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Optimizing the Social Network in the SCARDO Model

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Abstract: This article examines an optimal control problem focused on optimizing the structure of a social network to achieve a desired opinion distribution in the population within a finite time horizon. The opinion dynamics adhere to the SCARDO model, and the network structure is operationalized using a stochastic block model whose parameters are subject to adjustment. We derive an analytical result demonstrating that the problem under consideration can be reduced to a control problem in which the structure of the network is fixed, but the parameters of the ranking algorithm—integrated into the model—are optimized. The latter problem is already solved in the scholarly literature. This allows us to introduce a numerical algorithm for solving the initial control problem. Our findings have potential applications in designing friend recommendation systems for real-world social platforms.

Keywords: opinion dynamics models, SCARDO model, mean-field approximation, modular networks, network structure optimization

1. INTRODUCTION

Control over social systems is one of the key goals of sociology, as noted by N. Friedkin in his influential paper [5]. However, networked social systems—as most social systems are essentially networked—appear to be extremely complex entities with intricate internal organization, whereby small interventions may lead to unexpected knock-on outcomes [17]. Consequently, decision-makers must exercise caution when designing control strategies and should rely on robust frameworks capable of accounting for the substantial complexity inherent in social systems.

When considering social systems characterized by dynamic opinion formation, the challenge of influence maximization has been the subject of extensive research [14]. Formally, influence maximization involves selecting a set of optimal nodes in a network so that their subsequent dissemination of a manipulator's message leads to the maximal possible shift in the system's overall state, as evaluated from the manipulator's standpoint [10].

Another research direction of great importance is that of structure optimization—how to adjust the structure of a given social system to change its macroscopic behavior. In consensus models, for instance, the second largest (in absolute values) eigenvalue τ_2 of the underlying adjacency matrix shows the speed of convergence to a consensus state T, as follows [4]:

$$T \propto \frac{-1}{\log|\tau_2|},$$

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where $|\tau_2| < 1$ for systems that are able to reach consensus and therefore $\log|\tau_2| < 0$. Having this, one can try to adjust the adjacency matrix to speed up convergence. Alternatively, there is a large interest in how minor structural interventions—such as adding and edge between individuals sharing a friend—affect assortativity patterns and structural balance in networks [1, 8].

In this paper, we propose a model-dependent framework for network structure optimization.

As a workhorse model, we harness an opinion dynamics model – these models formalize the processes of opinion formation in social groups [7, 9, 16]. Concretely, we build upon the SCARDO model that describes opinion formation as a sequence of stochastic-output pairwise interactions unfolding on a given network [11]. Previously, this model was used to explore bot attacks [12] and to optimize filtering algorithms to achieve a desirable opinion distribution. In the current paper, we take a different perspective and try to investigate which network structure facilitates the attainment of what we call ideal opinion distributions. We leverage the stochastic block model framework, which enables us to model a social system as a modular network structure—a feature common to many real-world social networks. As a result, we reduce the problem of network optimization to finding the most appropriate parameters of the underlying stochastic block model, including those that relate to personalization algorithms [2, 15, 18].

2. MODEL SETUP

The model concerns N agents that are situated in an undirected social network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. We use V_i to denote the immediate neighbors of agent i. Each agent has their opinion and type. By saying "type", we imply any individual characteristic that can affect the level of the agent's openness to influence or its location in the network—such as age, gender, level of education, big-five rate, etc. The opinion of agent i is denoted by $x_i \in Z$, their type is given by $\xi_i \in \Xi$, where $Z = \{Z_1, \dots, Z_m\}$ and $\Xi = \{\Xi_1, \dots, \Xi_M\}$ are the opinion and type alphabets, respectively.

Opinion dynamics are represented as a sequence of consecutive pairwise stochastic interactions in discrete time $t = 1, 2, \dots$. At each time step, an agent is chosen by chance, with the probability of agent i being selected proportional to $\pi_i \geq 0$, where $\pi_i = f(\xi_i)$ is agent i's activity rate, which is uniquely determined by their type. After that, a neighbor of the chosen agent is also selected randomly. Again, the probability distribution is defined by the activity rates of the chosen agent's immediate neighbors—the larger π_i, the more often agent i participates in communication on both sides of information transmission. Next, the second chosen agent (influence sender) is ready to influence the agent chosen first (influence receiver).

However, once two agents are chosen for communication, a personalization system that is an integral part of the underlying online platform decides whether they will communicate. Here, we rely on a data-driven approach, which leaves room for interpretation. We model the occurrence of the communication between the agents as a single Bernoulli trial, with the probability of communication being a function of the chosen agents' attributes. Mathematically, for agents with opinions Z_s and Z_l, and types Ξ_f and Ξ_r (where the attributes Z_s and Ξ_f concern the influence receiver), the probability is $\Delta^{f,r}_{s,l}$.

If the interaction is allowed, then the receiver updates their opinion in accordance with the probability distribution $p^{f,r}_{s,l,1}, \dots p^{f,r}_{s,l,m}$, which is defined on the set of all possible opinion values. We require that $p^{f,r}_{s,l,1} + \dots + p^{f,r}_{s,l,m} = 1$.

3. NETWORK STRUCTURE

We assume that the underlying social network is built upon a stochastic block model. This family of network generation algorithms takes a set of nodes divided into disjoint blocks and then creates ties between nodes independently. For each pair of nodes, the probability of tie creation is a function of the blocks to which the nodes belong. These probabilities are the parameters of the model, resulting in a total of $k \times (k-1)/2 + k$ parameters for an unweighted network of k blocks.

Against this backdrop, we define how ties between agents form. We assume that blocks unambiguously correspond to agents' types. Thus, we end up with M blocks. We denote the population of agents in block f by $N_f = \{i \in \mathcal{V} \mid \xi_i = \Xi_f\}$. For each pair $f, r \in [M]$ of blocks, we denote the probability that a randomly chosen pair of vertices of the corresponding types Ξ_f and Ξ_r will be connected by $\rho_{f,r} \in [0;1]$.

In the rest of this paper, we will assume that the parameters of the stochastic block model are not constant but change over time τ (see the definition of this time below): $\rho_{f,r} = \rho_{f,r}(\tau)$. Note that, for an undirected network (which is our case), it holds that $\rho_{f,r}(\tau) = \rho_{r,f}(\tau)$.

4. MEAN-FIELD APPROXIMATION

Let us introduce macroscopic variables that we will use to describe the dynamics of our system. First, by $y_{a,f}(t)$ ($a \in [m], f \in [M]$), we denote the fraction of agents who have opinion Z_a and type Ξ_f at time t:

$$y_{a,f}(t) = \frac{|\{i \in \mathcal{V} \mid x_i(t) = Z_a, \xi_i = \Xi_f\}|}{N}.$$

Next, n_f will define the total fraction of agents with type Ξ_f:

$$n_f = \sum_{a \in [m]} y_{a,f}(t),$$

which is constant, and $y_a(t)$ will denote the total fraction of agents with opinion Z_a at time t:

$$y_a = \sum_{f \in [M]} y_{a,f}(t).$$

Let τ be a scaled transformation of the initial discrete time t:

$$\tau = \frac{t}{N}, \delta\tau = \frac{1}{N}.$$

Let us also introduce the following two quantities:

$$A = \sum_{f \in [M]} n_f \cdot \pi_f, \quad B_f(\tau) = \sum_{r \in [M]} n_r \cdot \pi_r \cdot \rho_{f,r}(\tau).$$

With all these notations, we can now write the mean-field approximation for the system under scrutiny, which was first introduced in [6]. In that paper, it was shown that the following nonlinear differential equation appears in the limit $N \to \infty$[†]:

[†] In fact, Ref. [6] presented a slightly more general model in which a special type of agents, who act strategically and are not susceptible to influence, was discussed. However, in this paper, we do not consider them in order to make our derivations less cumbersome and more illustrative.

$$\dot{y}_{a,f} = \frac{\pi_f}{A \cdot B_f(\tau)} \sum_{s\in[m]} y_{s,f} \sum_{l\in[m]} \sum_{r\in[M]} y_{l,r} \cdot \pi_r \cdot \rho_{f,r}(\tau) \cdot \Delta^{f,r}_{s,l} \cdot \sum_{k\in[m]} p^{f,r}_{s,l,a}(\delta_{k,a} - \delta_{s,a}), \tag{4.1}$$

$$a \in [M], f \in [M].$$

In the main equation (4.1), the quantities A and $B_f(\tau)$ serve as the normalization values, and $\delta_{k,a}$ is the Kronecker delta. Eq. (4.1) silently assumes that quantities $\rho_{f,r}$, which outline the structure of the social network, change over time. We will suppose that these dynamics are prescribed externally and are subject to optimization. Because of this, for equation (4.1) to be valid, it is necessary for $\rho_{f,r}$ to change "no faster" than the system evolves.

Building upon (4.1), we consider the following Cauchy problem:

$$\dot{y}_{a,f} = \frac{\pi_f}{A \cdot B_f(\tau)} \sum_{s\in[m]} y_{s,f} \sum_{l\in[m]} \sum_{r\in[M]} y_{l,r} \cdot \pi_r \cdot \rho_{f,r}(\tau) \cdot \Delta^{f,r}_{s,l} \cdot \sum_{k\in[m]} p^{f,r}_{s,l,a}(\delta_{k,a} - \delta_{s,a}), \tag{4.2}$$

$$y_{a,f}(0) = q_{a,f}, \quad a \in [M], f \in [M].$$

Let $\rho(\tau) = (\rho_{f,r}(\tau))_{f,r\in[M]}$ and $q = (q_{f,r})_{f,r\in[M]}$. We assert the following result (the proof follows the same line of reasoning as that for Statement 1 and Statement 2 of [13]):

Theorem 4.1:
Let $\rho(\tau)$ *be measurable on* $[0;T]$. *Let* $q \geq 0$ *and*

$$\sum_{a\in[m]} \sum_{f\in[M]} q_{a,f} = 1.$$

Then the Cauchy problem (4.2) *has a unique solution* $y(\tau)$, *which is extendable to* $[0;T]$. *Moreover, for* $\tau \in [0;T]$, *we have:*

$$y(\tau) \geq 0, \quad \sum_{a=1}^{m} y_{a,f}(\tau) = \sum_{a=1}^{m} q_{a,f}, \quad \sum_{f=1}^{M} \sum_{a=1}^{m} y_{a,f}(\tau) = 1.$$

5. CONTROL PROBLEM

Let us assume that a person (say, a platform owner) optimizes the network structure G in order to sway the opinion distribution. One can think that they are able to do this by virtue of tuning friend recommendation algorithms.

Recall that the opinion distribution is defined by the quantities $y_1, ..., y_m$. Following [12], we regard the linear functional

$$J = K \int_0^T \sum_{a=1}^{m} v_a y_a(\tau) d\tau + \sum_{a=1}^{m} v_a y_a(T),$$

where the weight vector $(v_1, ..., v_m) \geq 0$ determines the platform owner's preferences. The larger v_i, the less desirable the opinion Z_i. The interval $[0;T]$ defines a finite time horizon. Parameter $K > 0$ determines the relative importance of the integral term of the functional.

We are now in a position to formulate the following control problem:

$$
\begin{aligned}
& J \longrightarrow \min_{\rho(.)}, \\
& \frac{dy_{a,f}}{d\tau} = \frac{\pi_f}{A \cdot \underbrace{B_f(\tau)}_{\sum_{r\in[M]} n_r \cdot \pi_r \cdot \rho_{f,r}(\tau)}} \sum_{s\in[m]} y_{s,f} \sum_{l\in[m]} \sum_{r\in[M]} y_{l,r} \cdot \pi_r \cdot \rho_{f,r}(\tau) \cdot \Delta^{f,r}_{s,l} \\
& \cdot \textstyle\sum_{k\in[m]} p^{f,r}_{s,l,a}(\delta_{k,a} - \delta_{s,a}), \\
& y_{a,f}(0) = q_{a,f}, \quad a \in [m], \quad f, r \in [M], \\
& 0 < \rho_{min} \le \rho_{f,r}(\tau) \le \rho_{max} \le 1, \quad f, r \in [M],
\end{aligned}
\tag{5.3}
$$

where the initial point q satisfies the restrictions of Theorem 4.1.

6. MAIN RESULT

In this section, we present our main theoretical findings. We will show that the control problem (5.3), where the control parameters

Let us rewrite a component of the phase velocity in a slightly different form:

$$
F_{a,f} = \frac{\pi_f}{A \cdot B_f(\tau)} \sum_{s\in[m]} y_{s,f} \sum_{l\in[m]} \sum_{r\in[M]} y_{l,r} \cdot \pi_r \cdot \rho_{f,r}(\tau) \cdot \Delta^{f,r}_{s,l} \sum_{k\in[m]} p^{f,r}_{s,l,a}(\delta_{k,a} - \delta_{s,a})
$$

$$
= \frac{\pi_f}{A} \sum_{s=1}^{m} y_{s,f} \sum_{l=1}^{m} \sum_{r=1}^{M} y_{l,r} \cdot \pi_r \left[\frac{\rho_{f,r}(\tau)}{\sum\limits_{i=1}^{M} n_i \pi_i \rho_{f,i}(\tau)} \cdot \Delta^{f,r}_{s,l} \right] \sum_{k=1}^{m} p^{f,r}_{s,l,k} \cdot (\delta_{k,a} - \delta_{s,a}).
$$

The fractional-linear function

$$
\frac{\rho_{f,r}(\tau)}{\sum\limits_{i=1}^{M} n_i \pi_i \rho_{f,i}(\tau)} \cdot \Delta^{f,r}_{s,l}
$$

maps the set of admissible controls (which is a convex compact set) to a convex compact set in $\mathbb{R}$ (interval $\mathcal{I}^{f,r}_{s,l} \subset \mathbb{R}$). Let us consider the mapping

$$
\chi(\rho) = \left(\frac{\rho_{f,r}(\tau)}{\sum\limits_{i=1}^{M} n_i \cdot \pi_i \cdot \rho_{f,i}(\tau)} \Delta^{f,r}_{s,l} \right)^{f,r\in[M]}_{s,l\in[m]},
$$

which we define on the set of all admissible controls. The mapping χ projects the set of admissible controls to the Cartesian product of the intervals $\mathcal{I}^{f,r}_{s,l}$—a hyperrectangle Π.

We now fix some $\hat{\rho}$ from the admissible set and vary the parameters $\Delta_{s,l}^{f,r}$ instead. For the sake of presentation, we will change our notation and use $\sigma = \left(\sigma_{s,l}^{f,r}\right)_{s,l\in[m]}^{f,r\in[M]}$ instead of $\Delta_{s,l}^{f,r}$ to show that these parameters are now subject to change.

The following statement holds.

Lemma 6.1:
There exist numbers $\underline{\Delta}_{s,l}^{f,r} \in \mathbb{R}$ *and* $\overline{\Delta}_{s,l}^{f,r} \in \mathbb{R}$ *for* $s, l \in [m]$ *and* $f, r \in [M]$ *such that* $\underline{\Delta}_{s,l}^{f,r} \leq \overline{\Delta}_{s,l}^{f,r}$ *and the function:*

$$\psi(\sigma) = \left(\frac{\hat{\rho}_{f,r}(\tau)}{\sum\limits_{i=1}^{M} n_i \pi_i \hat{\rho}_{f,i}(\tau)} \cdot \sigma_{s,l}^{f,r} \right)_{s,l\in[m]}^{f,r\in[M]},$$

maps the set

$$\{\Delta \mid \underline{\Delta}_{s,l}^{f,r} \leq \sigma_{s,l}^{f,r} \leq \overline{\Delta}_{s,l}^{f,r},\ s,l \in [m],\ f,r \in [M]\}$$

to Π.

Proof
We first notice that the quantities $\sigma_{s,l}^{f,r}$ appear in ψ independently and linearly. As such, we argue that we can treat each of the dimensions of Π independently from the others by (uniquely!) selecting some values $\underline{\Delta}_{s,l}^{f,r}$ and $\overline{\Delta}_{s,l}^{f,r}$, ensuring that the image of our projection will fit the target set Π. Figure 6.1 presents a graphical explanation of this idea. □

Having this result, we now suggest considering the following control problem:

$$\begin{aligned}
&J \longrightarrow \min_{\sigma(.)},\\
&\frac{dy_{a,f}}{d\tau} = \frac{\pi_f}{A \cdot \hat{B}_f(\tau)} \sum_{s\in[m]} y_{s,f} \sum_{l\in[m]} \sum_{r\in[M]} y_{l,r} \cdot \pi_r \cdot \hat{\rho}_{f,r} \cdot \sigma_{s,l}^{f,r}(\tau) \sum_{k\in[m]} p_{s,l,a}^{f,r} \cdot (\delta_{k,a} - \delta_{s,a}),\\
&y_{a,f}(0) = q_{a,f}, \quad a \in [m], f \in [M],\\
&\underline{\Delta}_{s,l}^{f,r} \leq \sigma_{s,l}^{f,r}(\tau) \leq \overline{\Delta}_{s,l}^{f,r}, s,l \in [m], \quad f,r \in [M],
\end{aligned} \tag{6.4}$$

where $\underline{\Delta}_{s,l}^{f,r}$ and $\overline{\Delta}_{s,l}^{f,r}$ are determined in accordance with Lemma 6.1 and

$$\hat{B}_f = \sum_{r\in[M]} n_r \cdot \pi_r \cdot \hat{\rho}_{f,r}.$$

This control problem was thoroughly studied in [13], so we can safely assume that it can be resolved somehow (for instance, by virtue of finite-difference schemes). That is, we have some optimal $\sigma^*(\tau)$. This solution corresponds to a trajectory Γ in the rectangle Π, defined by $\sigma^*(\tau)$ and $\hat{\rho}$.

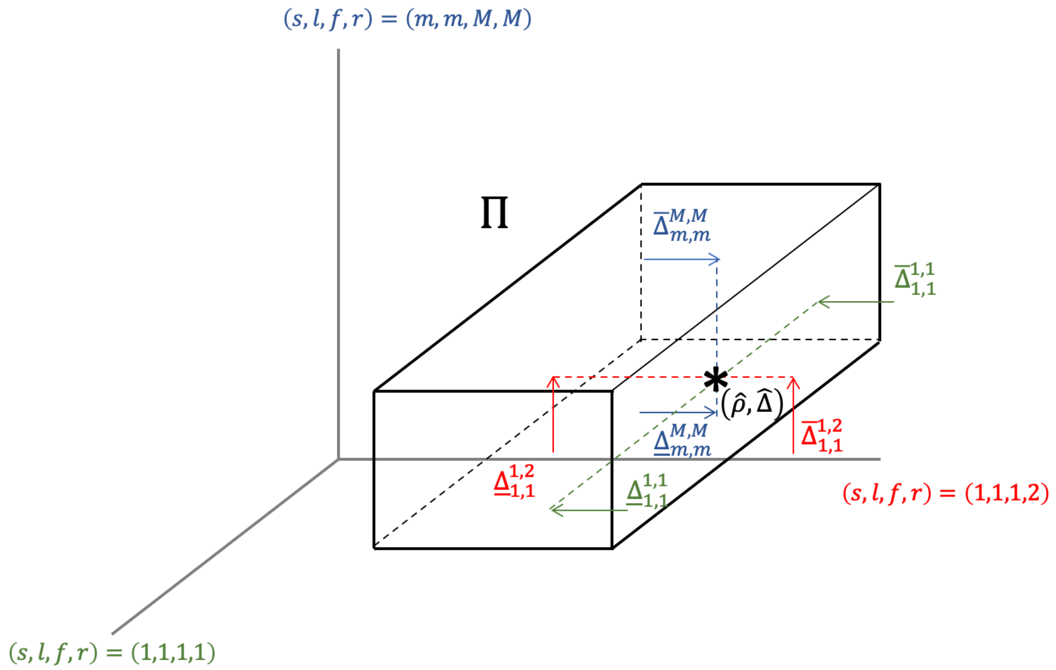

Fig. 6.1. Graphical sketch of the proof of Lemma 6.1. We fix a point $(\hat{\rho},\hat{\Delta})$ such that $\psi(\hat{\Delta})\in\Pi$ (marked by "*") and slide over each of the axes (highlighted in different colors) by fitting the image of the projection to Π.

To summarize, we have just demonstrated how to reduce the control problem (5.3) to a different control problem that we can already solve. The question is, however, how to go back to our initial notations and derive a solution in terms of the initial control parameters $\rho(\tau)$? We argue that this problem is not insurmountable and can be solved pointwise by addressing a constrained optimization problem.

As a result, the pipeline of our method can be formulated as follows:

1. For a given Δ, fix an admissible $\hat{\rho}$.
2. Calculate $\underline{\Delta}^{f,r}_{s,l}$ and $\overline{\Delta}^{f,r}_{s,l}$ for $s,l\in[m]$ and $f,r\in[M]$ in accordance with Lemma 6.1. Note that the values found may fall outside the initial interval $[0;1]$.
3. Solve the control problem 6.4 and find an optimal $\sigma^*(\tau)$.
4. For given Δ, $\hat{\rho}$, and $\sigma^*(\tau)$, find $\rho(\tau)$ that admits

$$\begin{cases} \dfrac{\rho_{f,r}(\tau)}{\sum\limits_{i=1}^{M} n_i\pi_i\rho_{f,i}(\tau)}\Delta^{f,r}_{s,l} = \dfrac{\hat{\rho}_{f,r}(\tau)}{\sum\limits_{i=1}^{M} n_i\pi_i\hat{\rho}_{f,i}(\tau)}\sigma^{f,r}_{s,l}, \quad s,l\in[m], \quad f,r\in[M], \\ \\ \rho_{min}\le\rho^{f,r}_{s,l}(\tau)\rho_{max}, \quad s,l\in[m], \quad f,r\in[M]. \end{cases} \tag{6.5}$$

Note that optimization problem (6.5) is solved pointwisely on a pre-defined grid.

Remark 6.1:
Our results are straightforwardly extended to the case of a system involving social bots (see [6]). We omit these derivations in the interest of space.

7. CONCLUSION

This paper concerns the problem of network structure optimization in the SCARDO model. The objective is to sway the macroscopic opinion of agents within a finite time horizon. We parametrize the network using a stochastic block model and formulate the problem under consideration in terms of the parameters of this model. After that, we show how to boil down the resulting control problem to a problem in which the structure of the network is fixed, but the parameters of the ranking algorithm are subject to optimization—the latter problem has been recently solved in [13].

Our results can be applied to crafting friend recommendation systems on real-world social platforms. With a solution to the control problem (5.3) in hand, we are able to provide a friend recommendation system with directives regarding the amplification or attenuation of ties between specific user groups.

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