Adv Syst Sci Appl 2026; **02**; 39-58
Published online at https://ijassa.ipu.ru

Firefly Optimization Analysis of Steel Manufacturing Plant under the Approach of RPGT

Shakuntla Singla[1], Ritu Rani[2*], Pooja Dhawan[2]

[1)] *Choudhary Ishwar Singh Kanya Mahavidyalaya, Fatehapur, Pundri, India*

[2)] *Maharishi Markandeshwar (Deemed to be University) Mullana, Ambala, India*

Abstract: This study presents a reliability and cost-based optimization framework for a steel manufacturing plant. It consists of four subsystems: "Grinding Machine," "Descaling Machine," "Hot Steckel Mill," and "Cutting Machine." A grinding machine is frequently utilized. If this machine malfunctions, it might lead to a serious system failure. Thus, we use a standby unit equipped with a grinding machine and a descaling machine that consists of two parallel machines operating simultaneously. This subsystem can operate with a single machine at a lower capacity. The failure in the hot steel mill and cutting machine leads to the complete breakdown of the system. The study's goal is to maximize manufacturing plant production by enhancing system dependability through better management, control, and maintenance. The analysis focuses on four key performance measures: Mean Time to System Failure (MTSF), steady-state availability, busy period, and the expected number of server visits under the approach of RPGT. A Markov-based simulation system model has been developed for performance analysis and solved using the firefly algorithm (FA) to improve the outcomes. The main tenets of this approach are as follows: subsystem repair and failure rates vary; repaired units function as reliably as new ones over a certain length of time; repair and failure rates operate independently; and a single repair facility meets all demands. These simplifications make it possible to analyze system performance and dependability in a targeted manner. In order to maximize industrial advantages and maximize dependability, the optimal values of all parameters based on the exponential distribution are taken into consideration. To examine the findings and make inferences about the behavior of the system, tabular and graphical analyses are provided.

Keywords: FA, RPGT, steady-state availability, server visits, standby unit

1. INTRODUCTION

Steel is extremely environmentally friendly due to its limitless recyclability, long life, and continual industry developments in decreasing emissions and resource consumption. Long-lasting steel structures require less maintenance and replacement, which reduces overall lifespan emissions—steel is more resistant to weather, pests, and wear than wood and concrete. Steel manufacturing equipment may be made more dependable by improved automation, tough materials, and smart monitoring systems, increasing uptime and precision in high-demand output like pipes. The research aims to optimize manufacturing plant production by improving system reliability via better management, control, and maintenance. Dahiya et al. [1] utilized Markov modeling to investigate the A-pan crystallization system in the sugar industry. Vanita et al. [2] emphasized how a briquette machine's availability and profit were affected by both minor and significant defects that were fixed by two repairmen. A reliability investigation of an elevator system under a resume repair strategy was carried out by Shristi Kharola et al. [3]. Kumar et al. [4] examined the elements that are important contributors to low availability in thermal power plants. According to the mathematical methods the authors used to calculate availability, one of the main causes of the system's low availability is boiler tube failure. It looks at a variety of studies on RAMS uses in both industry and research, including books, conference

*Corresponding author: riturani32941@gmail.com

papers, and articles. The study offers insights into the condition of RAMS practices both now and in the past by covering a variety of tools, strategies, and approaches that are helpful for both qualitative and quantitative research. Particle swarm optimization is used by Sunita et al. [5] to solve limited problems. Malik et al. [6] focused on optimizing coal ash management in a subcritical thermal power plant: performance and maintenance considerations. Lalji Munda et al. [7] conducted a reliability and economic study of a system consisting of three units: operating, warm standby, and hot standby. According to the mathematical methods the authors used to calculate availability, one of the main causes of the system's low availability is boiler tube failure. The efficacy of Safety, Availability, Reliability, and Maintainability techniques in a range of process industry mechanical systems was examined by Parkash & Tewari [8]. Parkash & Tewari [9] concentrated on modeling performance and suggested a probabilistic decision support system for maintenance system prioritization in a manufacturing line. Painting machine, shot peening, riveting machine, and assembly platform are some of the subunits that make up the system. Performance was modeled using a Markovian technique. Decision matrices and Differential equations with varying repair and failure rates were developed, and steady-state probabilities were computed using a transition diagram. Pinki et al. [10] concentrated on a two-unit gas turbine system's dependability was modeled while taking humidity into account. Singla et al. [11] concentrated on the reliability parameters of a parallel system under the cold standby unit. Singla et al. [12] use a genetic algorithm to investigate a problematic system to determine dependability measures that are impacted by the pace of deterioration and preventative maintenance. Singla et al. [13] concentrated on the stochastic evaluation and reliability modeling of a machine including multiple units with prompt repair of malfunctioning units. Singla et al. [14] used deep learning to optimize dependability parameters, rise in industrial paychecks, and establish a good 2:3 system. Tewari and Kumar [15] used Markov modeling to analyze the availability of the milling system in rice production facilities.

The work that is being presented aims to provide the most favorable operating condition for the steel manufacturing plant, balancing high reliability, low maintenance burden, and maximum long-run profit. The paper is organized as follows: Section 2 explains the model's specifics, including the model frame, notations, assumptions, and state summary. The system's mathematical representation is covered in Section 3. Section 4 presents the approach used to determine the impact of various rates. Section 5 discusses the results of numerical simulations. The observations discussion brings Section 6 to a close. The conclusion is given in Section 7.

2. MODEL DETAILS, ASSUMPTIONS AND NOTATIONS

2.1. System Description

Because of its great tensile strength, formability, and resistance to corrosion, steel can withstand extreme stress while yet being lighter than other materials. Since more than 90% of steel is reused worldwide, it is completely recyclable and sustainable. Compared to aluminum or other metals, production costs are minimal and energy requirements are significantly lower. This research focused on the "steel industry," which is comprised of the four subsystems depicted in the figure: "Grinding Machine," "Descaling Machine," "Hot Steckel Mill," and "Cutting Machine."

The structure of the steel manufacturing plant, presumptions, and notations needed to build the mathematical formulation for the plant's behavior analysis are now presented.

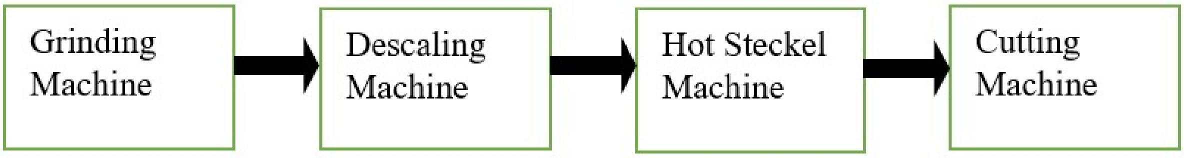

Fig. 2.1. Structure of Steel Manufacturing Plant

Sub-system G (Grinding Machine): For grinding, a grinding machine is frequently utilized. If this machine malfunctions, it might lead to a serious system failure. Thus, we use a standby unit equipped with a grinding machine.

Sub-system D (Descaling Machine): In actuality, a descaling machine consists of two parallel machines operating simultaneously. This subsystem can operate with a single machine at a lower capacity.

Sub-system H (Hot Steckel Mill): Its failure causes the system to completely fail.

Sub-system C (Cutting Machine): Its failure causes the system to completely fail.

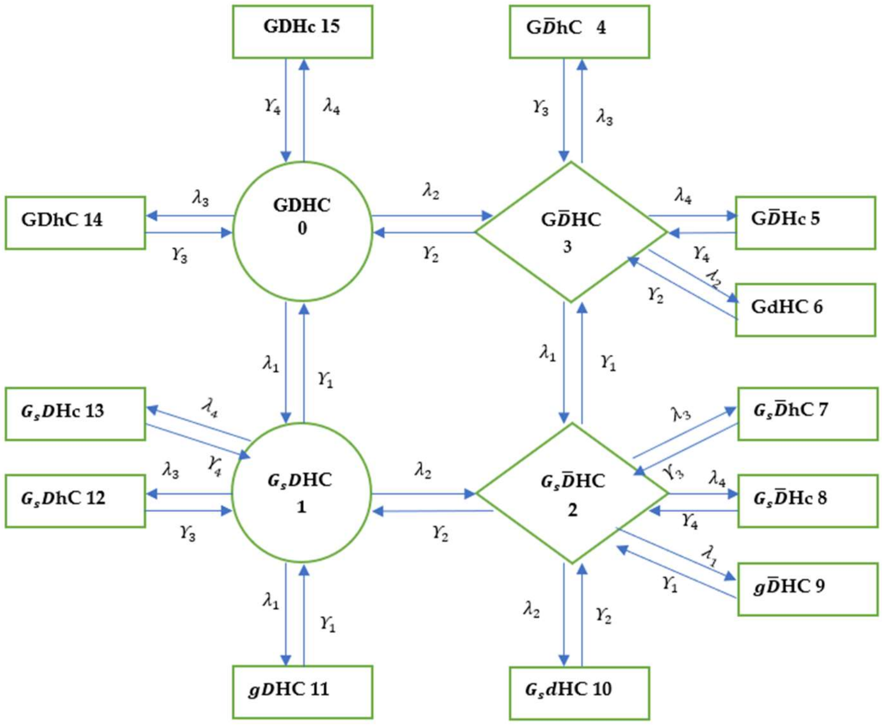

Figure 2.2: State Transition Diagram

2.2. Assumptions

- Repair rates and failure rates are independent of one another.
- For a certain amount of time, a repaired unit performs as well as new.
- Subsystem repair and failure rates are considered to be variable.
- There is just one repair facility available.
- There are no simultaneous failures.

2.3. Notations

G, D, H, C	Indicates that the subsystems are fully functional.
G_s	Indicates that the subsystem is operating on a standby unit.
$\bar{D}$	Indicates that subsystem D is a reduced configuration
g, d, h, c	Indicate the failure state of the subsystem.
$\lambda_i (i = 1,2,3,4)$	Subsystem failure rates are represented as G, D, H, and C.
$\Upsilon_i (i = 1,2,3,4)$	Repair rates for G, D, H, and C, correspondingly.
q_{ij}/p_{ij}	Probability density function / Steady state probability.
©/Ⓢ	Laplace convolution / Laplace Stieltjes convolution.

Q_{ij}	Probability cumulative distribution function.
$*$	Sign for Laplace Transform (LT).
$**$	Laplace Stieltjes convolution (LST).
A_0	Availability of the system.
$\mathrm{W_i(t)}$	The likelihood that the server will remain active in state S_i until time t without changing to another regenerative state or going back to the same state through one or more regenerative states.
$\mathrm{B_i(t)}$	The likelihood that the system will operate in state i instead of changing to a different state.
$'$	Symbol for the function's derivative.
$\mathrm{BJ_0(t)}$	Probability that a repair is being worked on by a repairman to the system at time t
V_0	The expected number of server's visit
μ_i	Stay time in particular sate i

3. MATHEMATICAL PRESENTATION OF THE SYSTEM

3.1. Transition Probabilities and Mean Sojourn Times

The system's transitions to various potential states are used to calculate transition probabilities and mean sojourn periods. From state i to state j, the probability density function $q_{i,j}(t)$ is defined as follows

$$q_{0,1}(t) = \lambda_1 e^{-\eta t} \tag{3.1}$$
$$q_{0,3}(t) = \lambda_2 e^{-\eta t} \tag{3.2}$$
$$q_{0,14}(t) = \lambda_3 e^{-\eta t} \tag{3.3}$$
$$q_{0,15}(t) = \lambda_4 e^{-\eta t} \tag{3.4}$$
$$q_{1,2}(t) = \lambda_2 e^{-(\eta+\Upsilon_1)t} \tag{3.5}$$
$$q_{1,0}(t) = \Upsilon_1 e^{-(\eta+\Upsilon_1)t} \tag{3.6}$$
$$q_{1,11}(t) = \lambda_1 e^{-(\eta+\Upsilon_1)t} \tag{3.7}$$
$$q_{1,12}(t) = \lambda_3 e^{-(\eta+\Upsilon_1)t} \tag{3.8}$$
$$q_{1,13}(t) = \lambda_4 e^{-(\eta+\Upsilon_1)t} \tag{3.9}$$
$$q_{2,3}(t) = \Upsilon_1 e^{-(\eta+\Upsilon_1+\Upsilon_2)t} \tag{3.10}$$
$$q_{2,1}(t) = \Upsilon_2 e^{-(\eta+\Upsilon_1+\Upsilon_2)t} \tag{3.11}$$
$$q_{2,7}(t) = \lambda_3 e^{-(\eta+\Upsilon_1+\Upsilon_2)t} \tag{3.12}$$
$$q_{2.8}(t) = \lambda_4 e^{-(\eta+\Upsilon_1+\Upsilon_2)t} \tag{3.13}$$
$$q_{2,9}(t) = \lambda_1 e^{-(\eta+\Upsilon_1+\Upsilon_2)t} \tag{3.14}$$
$$q_{2,10}(t) = \lambda_2 e^{-(\eta+\Upsilon_1+\Upsilon_2)t} \tag{3.15}$$
$$q_{3,0}(t) = \Upsilon_2 e^{-(\eta+\Upsilon_2)t} \tag{3.16}$$
$$q_{3,2}(t) = \lambda_1 e^{-(\eta+\Upsilon_2)t} \tag{3.17}$$
$$q_{3,4}(t) = \lambda_3 e^{-(\eta+\Upsilon_2)t} \tag{3.18}$$
$$q_{3,5}(t) = \lambda_4 e^{-(\eta+\Upsilon_2)t} \tag{3.19}$$
$$q_{3,6}(t) = \lambda_2 e^{-(\eta+\Upsilon_2)t} \tag{3.20}$$
$$q_{4,3}(t) = \Upsilon_3 e^{-\Upsilon_3 t} \tag{3.21}$$
$$q_{5,3}(t) = \Upsilon_4 e^{-\Upsilon_4 t} \tag{3.22}$$
$$q_{6,3}(t) = \Upsilon_2 e^{-\Upsilon_2 t} \tag{3.23}$$
$$q_{7,2}(t) = \Upsilon_3 e^{-\Upsilon_3 t} \tag{3.24}$$
$$q_{8,2}(t) = \Upsilon_4 e^{-\Upsilon_4 t} \tag{3.25}$$
$$q_{9,2}(t) = \Upsilon_1 e^{-\Upsilon_1 t} \tag{3.26}$$

$$q_{10,2}(t) = \Upsilon_2 e^{-\Upsilon_2 t} \tag{3.27}$$
$$q_{11,1}(t) = \Upsilon_1 e^{-\Upsilon_1 t} \tag{3.28}$$
$$q_{12,1}(t) = \Upsilon_3 e^{-\Upsilon_3 t} \tag{3.29}$$
$$q_{13,1}(t) = \Upsilon_4 e^{-\Upsilon_4 t} \tag{3.30}$$
$$q_{14,0}(t) = \Upsilon_3 e^{-\Upsilon_3 t} \tag{3.31}$$
$$q_{15,0}(t) = \Upsilon_4 e^{-\Upsilon_4 t} \tag{3.32}$$

where,

$$\eta = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4.$$

Now defining steady state probability,

$$p_{i,j} = \lim_{s\to 0} q_{i,j}^{*}(s) = \lim_{s\to 0} \int_0^{\infty} e^{-st}\, q_{i,j}(t)dt$$

$$p_{0,1} = \frac{\lambda_1}{\eta} \tag{3.33}$$
$$p_{0,3} = \frac{\lambda_2}{\eta} \tag{3.34}$$
$$p_{0,14} = \frac{\lambda_3}{\eta} \tag{3.35}$$
$$p_{0,15} = \frac{\lambda_4}{\eta} \tag{3.36}$$
$$p_{1,2} = \frac{\lambda_2}{\eta + \Upsilon_1} \tag{3.37}$$
$$p_{1,0} = \frac{\Upsilon_1}{\eta + \Upsilon_1} \tag{3.38}$$
$$p_{1,11} = \frac{\lambda_1}{\eta + \Upsilon_1} \tag{3.39}$$
$$p_{1,12} = \frac{\lambda_3}{\eta + \Upsilon_1} \tag{3.40}$$
$$p_{1,13} = \frac{\lambda_4}{\eta + \Upsilon_1} \tag{3.41}$$
$$p_{2,1} = \frac{\Upsilon_2}{\eta + \Upsilon_1 + \Upsilon_2} \tag{3.42}$$
$$p_{2,3} = \frac{\Upsilon_1}{\eta + \Upsilon_1 + \Upsilon_2} \tag{3.43}$$
$$p_{2,7} = \frac{\lambda_3}{\eta + \Upsilon_1 + \Upsilon_2} \tag{3.44}$$
$$p_{2,8} = \frac{\lambda_4}{\eta + \Upsilon_1 + \Upsilon_2} \tag{3.45}$$
$$p_{2,9} = \frac{\lambda_1}{\eta + \Upsilon_1 + \Upsilon_2} \tag{3.46}$$
$$p_{2,10} = \frac{\lambda_2}{\eta + \Upsilon_1 + \Upsilon_2} \tag{3.47}$$
$$p_{3,0} = \frac{\Upsilon_2}{\eta + \Upsilon_2} \tag{3.48}$$
$$p_{3,2} = \frac{\lambda_1}{\eta + \Upsilon_2} \tag{3.49}$$
$$p_{3,4} = \frac{\lambda_3}{\eta + \Upsilon_2} \tag{3.50}$$
$$p_{3,5} = \frac{\lambda_4}{\eta + \Upsilon_2} \tag{3.51}$$

$$p_{3,6} = \frac{\lambda_2}{\eta + \Upsilon_2} \tag{3.52}$$

$$p_{i,3} = 1; i = 4,5,6 \tag{3.53}$$

$$p_{j,2} = 1; j = 7,8,9,10 \tag{3.54}$$

$$p_{k,1} = 1; k = 11,12,13 \tag{3.55}$$

$$p_{l,0} = 1; l = 14,15 \tag{3.56}$$

Mean Sojourn Times:

The mean duration of time in the i^{th} state , denoted by μ_i, is given by:

$$\mu_i = E(t) = \int_0^\infty \text{t. (corresponding pdf for transitioning from the i}^{\text{th}}\text{ sate) dt}$$

$$\mu_0 = \frac{1}{\eta} \tag{3.57}$$

$$\mu_1 = \frac{1}{\eta + \Upsilon_1} \tag{3.58}$$

$$\mu_2 = \frac{\lambda_2}{\eta + \Upsilon_1 + \Upsilon_2} \tag{3.59}$$

$$\mu_3 = \frac{\lambda_2}{\eta + \Upsilon_2} \tag{3.60}$$

$$\mu_4 = \frac{1}{\Upsilon_3} \tag{3.61}$$

$$\mu_5 = \frac{1}{\Upsilon_4} \tag{3.62}$$

$$\mu_6 = \frac{1}{\Upsilon_2} \tag{3.63}$$

$$\mu_7 = \frac{1}{\Upsilon_3} \tag{3.64}$$

$$\mu_8 = \frac{1}{\Upsilon_4} \tag{3.65}$$

$$\mu_9 = \frac{1}{\Upsilon_1} \tag{3.66}$$

$$\mu_{10} = \frac{1}{\Upsilon_2} \tag{3.67}$$

$$\mu_{11} = \frac{1}{\Upsilon_1} \tag{3.68}$$

$$\mu_{12} = \frac{1}{\Upsilon_3} \tag{3.69}$$

$$\mu_{13} = \frac{1}{\Upsilon_4} \tag{3.70}$$

$$\mu_{14} = \frac{1}{\Upsilon_3} \tag{3.71}$$

$$\mu_{15} = \frac{1}{\Upsilon_4} \tag{3.72}$$

It is possible to calculate analytically the unconditional mean time needed for the system to pass through any regeneration state j, counting backward from the epoch of entry into state i:

$$m_{i,j} = \int_0^\infty t q_{i,j}(t) dt = {q'}^{*}_{i,j}(0)$$

$$m_{0,1} = \frac{\lambda_1}{\eta^2} \tag{3.73}$$

$$m_{0,3} = \frac{\lambda_2}{\eta^2} \tag{3.74}$$

$$m_{0,14} = \frac{\lambda_3}{\eta^2} \tag{3.75}$$

$$m_{0,15} = \frac{\lambda_4}{\eta^2} \tag{3.76}$$

$$m_{1,2} = \frac{\lambda_2}{(\eta + \Upsilon_1)^2} \tag{3.77}$$

$$m_{1,0} = \frac{\Upsilon_1}{(\eta + \Upsilon_1)^2} \tag{3.78}$$

$$m_{1,11} = \frac{\lambda_1}{(\eta + \Upsilon_1)^2} \tag{3.79}$$

$$m_{1,12} = \frac{\lambda_3}{(\eta + \Upsilon_1)^2} \tag{3.80}$$

$$m_{1,13} = \frac{\lambda_4}{(\eta + \Upsilon_1)^2} \tag{3.81}$$

$$m_{2,1} = \frac{\Upsilon_2}{(\eta + \Upsilon_1 + \Upsilon_2)^2} \tag{3.82}$$

$$m_{2,3} = \frac{\Upsilon_1}{(\eta + \Upsilon_1 + \Upsilon_2)^2} \tag{3.83}$$

$$m_{2,7} = \frac{\lambda_3}{(\eta + \Upsilon_1 + \Upsilon_2)^2} \tag{3.84}$$

$$m_{2,8} = \frac{\lambda_4}{(\eta + \Upsilon_1 + \Upsilon_2)^2} \tag{3.85}$$

$$m_{2,9} = \frac{\lambda_1}{(\eta + \Upsilon_1 + \Upsilon_2)^2} \tag{3.86}$$

$$m_{2,10} = \frac{\lambda_2}{(\eta + \Upsilon_1 + \Upsilon_2)^2} \tag{3.87}$$

$$m_{3,0} = \frac{\Upsilon_2}{(\eta + \Upsilon_2)^2} \tag{3.88}$$

$$m_{3,2} = \frac{\lambda_1}{(\eta + \Upsilon_2)^2} \tag{3.89}$$

$$m_{3,4} = \frac{\lambda_3}{(\eta + \Upsilon_2)^2} \tag{3.90}$$

$$m_{3,5} = \frac{\lambda_4}{(\eta + \Upsilon_2)^2} \tag{3.91}$$

$$m_{3,6} = \frac{\lambda_2}{(\eta + \Upsilon_2)^2} \tag{3.92}$$

$$m_{4,3} = \frac{1}{\Upsilon_3} \tag{3.93}$$

$$m_{5,3} = \frac{1}{\Upsilon_4} \tag{3.94}$$

$$m_{6,3} = \frac{1}{\Upsilon_2} \tag{3.95}$$

$$m_{7,2} = \frac{1}{\Upsilon_3} \tag{3.96}$$

$$m_{8,2} = \frac{1}{\Upsilon_4} \tag{3.97}$$

$$m_{9,2} = \frac{1}{\Upsilon_1} \tag{3.98}$$
$$m_{10,2} = \frac{1}{\Upsilon_2} \tag{3.99}$$
$$m_{11,1} = \frac{1}{\Upsilon_1} \tag{3.100}$$
$$m_{12,1} = \frac{1}{\Upsilon_3} \tag{3.101}$$
$$m_{13,1} = \frac{1}{\Upsilon_4} \tag{3.102}$$
$$m_{14,0} = \frac{1}{\Upsilon_3} \tag{3.103}$$
$$m_{15,0} = \frac{1}{\Upsilon_4} \tag{3.104}$$

It is simple to verify the following relationships:

$$m_{0,1} + m_{0,3} + m_{0,14} + m_{0,15} = \mu_0 \tag{3.105}$$
$$m_{1,0} + m_{1,2} + m_{1,11} + m_{1,12} + m_{1,13} = \mu_1 \tag{3.106}$$
$$m_{2,1} + m_{2,3} + m_{2,7} + m_{2,8} + m_{2,9} + m_{2,10} = \mu_2 \tag{3.107}$$
$$m_{3,0} + m_{3,2} + m_{3,4} + m_{3,5} + m_{3,6} = \mu_3 \tag{3.108}$$
$$m_{4,3} = \mu_4 \tag{3.109}$$
$$m_{5,3} = \mu_5 \tag{3.110}$$
$$m_{6,3} = \mu_6 \tag{3.111}$$
$$m_{7,2} = \mu_7 \tag{3.112}$$
$$m_{8,2} = \mu_8 \tag{3.113}$$
$$m_{9,2} = \mu_9 \tag{3.114}$$
$$m_{10,2} = \mu_{10} \tag{3.115}$$
$$m_{11,1} = \mu_{11} \tag{3.116}$$
$$m_{12,1} = \mu_{12} \tag{3.117}$$
$$m_{13,1} = \mu_{13} \tag{3.118}$$
$$m_{14,0} = \mu_{14} \tag{3.119}$$
$$m_{15,0} = \mu_{15} \tag{3.120}$$

3.2 Mean time to system failure (MTSF)

Let $\Phi_i(t)$ be the first passage time's cdf from the regenerating state to the failing state. We have the following recursive relations $(3.121) - (3.124)$, for $\Phi_i(t)$ when considering the failed state as an absorbing state:

$$\Phi_0(t) = Q_{0,1}(t) Ⓢ \Phi_1(t) + Q_{0,3}(t) Ⓢ \Phi_3(t) + Q_{0,15}(t) + Q_{0,14}(t) \tag{3.121}$$
$$\Phi_1(t) = Q_{1,0}(t) Ⓢ \Phi_0(t) + Q_{1,2}(t) Ⓢ \Phi_2(t) + Q_{1,11}(t) + Q_{1,12}(t) + Q_{1,13}(t) \tag{3.122}$$
$$\Phi_2(t) = Q_{2,1}(t) Ⓢ \Phi_1(t) + Q_{2,3}(t) Ⓢ \Phi_1(t) + Q_{2,7}(t) Ⓢ \Phi_1(t) + Q_{2,8}(t) + Q_{2,9}(t) + Q_{2,10}(t) \tag{3.123}$$
$$\Phi_3(t) = Q_{3,0}(t) Ⓢ \Phi_0(t) + Q_{3,2}(t) Ⓢ \Phi_2(t) + Q_{3,4}(t) + Q_{3,5}(t) + Q_{3,6}(t) \tag{3.124}$$

Solving for $\Phi_0^{**}(s)$, using the Laplace Stieltjes transform of equations $(3.121) - (3.124)$, we have

$$\Phi_0^{**}(s) = \frac{M_1(s)}{T_1(s)}$$

where,

$$M_1(s) = \begin{vmatrix} Q_{0,15}^{**}(s) + Q_{0,14}^{**}(s) & -Q_{0,1}^{**}(s) & 0 & -Q_{0,3}^{**}(s) \\ Q_{1,11}^{**}(s) + Q_{1,12}^{**}(s) + Q_{1,13}^{**}(s) & 1 & -Q_{1,2}^{**}(s) & 0 \\ Q_{2,7}^{**}(s) + Q_{2,8}^{**}(s) + Q_{2,9}^{**}(s) & -Q_{2,1}^{**}(s) & 1 & -Q_{2,3}^{**}(s) \\ Q_{3,4}^{**}(s) + Q_{3,5}^{**}(s) + Q_{3,6}^{**}(s) & 0 & -Q_{3,2}^{**}(s) & 1 \end{vmatrix}$$

And

$$T_1(s) = \begin{vmatrix} 1 & -Q_{0,1}^{**}(s) & 0 & -Q_{0,3}^{**}(s) \\ -Q_{1,0}^{**}(s) & 1 & -Q_{1,2}^{**}(s) & 0 \\ 0 & -Q_{2,1}^{**}(s) & 1 & -Q_{2,3}^{**}(s) \\ -Q_{3,0}^{**}(s) & 0 & -Q_{3,2}^{**}(s) & 1 \end{vmatrix}$$

The MTSF is determined by

$$MTSF = \lim_{s\to 0} \frac{1 - \Phi_0^{**}(s)}{s} = \lim_{s\to 0} \frac{1 - \frac{M_1(s)}{T_1(s)}}{s} = \lim_{s\to 0} \frac{T_1(s) - M_1(s)}{sT_1(s)} \tag{3.125}$$

Using the standard properties of the Laplace–Stieltjes transform, we take

$$Q_{i,j}^{**}(0) = p_{i,j} \ and \ Q_{i,j}^{**'}(s)|_{s=0} = -m_{i,j} \tag{3.126}$$

Substituting these values into the above expression, we obtain an indeterminate form. Therefore, L' Hospital's rule is applied to evaluate the limit and obtain the final expression for the MTSF we get

$$MTSF = \lim_{s\to 0} \frac{T_1'(s) - M_1'(s)}{sT_1'(s) + T_1(s)} = \frac{T_1'(0) - M_1'(0)}{T_1(0)} = \frac{N_1}{D_1} \tag{3.127}$$

The final numerical value of the MTSF is then computed in Mathematica by substituting the corresponding values of $p_{i,j}$ and $m_{i,j}$ into the derived expression we get

$$N_1 = (2\lambda_1{}^3 + 2\lambda_1(2\lambda_2 + \lambda_3 + \lambda_4 + \Upsilon_2)(2(\lambda_2 + \lambda_3 + \lambda_4 + \Upsilon_1) + \Upsilon_2) + \lambda_1{}^2(8\lambda_2 + 5\lambda_3 + 5\lambda_4 + \Upsilon_1 + 4\Upsilon_2) + (2\lambda_2 + \lambda_3 + \lambda_4 + \Upsilon_2)((\lambda_2 + \lambda_3 3 + \lambda_4 + \Upsilon_1)^2 + (\lambda_3 + \lambda_4 + \Upsilon_1)\Upsilon_2)$$

$$D_1 = (\lambda_1{}^4 + 2\lambda_1{}^3(2(\lambda_2 + \lambda_3 + \lambda_4) + \Upsilon_2) + 2\lambda_1(2(\lambda_2 + \lambda_3 + \lambda_4 + \Upsilon_1) + \Upsilon_2)((\lambda_2 + \lambda_3 + \lambda_4)^2 + (\lambda_3 + \lambda_4)\Upsilon_2) + ((\lambda_2 + \lambda_3 + \lambda_4)^2 + (\lambda_3 + \lambda_4)\Upsilon_2)((\lambda_2 + \lambda_3 + \lambda_4 + \Upsilon_1)^2 + (\lambda_3 + \lambda_4 + \Upsilon_1)\Upsilon_2) + \lambda_1{}^2(2(\lambda_2 + \lambda_3 + \lambda_4)(3(\lambda_2 + \lambda_3 + \lambda_4) + \Upsilon_1) + (4\lambda_2 + 6(\lambda_3 + \lambda_4) + \Upsilon_1)\Upsilon_2 + \Upsilon_2{}^2))$$

3.3. Steady State Availability

Let $A_i(t)$ denote the chance that the system is in upstate at moment t, assuming that it entered regenerative mode at t=0.The recursive relations for $A_i(t)$ are provided as

$$A_0(t) = B_0(t) + q_{0,1}(t)© A_1(t) + q_{0,3}(t)©A_3(t) + q_{0,14}(t)©A_{14}(t) + q_{0,15}(t)©A_{15}(t) \tag{3.128}$$

$$A_1(t) = B_1(t) + q_{1,2}(t)© A_2(t) + q_{1,0}(t)©A_0(t) + q_{1,11}(t)©A_{11}(t) + q_{1,12}(t)©A_{12}(t) + q_{0,13}(t)©A_{13}(t) \tag{3.129}$$

$$A_2(t) = B_2(t) + q_{2,1}(t)© A_1(t) + q_{2,3}(t)©A_3(t) + q_{2,7}(t)©A_7(t) + q_{2,8}(t)©A_8(t) + q_{2,9}(t)©A_9(t) + q_{2,10}(t)©A_{10}(t) \tag{3.130}$$

$$A_3(t) = B_3(t) + q_{3,0}(t)© A_0(t) + q_{3,2}(t)©A_2(t) + q_{3,4}(t)©A_4(t) + q_{3,5}(t)©A_5(t) + q_{3,6}(t)©A_6(t) \tag{3.131}$$

$$A_4(t) = q_{4,3}(t)© A_3(t) \tag{3.132}$$

$$A_5(t) = q_{5,3}(t)© A_3(t) \tag{3.133}$$

$$A_6(t) = q_{6,3}(t)©A_3(t) \tag{3.134}$$

$$A_7(t) = q_{7,2}(t)©A_2(t) \tag{3.135}$$

$$A_8(t) = q_{8,2}\ (t)©A_2(t) \tag{3.136}$$
$$A_9(t) = q_{9,2}\ (t)©A_2(t) \tag{3.137}$$
$$A_{10}(t) = q_{10,2}\ (t)©A_2(t) \tag{3.138}$$
$$A_{11}(t) = q_{11,1}\ (t)©A_1(t) \tag{3.139}$$
$$A_{12}(t) = q_{12,1}\ (t)©A_1(t) \tag{3.140}$$
$$A_{13}(t) = q_{13,1}\ (t)©A_1(t) \tag{3.141}$$
$$A_{14}(t) = q_{14,0}\ (t)©\ A_0(t) \tag{3.142}$$
$$A_{15}(t) = q_{15,0}\ (t)©\ A_0(t) \tag{3.143}$$

then solving the equations for $A_0^*(s)$ by using the Laplace transforms, the following equation (3.144) is obtained:

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2} \tag{3.144}$$

where,

$$\begin{aligned}N_2 = (&\Upsilon_1\Upsilon_2(\lambda_1{}^3(\lambda_2+\Upsilon_2)+\Upsilon_1(\eta-\lambda_1+\Upsilon_1)(\lambda_2+\Upsilon_2)(\eta-\lambda_1+\Upsilon_1+\Upsilon_2)+\lambda_1{}^2(2\lambda_2(\eta-\lambda_1)\\&+3\lambda_2\Upsilon_1+(4\lambda_2+2(\lambda_3+\lambda_4)+3\Upsilon_1)\Upsilon_2)+\lambda_1(\lambda_2(\eta-\lambda_1+\Upsilon_1)(\eta-\lambda_1+3\Upsilon_1)\\&+(3\lambda_2{}^2+4\lambda_2(\lambda_3+\lambda_4)+7\lambda_2\Upsilon_1+(\lambda_3+\lambda_4+\Upsilon_1)(\lambda_3+\lambda_4+3\Upsilon_1))\Upsilon_2+(\lambda_2\\&+\Upsilon_1)\Upsilon_2{}^2))\Upsilon_3\Upsilon_4)\end{aligned}$$

$$\begin{aligned}D_2 = (&(\eta+\Upsilon_1)(\eta+\Upsilon_1+\Upsilon_2)(\lambda_4\Upsilon_1(\lambda_1+\Upsilon_1)\Upsilon_2(\lambda_2+\Upsilon_2)\Upsilon_3+\lambda_3\Upsilon_1(\lambda_1+\Upsilon_1)\Upsilon_2(\lambda_2+\Upsilon_2)\Upsilon_4\\&+(\lambda_1{}^2\Upsilon_2(\lambda_2+\Upsilon_2)+\lambda_1\Upsilon_1(\lambda_2{}^2+\lambda_2\Upsilon_2+\Upsilon_2{}^2)+\Upsilon_1{}^2(\lambda_2{}^2+\lambda_2\Upsilon_2+\Upsilon_2{}^2))\Upsilon_3\Upsilon_4))\end{aligned}$$

3.4. Busy Period Analysis

Assuming the system enters the regenerative state at t=0, let $BJ_i(t)$ represent the likelihood of the server being busy at instant ‘t’ due to repair. The recursive relations for $BJ_i(t)$ are as follows:

$$\begin{aligned}BJ_0(t) = {}& q_{0,1}\ (t)©\ BJ_1(t) + q_{0,3}(t)©BJ_3(t) + q_{0,14}\ (t)©BJ_{14}(t)\\&+q_{0,15}\ (t)©BJ_{15}(t)\end{aligned} \tag{3.145}$$

$$\begin{aligned}BJ_1(t) = {}& W_1(t) + q_{1,2}\ (t)©\ BJ_2(t) + q_{1,0}(t)©BJ_0(t) +\\& q_{1,11}\ (t)©BJ_{11}(t) + q_{1,12}\ (t)©BJ_{12}(t) + q_{0,13}\ (t)©BJ_{13}(t)\end{aligned} \tag{3.146}$$

$$\begin{aligned}BJ_2(t) = {}& W_2(t) + q_{2,1}\ (t)©\ BJ_1(t) + q_{2,3}(t)©BJ_3(t) +\\& q_{2,7}\ (t)©BJ_7(t) + q_{2,8}\ (t)©BJ_8(t) + q_{2,9}\ (t)©BJ_9(t) +\\& q_{2,10}\ (t)©BJ_{10}(t)\end{aligned} \tag{3.147}$$

$$\begin{aligned}BJ_3(t) = {}& W_3(t) + q_{3,0}\ (t)©\ BJ_0(t) + q_{3,2}(t)©BJ_2(t) +\\& q_{3,4}\ (t)©BJ_4(t) + q_{3,5}\ (t)©BJ_5(t) + q_{3,6}\ (t)©BJ_6(t)\end{aligned} \tag{3.148}$$

$$BJ_4(t) = W_4(t) + q_{4,3}\ (t)©\ BJ_3(t) \tag{3.149}$$
$$BJ_5(t) = W_5(t) + q_{5,3}\ (t)©\ BJ_3(t) \tag{3.150}$$
$$BJ_6(t) = W_6(t) + q_{6,3}\ (t)©BJ_3(t) \tag{3.151}$$
$$BJ_7(t) = W_7(t) + q_{7,2}\ (t)©BJ_2(t) \tag{3.152}$$
$$BJ_8(t) = W_8(t) + q_{8,2}\ (t)©BJ_2(t) \tag{3.153}$$
$$BJ_9(t) = W_9(t) + q_{9,2}\ (t)©BJ_2(t) \tag{3.154}$$
$$BJ_{10}(t) = W_{10}(t) + q_{10,2}\ (t)©BJ_2(t) \tag{3.155}$$
$$BJ_{11}(t) = W_{11}(t) + q_{11,1}\ (t)©BJ_1(t) \tag{3.156}$$
$$BJ_{12}(t) = W_{12}(t) + q_{12,1}\ (t)©BJ_1(t) \tag{3.157}$$
$$BJ_{13}(t) = W_{13}(t) + q_{13,1}\ (t)©BJ_1(t) \tag{3.158}$$
$$BJ_{14}(t) = W_{14}(t) + q_{14,0}\ (t)©\ BJ_0(t) \tag{3.159}$$
$$BJ_{15}(t) = W_{15}(t) + q_{15,0}\ (t)©\ BJ_0(t) \tag{3.160}$$

Using the LT of relations $(3.145) - (3.160)$, solve for $BJ_0^*(s)$. In the long run, the system's repair time is determined by

$$BJ_0 = \lim_{s\to 0} sBJ_0^*(s) = \frac{N_3}{D_3} \tag{3.161}$$

where

$$N_3 = (\lambda_4 \Upsilon_1(\lambda_1+\Upsilon_1)\Upsilon_2(\lambda_2+\Upsilon_2)\Upsilon_3 + \lambda_3\Upsilon_1(\lambda_1+\Upsilon_1)\Upsilon_2(\lambda_2+\Upsilon_2)\Upsilon_4 + (\lambda_2{\Upsilon_1}^2(\lambda_2+\Upsilon_2) + {\lambda_1}^2\Upsilon_2(\lambda_2+\Upsilon_2) + \lambda_1\Upsilon_1({\lambda_2}^2+\lambda_2\Upsilon_2+{\Upsilon_2}^2))\Upsilon_3\Upsilon_4)$$

$$D_3 = (\lambda_4 \Upsilon_1(\lambda_1+\Upsilon_1)\Upsilon_2(\lambda_2+\Upsilon_2)\Upsilon_3 + \lambda_3\Upsilon_1(\lambda_1+\Upsilon_1)\Upsilon_2(\lambda_2+\Upsilon_2)\Upsilon_4 + ({\lambda_1}^2\Upsilon_2(\lambda_2+\Upsilon_2) + \lambda_1\Upsilon_1({\lambda_2}^2+\lambda_2\Upsilon_2+{\Upsilon_2}^2) + {\Upsilon_1}^2({\lambda_2}^2+\lambda_2\Upsilon_2+{\Upsilon_2}^2))\Upsilon_3\Upsilon_4)$$

3.5. Expected Number of the Visits by the Server

Considering that the system enters the regenerative state at t=0, let $\mathrm{V_i(t)}$ be the anticipated number of server visits in (0, t]. The recursive relations for $\mathrm{V_i(t)}$ are as follows:

$$\mathrm{V_0(t) = Q_{0,1}(t)Ⓢ[1+V_1(t)] + Q_{0,3}(t)Ⓢ[1+V_3(t)] + Q_{0,14}(t)Ⓢ[1+V_{14}(t)] + Q_{0,15}(t)Ⓢ[1+V_{15}(t)]} \tag{3.162}$$

$$\mathrm{V_1(t) = Q_{1,2}(t)Ⓢ V_2(t) + Q_{1,0}(t)Ⓢ V_0(t) + Q_{1,11}(t)Ⓢ V_{11}(t) + Q_{1,12}(t)©V_{12}(t) + Q_{0,13}(t)©V_{12}(t)} \tag{3.163}$$

$$\mathrm{V_2(t) = Q_{2,1}(t)Ⓢ V_1(t) + Q_{2,3}(t)ⓈV_3(t) + Q_{2,7}(t)Ⓢ V_7(t) + Q_{2,8}(t)Ⓢ V_8(t) + Q_{2,9}(t)Ⓢ V_9(t) + Q_{2,10}(t)Ⓢ V_{10}(t)} \tag{3.164}$$

$$\mathrm{V_3(t) = Q_{3,0}(t)Ⓢ V_0(t) + Q_{3,2}(t)ⓈV_2(t) + Q_{3,4}(t)Ⓢ V_4(t) + Q_{3,5}(t)Ⓢ V_5(t) + Q_{3,6}(t)Ⓢ V_6(t)} \tag{3.165}$$

$$\mathrm{V_4(t) = Q_{4,3}(t)Ⓢ V_3(t)} \tag{3.166}$$

$$\mathrm{V_5(t) = Q_{5,3}(t)Ⓢ V_3(t)} \tag{3.167}$$

$$\mathrm{V_6(t) = Q_{6,3}(t)Ⓢ V_3(t)} \tag{3.168}$$

$$\mathrm{V_7(t) = Q_{7,2}(t)Ⓢ V_2(t)} \tag{3.169}$$

$$\mathrm{V_8(t) = Q_{8,2}(t)Ⓢ V_2(t)} \tag{3.170}$$

$$\mathrm{V_9(t) = Q_{9,2}(t)Ⓢ V_2(t)} \tag{3.171}$$

$$\mathrm{V_{10}(t) = Q_{10,2}(t)Ⓢ V_2(t)} \tag{3.172}$$

$$\mathrm{V_{11}(t) = Q_{11,1}(t)Ⓢ V_1(t)} \tag{3.173}$$

$$\mathrm{V_{12}(t) = Q_{12,1}(t)Ⓢ V_1(t)} \tag{3.174}$$

$$\mathrm{V_{13}(t) = Q_{13,1}(t)Ⓢ V_1(t)} \tag{3.175}$$

$$\mathrm{V_{14}(t) = Q_{14,0}(t)Ⓢ V_0(t)} \tag{3.176}$$

$$\mathrm{V_{15}(t) = Q_{15,0}(t)Ⓢ V_0(t)} \tag{3.177}$$

Using the LST of relations $(3.162) - (3.177)$ and solving for $V_0^{**}(s)$, we get the expected number of server visits per unit time as

$$V_0 = \lim_{s\to 0} sV_0^{**}(s) = \frac{N_4}{D_4} \tag{3.178}$$

Where

$$N_4 = (\eta {\Upsilon_1}^2{\Upsilon_2}^2\Upsilon_3\Upsilon_4)$$

$$D_4 = (\lambda_4 \Upsilon_1(\lambda_1+\Upsilon_1)\Upsilon_2(\lambda_2+\Upsilon_2)\Upsilon_3 + \lambda_3\Upsilon_1(\lambda_1+\Upsilon_1)\Upsilon_2(\lambda_2+\Upsilon_2)\Upsilon_4 + ({\lambda_1}^2\Upsilon_2(\lambda_2+\Upsilon_2) + \lambda_1\Upsilon_1({\lambda_2}^2+\lambda_2\Upsilon_2+{\Upsilon_2}^2) + {\Upsilon_1}^2({\lambda_2}^2+\lambda_2\Upsilon_2+{\Upsilon_2}^2))\Upsilon_3\Upsilon_4)$$

3.6. Profit Analysis:

The difference between total value created and total expenses is known as profit. Therefore, the anticipated profit in a steady state is

$$\mathrm{Profit(P) = P_0(A_0) - P_1(BJ_0) - P_2(V_0)} \tag{3.179}$$

Where,

P_0 = Revenue per unit of system uptime

P_1 = cost per unit of time spent repairing the server

P_2 = cost of each server visits for maintenance

4. METHODOLOGY

Dr. Xin-She Yang of Cambridge University introduced the Firefly Algorithm (FA), a nature-inspired metaheuristic optimization approach, in late 2007 and 2008. It is based on three fundamental ideas: absorption, brightness, and attraction. These ideas work together to enable the swarm to explore and find the optimal solutions.

4.1. Fundamentals (Idealized Rules)

Three idealized criteria are used by the program to simulate firefly flashing behavior:
1. Fireflies are attracted to one another regardless of sex since they are all unisex.
2. Brightness and attractiveness are correlated; fireflies that are less luminous ravitate near those that are brighter.
3. It travels at random if no firefly is brighter than a certain one.

4.2. How It Works

Firefly Initialization: A colony of fireflies is dispersed at random around the search area. A possible solution to the optimization issue is represented by each firefly.
Evaluation of the Objective Function: The objective function (fitness function) is used to determine each firefly's "brightness" or light intensity. Higher brightness often corresponds to a better solution in a maximizing issue**.**
Firefly Ranking: The light intensity of fireflies determines their ranking.
Find The Best Function Right Now: The algorithm finds the firefly that is the brightest, which is the best option right now.
All Fireflies' Transition to a Better Solution: Fireflies are drawn to those who are more brilliant than they are. The movement is controlled by:
Attractiveness: This diminishes when two fireflies become farther apart.
Randomness: To guarantee exploration, take a tiny random step.
Is iteration = Max Generation? This decision node determines if the maximum number of iterations, or halting condition, has been reached.
NO: The procedure returns to the assessment stage for a fresh generation.
Yes: The Current Best results are printed and the procedure ends.

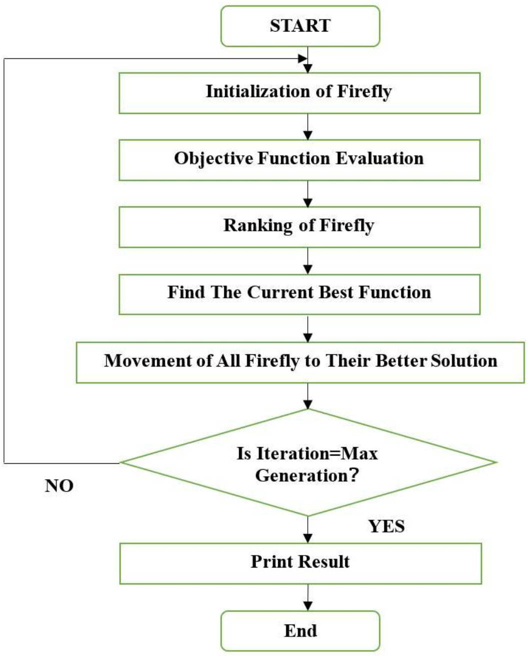

Fig. 4.1. Flowchart of the methodology used.

4.3. Objective Function

MTSF:

The objective is to find the value of $\lambda_1$that maximizes the Mean Time to System Failure (MTSF). The MTSF is given by

$$\mathrm{MTSF}(\lambda_1) = \frac{0.00066132 + 0.02464\,\lambda_1 + 0.309\lambda_1^2 + 2\lambda_1^3}{6.0601 \times 10^{-6} + 0.000225792\lambda_1 + 0.010224\lambda_1^2 + 0.128\lambda_1^3 + \lambda_1^4}$$

and the Firefly Algorithm is applied to minimize the objective function

$$\mathrm{f}\,(\lambda_1) = -\,\mathrm{MTSF}\,(\lambda_1), \lambda 1 \in [0.01, 0.05]. \tag{4.1}$$

Availability:

The Firefly Algorithm is also applied to maximize system availability, which is given by

$$\text{Availability}(\lambda_1) = \frac{0.0000777216 + \lambda_1(0.00216073 + (0.0215665 + 0.08\lambda_1)\lambda_1)}{0.0000908228 + \lambda_1(0.00281054 + \lambda_1(0.0405635 + \lambda_1(0.317485 + \lambda_1)))}$$

The corresponding objective function is

$$\mathrm{f_A}\,(\lambda_1) = -\text{Availability}\,(\lambda_1), \tag{4.2}$$

and the algorithm searches over λ_1 to find the value that maximizes system availability.

Busy Period:

The objective is to minimize the busy period, defined as

$$\text{Busy Period}\,(\lambda_1) = 1 - \frac{0.00595349}{0.00747882 + \lambda_1(0.0934853 + \lambda_1)}$$

The Firefly Algorithm is applied to minimize

$$\mathrm{f_B}\,(\lambda_1) = \text{Busy Period}\,(\lambda_1), \lambda_1 \in [0.01, 0.05] \tag{4.3}$$

Minimum Number of Server Visits:

Additionally, the expected number of server visits is minimized, where

$$\text{Server Visits}\,(\lambda_1) = \frac{0.0000714419 + 0.00595349\lambda_1}{0.007477882 + \lambda_1(0.0934853 + \lambda_1)}$$

The corresponding objective function is

$$\mathrm{f_V}\,(\lambda_1) = \text{Server Visits}\,(\lambda_1), \lambda_1 \in [0.01, 0.05] \tag{4.4}$$

Although a one-dimensional grid search over $\lambda_1 \in [0.01, 0.05]$ would also yield the optimal MTSF and availability for this specific case, the Firefly Algorithm is used to demonstrate a general optimization framework that can be extended to multi-parameter reliability models. In higher-dimensional settings, exhaustive grid search becomes computationally infeasible, whereas FA efficiently explores the solution space and can handle multimodal objective functions such as MTSF and availability.

In future extensions, penalties can be incorporated through a composite objective such as

$$f(\lambda_1) = -MTSF(\lambda_1) + \lambda Cost(\lambda_1), \#(4.5)$$

Where λ controls the trade-off between reliability and cost.

4.4. Firefly Algorithm Application Chart

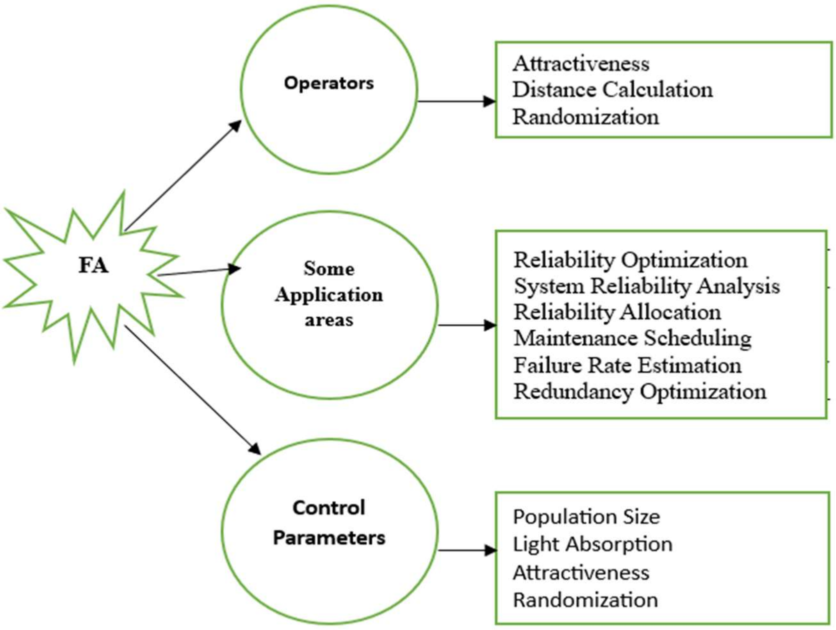

Fig. 4.2. Application chart of FA.

5. RESULTS OF NUMERICAL SIMULATIONS

5.1. System Failure Average Time

The results of applying the Firefly Algorithm to optimize the Mean Time to System Failure (MTSF) with regard to the parameter λ_1 are shown in Figure 5.1 and summarized in Table 5.1. Table 5.1 provides several λ_1 values between 0.0100 and 0.0467 along with the matching MTSF (λ_1) values. This indicates that MTSF reduces as λ_1 grows, with the greatest value of MTSF $\approx$ 99.23806 happening at $\lambda_1 = 0.0100$. The MTSF (λ_1) curve is plotted in Figure 5.1, where the FA optimal point is marked at $(\lambda_1,\ \text{MTSF}) = (0.01, 99.238)$. This visual confirmation shows that the Firefly algorithm properly determined the parameter value that maximizes system dependability.

Table 5.1. MTSF vs. Different Failure Rate.

Sr. No.	Value of Failure Rate (λ_1)	MTSF	Constant Parameters
1	0.0100	99.23806	$\lambda_2 = 0.003$ $\lambda_3 = 0.004$ $\lambda_4 = 0.005$ $\Upsilon_1 = 0.08$ $\Upsilon_2 = 0.04$ $\Upsilon_3 = 0.05$ $\Upsilon_4 = 0.06$
2	0.0141	92.17907	
3	0.0182	84.85693	
4	0.0222	77.79082	
5	0.0263	71.24211	
6	0.0304	65.31083	
7	0.0345	60.00644	
8	0.0386	55.29281	
9	0.0427	51.11438	
10	0.0467	47.41031	

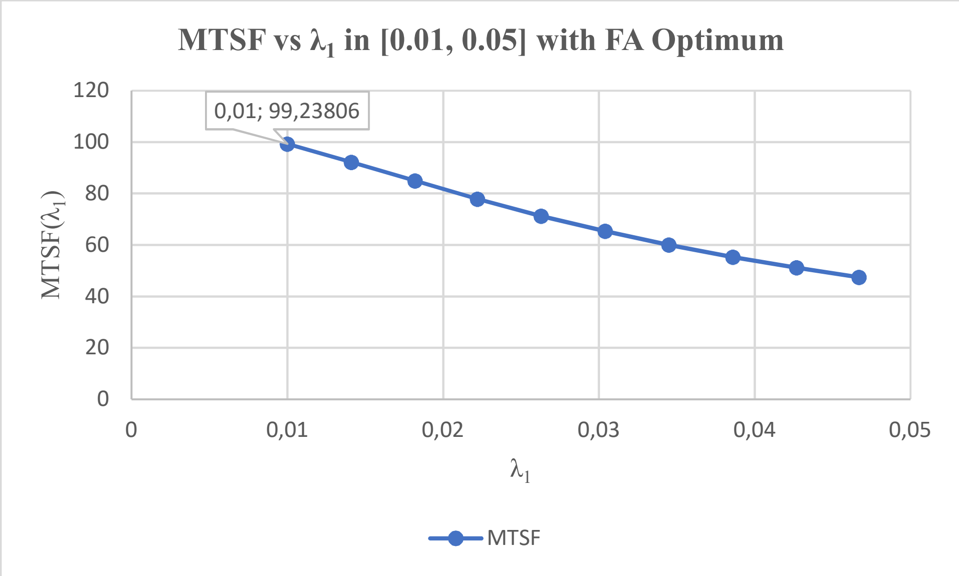

Fig. 5.1. MTSF vs λ_1

5.2. Operating Time (Accessibility)

The Firefly Algorithm is employed to optimize the parameter λ_1 throughout the interval $[0.01, 0.05]$ in order to maximize the steady-state availability (A). The results indicate that the search has converged since, by iteration 20, the algorithm has already discovered $\lambda_1 = 0.01$ with $A = 0.823647$. This optimal solution does not change over iterations $40, 60, 80,$ and 100. The table displays values of λ_1 from 0.01 to 0.05 along with the related availability $A\,(\lambda_1)$. It is evident that availability falls as λ_1 rises, showing that $\lambda_1 = 0.01$ produces the maximum availability of 0.8236 in this range.

Table 5. 2. Availability vs. Different Failure Rate.

Sr. No.	Value of Failure Rate (λ_1)	Availability	Constant Parameters
1	0.0100	0.823647	$\lambda_2 = 0.003$ $\lambda_3 = 0.004$ $\lambda_4 = 0.005$ $\Upsilon_1 = 0.08$ $\Upsilon_2 = 0.04$ $\Upsilon_3 = 0.05$ $\Upsilon_4 = 0.06$
2	0.0144	0.806860	
3	0.0189	0.789100	
4	0.0233	0.770701	
5	0.0278	0.751941	
6	0.0322	0.733050	
7	0.0367	0.714211	
8	0.0411	0.695570	
9	0.0456	0.677239	
10	0.0500	0.659303	

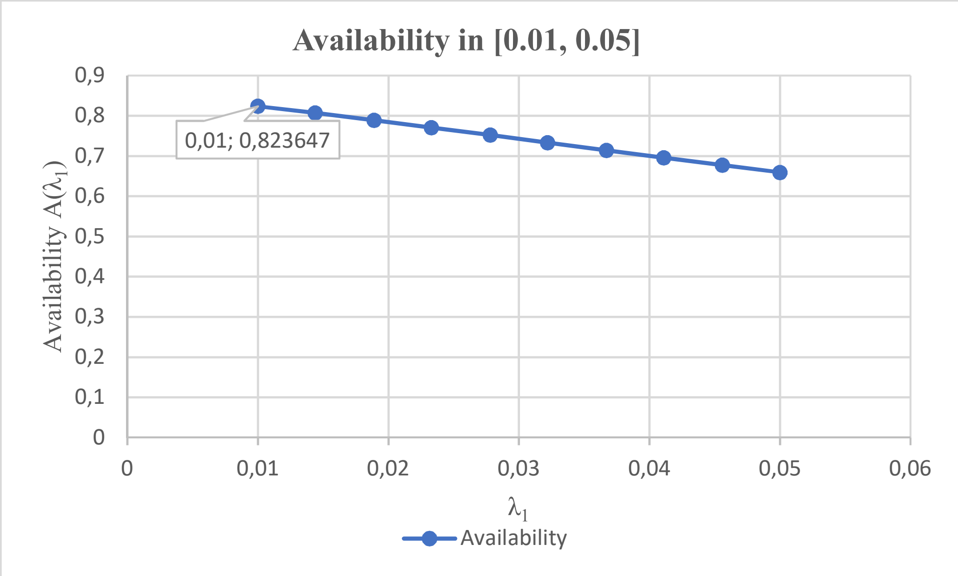

Fig. 5.2. Availability vs λ_1

5.3. Busy Period for Repairman

Using the Firefly Algorithm, the "Busy Period" (BP) is reduced by adjusting the parameter λ_1 within the range $[0.010, 0.050]$. The trace shows that by Generation 25, the algorithm has already discovered $\lambda_1 = 0.01000$ with a minimum busy period is 0.300714. This optimal solution remains unchanged at Generations 50, 75, and 100, indicating that the search has converged and FA believes that $\lambda_1 = 0.01$ provides the smallest Busy Period in this range. The values of λ_1 are listed in the table from 0.01000 to 0.05000 along with the matching Busy Periods, and it is evident that the Busy Period rises as λ_1 rises, so the FA result $\lambda_1 = 0.01000$ with Minimum busy period is 0.300714 is compatible with the data.

Table 5.3. Busy Period vs Different Failure Rates

Sr. No.	Value of Failure Rate (λ_1)	Busy Period	Constant Parameters
1	0.01000	0.300714	$\lambda_2 = 0.003$ $\lambda_3 = 0.004$ $\lambda_4 = 0.005$ $\Upsilon_1 = 0.08$ $\Upsilon_2 = 0.04$ $\Upsilon_3 = 0.05$ $\Upsilon_4 = 0.06$
2	0.01444	0.341268	
3	0.01889	0.379938	
4	0.02333	0.416587	
5	0.02778	0.451152	
6	0.03222	0.483625	
7	0.03667	0.514043	
8	0.04111	0.542470	
9	0.04556	0.568991	
10	0.05000	0.593704	

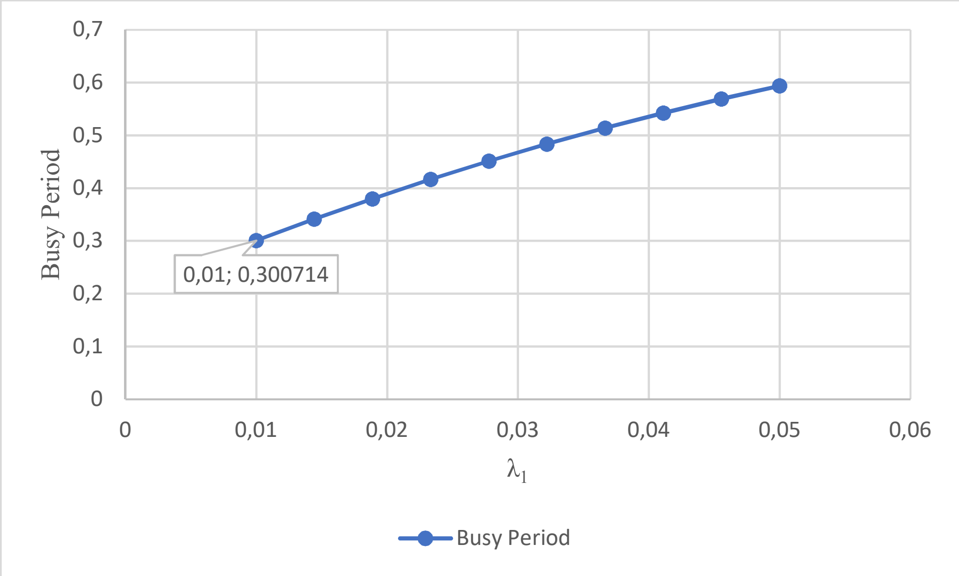

Fig. 5.3. Busy period vs λ_1

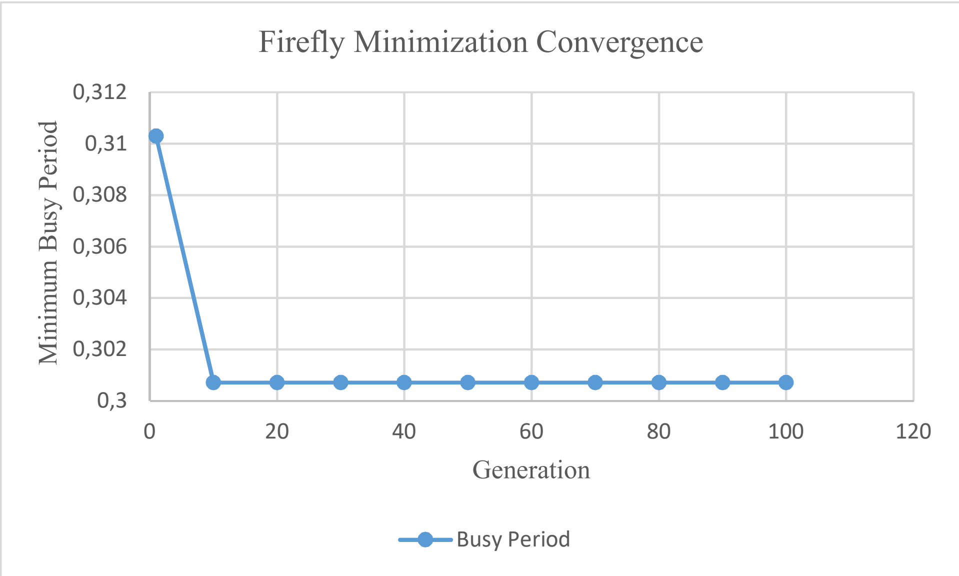

Fig. 5.4. Busy period vs Generation

5.4. Expected Frequency of Server Visit

By adjusting the parameter λ_1 within the range $[0.010, 0.050]$, the Firefly Algorithm minimizes a performance function $f(\lambda_1)$, which shows the anticipated number of server visits per unit time (or per task). The table shows a number of λ_1 values together with the accompanying $f(\lambda_1)$ values. You can see that $f(\lambda_1)$ rises as λ_1 grows, indicating that more server visits are required when λ_1 increases. The method determines that the value of λ_1 that minimizes the average server-visit load in the specified range is $\lambda_1 = \ 0.010000$, where the expected number of server visits is $f(\lambda_1) = \ 0.015384$.

TABLE 5. 4. Expected Frequency of Server Visits vs Different Failure Rates

Sr. No.	Value of Failure Rate (λ_1)	Server visits $f(\lambda_1)$	Constant Parameters
1	0.010000	0.015384	$\lambda_2 = 0.003$ $\lambda_3 = 0.004$ $\lambda_4 = 0.005$ $\Upsilon_1 = 0.08$
2	0.014444	0.017420	
3	0.018889	0.019153	

4	0.023333	0.020614	$\Upsilon_2 = 0.04$ $\Upsilon_3 = 0.05$ $\Upsilon_4 = 0.06$
5	0.027778	0.021832	
6	0.032222	0.022835	
7	0.036667	0.023650	
8	0.041111	0.024300	
9	0.045556	0.024807	
10	0.050000	0.025190	

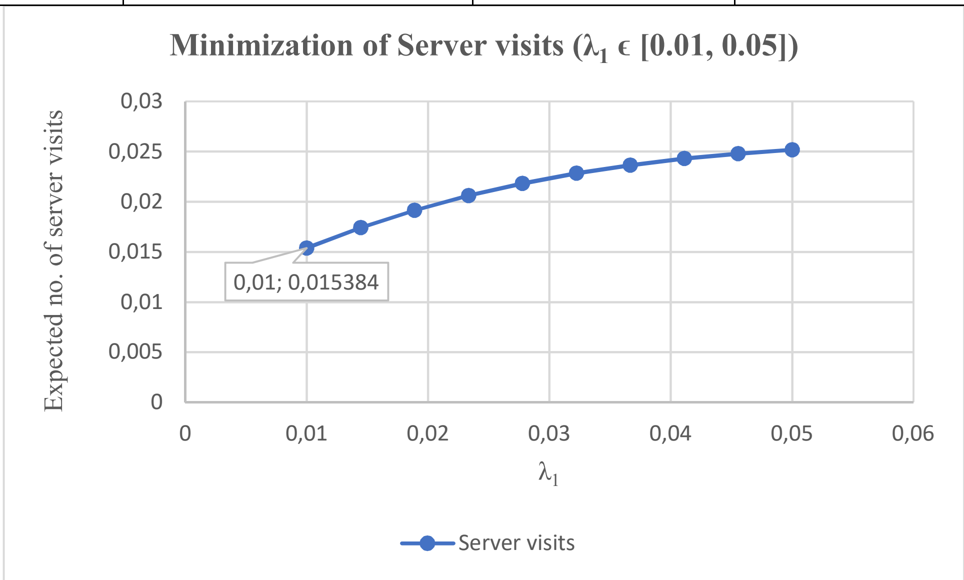

Fig. 5.5. Expected no. of server visits vs λ_1

5.5. Profit Analysis

Using the optimal configuration (maximum availability, minimum busy period, expected no. of server visits), the net profit is about 792.04 units. This number expresses how profitable the system is when all reliability factors are simultaneously optimized under the chosen cost structure.

6. OBSERVATIONS

Overall, it is determined that the parameter $\lambda_1 = 0.01$ provides the optimal trade-off for all four-performance metrics:

- Mean Time to System Failure, or MTSF: The maximum MTSF $\approx$ 99.24 is obtained when $\lambda_1 = 0.01$, which increases reliability by allowing the system to function longer before failing.
- Availability: Availability achieves its highest value of around 0.8236 at $\lambda_1 = 0.01$, meaning that the system operates in a steady state approximately 82.4% of the time.
- Busy Period: The system is continually busy (processing units) for the smallest amount of time at $\lambda_1 = 0.01$, where the minimum Busy Period of 0.3007 happens.
- Anticipated frequency of server visits: The lowest frequency of server visits is 0.015384 at $\lambda_1 = 0.01$, indicating that preventive-maintenance action is least frequent in this arrangement, which lowers maintenance burden.

Profit: When the cost parameters are paired with these optimal values with $P_0 = 1000, P_1 = 100$, and $P_2 = 100$, the calculated profit is around 792.04, the greatest under the selected dependability and maintenance structure. Therefore, choosing $\lambda_1 = 0.01$ is the system's optimal operating point since it simultaneously maximizes dependability (MTSF and availability), lowers system load (busy period and server visits), and maximizes long-term profit.

CONCLUSION

This study presents a reliability and cost-based optimization framework for a steel manufacturing plant using the Firefly Algorithm to tune the critical parameter λ_1 over the interval $[0.01, 0.05]$. The analysis focuses on four key performance measures: Mean Time to System Failure ($MTSF$), steady-state availability, busy period, and the expected number of server visits. The optimization results show that $\lambda_1 = 0.01$ maximizes $MTSF$ (≈ 99.24) and steady-state availability (≈ 0.8236), while simultaneously minimizing the busy period (≈ 0.3007) and the expected frequency of server visits (≈ 0.0154), thus reducing both downtime and maintenance load. By integrating these optimized reliability indices with cost parameters $C_0 = 1000$ (revenue-related gain), $C_1 = 100$ (repair cost), and $C_2 = 100$ (server visit cost), a profit model is formulated, yielding a net profit of approximately 792.04 units. The results indicate that setting $\lambda_1 = 0.01$ provides the most favorable operating condition for the steel manufacturing plant, balancing high reliability, low maintenance burden, and maximum long-run profit.

REFERENCES

[1] Dahiya, O., Kumar, A. & Saini, M. (2019). Mathematical modeling and performance evaluation of A-pan crystallization system in a sugar industry, *SN Applied Sciences*, **1**(4), 339. doi: 10.1007/s42452-019-0348-0

[2] Garg, D., Garg, R. & Garg, V. (2022). Inspecting briquette machine with different faults, *Recent Advances in Computer Science and Communications*, **15**(4), 481–486. doi: 10.2174/2666255813999200909110603

[3] Kharola, S., Ram, M. & Singh, S. (2026). Reliability Analysis of an Elevator System Under Resume Repair Policy, *Journal of Graphic Era University*, **14**(1), 251–270. doi: 10.13052/jgeu0975-1416.1418

[4] Kumar, P., Ali, A. & Kumar, S. (2020). Coal-fired thermal power plant performance optimization using Markov and CFD analysis, *SN Applied Sciences*, **2**; 1–10. doi: 10.1007/s42452-020-2003-1

[5] Kumari, S., Khurana, P., Singla, S. & Kumar, A. (2021). Solution of constrained problems using particle swarm optimization, *International Journal of System Assurance Engineering and Management*, 1688–1695. doi: 10.1007/s13198-021-01524-x

[6] Malik, S. & Tewari, P. C. (2023). Performability and maintenance decisions for coal ash handling system of a subcritical thermal power plant, *International Journal of System Assurance Engineering and Management*, **14**(1), 45–54. doi: 10.1007/s13198-021-01097-9

[7] Munda, L., Taneja, G. & Sachdeva, K. (2025). Reliability and economic analysis of a system comprising three units i.e. operative, hot standby and warm standby, *Journal of mechanics of continua and mathematical sciences*, **20**(1), 17–33. doi: 10.26782/jmcms.2025.01.00002

[8] Parkash, S. & Tewari, P. C. (2021). Critical Review of Rams Tools and Techinques for the Analysis of Multi Component Complex Systems, *Reliability: Theory & Applications*, **16**(3(63)), 220–227.

[9] Parkash, S. & Tewari, P. C. (2022). Performance modeling and dss for assembly line system of leaf spring manufacturing plant, *Reliability: Theory & Applications*, **17**(2(68)), 403–412.

[10] Pinki & Singh, D. (2023). Reliability modeling of two-unit gas turbine system considering the effect of humidity, *Reliability Theory and Applications*, **18**(1(72)), 607–617.

[11] Singla, S., Rani, R. & Mangla, D. (2025). Mathematical Computing of Reliability Parameters of a Parallel System Under the Cold Standby Unit, *Proc. of 12th International*

Conference on Computing for Sustainable Global Development (INDIACom) (Delhi, India), 1–6. doi: 10.23919/INDIACom66777.2025.11115561.

[12] Singla, S., Mangla, D., Panwar, P. & Taj, S. Z. (2024). Reliability Optimization of a Degraded System under Preventive Maintenance using Genetic Algorithm, *Journal of Mechanics of Continua and Mathematical Sciences*, **19**(1), 1-14. doi: 10.26782/jmcms.2024.01.00001

[13] Singla, S., Rani, R., Mangla, D. & Modibbo, U. M. (2024). Reliability modeling and stochastic evaluation of a machine of various units with immediate repair of failed unit, *Journal of Mechanics of Continua and Mathematical Sciences*, **19**(12), 159–173. doi: 10.26782/jmcms.2024.12.00010

[14] Singla, S., Rani, S., Modibbo, U. M. & Ali, I. (2023). Optimization of System Parameters of 2:3 Good Serial System using Deep Learning, *Reliability Theory and Applications*, **18**(4(76)), 670–679. doi: 10.24412/1932-2321-2023-476-670-679

[15] Tewari, P. C. & Kumar, P. (2016) Availability Analysis of Milling System in a Rice Milling Plant, *International Journal of Industrial and Manufacturing Engineering*, **10**(8), 1600–1609. doi: 10.5281/zenodo.1127166.