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Reliability Assessment and Cost Optimization of a Fault-Tolerant Rice Processing System under Environmental Disturbances

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Abstract: Modern rice processing plants operate in harsh environment conditions characterized by dust accumulation, humidity level, and unstable power supply, all of which significantly influence system reliability. However, such systems are exposed not only to internal component failures but also to environmental disturbances such as humidity, temperature fluctuations, dust, and power-quality variations. This paper develops a comprehensive reliability and cost optimization model for a four-unit rice industry system consisting of three active units and one warm standby unit supported by a single repair mechanism. An additional environmental error stage is explicitly incorporated into the state transition diagram to capture real-world operating conditions. The system behaviour is modelled as a continuous-time Markov process with lacking fault exposure, restart mechanism, and repair capacity subject to service interruption. Transient state probabilities are derived using Laplace transform and matrix-analytical techniques. Key performance indicators, including Mean Time to Failure, system reliability, availability and Average number of component failures, are evaluated. A Cost-effective ratio (CER) is further formulated by combining operational, repair, environmental recovery, and downtime costs. Numerical illustrations demonstrate how environmental disturbance intensity and maintenance parameters influence system performance. The results show that integrating fault-tolerant design with environmental recovery strategies significantly improves operational reliability while reducing long-term system cost in rice processing industries.

Keywords: Rice industry system, fault tolerance, environmental error, Markov process, reliability analysis, cost optimization

1. INTRODUCTION

Rice processing industries in developing economies generally depends on continuous functioning of interconnected devices such as cleaners, dehuskers, polishers, graders, and packaging units. For the reason that these machines work in series, the failure of one unit can disrupt the entire production line, resulting in production loss and quality degradation. To minimize such interruptions, rice mills usually use fault-tolerant architectures with standby units and limited maintenance resources.

Classical reliability models for fault-tolerant systems concentrate on component failures, maintenance strategies, backup redundancy, and service disruptions. However, real rice mills operate in harsh indoor environments. Dust from husk and bran accumulates in machine parts, electrical components, and voltage fluctuations frequently cause machines to shut down or reboot.

Existing reliability literature has considered common-cause failures, disaster queues, server breakdowns, and vacation policies. Nevertheless, very limited attention has been given

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to modelling plant-level environmental disturbances as a distinct operational state in fault-tolerant repairable systems. In practice, environmental effects do not permanently damage machines but temporarily force the system into a disturbed condition from which recovery is possible through cleaning, ventilation, stabilization, or resetting operations.

This gap motivates the present study, where environmental disturbance is explicitly incorporated as a separate state in the Markov model of a fault-tolerant rice processing system. The model also integrates reboot mechanisms, working vacation repair policies, imperfect fault coverage, and degraded repair conditions to realistically reflect industrial practice.

By incorporating a generalized retrial mechanism, M. Jain and S. S. Sanga [7] examined characteristics of Fault-tolerance system under redundant repairable system operating under an enter control policy into a retrial orbit instead of receiving immediate repair. By implementing technique of supplementary variable, authors established Chapman – Kolmogorov differential equations and subsequently determined steady-state probability distribution representing number of failed units waiting for repair. In another study, M. Jain and R. Gupta [4] studied the reliability characteristics of a repairable redundant system equipped by mixed stand-in configuration with two repair workers operating under a threshold N-policy. Their model incorporates practical considerations such as reserve switching letdown, component deprivation, and ordinary basis failure for better representation of service of real-time repairable systems. To assess structure performance, the authors utilized Adaptive Neuro-Fuzzy inference system technique and compared the result with analytical solutions attained through a supervised learning-based approach. To address environmental influences, Wang and co-authors [23] presented an analytical framework to assess the reliability of offshore wind influence plants with particular attention to the impact of environmental conditions on component failures. The methodology is based on the integration of Protected Zone modeling, corresponding power unit representation, and analysis of Common Cause Failure for evaluating system reliability. The study demonstrated, through a case analysis, that incorporating common cause failure mechanisms significantly improves the realism of reliability modelling. Furthermore, Sensitivity analysis was perform to assess impact of uncertainty present in failure state. A stochastic reliability model for a combined hardware–software system was proposed by Kumar [14], where multiple failure sources were considered. The model includes the effects of human errors as well as common cause failures. In this framework, common cause failures result in complete system collapse, whereas human errors may trigger either hardware or software failures. Key reliability indicators such as Mean Time to System Failure, mean uptime and degradation duration were evaluated using a Markov process formulation. A routine evaluation of a Fault-tolerance with a failure-flat server worked with warm standby units was studied by C. Shekhar and R. K. Meena [8]. In their model, a working vacation policy is introduced so that the idle periods of the server can be utilized effectively, thereby improving the cost efficiency of the system. Further, to enhance fault tolerance, additional mechanisms such as reboot and recovery are incorporated alongside the standby configuration in the development of the Markov model. The resulting performance indicators of system are derived through numerical computation using Runge-Kutta method. Adefarati, T., et. al. [1], have paid attention on evaluation of the reliability, economic and ecological benefits of renewable energy resources in a micro grid system. A lifecycle assessment was performed to evaluate the feasibility of deploying such systems in rural regions that lack reliable access to electricity. A Markovian queueing system incorporating working vacations, a retrial mechanism, and customer impatience, along with imperfect service during the working vacation period, was investigated by M. Jain and S. Dhibar [6]. Through sensitivity analysis, the authors evaluated the outcome of system parameter on the performance characteristics of model. Furthermore, cost optimization was performed using Genetic Algorithm and Quasi-Newton methods both. In another study, V. P. Singh et al. [5] investigated Availability and reliability of Machining system that may endure multiple stages of dreadful conditions, while also considering the effects of common cause failures and catastrophic failures. In a multi-degraded system, catastrophic failures may also result from human errors caused by inexperienced workers, and the repair process is modeled using a general distribution. To develop a sustainable blood provides string that includes manifold donors, group centers, allocation centers and hospitals, a comprehensive model was formulated. The proposed model was solved using an enhanced ϵ -constraint method, while single-objective version of the trouble was addressed through the application of imperialist competitive algorithm. The results were further validated using the CPLEX optimization solver [3]. M. Kirti, A. K. Maurya, and their co-authors [10] presented a review of diverse fault-tolerance techniques used in dispersed and cloud computing environments and discussed about their catalogue of four types; reactive approaches, adaptive, proactive and hybrid methods. The study presents a comparative evaluation of these methods with respect to system reliability and overall performance. Environmental stress conditions may significantly influence the reliability of measurement devices. Ma and collaborators [16] evaluated the reliability of electric metering devices by analyzing environmental stress effects and measurement exactness. To detect outliers, a bidirectional technique grounded in minimum covariance determinant (MCD) robust analysis was developed, whereas the Thompson tau test was utilized to recognize abnormal observations across horizontal and longitudinal dimensions. Singh, M., Jain, M., 'I&' Azhagappan, A. [21] have examined a transient Markov queue that allows two distinct types of server vacations. The transient queue length distribution and additional system properties were determined using analytical approaches involving continued fractions and probability generating functions. The steady-state behavior was subsequently derived from the transient solution, and numerical experiments were performed to illustrate the model's practical applicability. Shakuntla et. al. [17] have discussed system working with the concept of preventive maintenance under two different approaches. Proposed model reliability is optimized through Genetic Algorithm. A sensitivity analysis is performed on three identical units with one standby unit. Kumar, A., Kumar, P.[12] have analyzed performance of Sugar mill incorporated with human errors, modeling and reliability approach by proposing framework. Sensitivity analysis is performed to identify impact of components' failure on the performance. Derived results demonstrate the potential of the proposed framework and provides valuable insights into the performance of the Sugar Mill under different scenarios, including the impact of human error. Singla, S., Rani, R. [18] have examined the reliability and quality performance of a dairy plant consisting of two units. Both units operate simultaneously. Using regenerative and semi-Markov analysis with Laplace transforms, measures such as mean time to failure, availability, repair activity, and profit are evaluated using real plant data. Singla, S., et. Al. [19] have evaluated reliability behavior of three-unit system using the Regenerative Point Graphical Technique. Each unit contains parallel components, allowing operation at reduced capacity after a single failure, while two reduced units cause system failure. Failure rates follow exponential distributions and repairs are generalized. Fuzzy logic identifies system states, and parameters are optimized using deep-learning algorithms such as Adam, SGD, and RMSprop. Mishra, S.K., Kumar, R. and Mohanta, D. K. [15] have discussed wind power oscillations which affect the resilience and consistency of power system, highlighting the role of situational awareness in integration of air energy. The authors developed a multi-state wind speed representation to better capture fluctuations in energy production. The proposed approach was implemented within a real-time platform integrating Internet-of-Things technology and the dSPACE system. Customer behavior and server operational policies in queueing systems were investigated by Kumar and M. Jain [13]. In their study, the authors developed Single-server Queueing model incorporating with customer balking, Server vacation periods, and breakdown during working conditions. The model assumes that the decision of customers to join the system may depend on prevailing conditions whether these are normal or degraded. Important performance metrics, including the average system size, throughput rate, and total expected cost, were obtained for different system states. Similarly, Vijayashree and Pavithra [22] have exercised on queueing system that can be regulated with N-policy where operational breakdowns may occur. In their framework, the system begins service only after a predetermined number of customers have arrived. Service is maintained until the system returns to an empty state. To determine the optimal cost, Particle swarm optimization algorithm was employed to minimize overall cost. A fluid approximation approach for analyzing a Markovian disaster queue was proposed by Singh and Jain [20]. Their model incorporates reboot and repair mechanisms following catastrophic failures. Analytical solutions based on continued fractions and probability generating functions were derived, and numerical results were compared with predictions obtained using the ANFIS technique. To analyze the dynamic performance of a machine repair system considering balking, reneging, and working breakdown scenarios, Kolledath and Kumar [11] utilized the Runge–Kutta numerical method of order (4,5). A cost function was formulated and further Sensitivity analysis was carried out to examine impact of different parameters on system indicators by. In the context of agricultural sustainability, Josan and Kaur [9] studied the factors influencing wheat growth and production of rice–wheat crop system at Indo-Gangetic plains. Their work highlighted role of genotype variety, sow time and effective residue supervision to improve crop yield while reducing environmental damage caused by residue burning. Finally, S. Dhibar and M. Jain [2] examined the performance characteristics of an queueing system operating under a hybrid vacation strategy with Bernoulli-type service interruptions. The stationary likelihood distribution of queue length was obtained using iterative procedures along with difference–differential equation method.

Motivated by these developments, the present study develops a comprehensive framework for analyzing the reliability and cost efficiency for a four-unit rice processing system operating under an uncertain environment. The proposed model preserves the analytical rigor and structural standards of established fault-tolerant reliability studies while extending them to a realistic agro-industrial setting. A Continuous-time Markov process is employed to represent the system dynamics and to evaluate steady-state performance indicators that includes system reliability, MTTF and Availability and expected number of failed units. Furthermore, to achieve economic sustainability, a cost-effective ratio is formulated by integrating operational, maintenance, downtime, and repair costs. This study makes the following key contributions:

- Development of a four-unit fault-tolerant rice processing system with one standby unit.
- Explicit incorporation of an environmental error stage in the state transition diagram.
- Transient and steady-state reliability analysis using Markovian modelling.
- Formulation of a cost-effective ratio considering environmental recovery costs.
- Numerical investigation highlighting the managerial significance of environmental control.

The structure of the present study is organized in a sequential manner to provide clarity in the development of the proposed model. Section 2 describes the conceptual framework of the system in detail, including its operating environment, underlying assumptions, and the mathematical notations adopted for the analysis. Section 3 develops the mathematical formulation that governs the behavior of the system. This section employs MAA to analyze the transitory dynamics of model and to compute the probabilities of different system states over time. Section 4 derives the principal reliability and queueing performance measures that characterize the operational effectiveness of the system. Section 5 introduces an economic evaluation by formulating a cost function along with a cost-effective ratio (CER), which helps to assess the financial feasibility of the system configuration. Section 6 presents numerical investigation to demonstrate the practical applicability of the developed model. Finally, Section 7 summarizes the key outcomes of the research and outlines possible directions for further study.

2. MODEL DESCRIPTION

We consider a rice processing system consisting of four identical processing units operating under fault-tolerant control. At any time, three units are active and one unit is in warm standby mode under the care of a single repairman. Active units perform rice processing operations such as cleaning, milling, or polishing whereas Standby unit immediately replaces a failed active unit. Repair facility repairs failed units and restores hem to standby condition. Environmental control mechanism acts to recover the system from environmental error states. To evaluate the system's operational performance, the stochastic model is constructed under the following assumptions:

2.1. Assumptions

1. The system comprises of L identical units operating in the active mode and W additional units kept as standby components. At any time, the active units perform the required operations while standby units replace failed active units whenever necessary.
2. Component failures are considered to be exponentially distributed, where the failure rate is affected by system load and varies according to fail units.
3. A single repairman is available to repair failed components which operate with two manners: normal busy state and working vacation state. During working vacation period, repair activities are comparatively slower than usual repair rate.
4. The fault tolerant system may occasionally enter a reboot state due to operational disturbances or system instability. Reboot states can occur when the repairman is in different operational conditions such as normal busy, working vacation, or breakdown states.
5. External environmental factors may impact the performance of the system. In such cases, the system temporarily enters an environmental error state, from which it recovers after corrective action.
6. Repair times, reboot times, and recovery times from environmental errors are expected to follow exponential distributions and are mutually self-governing.
7. All random processes involved in the model, including failure, repair, reboot, and environmental recovery processes, are believed to be statistically independent.
8. System stochastic behavior is modeled as two-dimensional continuous-time Markov process, capturing both status of the repairman and the number of failed units in the system.
9. To represent this plant-level phenomenon, aggregated environmental disturbance rate γ drives the system into a distinct environmental error state, which results in temporary degradation of system performance. Recovery from this state occurs at rate δ, reflects the effectiveness of environmental control measures such as dust extraction, ventilation, humidity regulation, and electrical stabilization.

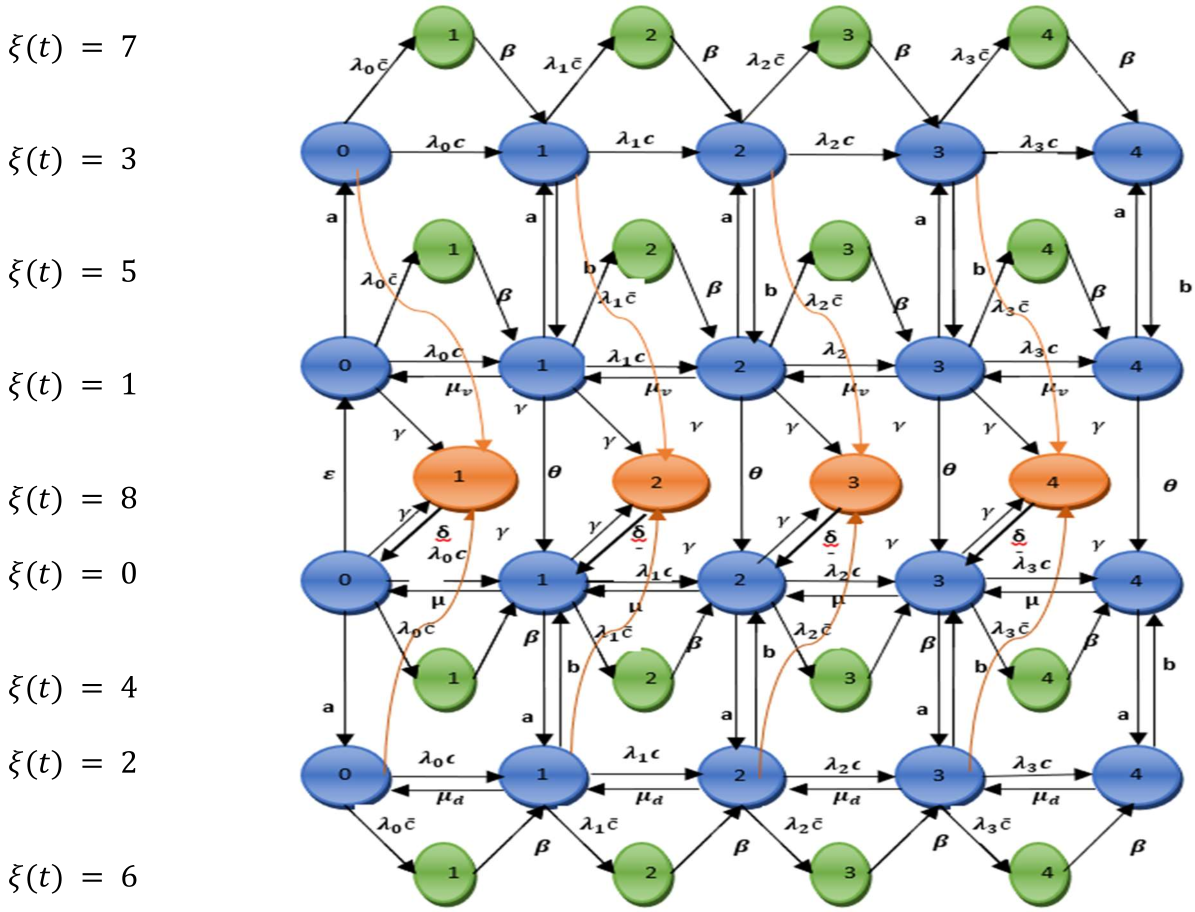


Fig 1 : Markov State Transition Representation of Rice Processing System

Let t denotes time, the variable $\xi(t)$ represents operational condition of repairmen and operational state of fault-tolerant control system at any time. The corresponding states for the repairman and the FTCS are specified below:

$$\xi(t) = \begin{cases} 0, \text{ maintenance technician at Normal busy state} \\ 1,2,3; \text{ maintenance technician experiences working vacation (WV),} \\ \text{failed state (DNB) from Normal Busy state,} \\ \text{failed state (DWV)from Working vacation state, at any time} \\ 45,6,7; \text{ when the Fault tolerance control system state enters into reboot state from Normal busy,} \\ \text{Working Vacation, working breakdown from Normal busy,} \\ \text{Working breakdown form Working Vacation time at time } t \textit{ respectively} \\ 8, \textit{ when the Fault Tolernat standby system encounters an environment error state at time } t \end{cases}$$

The system behavior is represented through a two-dimensional continuous-time Markov process $(\xi(t), \chi(t))$, where these two variables describe different aspects of system operation at any time $t \geq 0$.

- $\xi(t)$ represents the operational regime of the repair facility and control system (normal repair, working vacation, breakdown, reboot, and environmental disturbance).
- $\chi(t)$ represents the **number of failed processing units** in the system, where $0 \leq \chi(t) \leq 4$. Thus, each state of the Markov process is identified by a pair (j, n), where j denotes the service/control regime and n denotes the failure level of the processing units.

The corresponding function becomes $S = \left\{(j, n); \begin{matrix} j = 0, 1, 2, 3, \dots, 8 \\ n = 0, 1, 2, 3, 4 \end{matrix}\right.$; where j represents the operational mode of the repairmen / FTCS and n indicates the no. of failed components. The following define probabilities describes the system state at any time $t \geq 0$.

i.) Normal Busy (NB) state: $P_n^B = P\{\xi(t) = 0, \chi(t) = n\}$

ii.) Working Vacation WV state: $P_n^V(t) = P\{\xi(t) = 1, \chi(t) = n\}$

iii.) Breakdown during Normal Busy DNB state: $P_n^D(t) = P\{\xi(t) = 2, \chi(t) = n\}$

iv.) Breakdown during Working vacation DWV state: $P_n^F(t) = P\{\xi(t) = 3, \chi(t) = n\}$

v.) Reboot during Normal Busy (RNB) state: $R_n^B = P\{\xi(t) = 4, \chi(t) = n\}$

vi.) Reboot during Working Vacation (RWV) state: $R_n^V(t) = P\{\xi(t) = 5, \chi(t) = n\}$

vii.) Reboot during Breakdown from Normal Busy (RDB) state: $R_n^D(t) = P\{\xi(t) = 6, \chi(t) = n\}$

viii.) Reboot during Breakdown form Working Vacation (RDV) state: $R_n^F(t) = \{\xi(t) = 7, \chi(t) = n\}$

ix.) Environment error state: $E_n(t) = P\{\xi(t) = 8, \chi(t) = n\}$ where $0 \leq n \leq 4$.

The Laplace transformations of the corresponding likelihoods are defined as under:

$$P^{E^*}_n(s) = \int_0^\infty e^{-st} * P_n^E(t)dt$$

and

$$R^{E^*}_n(s) = \int_0^\infty e^{-st} * R_n^E(t)dt$$

where E indicates the operational state of the server, namely {B,V,D,F}.

The load-dependent malfunction rate for the various mechanism of the FTCS system is expressed as under.: $\lambda_n = \begin{cases} L\lambda + (W - 4)\alpha\,; & 0 \leq n < W \\ (L + W - 4)\lambda\,; & W \leq 4 < L + W = K \end{cases}$; where L is identical online units and W is standby units at any time t.

2.2. Table

A table is generated to show the meaning of parameters used in state transition diagram:

Table 2.2.1. Parameters used in the diagram

Symbol	Meaning	Symbol	Meaning
λ	Failure rate of active unit	β	Reboot frequency
α	Failure rate of standby unit	γ	Environmental disturbance rate
μ	Normal repair rate	δ	Environmental recovery rate
μ_v	Repair rate during working vacation	a	Repairman breakdown rate
μ_d	Repair rate during working breakdown	b	Repairman repair rate
c	Fault coverage probability	ε	Vacation initiation rate
		θ	Vacation completion rate

2.3. Equations

By applying the Chapman–Kolmogorov differential equations, the transient state equations describing the structure are formulated are follows.

i. $\xi(t) = 0, 0 \leq n \leq 4$.

$$\frac{d}{dt}P_0^B(t) = -(\varepsilon + \gamma + \lambda_0 + a)P_0^B(t) + \delta E_1^B(t) + \mu P_{1(t)}^B \tag{2.1}$$

$$\frac{d}{dt}P_n^B(t) = -(\lambda_n + a + \mu + \gamma)P_n^B(t) + \lambda_{n-1}cP_{n-1}^B + \theta P_n^V(t) + \delta E_n^B(t) + \mu P_{n+1}^B(t) + bP_n^D(t) + \beta R_n^B(t) \quad (2.2)$$

$$\frac{d}{dt}P_4^B(t) = -(a + \mu)P_4^B(t) + \lambda_3 cP_3^B + \theta P_4^V(t) + bP_4^D(t) + \beta R_4^B(t) \quad (2.3)$$

ii. $\xi(t) = 1, 0 \le n \le 4$

$$\frac{d}{dt}P_0^V(t) = -(\gamma + \lambda_0 + a)P_0^V(t) + \varepsilon P_0^B(t) + \mu_v P_{1(t)}^V \quad (2.4)$$

$$\frac{d}{dt}P_n^V(t) = -(\lambda_n + a + \mu_v + \theta + \gamma)P_n^V(t) + \lambda_{n-1}cP_{n-1}^V(t) + \mu_v P_{(n+1)}^V(t) + bP_n^F(t) + \beta R_n^V(t) \quad (2.5)$$

$$\frac{d}{dt}P_4^V(t) = -(a + \mu_v + \theta)P_4^V(t) + \lambda_3 cP_3^V(t) + bP_4^F(t) + \beta R_4^V(t) \quad (2.6)$$

iii. $\xi(t) = 2, 0 \le n \le 4$

$$\frac{d}{dt}P_0^D(t) = -(\gamma + \lambda_0)P_0^D(t) + aP_0^B(t) + \mu_d P_1^D(t) \quad (2.7)$$

$$\frac{d}{dt}P_n^D(t) = -(\lambda_n + \mu_d + b + \gamma)P_n^D(t) + \lambda_{n-1}cP_{n-1}^D(t) + aP_n^B(t) + \mu_d P_{(n+1)}^D(t) + \beta R_n^D(t) \quad (2.8)$$

$$\frac{d}{dt}P_4^D(t) = -(\mu_d + b)P_4^D(t) + \lambda_3 cP_3^D(t) + aP_4^B(t) + \beta R_4^D(t) \quad (2.9)$$

iv. $\xi(t) = 3, 0 \le n \le 4$

$$\frac{d}{dt}P_0^F(t) = -(\gamma + \lambda_0)P_0^F(t) + aP_0^V(t) \quad (2.10)$$

$$\frac{d}{dt}P_n^F(t) = -(\lambda_n + b + \gamma)P_n^F(t) + \lambda_{n-1}cP_{n-1}^F(t) + aP_n^V(t) + \beta R_n^F(t) \quad (2.11)$$

$$\frac{d}{dt}P_4^F(t) = -bP_4^F(t) + \lambda_3 cP_3^F(t) + aP_4^V(t) + \beta R_4^F(t) \quad (2.12)$$

v. $\xi(t) = 4, 1 \le n \le 4$

$$\frac{d}{dt}R_n^B(t) = \lambda_{n-1}\bar{c}P_{n-1}^B(t) - \beta R_n^B(t) \quad (2.13)$$

vi. $\xi(t) = 5, 1 \le n \le 4$

$$\frac{d}{dt}R_n^V(t) = \lambda_{n-1}\bar{c}P_{n-1}^V(t) - \beta R_n^V(t) \quad (2.14)$$

vii. $\xi(t) = 6, 1 \le n \le 4$

$$\frac{d}{dt}R_n^D(t) = \lambda_{n-1}\bar{c}P_{n-1}^D(t) - \beta R_n^D(t) \quad (2.15)$$

viii. $\xi(t) = 7, 1 \le n \le 4$

$$\frac{d}{dt}R_n^F(t) = \lambda_{n-1}\bar{c}P_{n-1}^F(t) - \beta R_n^F(t) \quad (2.16)$$

ix. $\xi(t) = 8, 1 \le n \le 4$

$$\frac{d}{dt}E_n^B(t) = -\delta E_n^B(t) + \gamma\left(P_{n-1}^B(t) + P_{n-1}^V(t) + P_{n-1}^D(t) + P_{n-1}^F(t)\right) \quad (2.17)$$

Initially, all the units of system are active so initial condition is $P_0^B(0) = 1$ and that time other state probabilities are zero as no unit is failed.

2.4 Practical Justification

Modern rice processing plants operate through interconnected machines such as cleaners, de-huskers, polishers, graders, and packaging units that function in a continuous production line. Since the failure of a single critical machine can interrupt the entire processing chain, rice mills often adopt fault-tolerant configurations to maintain uninterrupted production. These configurations typically include redundant machines and standby arrangements that allow the system to continue operating even when a component fails.

The proposed model represents this industrial configuration by considering a four-unit rice processing system in which three machines operate actively while one unit remains in warm standby mode. In practical milling plants, the standby unit immediately replaces a failed operating machine to avoid production stoppage. Machine failures may occur due to mechanical wear, belt misalignment, motor overheating, or electrical malfunction. The standby mechanism therefore ensures continuity of processing while the failed unit is sent for repair, which are not always detected perfectly. Operators may initially attempt to restart the machine before performing detailed maintenance. For example, temporary electrical disturbances or controller errors are often corrected by resetting the machine. This operational practice is represented in the model through the reboot state, where a system restart is attempted before initiating full repair.

Maintenance resources in rice processing plants are usually limited. Often a single technician supervises several machines and may perform preventive maintenance during idle periods. This situation is represented by the working vacation state, where repair service is available but operates with reduced efficiency. In addition, maintenance activities themselves may be disrupted due to tool unavailability or power interruptions, which is represented through degraded repair states in the model.

Environmental conditions inside rice mills also play a major role in system performance. Rice milling generates large amounts of dust, bran particles, and husk fragments that accumulate in machine components. High humidity, temperature variations, and unstable power supply can further disturb machine operation. These conditions may temporarily degrade system performance even when no mechanical failure occurs. To capture this realistic industrial phenomenon, the model incorporates an environmental error state representing disturbances caused by environmental stress. The environmental recovery mechanism reflects practical corrective measures such as dust extraction systems, improved ventilation, humidity control, and electrical stabilization. Improvements in these environmental control mechanisms increase the recovery rate and help restore normal system operation.

Thus, the proposed fault-tolerant model realistically reflects the operational behavior of rice processing plants by incorporating redundancy, imperfect fault detection, maintenance limitations, reboot mechanisms, and environmental disturbances. This representation allows industry managers to evaluate system reliability and develop cost-effective maintenance strategies for stable rice processing operations.

3. TRANSIENT ANALYSIS

To solve transient controlling Eqs. $(2.1) - (2.17)$, a matrix analytic method is used to create the queueing, dependability, and availability indices.

By applying Laplace transforms to Equations (2.1) through (2.17), we obtain

$$(\varepsilon + \gamma + \lambda_0 + a + s)P_0^{B*}(s) - \delta E_1^{B*}(s) - \mu P_1^{B*}(s) = 1 \quad (3.1)$$

$$(\lambda_n + a + \mu + \gamma + s)P_n^{B*}(s) - \lambda_{n-1}cP_{n-1}^{B*}(s) - \theta P_n^{V*}(s)$$
$$-\delta E_n^{B*}(s) - \mu P_{(n+1)}^{B*}(s) - bP_n^{D*}(s) - \beta R_n^{B*}(s) = 0 \quad (3.2)$$
$$(a + \mu + s)P_4^{B*}(s) - \lambda_3 cP_3^{B*}(s) - \theta P_4^{V*}(s) - bP_4^{D*}(s) - \beta R_4^{B*}(s) = 0 \quad (3.3)$$
$$(\gamma + \lambda_0 + a + s)P_0^{V*}(s) - \varepsilon P_0^{B*}(s) - \mu_v P_1^{V*}(s) = 0 \quad (3.4)$$
$$(\lambda_n + a + \mu_v + \theta + \gamma + s)P_n^{V*}(s) - \lambda_{n-1}cP_{n-1}^{V*}(s)$$
$$-\mu_v P_{(n+1)}^{V*}(s) - bP_n^{F*}(s) - \beta R_n^{V*}(s) \quad = 0 \quad (3.5)$$
$$(a + \mu_v + \theta + s)P_4^{V*}(s) - \lambda_3 cP_3^{V*}(s) - bP_4^{F*}(s) - \beta R_4^{V*}(s) = 0 \quad (3.6)$$
$$(\gamma + \lambda_0 + s)P_0^{D*}(s) - aP_0^{B*}(s) - \mu_d P_1^{D*}(s) = 0 \quad (3.7)$$
$$(\lambda_n + \mu_d + b + \gamma + s)P_n^{D*}(s) - \lambda_{n-1}cP_{n-1}^{D*}(s) - aP_n^{B*}(s)$$
$$-\mu_d P_{(n+1)}^{D*}(s) - \beta R_n^{D*}(s) = 0 \quad (3.8)$$
$$(\mu_d + b + s)P_4^{D*}(s) - \lambda_3 cP_3^{D*}(s) - aP_4^{B*}(s) - \beta R_4^{D*}(s) = 0 \quad (3.9)$$
$$(\gamma + \lambda_0 + s)P_0^{F*}(s) - aP_0^{V*}(s) = 0 \quad (3.10)$$
$$(\lambda_n + b + \gamma + s)P_n^{F*}(s) - \lambda_{n-1}cP_{n-1}^{F*}(s) - aP_n^{V*}(s) - \beta R_n^{F*}(s) = 0 \quad (3.11)$$
$$(b + s)P_4^{F*}(s) - \lambda_3 cP_3^{F*}(s) - aP_4^{V*}(s) - \beta R_4^{F*}(s) = 0 \quad (3.12)$$
$$-\lambda_{n-1}\bar{c}P_{n-1}^{B*}(s) + (\beta + s)R_n^{B*}(s) = 0 \quad (3.13)$$
$$-\lambda_{n-1}\bar{c}P_{n-1}^{V*}(s) + (\beta + s)R_n^{V*}(s) = 0 \quad (3.14)$$
$$-\lambda_{n-1}\bar{c}P_{n-1}^{D*}(s) + (\beta + s)R_n^{D*}(s) = 0 \quad (3.15)$$
$$-\lambda_{n-1}\bar{c}P_{n-1}^{F*}(s) + (\beta + s)R_n^{F*}(s) = 0 \quad (3.16)$$
$$(\delta + s)E_n^{B*}(s) - \gamma\left(P_{n-1}^{B*}(s) + P_{n-1}^{V*}(s) + P_{n-1}^{D*}(s) + P_{n-1}^{F*}(s)\right) = 0 \quad (3.17)$$

Equations $(3.1) - (3.17)$ can be rewritten in a concise matrix form given by

$$Q(s) * P(s) = P\,(0) \quad (3.18)$$

subject to the initial conditions. Here, $P(0)$ and $P(s)$ represent column entities, and their components be described below.

$$P(s)\ [\,H_0(s), H_1(s), H_2(s), H_3(s), H_4(s)]^T \quad (3.19)$$

and

$$P(0) = [\,0\ 0\ 1\ 0\ \cdots\ 0\ 0\,]^T \quad (3.20)$$

Here $H_0(s) = [P_0(s)]$ and $H_n(s) = [R_n(s)E_n(s)P_n(s)]$,

$$Rn(s) = [R_n^{F*}(s)R_n^{V*}(s)R_n^{B*}(s)R_n^{D*}(s)],$$
$$P_n(s) = [P_n^{F*}(s)P_n^{V*}(s)P_n^{B*}(s)P_n^{D*}(s)], 0 \le n \le 4.$$

The notation T signifies matrix transpose. The initial probability vector $P\,(0)$ is a column vector of order $(9K + 4) \times 1$, indicating that the system comprises $(9K + 4)$. The coefficient matrix is defined $by\ Q(s) = Q(0) + sI$, which is of dimension 40×40

where $Q(0) = \begin{bmatrix} D_0 & E & Z_2 & Z_2 & Z_2 \\ C_0 & D_1 & E_1 & Z_3 & Z_3 \\ Z_1 & C_1 & D_2 & E_1 & Z_3 \\ Z_1 & Z_3 & C_2 & D_3 & E_1 \\ Z_1 & Z_3 & Z_3 & C_3 & D_4 \end{bmatrix}$ which is a 40 order square matrix in which $Z_1, Z_2\ and\ Z_3$ are matrices of order $9x4, 4x9$ and of order $9x9$ respectively. Matrices $Dn, Cn\ , E\ and\ E1$ are defined as under:

$$D_0 = \begin{bmatrix} \lambda_0 + \gamma & 0 & 0 & 0 \\ 0 & \lambda_0 + \gamma + a & 0 & 0 \\ 0 & 0 & \epsilon + \gamma + \lambda_0 + a & 0 \\ 0 & 0 & -a & \lambda_0 + \gamma \end{bmatrix}_{4\times 4}$$

$$D_n = \begin{bmatrix} G_n & 0 & -M \\ 0 & \delta & 0 \\ 0 & 0 & M \end{bmatrix},$$

$$G_n = \begin{bmatrix} b+\lambda_n+\gamma & -a & 0 & 0 \\ -b & a+\lambda_n+\gamma+\theta+\mu_v & 0 & 0 \\ 0 & -\theta & \lambda_n+a+\mu+\gamma & -b \\ 0 & 0 & -a & \mu_d+b+\lambda_n+\gamma \end{bmatrix}, \text{ for}$$

$1 < n < 3M = \beta I_4$, where I_4 is identity matrix of order 4.

$$D_4 = \begin{bmatrix} G_4 & 0 & -M \\ 0 & \delta & 0 \\ 0 & 0 & M \end{bmatrix}, G_4 = \begin{bmatrix} b & -a & 0 & 0 \\ -b & \mu_v+a+\theta & 0 & 0 \\ 0 & -\theta & \mu+a & -b \\ 0 & 0 & -a & \mu_d+b \end{bmatrix}$$

Now, $C_0 = \begin{bmatrix} A_n \\ -\gamma J \\ \overline{A_n} \end{bmatrix}_{9\times 4}$, where $A_n = -\lambda_0 c I_4$ and $\overline{A_n} = -\lambda_0 \bar{c} I_4$ and J is a row matrix of order 4 with each entry is unity.

$$C_n = \begin{bmatrix} A_n & 0 & 0_4 \\ -\gamma J & 0 & 0 \\ \overline{A_n} & 0 & 0_4 \end{bmatrix}_{\{9\times 9\}}$$

Also $E = [F \quad 0_{4\times 1} \quad 0_4]_{4\times 9}$, where $F = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\mu_v & 0 & 0 \\ 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & -\mu_d \end{bmatrix}_{4\times 4}$ and

$$E_1 = \begin{bmatrix} E & 0 & 0_4 \\ 0 & 0 & 0 \\ 0_4 & 0 & 0_4 \end{bmatrix}_{9\times 9}.$$

The model is formulated as a continuous-time Markov process with a finite state space comprising 40 states. For indexing purposes, the states are defined as follows.

$H_0(s) = [h_1^*(s), h_2^*(s), h_3^*(s), h_4^*(s)]$ and

$H_n(s) = [P_n^{B*}(s), P_n^{V*}(s), P_n^{D*}(s), P_n^{F*}(s), R_n^{B*}(s), R_n^{V*}(s), R_n^{D*}(s), R_n^{F*}(s), E_n^*(s)]$ for $0 < n < 4$; Or

$[h_{9n-4}^*(s), h_{9n-3}^*(s), h_{9n-2}^*(s), h_{9n-1}^*(s), h_{9n}^*(s), h_{9n+1}^*(s), h_{9n+2}^*(s), h_{9n+3}^*(s), h_{9n+4}^*(s)]$

Now, using Cramer's rule is employed on equation (35), we get

$$h_m^*(s) = \frac{|Q_m s|}{|Q s|} \tag{3.21}$$

Also $P(0) = [1,0,0 \ldots\ldots\ldots\ldots\ldots\ldots.]_{40\times 1}$

By using $s = -r$ (arbitrary) we have $|Q(-r)| = |Q - rI|$ to find characteristic roots we will use

$$|Q(-r)| = 0 \tag{3.22}$$

Assume that we have k real characteristic roots $(u_1, u_2, u_3, \ldots\ldots\ldots. u_k)$, excluding zero and suppose that there are l additional complex roots $(u_{k+1}, u_{k+2}, \ldots\ldots. u_{k+l})$ where complex roots occur in pairs. So, we have $k + 2l = 39$ (as 0 is also characteristic root of the equation $|Q(s)| = 0$.

Furthermore, when l=0, then all characteristic roots are real; however, if k=0, then all roots occur in complex conjugate pairs. Now, we can write

$$|Q(s)| = \{s \prod_{i=1}^{k}(s+u_i) \prod_{i=1}^{l}(s^2 + (u_{k+i} + \overline{u_{k+\iota}})s + u_{k+i}\overline{u_{k+\iota}})\} \quad \ldots\ldots(3.23)$$

Now, by using equation (40) in equation (38), $h_m^*(s)$ can therefore be represented as partial fraction type as under:

$$h_m^* = \frac{d_{0,m}}{s} + \sum_{\{i=1\}}^{\{k\}} \frac{d_{i,m}}{s+\mu_i} + \sum_{\{i=1\}}^{\{l\}} \frac{e_{i,m}s + f_{i,m}}{s^2 + (u_{k+i} + \overline{u_{k+\iota}})s + u_{k+i}\overline{u_{k+\iota}}} \tag{3.24}$$

in which $d_{0,m} = \frac{|Q(0)|}{\prod_{i=1}^{j}(u_i)\prod_{i=1}^{r} u_{k+i}\overline{u_{k+\iota}}}$,

$d_{i,m} = \frac{|Q(-u_i)|}{u_i \prod_{k=1}^{j}(u_k - u_i)\prod_{k=1}^{r}(u_i^2 + (u_{j+k} + \overline{u_{j+k}})(-u_i) + u_{j+k}\overline{u_{j+k}}}$ and

$$e_{i,m}s + f_{i,m} = \frac{|Q_m(-u_{j+k})|}{(-u_{j+i})\prod_{k=1}^{j}(u_k - u_{j+i})\prod_{k=1}^{r}((-u_{j+i}^2) + (u_{j+k} + \overline{u_{j+k}})(-u_{j+i}) + u_{j+k}\overline{u_{j+k}})}$$

Let v_t and w_t denotes real and imaginary part of complex root u_{k+t} respectively. Then probability values can be obtained by taking the inverse Laplace transform of Eq. (3.24)

$$h_m(t) = d_{0,m} + \sum_{i=1}^{j} d_{i,m}\exp(-u_i t) + \sum_{i=1}^{j} d_{i,m}\exp(-v_i t)\cos(w_i t) + \frac{f_{i,m} - e_{i,m}v_i}{w_i}\exp(-v_i t)\sin(w_i t) \tag{3.25}$$

It is generally recognized that system failures tend to appear after the system has been operating for long time, i.e., as $t\to\infty$. Hence, in the steady-state regime, the normalization condition can be written as follows.

$$\lim_{t\to\infty}\sum_{n=0}^{4}\left(P_n^F(t) + P_n^V(t) + P_n^B(t) + P_n^D(t)\right) + \sum_{n=1}^{4}\left(R_n^F(t) + R_n^V(t) + R_n^B(t) + R_n^D(t)\right) + \sum_{\{n=0\}}^{\{4\}}\left(E_n^F(t)\right) = 1 \tag{3.26}$$

Following the derivation of the transient probabilities for system states, the relevant performance indicators are determined in subsequent section

1. The average number of failed machines present in the FTCS is expressed as

$$E_F(t) = \sum_{\{n=0\}}^{\{4\}} n\left(P_n^F(t) + P_n^V(t) + P_n^B(t) + P_n^D(t)\right) + \sum_{\{n=1\}}^{\{4\}} n\left(R_n^F(t) + R_n^V(t) + R_n^B(t) + R_n^D(t)\right) + \sum_{\{n=0\}}^{\{4\}} n\left(E_n^F(t)\right)$$

2. Availability of machines can be determined as

$$A_M(t) = 1 - \frac{E_F(t)}{4}$$

3. System State probabilities where the repair mechanism is in normal busy, Vacation during working, breakdown during Normal busy, and vacation breakdown states are obtained as follows.

$$B_P(t) = \sum_{n=0}^{\{4\}} P_n^B(t) \ ; V_P(t) = \sum_{n=0}^{\{4\}} P_n^V(t) \ ; D_P(t) = \sum_{n=0}^{\{4\}} P_n^D(t) \ ; F_P(t) = \sum_{n=0}^{\{4\}} P_n^F(t)$$

4. Probability of the system at reboot state

$$R_F(t) = \sum_{\{n=1\}}^{\{4\}} \left(R_n^F(t) + R_n^V(t) + R_n^B(t) + R_n^D(t)\right)$$

5. System reliability is determined as:

$$R_I(t) = 1 - P_4^F(t) - P_4^V(t) - P_4^B(t) - P_4^D(t) - \sum_{\{n=1\}}^{\{4\}} \left(R_n^F(t) + R_n^V(t) + R_n^B(t) + R_n^D(t)\right) - \sum_{\{n=1\}}^{\{4\}} \left(E_n(t)\right)$$

6. Probability of system at environment state

$$E_n(t) = \sum_{\{n=1\}}^{\{4\}} \left(E_n(t)\right)$$

7. MTTF is calculated as

$$MTTF = F_M(t) = \int_0^{\infty} R_I(t)dt$$

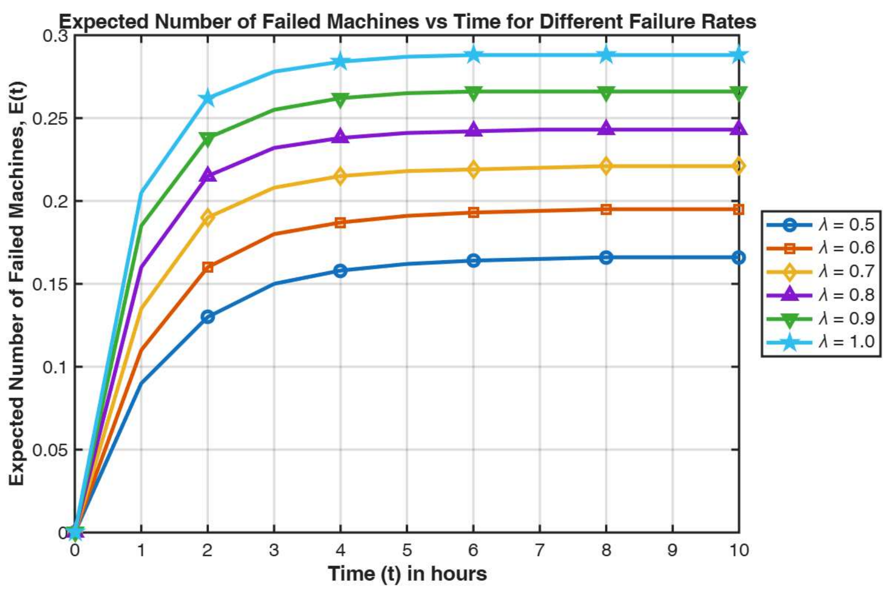
$$= \lim_{s\to 0} R_I^*(s) = \lim_{s\to 0} [\frac{1}{s} - P_4^{F*}(s) - P_4^{V*}(s) - P_4^{B*}(s) - P_4^{D*}(s) - \sum_{\{n=1\}}^{\{4\}} \left(R_n^{F*}(s) + R_n^{V*}(s) + R_n^{B*}(s) + R_n^{D*}(s)\right) - \sum_{\{n=1\}}^{\{4\}} \left(E_n^*(s)\right)$$

4. PERFORMANCE MEASURES

Here, several important reliability indicators, with the total cost of the system, are attained to enable a quantitative evaluation of fault-tolerant control system (FTCS). Key queuing-based measures, including the average number of failed units, Availability of the system and probabilities associated with different system states, are formulated and analyzed to assess the operational behaviour of the system.

Table 4.1 Variation in different parameters due to repair rate (μ)

μ (*Repair rate*)	t (*Time in hours*) t)	$A_M(t)$ (*Aailability of Machines*)	$E_F(t)$ (Expected No. of failed Machines)	$B_P(t)$ (Probability of the system at Normal Busy state)	$V_P(t)$ (Probability of the system at *Working Vacation* state)	$D_P(t)$(Probability of the system at Breakdown during Normal Busy state)	$F_P(t)$ (Probability of the system at Breakdown from working vacation sta	$R_F(t)$(Probability of the system at Reboot state)	$E_n(t)$(Probability of the system at Environment error state)	$R_I(t)$ *Reliability of the system*
1.5	0	1	0	1	0	0	0	0	0	1
	1	0.5114	0.3725	0.3697	0.1418	0.2873	0.0531	0.07	0.0781	0.9997
	2	0.3757	0.4836	0.2347	0.1415	0.3324	0.1061	0.0868	0.0985	0.9989
	3	0.3369	0.5461	0.1964	0.1416	0.3269	0.1406	0.0897	0.1048	0.9980
	4	0.3260	0.5937	0.1808	0.147	0.3115	0.1651	0.0892	0.1064	0.9970
	5	0.3234	0.635	0.1715	0.1544	0.2954	0.1844	0.0882	0.1061	0.9959
2.5	0	1	0	1	0	0	0	0	0	1
	1	0.5114	0.3281	0.3652	0.1462	0.2864	0.054	0.0703	0.0778	0.9998
	2	0.3761	0.4223	0.2273	0.1491	0.3286	0.1099	0.0875	0.0975	0.9994
	3	0.3377	0.4774	0.188	0.1505	0.3206	0.1473	0.0906	0.1029	0.9987
	4	0.327	0.5204	0.1718	0.1567	0.3032	0.1741	0.0904	0.1039	0.9979
	5	0.3246	0.5589	0.1618	0.165	0.2853	0.1955	0.0894	0.103	0.9969
3.5	0	1	0	1	0	0	0	0	0	1
	1	0.5114	0.2994	0.362	0.1494	0.2856	0.0547	0.0705	0.0776	0.9999
	2	0.3763	0.3888	0.2229	0.1537	0.3262	0.1125	0.0879	0.0968	0.9995
	3	0.3381	0.443	0.1835	0.1554	0.3168	0.1514	0.0912	0.1018	0.9989
	4	0.3276	0.4859	0.1671	0.1619	0.2983	0.1794	0.0909	0.1024	0.9981
	5	0.3252	0.5247	0.1569	0.1706	0.2798	0.2017	0.09	0.1012	0.9971



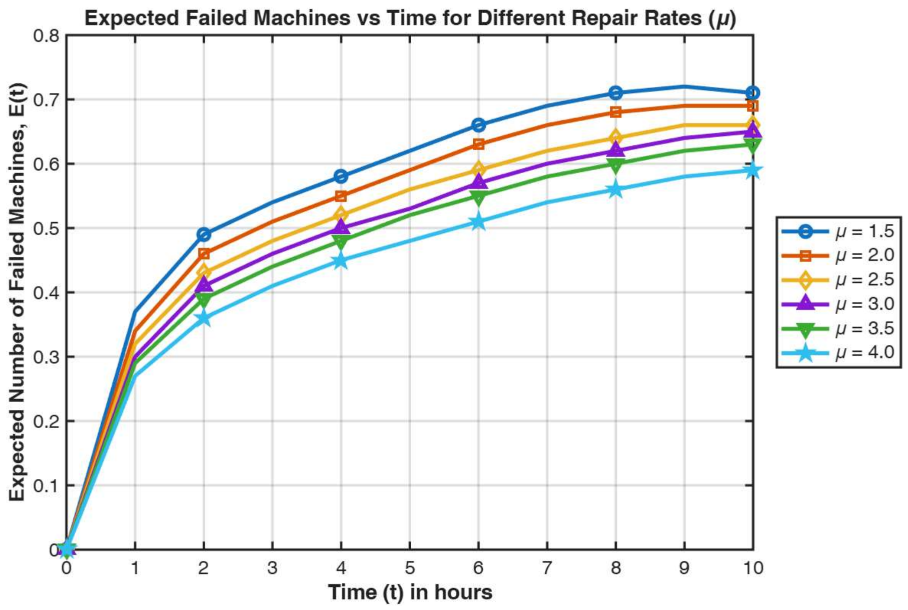
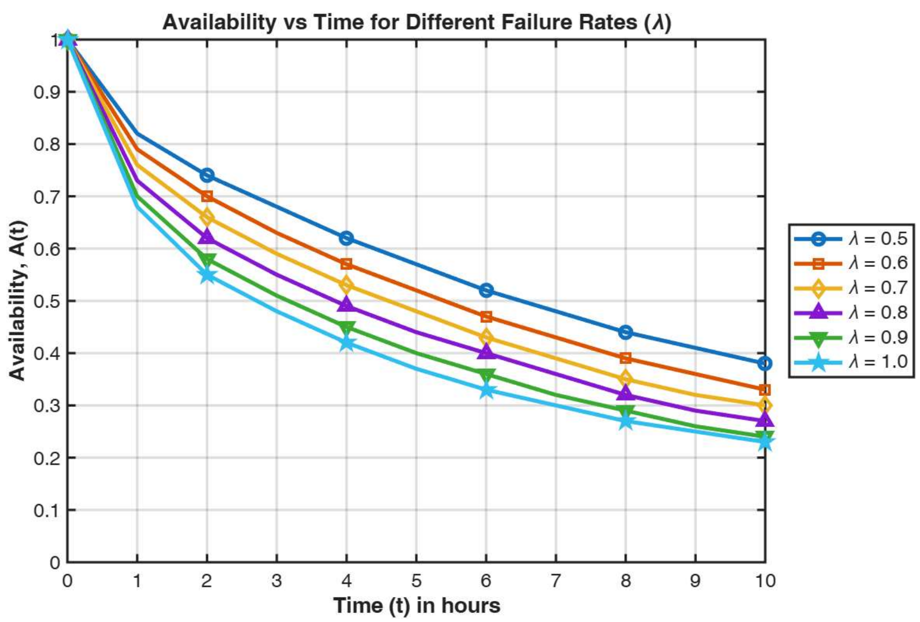


Fig 4.1. Time-dependent variation in the expected number of fail units for different values of the parameters (i) λ and (ii) μ.

Table 4.2 MTTF for distinct λ and μ_d

μ \ λ	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
1.5	32.084	27.97	25.026	22.815	21.093	19.715
2	39.197	33.915	30.136	27.298	25.086	23.315
2.5	46.274	39.828	35.217	31.753	29.056	26.894
3	53.329	45.719	40.277	36.19	33.007	30.457
3.5	60.371	51.596	45.323	40.612	36.945	34.007
4	67.404	57.464	50.359	45.025	40.873	37.548



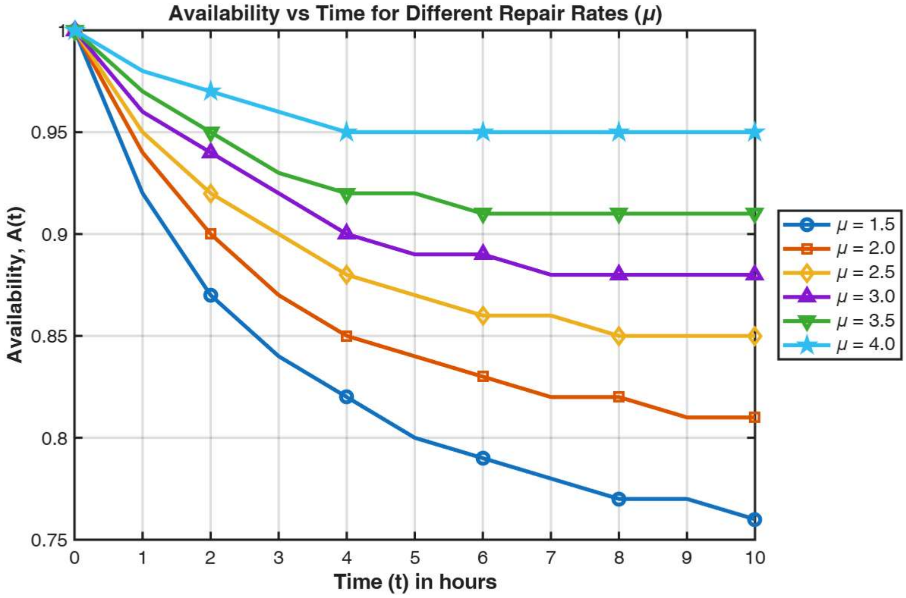


Fig 4.2. Time-dependent behavior of system availability for distinct rates of parameters (i) λ and (ii) μ.

Table 4.3 MTTF for distinct λ $(Failure\ rate)$ and μ_d $(Repair\ during\ breakdown)$

μ_d \ λ	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
0.6	20.43	18.388	16.916	15.802	14.928	14.222
0.8	23.16	20.656	18.855	17.495	16.43	15.571
1	25.885	22.918	20.788	19.181	17.924	16.913
1.2	28.607	25.177	22.717	20.863	19.415	18.251
1.4	31.327	27.434	24.643	22.542	20.902	19.585
1.6	34.047	29.689	26.568	24.219	22.387	20.917

Table 4.4 MTTF for distinct λ and μ_v (Repair during Working vacation)

μ_v \ λ	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
0.6	26.752	23.351	20.923	19.106	17.695	16.571
0.8	27.566	24.143	21.692	19.851	18.417	17.269
1.0	28.158	24.728	22.269	20.418	18.973	17.813
1.2	28.607	25.177	22.717	20.863	19.415	18.251
1.4	28.96	25.534	23.075	21.223	19.774	18.609
1.6	29.245	25.823	23.369	21.519	20.073	18.909

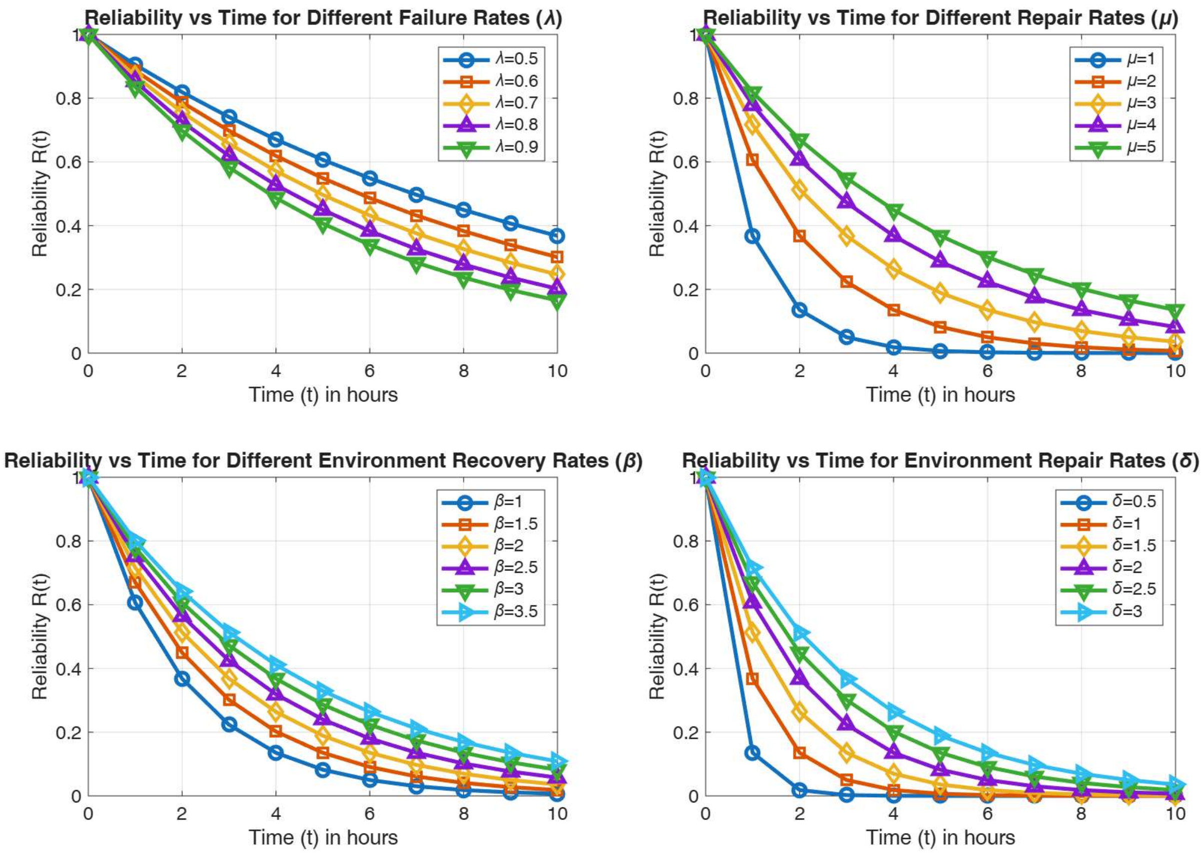


Fig. 4.3: Reliability behaviour of the system for various parameter values of (i) λ, (ii) μ, (iii) β, and (iv) δ.

Table 4.5. MTTF for distinct λ $(Failure\ rate)$ and 'a' (Repairman breakdown rate)

a \ λ	0.5	0.6	0.7	0.8	0.9	1
0.3	14.746	13.282	12.18	11.319	10.626	10.057
0.4	14.11	12.8	11.803	11.015	10.378	9.8493
0.5	13.552	12.372	11.463	10.74	10.15	9.6583
0.6	13.059	11.988	11.155	10.488	9.9405	9.4819
0.7	12.619	11.642	10.876	10.258	9.7476	9.3183
0.8	12.225	11.329	10.621	10.046	9.5692	9.1663

Table 4.6. MTTF for distinct λ (Failure rate) and 'b' (Repairman repair rate)

b \ λ	0.5	0.6	0.7	0.8	0.9	1
2	8.0815	7.4893	7.0592	6.735	6.4833	6.2833
3	7.4943	6.9681	6.5905	6.3094	6.0938	5.9247
4	7.192	6.7006	6.3505	6.0917	5.8948	5.7416
5	7.0078	6.5379	6.2046	5.9595	5.774	5.6305
6	6.8838	6.4284	6.1066	5.8707	5.6929	5.5559
7	6.7946	6.3498	6.0361	5.8069	5.6346	5.5023

Table 4.7. MTTF for distinct λ $(Failure\ rate)$ and β (Reboot Recovery rate)

β \ λ	0.5	0.6	0.7	0.8	0.9	1
0.5	9.3585	8.5318	7.8864	7.3652	6.9329	6.5668
0.6	9.2261	8.3763	7.7097	7.1689	6.7187	6.3364
0.7	9.1119	8.2435	7.56	7.004	6.5403	6.1459
0.8	9.0125	8.1287	7.4315	6.8636	6.3893	5.9859
0.9	8.9251	8.0284	7.3201	6.7425	6.26	5.8495

1	8.8477	7.9401	7.2225	6.637	6.1479	5.732

Table 4.8. MTTF for distinct λ $(Failure\ rate)$ and δ $(Environment\ Recovery\ rate)$

δ \ λ	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
0.5	7.027	6.6362	6.3109	6.0299	5.7806	5.5553
0.6	7.4079	6.9676	6.6012	6.2848	6.0048	5.7526
0.7	7.7825	7.2921	6.8840	6.5320	6.2211	5.9420
0.8	8.1509	7.6098	7.1595	6.7716	6.4298	6.1239
0.9	8.5132	7.9207	7.4278	7.0039	6.6311	6.2985
1	8.8695	8.2250	7.6891	7.2290	6.8253	6.4662

Table 4.9. Validation range of Different Parameters

Units (Per day)	**Meaning**	**Parameter Range**	**Operational Interpretation**
λ	Failure rate of active unit	$0.2-0.6$	Represents mechanical/electrical failure of milling machines due to wear, overheating, belt slip, motor fault
α	Failure rate of standby unit	$0.05-0.2$	Standby unit fails less frequently as it operates under no load
μ	Normal repair rate	$0.3-0.8$	Typical repair/restore time of 1–3 days by plant technician
μ_v	Repair rate during working vacation	$0.15-0.4$	Slower repair when technician performs preventive/secondary tasks
μ_d	Repair rate during working breakdown	$0.1-0.3$	Repair slowed due to tool/power/maintenance interruption
c	Fault coverage probability	$0.5-0.9$	Probability that standby successfully replaces failed unit
β	Reboot frequency	$0.2-0.5$	Controller reset/restart attempts before full repair
γ	Environmental disturbance rate	$0.1-0.4$	Disturbances from dust, humidity, voltage fluctuation causing temporary halt
δ	Environmental recovery rate	$0.4-1.2$	Effectiveness of cleaning, ventilation, and power stabilization
a	Repairman breakdown rate	$0.1-0.3$	Maintenance interruption due to tool/power/personnel issue
b	Repairman repair rate	$0.3-0.7$	Restoration of maintenance facility to normal condition
ϵ	Vacation initiation rate	$0.1-0.3$	Technician shifts to preventive/auxiliary work
θ	Vacation completion rate	$0.3-0.6$	Technician returns to normal repair duty

The parameter ranges shown above are chosen to reflect realistic operating conditions observed in medium-scale rice processing plants, where machine failures and repairs are typically recorded on a daily basis. Active units operate continuously under load and therefore exhibit higher failure rates compared to standby units. Repair rates correspond to average restoration times of one to three days depending on maintenance conditions. Environmental disturbance and recovery rates represent the frequency and mitigation of dust accumulation, humidity effects, and power-quality issues commonly observed in milling environments. These ranges are consistent with reliability studies of agro-processing and machining systems reported in the literature.

5. COST FUNCTION

In modern rice processing industries, uninterrupted system operation is essential because equipment downtime directly affects production yield, grain quality, and operational profitability. The proposed fault-tolerant rice industry system includes multiple operational modes such as normal working, degraded operation, server vacation, reboot, and the newly introduced environmental disturbance state. Since each operational state of the system carries a different cost implication, it is important to formulate an overall cost framework that captures the financial effects of every possible system state. To assess the economic performance of system, expected cost per unit time is formulated under consideration of expenses arising from machine failures, repair activities, server states, recovery operations, and environmental interruptions. In actual plant operation, the expenditure incurred by the system depends on the duration of time for which the system remains in various operational conditions. Therefore, the cost model is formulated using the state probabilities $P_{\{j,n\}}(t)$ obtained from the Markov process $\{\xi(t), \chi(t)\}$ rather than using transition rates.

Let the following cost elements (per day basis) be defined:

Symbol	Description
C_0	$Cost\ when\ the\ system\ is\ in\ normal\ busy\ state\ \xi(t) = 0$
C_1	$Cost\ during\ working\ vacation\ state\ \xi(t) = 1$
C_2	$Cost\ during\ degraded\ repair\ state\ \xi(t) = 2$
C_3	$Cost\ during\ breakdown\ repair\ state\ \xi\ (t) = 3$
C_4	$Cost\ during\ reboot\ from\ normal\ busy\ \xi(t) = 4$
C_5	$Cost\ during\ reboot\ from\ working\ vacation\ \xi(t) = 5$
C_6	$Cost\ during\ reboot\ from\ degraded\ busy\ \xi(t) = 6$
C_7	$Cost\ during\ reboot\ from\ degraded\ vacation\ \xi(t) = 7$
C_8	$Cost\ during\ environmental\ disturbance\ state\ \xi(t) = 8$
C_l	$Production\ loss\ cost\ when\ the\ system\ is\ unavailable$

Here, $P_{j,n}\ (t)$ denotes the probability that at time t, the system is in state $\xi(t) = j\ with\ \chi(t) = n$ failed units.

Total Expected Cost at Time t

$$T_C(t) = \sum_{n=0}^{4} \{C_0 P_{0,n}(t) + C_1 P_{1,n} + C_2 P_{2,n} + C_3 P_{3,n}\} + \sum_{n=1}^{4} \{C_4 P_{4,n} + C_5 P_{5,n} + C_6 P_{6,n} + C_7 P_{7,n} + C_8 P_{8,n}\} + C_l\big(1 - A(t)\big),$$

where, A(t) represents the availability of the system at any time t (in hours)

Steady-State Cost

As $t \to \infty$, the steady-state expected cost per day becomes

$$T_C(t) = \sum_{n} \{C_0 P_{0,n} + C_1 P_{1,n} + C_2 P_{2,n} + C_3 P_{3,n}\} + \sum_{n} \{C_4 P_{4,n} + C_5 P_{5,n} + C_6 P_{6,n} + C_7 P_{7,n} + C_8 P_{8,n}\} + C_l(1 - A(t))$$

5.1. Cost Effectiveness Ratio

In industrial systems such as rice processing plants, performance evaluation must account for both operational efficiency and the financial resources required to maintain that efficiency. Although system availability reflects the proportion of time during which the production facility remains functional, it does not independently capture the economic burden associated with maintaining that operational state. For this reason, a combined performance indicator known as the cost–effectiveness ratio is introduced to provide a more realistic assessment of system behavior from an economic perspective. The cost–effectiveness ratio represents the expenditure incurred per unit of available system performance. This measure is particularly useful for comparing alternative maintenance strategies, repair policies, or system configurations because it incorporates both reliability characteristics and economic considerations into a single index.

Assume that $T_c(t)$ represents the expected total operational cost per unit time, while $A_M(t)$ denotes the system availability at any instant t. According to the selected performance measure, the cost–effectiveness ratio, expressed as $CER(t)$, can be defined as: $CET = \frac{T_C(t)}{A_M(t)}$.

Cost–effectiveness ratio is influenced by several stochastic factors, including machine failure behavior, repair efficiency, server operating conditions, recovery mechanisms, and the newly incorporated environmental disturbance state. Environmental effects such as humidity, dust exposure, temperature variation, and power instability may increase maintenance frequency and associated costs, thereby affecting the overall economic performance of the system. By integrating these practical considerations into the analysis, the resulting cost–effectiveness ratio provides a more accurate representation of real production environments. The developed measure can assist system planners and industry managers in identifying parameter combinations that minimize operational cost while maintaining acceptable levels of availability. Consequently, it serves as a valuable decision-support tool for optimizing maintenance policies and improving the economic sustainability of rice processing operations.

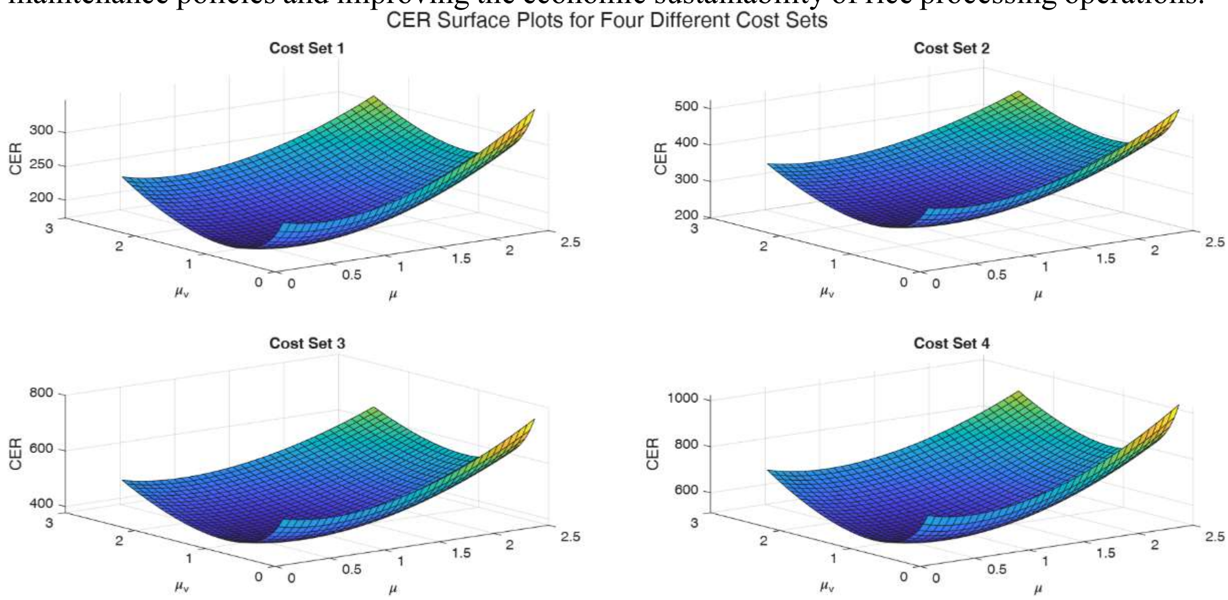


Fig. 5.1. Variation in Cost–Effectiveness Ratio w.r.t μ and μ_v under four different cost structures: (i) I, (ii) II, (iii) III, and (iv) IV.

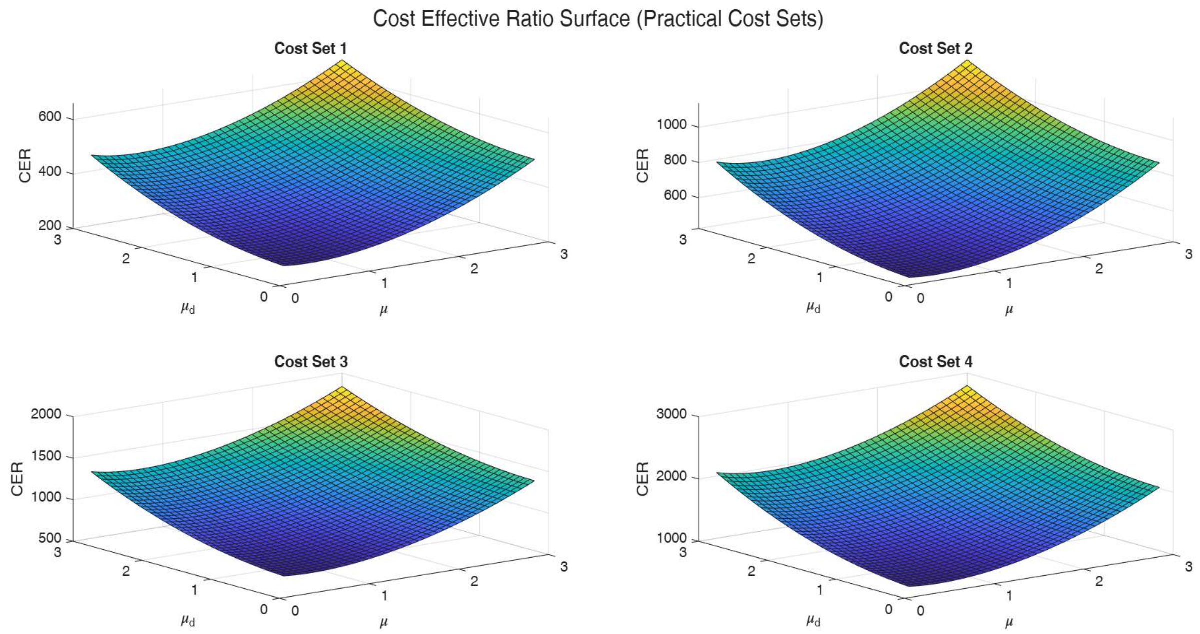


Fig. 5.2. Variation in Cost–Effectiveness Ratio w.r.t μ and μ_d under four different cost structures: (i) I, (ii) II, (iii) III, and (iv) IV.

5.2. Managerial Sensitivity Analysis

The numerical experiments provide useful insight into how different parameters influence system reliability and economic performance. From a managerial perspective, it is important to understand which improvement yields the greatest benefit under limited maintenance budget.

The sensitivity results indicate three key observations:

(i). Influence of Repair rate μ: Increasing the normal repair rate significantly reduces the probability of degraded and breakdown repair states. This directly improves availability and reduces repair-related cost. Investment in technician training, better tools, and spare part availability therefore has a strong positive impact on system performance.

(ii). Influence of environmental recovery rate δ and disturbance rate γ: Environmental control plays a crucial role in maintaining uninterrupted operation. Improving ventilation, dust extraction, humidity regulation, and power stabilization reduces the time spent in environmental disturbance state, thereby improving both availability and cost-effectiveness.

(iii). Influence of fault coverage probability c : Higher standby reliability ensures that failure of an active unit does not immediately reduce system performance. This increases system availability without increasing repair effort.

The comparative impact of these parameters suggests that, for rice processing plants operating in harsh indoor conditions, investment in environmental control measures and repair efficiency yields greater benefit than solely improving standby reliability. This provides a practical decision guideline for plant managers seeking cost-effective reliability improvement.

6. NUMERICAL INVESTIGATION

To assess the practical performance of projected rice processing system, numerical experiments are conducted by assigning representative parameter values that reflect realistic industrial conditions. The default values used for computation are $\lambda = 0.4, \alpha = 0, \beta = 0.25, \gamma = 0.2, \mu = 0.35$ and $a = 0.5, b = 0.45, c = 0.6$ representing other transition intensities of the system. These values simulate a typical rice milling environment where machines experience mechanical failures, temporary malfunctions, and environmental disturbances.

The transient probabilities presented in Table 4.1 show how the system behavior evolves with time. Initially the system starts in a fully operational state and gradually moves toward a stable operating condition. The availability values remain very high during the observed period; however, a small reduction can be noticed as time progresses due to the existence of letdowns and disturbance events.

Graphical results further illustrate the effect of parameter variations. The plots show that increasing repair efficiency improves system availability and reliability because failed units are restored more quickly. Conversely, higher failure or disturbance rates reduce system reliability since machines spend more time in failed or degraded state. The Mean Time to Failure ($MTTF$) values reported in Tables $4.1\ to\ 4.8$ provide further insight into system stability. The results clearly indicate that $MTTF$ increases when repair and recovery rates improve. For example, increasing the repair parameter significantly raises the expected operating duration of the system. In contrast, when the environmental disturbance parameter increases, the $MTTF$ values decline, indicating a shorter system lifetime under unstable environmental conditions. The financial behaviour of system is examined through Cost Effectiveness Ratio (CER). The CER represents the ratio between the total operational cost and system availability, providing a practical measure of economic performance. The computed results show that CER decreases when repair efficiency improves, since faster repair reduces downtime and maintenance backlog. However, the presence of environmental disturbances increases the CER value. When environmental error occurs, machines frequently enter disturbance or recovery states, leading to higher maintenance effort and increased operational interruptions. As a result, the cost per unit availability becomes larger compared with the normal operating condition. The results also demonstrate that increasing the environmental recovery rate reduces the CER value. Faster recovery shortens the disturbance duration and restores the system to productive operation more quickly. Therefore, environmental management measures such as dust removal systems, ventilation improvement, and power stabilization can reduce operational cost while improving system reliability. Inclusively, the numerical investigation confirms that environmental disturbances significantly influence both reliability and economic efficiency of rice processing systems.

7. CONCLUSION

This study develops a reliability and cost evaluation model for a fault-tolerant rice processing system working under environmental disturbances. The proposed model incorporates practical features such as standby redundancy, reboot mechanisms, repair activities, and environmental error states to represent real industrial operating conditions. The numerical results show that system performance strongly depends on repair efficiency and environmental stability. Higher repair rates improve system availability and increase the Mean Time to Failure, thereby enhancing operational continuity. At the same time, the standby configuration helps maintain production even when a machine failure occurs. The study also highlights the critical role of environmental disturbances in rice processing plants. Dust accumulation, humidity variations, and power fluctuations can push the system into disturbance states, reducing system lifetime and increasing the frequency of recovery operations. The numerical results indicate that higher disturbance intensity decreases $MTTF$ and negatively affects reliability. From an economic perspective, environmental disturbances increase the Cost Effectiveness Ratio because additional recovery and repair activities raise the operational cost while reducing effective system availability. In contrast, when environmental recovery is faster, the system returns quickly to normal operation, which lowers downtime and improves cost efficiency. Therefore, maintaining stable environmental conditions within rice processing facilities is essential for both reliability and economic performance. Investments in environmental control technologies can reduce disturbance frequency, increase system lifetime, and improve overall cost effectiveness. In summary, the proposed model provides a useful analytical framework for understanding how environmental disturbances influence reliability and operational cost in rice processing industries, enabling managers to design more efficient maintenance and environmental management strategies.

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8. APPENDIX

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