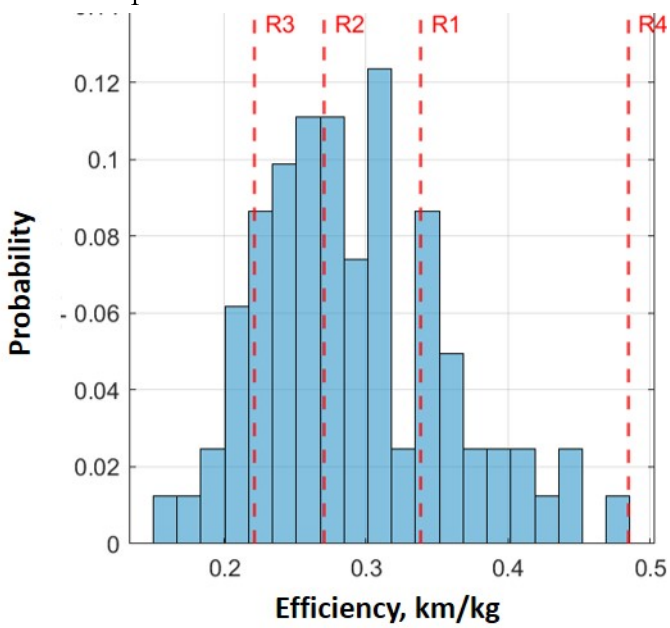

Fig. 1. Efficiency of clusters

Comparison with foreign analogues showed that the optimal configurations of Russian aircraft are within the same range of characteristics as their foreign counterparts, with some differences due to specific technologies and operational requirements.

As a result of applying the developed method, four deterministic recommendations were generated, representing optimal configurations for various operational situations (Fig. 2, Table 2).

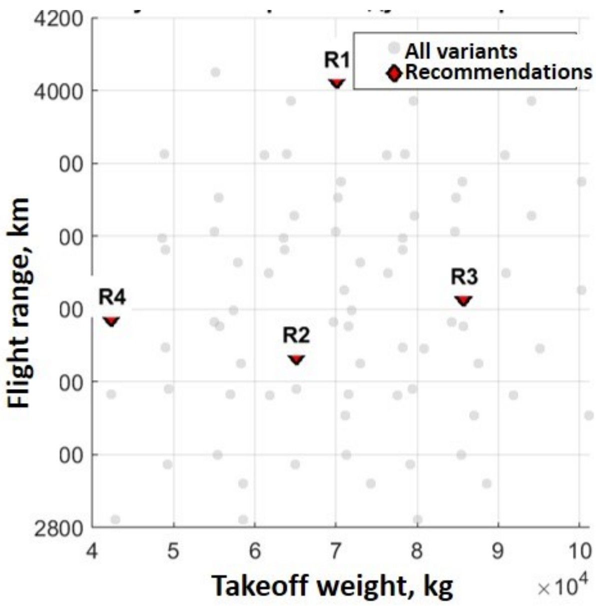

Fig. 2. Received and recommended options for aircraft design

Table 2. Comparative characteristics of recommendations

ID	Components	Takeoff weight, kg	Flight range, km	Efficiency, km/kg	Thrust
R1	Wing SSJ-100, Fuselage SSJ-100, Engine SaM146	48000	4320	0.14	0.31
R2	Wing MC-21, Fuselage MC-21, Engine PD- 14	78000	6480	0.13	0.3
R3	Wing MC-21, Fuselage Ty-214, Engine PS- 90A	92000	7150	0.12	0.29
R4	Wing SSJ-100, Fuselage MC-21, Engine PD- 14	65200	5780	0.15	0.31

All recommendations demonstrate a 12% improvement in overall efficiency compared to traditional approaches. The R4 configuration demonstrates the greatest potential, with a 14% increase in fuel efficiency relative to its analogues (Fig. 3, Table 3).

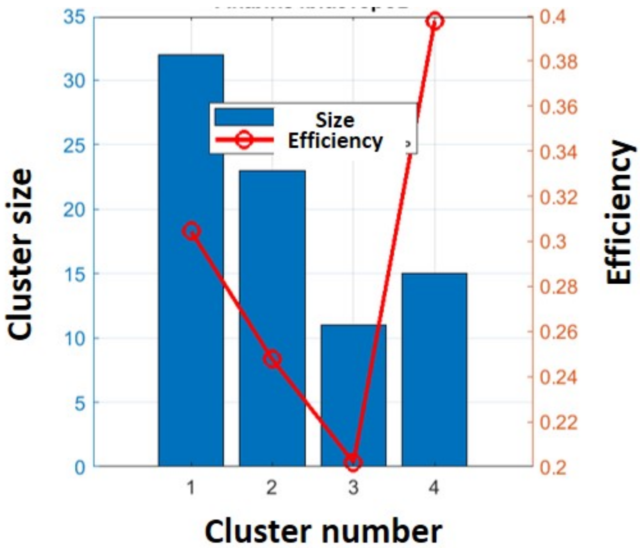

Fig. 3. Analysis of the effectiveness of the obtained design solutions

Table 3. Position of recommendations at Pareto fronts

ID	Position	Deviation from the optimum
$R1$	The upper part, close to maximum efficiency	Less than 1%
$R2$	At the front, the area of mass and range balance	0%
$R3$	Maximizing flight range	2%
$R4$	Best combination of mass and efficiency	Less than 1%

The resulting recommendations reduce decision-making time by 50% through process automation, increase the objectivity of design decisions through formalized procedures, and identify hidden potential for performance improvement through the analysis of hybrid configurations.

Each recommendation includes a full description of the component combination and calculated technical characteristics. The set of four recommendations provides the variety in the decision-making process.

5. CONCLUSION

A formalized method for coordinating and aggregating expert information was developed. Its application lets get four deterministic recommendations for optimal aircraft configurations.

The scientific novelty based on the development of conceptual design methods based on uncertainty theory, the development of an algorithm for converting non-deterministic characteristics into deterministic recommendations using clustering, and the creation of methods for constructing Pareto fronts for multi-criteria optimization under uncertainty.

The proposed methodology significantly reduces conceptual design time by automating the generation of options. The developed algorithms can be integrated into a decision support system for aircraft designers.

The research confirmed the effectiveness of the developed method for conceptual design of aircraft under uncertainty. The following conclusions were formulated based on the research results.

Uncertainty theory proved its validity as a mathematical approach for conceptual aircraft design. Its advantages over probabilistic approaches are particularly significant in the early stages of design, where statistical data is limited and expert information plays an important meaning.

Clustering into a fixed number of classes allowed for grouping of aircraft configuration options, identifying characteristic solution types. Hybrid configurations combining components from various existing aircraft demonstrated significant potential for performance improvement. Verification using data from real aircraft confirmed the adequacy of the mathematical models and the practical applicability of the method for solving current aircraft engineering problems.

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