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Optimization of Multi-Currency Deposit Structure by Two Indicators (Income and Risk) under Uncertainty

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Abstract. A two-criteria vector optimization problem – finding Pareto-optimal solutions in linear systems with interval uncertainty of coefficients – is considered. The problem of resource allocation to multiple activities is investigated. The uncertainty-adjusted income is a bilinear function, linear by strategy under fixed uncertainty and by uncertain parameters under fixed strategy. Guaranteed income is a linear function of variables and guaranteed risk is a piecewise linear function. Finding the optimal guaranteed risk is reduced to a linear programming problem by piecewise linear programming methods. To solve the two-criteria problem of the optimal allocation of financial resource on three currency deposits, the parameterization of the Pareto set by the value of the guaranteed income criterion is applied. Thus, the construction of a representative subset of Pareto-optimal solutions is reduced to solving a finite number of linear programming problems. The results can be used in analyzing the problems of financial management under conditions of incomplete information.

Keywords: *deposit diversification, bicriteria optimization, incomplete information, minimax regret solution.*

1. INTRODUCTION

Decision-making tasks in most cases are complicated by the factor of incomplete information. We can distinguish some typical situations by the nature of information available to the decision maker (DM) or to the researcher of the problem, the developer of appropriate models.

A deterministic version in which the values of all model parameters are known completely and accurately.

A stochastic variant in which some parameters of the model are not known exactly, but the stochastic characteristics of these non-deterministic parameters are specified.

A case of substantial uncertainty, when neither exact values nor any stochastic characteristics are known for some model parameters. In this case, the uncertain parameters are known only to the precision of some known set.

The first case is most fully developed both in theory and in terms of applications. The stochastic variant in optimization problems is usually reduced to the deterministic case by considering the corresponding deterministic problem for the mean values and variance of the optimized indicators.

In problems with substantial uncertainty, the following situation is typical: only the boundaries of possible values for uncertain parameters are known, often – two-sided ranges. The principle of the best-guaranteed result (Wald principle [1]) is fruitful here. It is applied either to the initial target indicator or to some secondary indicators, for example, to the risk function according to Savage [2]. This risk can be interpreted as a function of losses due to ignorance – incomplete information.

Savage optimization and other approaches in optimization problems under uncertainty were considered in [3-6]. In particular, the paper [3] studied the case of "mixed type" uncertainty, in which statistical distributions are known for nondeterministic parameters of the problem, but some characteristics of these distributions (e.g., mathematical distribution or dispersion) are uncertain parameters with known ranges of possible but unknown in advance values. The works [5, 6]

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consider in detail the problems of optimal distribution of deposits in three currencies according to one criterion (guaranteed income or guaranteed (minimal) Savage risk), algorithms for calculating the optimal deposit structure were obtained, and multivariate calculations were carried out.

This paper considers a two-criteria variant of the optimization of a three-currency deposit, in which both of the mentioned criteria are used simultaneously.

2. STATEMENT OF THE PROBLEM

Decision-making problems are characterized by two features. First, for some uncontrollable parameters only the limits of their possible values are known (they are known with the accuracy of some set). Second, in order to assess the quality of decisions, it is necessary to consider simultaneously several quality indicators – criteria. For example, in the problems of rational use of investments, one of the main criteria is income. At the same time, the uncertainty of the future economic environment entails uncertainty of expected financial results. This leads to the need to consider risks, understood as the difference between the desired or expected results and the results obtained.

In the field of single-criteria optimization of income under uncertainty, the most convincing and elaborated, in our opinion, is the Wald principle [1] – the principle of the best-guaranteed result. To measure risk, the risk function (loss function) according to Savage [2] is often used, i.e. the difference between the best result (at a known ahead value of uncertainty) and the actual result at a particular real strategy. Applying the principle of the best-guaranteed outcome to the risk function leads to the concept of guaranteed risk and optimal guaranteed risk.

The problem of multi-currency deposit optimization under uncertainty in terms of each these indicators (income and risk) separately was considered in [5, 6]. This paper develops a two-criteria approach to the above problem. The generally recognized concept of Pareto (or Slater) optimality is used. Guaranteed income (according to Wald) and guaranteed risk (according to Savage) are chosen as criteria.

The Pareto principle identifies a set of efficient solutions (synonyms – non-dominated, non-improvable, efficient, rational), but the DM must choose a specific solution. There are many approaches, recommendations and procedures proposed in the vast literature on multicriteria optimization regarding the choice of a single solution. Some of them, despite their apparent logical simplicity (e.g., minimization of linear convolution of criteria with given weight "importance coefficients"), have been subjected to reasonable criticism.

One of the possible solutions here is to visualize the whole set of Pareto-optimal solutions or at least a sufficiently representative discrete subset of this set and leave the choice of a single solution to the discretion of the DM. It is desirable that the parameterization of the set of Pareto-optimal solutions be transparent and understandable for the DM.

The following property of two-criteria problems is used below. If the constraint "no worse than P= const" is imposed on one criterion and optimization is performed on the second criterion, then the obtained solution (if it exists) is Pareto optimal or, in the extreme case, Slater optimal. In case the obtained solution is unique in terms of the value of the second criterion, it is also Pareto optimal [7]. If the minimum and maximum values of the criteria are finite (in our deposit problem it is so), then the procedure with a "sufficiently detailed" partitioning by the parameter P will give a "sufficiently detailed" representation of the set of optimal solutions.

Within the framework of this approach, we propose an algorithm for constructing a subset of Pareto-optimal solutions for the problem of optimal deposit diversification.

3. BILINEAR RESOURCE ALLOCATION PROBLEMS UNDER UNCERTAINTY

Consider the problem of allocating a unit of some resource to m types of activity. The profitability of the latter depends on uncertain factors.

Let the initial efficiency indicator (income) be given by a bilinear function of the form

$$f(x,y)=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_iy_j\,,x\in X,y\in Y,all\ c_{ij}\geq 0. \tag{1}$$

Here, the vector $x=(x_1,\dots,x_m)$ is the DM's strategy of allocating the resource to m activities,

$$X=\{x=(x_1,\dots,x_m):x_i\geq 0\ (i=1,\dots,m),x_1+\dots+x_m=1\} \tag{2}$$

is the set of DM's strategies whose components are non-negative and satisfy the budget constraint (canonical simplex in m-dimensional space),

$$Y=\{y=(y_1,\dots,y_n):0\leq a_j\leq y_j\leq b_j,(j=1,\dots,n)\} \tag{3}$$

is the set of possible states of uncertainty (a parallelepiped in n-dimensional space given by interval boundaries of uncertain factors). The coefficients c_{ij} have the sense of income at full utilization of a resource unit only for the i-th activity under conditions when the uncertainty vector has the form $y=(0,\dots,y_j=1,\dots,0)$

The upper and lower bounds of the possible values of the uncertain parameters $a_{j,}$ b_j (j= 1,...,n) are assumed to be known.

By virtue of compactness of the sets X and Y and continuity of the function$f(x,y)$, minima and maxima occurring further exist [9].

The problem is to find strategies that are either Wald- or Savage-optimal.

In the first case, it is about maximizing guaranteed income

$$f[x]=\min_{y\in Y}f(x,y)=\min_{y\in Y}\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_iy_j \tag{4}$$

on the set of the strategies X (risk is excluded).

Let us rewrite the guaranteed income function in the form

$$f[x]=\min_{y\in Y}f(x,y)=\min_{y\in Y}\sum_{i=1}^{m}\left(\sum_{j=1}^{n}c_{ij}y_j\right)x_i\,. \tag{5}$$

Since by assumption $c_{ij}\geq 0$, the minima of the expressions in the last brackets in formula (5) are reached at $y_j=a_j$ (j=1,...,n). Therefore, taking into account the non-negativity of variables x_i, the guaranteed income is a linear function

$$f[x]=\min_{y\in Y}\sum_{i=1}^{m}\left(\sum_{j=1}^{n}c_{ij}a_j\right)x_i=\sum_{i=1}^{m}d_ix_i\,, \tag{6}$$

where $d_i=\sum_{j=1}^{n}c_{ij}a_j$.

Let $d=\max_i d_i$. Then the optimal (maximal) guaranteed revenue $f^w=$ d is attained for any DM strategy x^w with arbitrary full allocation of the unit of resource to the activities with $d_i=d$.

The problem of Savage risk optimality is more complicated. Here the basic concept of risk function (regret function or loss function), defined by the formula

$$\Phi(x,y)=\max_{z\in X}f\,(z,y)-f(x,y). \tag{7}$$

It means the loss of income due to incomplete information as the difference between the best outcome that could have been obtained with a known forward uncertainty value and the actual outcome when the strategy x is chosen. The concept of the best-guaranteed outcome can also be applied to this secondary indicator according to the following definition.

Definition 1:

An admissible solution $x^r \in X$ *is called a risk-guaranteed optimal solution if it delivers a minimum guaranteed risk:*

$$\min_{x \in X} \max_{y \in Y} \Phi(x, y) = \max_{y \in Y} \Phi(x^r, y) = \Phi^r, \tag{8}$$

where $\max_{y \in Y} \Phi(x, y)$ *is the guaranteed risk to the DM when he uses the strategy* $x \in X$.

Let us call the value Φ^r the optimal guaranteed risk. For brevity, we will also speak in this case of risk and Savage-optimal solutions.

The above definition applies to arbitrary income function$f(x, y)$, the set of DM's strategies X and the set of possible values of uncertainty Y.

The considered bilinear income functions of the form (1) are linear by x at each fixed y and linear by y at each fixed x, hence continuous over the set of variables. The sets of strategies X and uncertainties Y are polyhedra, hence compact. Therefore, all maxima and minima in (8) are attained and the optimal solution and the optimal risk exist. The guaranteed risk $\Phi[x] = \max_{y \in Y} \Phi(x, y)$ depends continuously on x.

4. SAVAGE GUARANTEED RISK OPTIMIZATION AND GOAL PROGRAMMING

Preliminarily consider the case where the DM has a finite number of strategies and the uncertainty can take a finite number of states:

$$X = \{u_1, \dots, u_k\}, \qquad Y = \{s_1, \dots, s_l\}. \tag{9}$$

Then the income is defined by the income $(k \times l)$-matrix $C = (c_{ij})$, whose element c_{ij} is equal to the income of the DM when he chooses a strategy u_i and the uncertainty state s_j realizes.

In this case, finding the optimal guaranteed income and optimal guaranteed risk does not cause computational difficulties – it is reduced to choosing maximal and minimal values from finite sets of numbers. Thus, finding a Wald solution consists in finding a maximin strategy for matrix game in pure strategies with a payoff matrix C. The DM's use of mixed strategies can improve his expected payoff.

The task of calculating the Savage solution can be interpreted in terms of goal programming [8]. Let us associate to each uncertainty state s_j *j-th* criterion, the values of which are specified by the *j-th* column of the matrix C . Consider the corresponding multi-criteria optimization problem with the set X of admissible strategies of the DM. As a target (or ideal, utopian) point in the *l-dimensional* space of criteria, let us assume a point – a set of maximum values of income at a known uncertainty state:

$$c^* = \left(\max_{1 \le i \le k} c_{i1}, \dots, \max_{1 \le i \le k} c_{il}\right). \tag{10}$$

The risk function in this case is $(k \times l) -$ matrix

$$(R_{ij}) = \left(\max_{1 \le i \le k} c_{ij} - c_{ij}\right). \tag{11}$$

The application of the Savage minimax approach to the risk matrix formally coincides with the search for an admissible strategy that gives the value of the vector criterion that is as close as possible to the target point in the uniform Chebyshev metric. Consequently, the search for a Savage-optimal solution can be interpreted as a goal-programming problem.

In the main considered continuous case (1) - (3), the construction of the risk function (1) in explicit form is much more complicated. The analogy with goal programming is preserved if we consider the corresponding multi-criteria problem with an infinite continuous set of criteria $\{f(x, y)\}_{y \in Y}$, in which each possible value of uncertainty corresponds to its own criterion function. However, this analogy has little computational value here.

5. OPTIMIZATION OF GUARANTEED RISK AND LINEAR PROGRAMMING

Let us consider step-by-step the process of constructing the optimal risk-guaranteed solution in problem (1) - (3), (8) in accordance with the definition in formula (8). To construct the risk function, we first calculate the function

$$f[y] = \max_{x \in X} f(x, y) \tag{12}$$

of the best results for the DM at each known value of uncertainty y$\in Y$. Due to the compactness of the sets X and Y and the continuity of the function $f(x, y)$ the function (12) is continuous on the set *Y*. Moreover, since X is a polyhedron, by virtue of the known extremal property of linear functions on a polyhedron, at any $y \in Y$ the maximum in (12) is also reached at one of the vertices of this polyhedron. Therefore, the function $f[y]$ is a piecewise linear function of the form

$$f[y] = \max_{1 \le k \le K} f\left(x^{(k)}, y\right) = \max_{1 \le k \le K} \sum_{j=1}^{n} c_{ij}\, x_i^{(k)} y_j \,, y \in Y, \tag{13}$$

where $\{x^{(1)}, \dots, x^{(K)}\}$ is the set of vertices of the polyhedron X. The risk function (3) will also be at each $x \in X$ a piecewise linear convex function on the argument *y* as the sum of linear and piecewise linear functions.

The next step is to find the guaranteed risk

$$\Phi[x] = \max_{y \in Y} \Phi(x, y)\,. \tag{14}$$

The piecewise linear property for $\Phi[x]$ is preserved given that Y is a polyhedron. The form of the function $\Phi[x]$ depends on the configuration of the set *Y* of possible uncertainty values. In the case of bilateral constraints, it is found explicitly in specific cases (see, for example, the next section).

Suppose that an explicit form of the guaranteed risk function $\Phi[x]$ is established:

$$\Phi[x] = \max_{1 \le k \le L} l_k(x), \tag{15}$$

where $l_k(x)$ $(i = 1, \dots, L)$ are linear functions.

Then the problem of minimization of the piecewise linear function of guaranteed risk can be [9] reduced to the following linear programming (LP) problem:

$$\left\{ \begin{array}{c} z \to \min \\ x \in X \\ z - l_k(x) \ge 0 \ \ (k = 1, \dots, L) \end{array} \right\} \tag{16}$$

with *m+1* variable and L additional constraints. The optimal value of the variable z is equal to the optimal guaranteed risk; the minimum point of this problem determines the optimal allocation of a unit of resource by activity.

The solution of the LP problem (16) can be performed by the simplex method, including the regular means of the "Solution Search" section in Excel, while the main difficulty is to construct the guaranteed risk function $\Phi[x]$

An alternative method of solving the original problem is to find the minimum of the function $\Phi[x]$ by investigating its extremal properties on separate parts of the boundary and inside the set X. This approach was used in [4, 5] in the case of three types of activity. We considered the problem of distributing a monetary unit resource over deposits in three types of currencies with uncertain future rates. The advantage of the method is to obtain the solution in analytical form, which made it possible to perform multivariate calculations in Excel and present their results in tabular and graphical form. However, the capabilities of the method are limited by low dimensions of the problems.

On the other hand, the use of piecewise linear programming methods allows to deal with problems of higher dimensionality, but there remains the need to pre-calculate the guaranteed risk function. In addition, this method is less explicit and it is more difficult to perform multivariate calculations. Therefore, a combination of these two approaches seems promising.

5. SOLVING THE PROBLEM OF OPTIMAL RESOURCE ALLOCATION FOR THREE TYPES OF DEPOSITS

The problem of the optimal structure of a multi-currency deposit [4] is of great practical interest.

Let a unit of money in the currency V_3 be distributed over three deposits in the currencies $V_1, V_{2,}V_3$ with known interest rates d_1, d_2, d_3. One of the currencies, V_3, is used to calculate the total income at the end of the period of deposits:

$$f(x, y) = \frac{1 + d_1}{K_1} x_1 y_1 + \frac{1 + d_2}{K_2} x_2 y_2 + (1 + d_3) x_3, \tag{17}$$

where x_i is the amount of currency V_3, converted at the initial rate K_i to currency V_i $(i = 1{,}2)$, x_3 is the remaining resource to deposit in currency V_3. The initial rates K_1, K_2 are assumed to be known, while y_1, y_2 are the unknown rates of currencies V_1, V_2 relative to currency V_3 at the end of the deposit period.

Regarding uncertain rates y_1, y_2 we assume that they can take any value from the given intervals

$$y_i \in [a_i, b_i], (i = 1{,}2). \tag{18}$$

This problem is a special case of the problem (1)–(3):

$$m = n = 3;\ c_{ij} = 0, if\ \ i \neq j;\ c_{ii} = \frac{1 + d_i}{K_i}\ (i = 1{,}2), c_{33} = (1 + d_3), K_3 = 1 \tag{19}$$

and the set of acceptable strategies of the DM and the set of possible states of uncertainty are given by linear constraints

$$\begin{cases} x_1 + x_2 + x_3 = 1, \\ x_i \geq 0\ (i = 1{,}2{,}3), \\ a_j \leq y_j \leq b_j\ \ (j = 1{,}2),\ y_3 = 1. \end{cases} \tag{20}$$

The dimensionality of the problem can be reduced by excluding the variable x_3 from the constraints and the target function, using the constraint-equality $x_3 = 1 - x_1 - x_2$.

We obtain a problem with two variables (x_1, x_2) as DM's decisions and two uncertain exchange rates (y_1, y_2):

$$f(x_1, x_2, y_1, y_2) = \frac{1 + d_1}{K_1} x_1 y_1 + \frac{1 + d_2}{K_2} x_2 y_2 + (1 + d_3)(1 - x_1 - x_2), \tag{21}$$

$$X = \begin{Bmatrix} x_1 + x_2 \leq 1 \\ x_1, x_2 \geq 0 \end{Bmatrix}, \tag{22}$$

$$Y = \begin{Bmatrix} a_1 \leq y_1 \leq b_1 \\ a_2 \leq y_2 \leq b_2 \end{Bmatrix}. \tag{23}$$

In [5] an explicit form of the piecewise linear risk function was obtained. In the same article, the expression for the guaranteed risk was established:

$$\Phi[x] = \max\{\alpha_1 x_1 + \alpha_2 x_2,\ \beta_1(1 - x_1) + \alpha_2 x_2\ ,\ \beta_2(1 - x_2) + \alpha_1 x_1\}, \qquad x \in X. \tag{24}$$

Here, the coefficients of the linear functions forming the piecewise linear function (24) are determined by the initial parameters of the problem:

$$\alpha_i = \left[(1 + d_3) - \frac{1 + d_i}{K_i} a_i\right]\ (i = 1{,}2), \tag{25}$$

$$\beta_i = \left[\frac{1+d_i}{K_i} b_i - (1+d_3)\right] \quad (i = 1,2). \tag{26}$$

In accordance with the results of the previous section, the optimal risk-guaranteed solution can be found from the following linear programming problem:

$$\begin{cases} z \longrightarrow \min, \\ x_1 + x_2 \le 1, \\ x_1, x_2 \ge 0, \\ z - \alpha_1 x_1 - \alpha_2 x_2 \ge 0, \\ z + \beta_1 x_1 - \alpha_2 x_2 \ge \beta_1, \\ z - \alpha_1 x_1 + \beta_2 x_2 \ge \beta_2. \end{cases} \tag{27}$$

The optimal value z_{min} equals the optimal guaranteed risk, the minimum point of $x^* = (x_1^*, x_2^*)$ together with $x_3^* = 1 - x_1^* - x_2^*$ sets the optimal allocation of a unit of resource V_3 to the three contributions.

In [6] the method of direct solution calculation was used through the analysis of extreme properties of the guaranteed risk function on the boundary and interior parts of the set X. The corresponding algorithm and its implementation in Excel were used for multivariate one- and two-parameter calculations in the problem with three types of currencies. In parallel, "pointwise" control calculations by the method of piecewise linear programming were carried out selectively, in which the methods gave matching results.

6. THE MULTI-CRITERIA APPROACH: PRELIMINARY INFORMATION

Consider a general two-criteria optimization problem:

$$G = \langle X, g_1(x), g_2(x) \rangle, \tag{28}$$

where X is the set of admissible solutions (alternatives), $g_1(x)$ and $g_2(x)$ are the target functions to be maximized by choosing the alternative $x \in X$.

Suppose that the set X is closed and bounded (compact), and the functions $g_1(x)$ and $g_2(x)$ are continuous. Then there are finite limits (partial maxima) of the target functions.

Definition 1:

An admissible solution $x' \in X$ is called a Pareto-improvement for an admissible solution $x \in X$, if $g_i(x') \ge g_i(x)$ $(i = 1,2)$ and at least one inequality is strong.

Definition 2:

An admissible solution $x^P \in X$ is called Pareto optimal (P-optimal) if no Pareto-improvement exists for it.

We denote the set of all Pareto optimal solutions by X^P , the set of corresponding optimal estimates $g(x) = (g_1(x), g_2(x))$ by Y^P . This set forms the so-called northeast boundary of the set of all estimates $Y = G(X)$.

Similarly to the above, the notion of Slater optimality is defined and sets X^S, Y^S are introduced. By Slater-improvement here we mean the improvement of all criteria at once.

From Pareto optimality follows Slater optimality: $X^S \supseteq X^P, Y^S \supseteq Y^P$. The converse, generally speaking, is not true.

Note that the sets Y^S and Y^P, X^S and X^P in many problems coincide or differ insignificantly.

As for the choice of the final optimal solution, there is a wide range of recommendations and procedures, significant part of which consists in "scalarization" of the problem, i.e., in replacing the vector criterion by a single scalar criterion. As such, linear convolutions, nonlinear monotonic convolutions (in particular, Leontief functions), and the lexicographic criterion are used. Various iterative procedures, such as the method of successive concessions and others, are also proposed.

One approach is to make available to the DM the entire set of P- or S-optimal estimates or at least some sufficiently representative subset, a kind of ε-network. As a rule, the Pareto frontier has a complex structure. If it is possible to construct it in a visual form and with a transparent for the DM algorithm of construction, it facilitates the task of choosing the final decision.

Let us use the following method of explicitly constructing the set *S*. Let us fix the minimum acceptable level of one criterion by the constraint $g_1(x) \geq C$ and solve the single-criteria optimization problem

$$\left\{\begin{matrix} g_2(x) \to \max \\ x \in X \\ g_1(x) \geq \mathrm{C} \end{matrix}\right\}. \tag{29}$$

Prorosition:

Let $C \leq \max_{x \in X} g_1(x)$ and x^* be a solution of the problem (29). Then x^* is Slater-optimal.

Proof. Indeed, let $Y = g(X) = \{g(x) | x \in X\}$ be the set of all achievable criterion values, and $X_C = \{x \in X | g_1(x) \geq C\}$. The alternatives from the set $X \backslash X_C$ cannot be Slater-improvements for x^* , since they have $g_1(x) < C \leq g_1(x^*)$. On the other hand, for all alternatives in the setX_C, by definition the solution x^*, $g_2(x) \leq g_2(x^*) = \max_{x \in X_C} g_2(x)$. Hence, X_C doesn't contain be a Slater-improvement for x^*. Since $X = (X \backslash X_C) \cup X_C$, the proposition is proved.

Remark:

In nondegenerate cases, the solution x^* and the estimate $g^*(x)$ are Pareto optimal well. For this, in particular, it is sufficient that the maximum in problem (29) is unique in the value of the function $g_2(x)$ [7].

8. ALGORITHM FOR CONSTRUCTING THE SET OF PARETO-OPTIMAL SOLUTIONS

Let us return to the problem of the optimal deposit structure given two criteria: guaranteed income and guaranteed risk (Section 5).

The formula for the guaranteed income in accordance with expressions (21) and (23) takes the form:

$$f[x] = \frac{1 + d_1}{K_1} a_1 x_1 + \frac{1 + d_2}{K_2} a_2 x_2 + (1 + d_3)(1 - x_1 - x_2) \Rightarrow \max. \tag{30}$$

or

$$f[x] = \left[\frac{1 + d_1}{K_1} a_1 - (1 + d_3)\right] x_1 + \left[\frac{1 + d_2}{K_2} a_2 - (1 + d_3)\right] x_2 + (1 + d_3) \Rightarrow \max. \quad (31)$$

Taking into account definitions (25) we finally have:

$$f[x] = -\alpha_1 x_1 - \alpha_2 x_2 + (1 + d_3) \Rightarrow \max. \quad (31)$$

Secondary coefficients α_i $(i = 1,2)$ in (31) depend on initial parameters and have the following meaningful interpretation. The value α_i specifies the difference between the direct income of a unit of currencyV_3 and the income when using this unit to buy currencyM_i at the beginning of the period at the rate K_i and selling the total $(1 + d_3)$ at the minimum rate a_i at the end of the period.

The expression for the guaranteed risk is given above in (24):

$$\Phi[x] = \max\{\alpha_1 x_1 + \alpha_2 x_2,\ \beta_1(1 - x_1) + \alpha_2 x_2,\ \beta_2(1 - x_2) + \alpha_1 x_1\} \Rightarrow \min. \quad (31)$$

The coefficient β_1 (β_2) specifies the difference between the return on a unit of currencyV_3 , deposited via currency V_1 (V_2), and the direct return on currencyV_3 at the highest final rate V_1 (V_2).

Consider the vector maximization problem

$$\langle X, f[x], -\Phi[x]\rangle, \tag{32}$$

in which risk minimization is replaced by risk maximization with minus sign, the set of admissible alternatives is given by conditions (22). Let us apply to it the ideas from the previous section. As it was shown above, the minimization of guaranteed risk can be performed by solving the LP problem (27). Let us supplement it with a restriction on the value of the first criterion – guaranteed income:

$$\begin{cases} z \longrightarrow \min, \\ x_1 + x_2 \le 1, \\ x_1, x_2 \ge 0, \\ z - \alpha_1 x_1 - \alpha_2 x_2 \ge 0, \\ z + \beta_1 x_1 - \alpha_2 x_2 \ge \beta_1, \\ z - \alpha_1 x_1 + \beta_2 x_2 \ge \beta_2, \\ f[x] = -\alpha_1 x_1 - \alpha_2 x_2 + (1 + d_3) \ge C. \end{cases} \tag{33}$$

The parameter C has the meaning of the minimum acceptable guaranteed income. Potentially, the solution of the LP problem (33) at all values of C from $C_{max} = \max_{x \in X} f(x)$ to $C_{min} = \min_{x \in X} f(x)$ will give the whole set X^p of Pareto-optimal solutions (more precisely, we obtain the set of all Slater-optimal solutions $X^S \supseteq X^P$). The interval for the parameter C can be reduced until the moment of reaching the extreme value of the second criterion, because further reduction of this parameter will only lead to deterioration (reduction) of the guaranteed income without improving the value of the guaranteed risk. Therefore, new neither Slater-optimal nor Pareto-optimal solutions will be obtained.

In practice, of course, one has to consider a discrete finite set of values of the parameter C . For example, we can use the sequence $\{C_k\}$:

$$C_k = C_{max}(1 - \Delta_k), \qquad k = 0,1,2, \dots k_{max}, \tag{34}$$

where k_{max} is defined from the condition $\Phi[x_{k_{max}}] = \min_{x \in X} \Phi[x]$, $\{x_k\}$ is the sequence of solutions of problem (33).

It follows from the results of Section 5 that

$$C_{max} = \max_{x \in X} f[x] = \max(-\alpha_1, -\alpha_2) + (1 + d_3), \tag{35}$$

$$C_{min} = \min_{x \in X} f[x] = \min(-\alpha_1, -\alpha_2) + (1 + d_3), \tag{36}$$

and $\min_{x \in X} \Phi[x]$ is found from the solution of the single-criteria LP problem (27) about the minimum guaranteed risk.

The result of solving the problem (33) $(x^{(k)}, z^{(k)})$ has a transparent and understandable interpretation for the DM. Let the DM agrees to reduce the guaranteed income by no more than $100\Delta_k\%$ of the maximum value. Under this condition, the solution $x^{(k)}$ guarantees the risk of no more than $\Phi[x^{(k)}] = z^{(k)}$.

The sequence of points $\{x^{(k)}\} = \{(x_1^{(k)}, x_2^{(k)})\}$ generates a sequence of values of the vector criteria $\{(f[x^{(k)}], \Phi[x^{(k)}])\}$. Recall that the volume of currency $x_3 = 1 - x_1 - x_2$.

This information, presented in tabular and graphical form, can be used by the DM, along with other considerations available to him, to make the final decision on the deposit structure.

CONCLUSION

The problems of decision making under complete information for linear systems are sufficiently developed both in theory and in applications. However, as a rule, in practice, decision making occurs under uncertainty of a number of parameters. In economic research, these are usually future values of prices, exchange rates, demand, terms of transactions, etc. The two most commonly used performance indicators in this situation are income and Savage risk [2], understood as loss of income due to incomplete information. The maximizing (for income) or minimizing (for risk) strategies of DM are considered as the optimal solution in accordance with the principle of the best-guaranteed result [1].

The possibility of obtaining optimal solutions in linear systems with interval uncertainty of coefficients by methods of piecewise linear programming was considered within the framework of the above approach. The problem of resource allocation by several types of activity was studied. The income given uncertain factors is a bilinear function, linear on strategy under fixed uncertainty and on uncertain parameters under fixed strategy. The sets of admissible strategies and possible values of uncertainty are assumed to be polyhedra. Because of these properties, the guaranteed return and risk are piecewise linear functions defined on the polyhedron of strategies. This allowed us to apply the piecewise linear programming method and reduce the problem of finding the optimal guaranteed risk to some linear programming problem. The obtained results are applied to the solution of the problem of optimal allocation of financial resource on three currency deposits.

As a rule, the higher the return on investment, the higher the risk. The attempt to find a compromise between these two indicators leads to the vector optimization problem and the concept of Pareto optimality. We propose a method for solving the two-criterion problem of optimal allocation of financial resource to three currency deposits, which uses the parameterization of the Pareto set by the value of the guaranteed income criterion. Thus, the construction of a representative subset of Pareto-optimal solutions is reduced to solving a finite number of linear programming problems. The obtained information, presented in tabular and graphical form, can be used by the DM along with other available considerations to make the final decision on the allocation of financial resources.

The results can be used in analyzing financial management problems under conditions of incomplete information.

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