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Adaptive Disturbance Rejection Control of 2D Crane Based on DREM with Instrumental Variables

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Abstract: A problem of control of uncertain 2D overhead crane affected by disturbances with unknown model is considered. An adaptive controller is proposed to solve the above-mentioned problem, which main salient feature is that it ensures asymptotic convergence of errors for both trolley position and payload swing angle not to a bounded set, but to zero without requirement of *a priori* knowledge on bounds of the system parameters and disturbance. First of all, the crane is represented in the feedback linearizable form via coordinate transformations under mild assumptions. Then such representation is parameterized in the form of perturbed linear regression equation, which parameters are exactly identified via application of procedure, which has been recently proposed by the authors and is based on the dynamic regressor extension and mixing method and instrumental variables approach. Obtained parameters estimates are substituted into derived ideal control law, and asymptotic stability of the proposed adaptive control system is rigorously proved. All theoretical results are validated via numerical experiments.

Keywords: 2D crane, parameter uncertainty, asymptotic stability, dynamic regressor extension and mixing, instrumental variables

1. INTRODUCTION

Overhead cranes are widely used to manipulate heavy objects at different enterprises and construction cites. They usually consist of a cart moving on a line/plane and a hook suspended to the cart via cables. On the one hand, as one of the main goals of any enterprise is to increase productivity, then cranes are expected to move the payload from one position to another in minimal time. On the other hand, such plants are underactuated, thus cart movement, as well as exogenous disturbances, cause payload swing, which deteriorates positioning and overall crane performance. Therefore, better productivity can be achieved via application of control systems that ensure high control quality under above-mentioned conditions [7, 22].

In this study we restrict our attention to 2D single pendulum cranes. Problem of such plants control has been paid considerable attention by the control community during several past decades, and main obtained results are condensed in [7]. As sway reduction is of high priority, open-loop [16] and collocated control [11] strategies are not considered in this study, and further only non-collocated ones are of interest.

Plenty of closed-loop control methods have been exploited for crane systems, *e.g.*, gain-scheduling [10], feedback linearization [19], singular perturbation analysis [21], predictive control [5, 12]. All these methods are based on at least one of the following restrictive assumptions: the system parameters are known, disturbance-free case is considered.

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At present, as far as parameter uncertainty and perturbations are concerned, the crane control problem is still a fairly open topic. Without being exhaustive, further some techniques to cope with the above-mentioned case are considered.

In [14] a new passivity-based control law is elucidated, which includes both actuated cart and underactuated payload positions, and the asymptotic stability is proved via application of Lyapunov and LaSalle's invariance theorem, but the robustness to disturbances is shown only via numerical experiments.

Parameter uncertainty and unknown perturbations are coped with simultaneously using sliding mode control (SMC) technique [4] in conjunction with sigmoid function to avoid chattering [1] and adaptive control [22] to relax assumption of *a priori* known bounds of the system parameters and disturbance. Particularly in [22], using a coordinate transformation, a new sliding surface is proposed, and uncertainty is parametrized as a linear regression equation, which parameters are then estimated (exactly, as no disturbances were included into initial model description, while friction was also parametrized as a linear regression). Then obtained uncertainty estimate is added to the SMC-based control law with a negative sign. Robustness to perturbations is again shown only via simulation.

In [9], using another coordinate transformation, the original system is represented as a chain of integrators with unmatched uncertainty. Then controller is designed with the help of backstepping approach for disturbance free case.

This result has been further improved in [20], [23], where the transformation from [9] is augmented with an additional assumption to represent the crane model in the feedback linearizable form with matched uncertainty. Then SMC- and backstepping-based controllers with finite-time and extended observers of bounded disturbance were designed. One of the main drawbacks of such designs from [20], [23] is the requirement to know the system parameters for control signal calculation.

The fact that the overhead crane model can be represented in feedback linearizable form means that other methods apart from SMC and backstepping can be applied to solve the control problem under consideration, particularly, adaptive control methods [15]. Considering them, firstly, an ideal control law is derived, which guarantees the control goal achievement under the assumption that the system parameters are known. Secondly, using *certainty equivalence* principle, the control law unknown parameters are substituted with their dynamic estimates obtained with the help of adaptive laws from initial system parametrizations represented in the form of linear regression equations. This principle has already been mentioned in this section when approach from [22] was described. To ensure convergence of cart and payload position errors to zero, exact estimates of the system parameters are required, otherwise only uniform ultimate boundedness of mentioned errors can be shown. But existing online continuous-time parameter estimation laws provide zero identification error only if disturbance-free case is considered (like in [22]) or the external perturbations are vanishing to zero or independent with the regressor of the system, which are far away from practical scenarios [2, 8, 18].

In [6] the authors have recently proposed a new method of online asymptotic identification of perturbed linear systems, which is based on dynamic regressor extension and mixing procedure [3] and instrumental variables approach [13] and ensures online exact asymptotic estimation of the unknown parameters of linear systems in the perturbed case even if the perturbation and regressor of the system are dependent. Such method became an inspiration for this study, as it allows one to overcome the hindrances mentioned in the previous paragraph.

Therefore, in order to overcome the above-mentioned limitations of the existing methods for the crane control, in this study we apply the approach from [6] to propose a control method, which exhibits simultaneously the following remarkable properties:

P1) it ensures asymptotic convergence to zero of both cart/payload position and parametric errors in case when non-vanishing bounded disturbance with unknown model affects the overhead crane,

P2) it does not require prior knowledge of system real/nominal parameters and disturbance or their bounds,

P3) it does not use SMC approach and free from complexity of the backstepping-based control laws.

The remaining parts of this paper are organized as follows. The rigorous problem statement is given in in Section II. Main result is elucidated in Section III. Section IV presents the results of the numerical experiments followed by the conclusions in Section V. Finally, proofs are postponed to Appendix.

Notation. Further the following notation is used: $|.|$ is the absolute value, $\|.\|$ is the suitable norm of $(.)$, $I_{n\times n} = I_n$ is an identity $n \times n$ matrix, $0_{n\times n}$ is a zero $n \times n$ matrix, 0_n stands for a zero vector of length n, $\det\{.\}$ stands for a matrix determinant, $\mathrm{adj}\{.\}$ represents an adjoint matrix. We also use the fact that for all (possibly singular) $n \times n$ matrices M the following holds: $\mathrm{adj}\{M\}M = \det\{M\}I_{n\times n}$ and say that $f \in L_q$ if $\sqrt[q]{\int\limits_{t_0}^{t} |f(s)|^q ds} < \infty$ for all $t \geq t_0$.

2. PROBLEM STATEMENT

A mathematical model of a 2D overhead crane is considered:

$$\begin{array}{l} (M+m)\,\ddot{q}_1 + ml\ddot{q}_2\cos(q_2) - ml\dot{q}_2^2\sin(q_2) = \tau + \tau_d, \\ ml^2\ddot{q}_2 + ml\ddot{q}_1\cos(q_2) + mgl\sin(q_2) = 0, \end{array} \tag{2.1}$$

where $q_1 \in \mathbb{R}$ stands for a trolley position, $q_2 \in \mathbb{R}$ denote a payload swing angle, $\tau \in \mathbb{R}$ is a control signal, $\tau_d \in \mathbb{R}$ is an unknown external perturbation. The masses of the trolley M and payload m are unknown, the gravity constant g and the cable length l are known.

The following assumptions are adopted for the system (2.1).

Assumption 1. The payload swing angle q_2 is mechanically bounded, *i.e.*, $q_2 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Assumption 2. It holds that $\frac{l}{g\cos(q_2)}\dot{q}_2^2 \ll 1$.

Assumption 3. It holds that $\dot{\tau}_d(t) \in L_p \cap L_\infty$ for $1 \leq p < \infty$.

The control goal for (2.1) is formulated as simultaneous stabilization of the payload swing angle q_2 at zero and asymptotic tracking of the reference position signal by the trolley. Considering the states $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$, $x_4 = \dot{q}_2$, this goal can be written as follows:

$$\lim_{t\to\infty} \|x(t) - x_d\| = 0, \tag{2.2}$$

where $x_d = [x_{1d} \quad 0 \quad 0 \quad 0]^{\mathrm{T}}$.

3. MAIN RESULT

The main result of the study is divided into three parts. In the first one, using [9] and Assumptions 1 and 2, the system is transformed into a feedback linearizable form. In the second part, an ideal control law is derived that allows one to achieve the stated goal in case that all plant parameters are known. In the third part, a procedure from [6] is used to form an estimation law for exact identification of parameters of the overhead crane affected by perturbations, and then an adaptive control system is designed that ensures the goal (2.2) achievement in case of unknown crane parameters.

3.1. System Parametrization

In this subsection transformations from [9] are described to convert the system into a feedback linearizable form.

The term $\ddot{q}_1$ is expressed from the second equation of (2.1):

$$\ddot{q}_1 = \frac{-ml^2\ddot{q}_2 - mgl\sin(q_2)}{ml\cos(q_2)} = -g\tan(q_2) - l\ddot{q}_2\sec(q_2). \tag{3.3}$$

Equation (3.3) is substituted into (2.1), and after series of simple transformations we have:

$$\begin{aligned} &-l\left[M + m\sin^2(q_2)\right]\ddot{q}_2 - (M+m)\,g\sin(q_2) - ml\dot{q}_2^2\sin(q_2)\cos(q_2) = \\ &= \cos(q_2)\left[\tau + \tau_d\right]. \end{aligned} \tag{3.4}$$

The left- and right-hand sides of (3.4) is divided by $-l\left[M + m\sin^2(q_2)\right]$ to obtain:

$$\ddot{q}_2 = \frac{\cos(q_2)}{-l\left(M + m\sin^2(q_2)\right)}\left[\tau + \tau_d + (M+m)\,g\tan(q_2) + ml\dot{q}_2^2\sin(q_2)\right]. \tag{3.5}$$

The following notation is introduced:

$$\begin{gathered} \theta^{\mathrm{T}} = [M \quad m], \\ \phi^{\mathrm{T}}(t) = [-l \quad -l\sin^2(q_2)], \\ \omega^{\mathrm{T}}(t) = [g\tan(q_2) \quad l\dot{q}_2^2\sin(q_2) + g\tan(q_2)], \end{gathered} \tag{3.6}$$

in order to rewrite (3.5) in a simpler form:

$$\ddot{q}_2 = \frac{\cos(q_2)}{\phi^{\mathrm{T}}(t)\,\theta}\left[\tau + \tau_d + \omega^{\mathrm{T}}(t)\,\theta\right]. \tag{3.7}$$

Equations (3.3) and (3.7) are written in a state space form:

$$\begin{aligned} &\dot{x}_1 = x_2, \\ &\dot{x}_2 = -g\tan(x_3) - lU\sec(x_3), \\ &\dot{x}_3 = x_4, \\ &\dot{x}_4 = U,\, U = \tfrac{\cos(x_3)}{\phi^{\mathrm{T}}(t)\theta}\left[\tau + \tau_d + \omega^{\mathrm{T}}(t)\,\theta\right]. \end{aligned} \tag{3.8}$$

In accordance with [9], the following coordinate transformation $\mathcal{T}\colon \mathbb{R}^4 \mapsto \mathbb{R}^4$ is introduced:

$$p = \mathcal{T}(x) = \begin{bmatrix} x_1 + \chi(x_3) \\ x_2 + \frac{lx_4}{\cos(x_3)} \\ x_3 \\ x_4 \end{bmatrix}, \tag{3.9}$$

where

$$\chi(x_3) = l\int_0^{x_3} \tfrac{1}{\cos(s)}ds = l\cdot\log\left(\tfrac{1+\tan\left(\frac{x_3}{2}\right)}{1-\tan\left(\frac{x_3}{2}\right)}\right).$$

Then, owing to

$$\tfrac{d}{dt}\left[\chi(x_3)\right] = l\frac{\dot{x}_3\left(\tan^2\left(\frac{x_3}{2}\right) + 1\right)}{\left(\tan^2\left(\frac{x_3}{2}\right) - 1\right)} = l\frac{x_4}{\cos(x_3)}, \tag{3.10}$$

it is written with the help of the new coordinates that:

$$\begin{array}{l} \dot{p}_1 = p_2, \\ \dot{p}_2 = -g\tan\left(p_3\right) - lU\sec\left(p_3\right) + l\frac{U + p_4^2\tan(p_3)}{\cos(p_3)} = \tan\left(p_3\right)\left(-g + \frac{lp_4^2}{\cos(p_3)}\right), \\ \dot{p}_3 = p_4, \\ \dot{p}_4 = U. \end{array} \tag{3.11}$$

Then, following [9], another one coordinate transformation $\mathcal{S}$: $\mathbb{R}^4 \mapsto \mathbb{R}^4$ is introduced:

$$\xi = \mathcal{S}\left(p\right) = \begin{bmatrix} p_1 \\ p_2 \\ -g\tan\left(p_3\right) \\ -g\left(1 + \tan^2\left(p_3\right)\right)p_4 \end{bmatrix}, \tag{3.12}$$

which allows one to rewrite (3.11) as follows:

$$\begin{array}{l} \dot{\xi}_1 = \xi_2, \\ \dot{\xi}_2 = \xi_3 - \frac{l\xi_3\xi_4^2}{\left(g^2 + \xi_3^2\right)^{\frac{3}{2}}}, \\ \dot{\xi}_3 = \xi_4, \\ \dot{\xi}_4 = V, \end{array} \tag{3.13}$$

where the following notation is used:

$$\begin{array}{c} \xi_3 - \frac{l\xi_3\xi_4^2}{\left(g^2 + \xi_3^2\right)^{\frac{3}{2}}} = -g\tan\left(p_3\right) + \frac{lg\tan(p_3)g^2\left(1 + \tan^2(p_3)\right)^2p_4^2}{\left(g^2 + g^2\tan^2(p_3)\right)^{\frac{3}{2}}} = -g\tan\left(p_3\right) + l\frac{\tan(p_3)p_4^2}{\cos(p_3)}, \\ V = -g\left(\tan^2\left(p_3\right) + 1\right)\left[U + 2p_4^2\tan\left(p_3\right)\right] = \\ = g\left(\xi\right)\left[\frac{\cos(x_3)}{\phi^{\mathrm{T}}(t)\theta}\left[\tau + \tau_d + \omega^{\mathrm{T}}\left(t\right)\theta\right] + F\left(\xi\right)\right], \\ F\left(\xi\right) = 2\left(\tan\left(\operatorname{atan}\left(-g^{-1}\xi_3\right)\right)\right)\left(\frac{\xi_4}{-g\left(1 + \tan^2\left(\operatorname{atan}\left(-g^{-1}\xi_3\right)\right)\right)}\right)^2, \\ g\left(\xi\right) = -g\left(\tan^2\left(\operatorname{atan}\left(-g^{-1}\xi_3\right)\right) + 1\right). \end{array} \tag{3.14}$$

If Assumptions 1 and 2 are met, then it holds for the second term from the second equation of (3.13) that:

$$\frac{l\xi_4^2}{\left(g^2 + \xi_3^2\right)^{\frac{3}{2}}} = \frac{lg^2\left(1 + \tan^2\left(p_3\right)\right)^2p_4^2}{\left(g^2 + g^2\tan^2\left(p_3\right)\right)^{\frac{3}{2}}} = \frac{lp_4^2}{g\cos\left(p_3\right)} = \frac{l}{g}\frac{\dot{q}_2^2}{\cos\left(q_2\right)} \ll 1, \tag{3.15}$$

which guarantees that

$$\zeta_3 - \frac{l\zeta_3\zeta_4^2}{\left(g^2 + \zeta_3^2\right)^{\frac{3}{2}}} \triangleq \zeta_3$$

and allows one to rewrite (3.13) as:

$$\dot{\xi}\left(t\right) = A_0\xi\left(t\right) + b_0 g\left(\xi\right)\left[\frac{\cos\left(x_3\right)}{\phi^{\mathrm{T}}\left(t\right)\theta}\left[\tau\left(t\right) + \tau_d\left(t\right) + \omega^{\mathrm{T}}\left(t\right)\theta\right] + F\left(\xi\right)\right], \tag{3.16}$$

where

$$A_0 = \begin{bmatrix} 0_4 & \begin{matrix} I_3 \\ 0_{1\times 3} \end{matrix} \end{bmatrix}, b_0 = \begin{bmatrix} 0_3 \\ 1 \end{bmatrix}.$$

Therefore, in case Assumptions 1 and 2 are met, the original problem (2.2) is reduced to tracking of the following reference signal by the new coordinates ξ:

$$\xi_d = \mathcal{S}\left(\mathcal{T}\left(x_d\right)\right) = \begin{bmatrix} x_{1d} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

in such a way that

$$\lim_{t \to \infty} \left\| \xi\left(t\right) - \xi_d \right\| = 0. \tag{3.17}$$

Since, under Assumption 1, the transformations (3.9) and (3.12) are diffeomorphisms, the objectives (2.2) and (3.17) are equivalent.

3.2. Ideal Control Law Derivation

To define the required control quality for the system (3.16), the following reference model is introduced:

$$\dot{\xi}_{ref}\left(t\right) = A_K \xi_{ref}\left(t\right) + b_0 e_1^{\mathrm{T}} K x_{1d}, \tag{3.18}$$

where $K \in \mathbb{R}^n$ is a parameter vector, which defines a Hurwitz matrix $A_K = A_0 + b_0 K^{\mathrm{T}}$.

Then the tracking error $\tilde{\xi} = \xi\left(t\right) - \xi_{ref}\left(t\right)$ is described by:

$$\begin{array}{c} \dot{\tilde{\xi}}\left(t\right) = A_K \tilde{\xi}\left(t\right) + b_0 g\left(\xi\right)\left[\frac{\cos(x_3)}{\phi^{\mathrm{T}}(t)\theta}\left[\tau\left(t\right) + \tau_d\left(t\right) + \omega^{\mathrm{T}}\left(t\right)\theta\right] + F\left(\xi\right)\right] - \\ -b_0\left(K^{\mathrm{T}}\xi\left(t\right) + e_1^{\mathrm{T}} K x_{1d}\right). \end{array} \tag{3.19}$$

To compensate for all perturbations in equation (3.19), the control law is chosen as:

$$\begin{array}{l} \tau\left(t\right) = \frac{\phi^{\mathrm{T}}(t)\theta}{\cos(x_3)}\tau_b\left(t\right) - \tau_{df}\left(t\right) - \omega^{\mathrm{T}}\left(t\right)\theta, \\ \tau_b\left(t\right) = \frac{1}{g(\xi)}\left(K^{\mathrm{T}}\xi + e_1^{\mathrm{T}} K x_{1d}\right) - F\left(\xi\right), \end{array} \tag{3.20}$$

where $\tau_{df}\left(t\right)$ denotes the filtered disturbance, $\tau_b\left(t\right)$ is a baseline component of the control signal.

In order to obtain the filtered version of $\tau_d\left(t\right)$, the stable first order filter $\frac{k}{s+k}\left[.\right],\ k > 0$ is applied to the left- and right-hand sides of the first equation from (2.1):

$$\left(M + m\right)\frac{k}{s+k}\left[\ddot{q}_1\right] + ml\frac{k}{s+k}\left[\ddot{q}_2 \cos\left(q_2\right)\right] - ml\frac{k}{s+k}\left[\dot{q}_2^2 \sin\left(q_2\right)\right] = \frac{k}{s+k}\left[\tau\right] + \frac{k}{s+k}\left[\tau_d\right], \tag{3.21}$$

Having applied the swapping lemma:

$$\frac{k}{s+k}\left[xy\right] = x\frac{k}{s+k}\left[y\right] - \frac{k}{s+k}\left[\dot{x}\frac{k}{s+k}\left[y\right]\right],$$

we have from (3.21) that:

$$\begin{array}{l} \tau_{df}\left(t\right) = \varphi^{\mathrm{T}}\left(t\right)\theta - z\left(t\right), \\ \varphi^{\mathrm{T}}\left(t\right) = \left[\frac{ks}{s+k}\left[x_2\right] \quad \frac{ks}{s+k}\left[x_2\right] + l\overline{\varphi}(x)\right], \\ z\left(t\right) = \frac{k}{s+k}\left[\tau\right], \end{array} \tag{3.22}$$

where

$$\tau_{df}\left(t\right) := \frac{k}{s+k}\left[\tau_d\left(t\right)\right],$$

$$\overline{\varphi}(x) = \cos\left(x_3\right)\frac{ks}{s+k}\left[x_4\right] + \frac{k}{s+k}\left[\sin\left(x_3\right)\left(x_4\frac{ks}{s+k}\left[x_4\right] - x_4^2\right)\right].$$

The substitution of (3.22) into (3.20) yields the final equation for the ideal control law, which is applicable in case all overhead crane parameters are known:

$$\tau(t) = \frac{\phi^{\mathrm{T}}(t)\theta}{\cos(x_3)}\tau_b(t) - \Omega^{\mathrm{T}}(t)\theta + z(t), \tag{3.23}$$

where $\Omega(t) = \omega(t) + \varphi(t)$.

If the crane parameters are unknown, they are required to be estimated to design an adaptive control system.

3.3. Adaptive Control System Design

The unknown parameters are proposed to be identified using the estimation law proposed in [6]. Following the procedure from [6], an instrumental variable is introduced on the basis of the reference model (3.18) states:

$$\zeta_{iv}(t) = \begin{bmatrix} \frac{ks}{s+k}[x_{2ref}] \\ \frac{ks}{s+k}[x_{2ref}] + l\overline{\varphi}(x_{ref}) \end{bmatrix} \tag{3.24}$$

where

$$x_{ref}(t) = \begin{bmatrix} x_{1ref}(t) \\ x_{2ref}(t) \\ x_{3ref}(t) \\ x_{4ref}(t) \end{bmatrix} = \mathcal{S}^I\left(\mathcal{T}^I(\xi_{ref})\right).$$

Using the instrumental variable (3.24), the regression equation (3.22) is extended with the help of a filter with averaging and a sliding window filter:

$$\begin{aligned} &\dot{\vartheta}(t) = \zeta_{iv}(t)z(t) - \zeta_{iv}(t-T)z(t-T), \vartheta(t_0) = 0, \\ &\dot{\psi}(t) = \zeta_{iv}(t)\varphi^{\mathrm{T}}(t) - \zeta_{iv}(t-T)\varphi^{\mathrm{T}}(t-T), \psi(t_0) = 0_{2\times 2}, \end{aligned} \tag{3.25}$$

$$\begin{aligned} &\dot{Y}(t) = -\frac{1}{F(t)}\dot{F}(t)(Y(t) - \vartheta(t)), Y(t_0) = 0, \\ &\dot{\Phi}(t) = -\frac{1}{F(t)}\dot{F}(t)(\Phi(t) - \psi(t)), \Phi(t_0) = 0_{2\times 2}, \\ &\dot{F}(t) = pt^{p-1}, F(t_0) = F_0 > 0, \end{aligned} \tag{3.26}$$

where $T > 0$ is a sliding window width, $p \geqslant 1$, $F_0 \geqslant t_0^p$ stand for the parameters of the filter with averaging.

Considering that $\theta = const$ and applying the filters (3.25) and (3.26), the following regression equation is obtained [6, Proposition 4]:

$$Y(t) = \Phi(t)\theta + W(t), \tag{3.27}$$

where the disturbance satisfies following equations:

$$\begin{aligned} &\dot{W}(t) = -\frac{1}{F(t)}\dot{F}(t)(W(t) - \varepsilon(t)), W(t_0) = 0, \\ &\dot{\varepsilon}(t) = \zeta_{iv}(t)w(t) - \zeta_{iv}(t-T)w(t-T), \varepsilon(t_0) = 0, \end{aligned} \tag{3.28}$$

where, in order to follow notation from [6], we introduce $w(t) := \frac{k}{s+k}[\tau_d(t)]$.

Multiplication of (3.27) by adj $\{\Phi(t)\}$ yields a set of scalar regression equations:

$$\mathcal{Y}(t) = \Delta(t)\theta + \mathcal{W}(t), \tag{3.29}$$

where

$$\mathcal{Y}(t) := \operatorname{adj}\left\{\tfrac{\Phi(t)}{1+\|\Phi(t)\|}\right\} Y(t)\,,\ \Delta(t) := \det\left\{\tfrac{\Phi(t)}{1+\|\Phi(t)\|}\right\},$$
$$\mathcal{W}(t) := \operatorname{adj}\left\{\tfrac{\Phi(t)}{1+\|\Phi(t)\|}\right\} W(t)\,.$$

We are in position to prove the following proposition for the disturbance from (3.29).

Proposition 3.1:
If the following inequality holds:

$$\forall t \geqslant t_0 \left|\int\limits_{t_0}^{t} \zeta_i(s)\, w(s)\, ds\right| \leqslant c < \infty\ \forall i = \{1{,}2\}\,, \tag{3.30}$$

then

i) $\mathcal{W}_i \in L_l$ *for all* $l \in (1, \infty)$,

ii) there exists a scalar $c_{\mathcal{W}} > 0$ *such that* $|\mathcal{W}_i(t)| \leqslant c_{\mathcal{W}} \frac{\dot{F}(t)}{F(t)} < \infty$.

Proof
Proof of Proposition 3.1 coincides with the one for Proposition 5 from [6]. □

The second Lyapunov method, the perturbation properties from Proposition 3.1, and the results from [6] jointly motivate introduction of the following adaptive control law:

$$\tau(t) = \tfrac{\phi^{\mathrm{T}}(t)\hat{\theta}(t)}{\cos(x_3)} \tau_b(t) - \Omega^{\mathrm{T}}(t)\,\hat{\theta}(t) + z(t)\,, \tag{3.31}$$

where

$$\dot{\hat{\theta}}(t) = -\gamma\Delta(t)\left(\Delta(t)\,\hat{\theta}(t) - \mathcal{Y}(t)\right), \tag{3.32}$$

where $\gamma > 0$ are adaptive gains.

The properties of the closed-loop adaptive control system are described in the following theorem.

Theorem 3.1:
Let Assumptions 1-3 be met and inequality (3.30) *hold, then the goal* (2.2) *is achieved.*

Proof
Proof of Theorem 3.1 is postponed to Appendix. □

Owing to the chosen structure of the reference model, boundedness (asymptotic stability) of the error $\tilde{\xi}$ leads to boundedness (asymptotic stability) of the error $\xi(t) - \xi_d$. Thus, the control law (3.31) + (3.32) ensures that the goal (2.2) is achieved even when the overhead crane parameters are unknown.

4. NUMERICAL EXPERIMENTS

The overhead crane model parameters have been picked from the state standard of the Russian Federation 'Electric single girder overhead cranes' and had the following values:

$$m = 800,\ M = 4660,\ l = 1,\ g = 9.81. \tag{4.33}$$

The disturbance was defined as:

$$\tau_d(t) = 1000e^{-0.001(t-t_0)}\sin(0.2\pi t) + 500, \tag{4.34}$$

where the exponential multiplier formally allows one to meet Assumption 3.

The reference signal was formed using the following function:

$$\begin{gathered} r\left(t\right) = 10\text{sign}\left\{\sin\left(0.05\pi\right)\right\}, \\ \dot{x}_{1d}\left(t\right) = \text{sat}|_{-2}^{+2}\left\{100\left(r\left(t\right) - x_{1d}\left(t\right)\right)\right\}. \end{gathered} \tag{4.35}$$

The parameters of the control law (3.23) were chosen as follows:

$$\begin{gathered} \hat{\theta}(t_0) = [0.5M \quad 0.1m]^{\mathrm{T}},\, K^{\mathrm{T}} = [-16 \quad -32 \quad -24 \quad -8], \\ k = 1,\, p = 1,\, T = 10,\, \gamma = 10^4. \end{gathered} \tag{4.36}$$

Figure 1 shows behavior of the unknown parameter estimates.

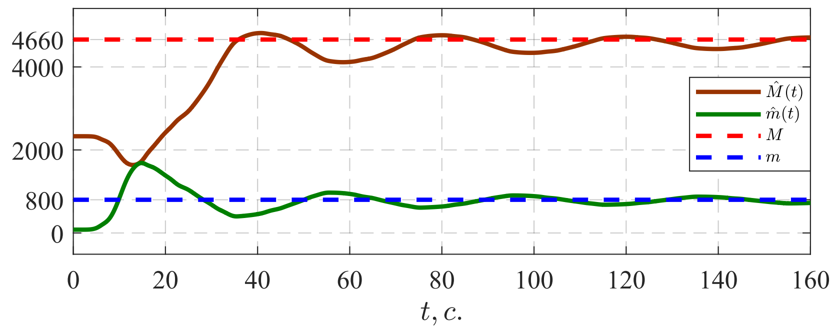

Fig. 4.1. Behavior of $\hat{\theta}(t)$.

Figure 2 presents behavior of x_1, x_{1d}, $\tilde{\xi}_1$, x_3, $\frac{l\xi_3\xi_4^2}{\left(g^2+\xi_3^2\right)^{\frac{3}{2}}}$ and f, f_f.

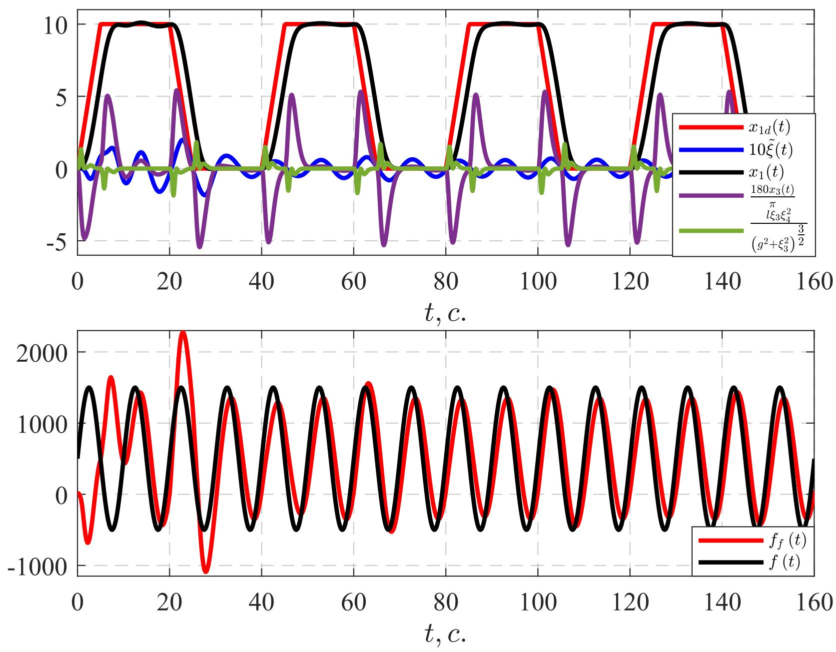

Fig. 4.2. Behavior of x_1, x_{1d}, $\tilde{\xi}_1$, x_3, $\frac{l\xi_3\xi_4^2}{\left(g^2+\xi_3^2\right)^{\frac{3}{2}}}$ and f, f_f.

The results of conducted experiments allow one to conclude that the proposed control system ensures goal (2.2) achievement under conditions of parametric uncertainty. Owing to the estimation of unknown parameters of the system, compensation of the disturbance is ensured without oscillations of the payload swing angle. The above-given transients of $\frac{l\xi_3\xi_4^2}{\left(g^2+\xi_3^2\right)^{\frac{3}{2}}}$ allow one to conclude that Assumption 2 and inequality (15) are met.

5. CONCLUSION

In this study a new adaptive control system for perturbed uncertain overhead crane is proposed, which, adopting non-strict assumptions, simultaneously (*i*) ensures asymptotic convergence to zero of both cart/payload position and parametric errors in case when non-vanishing bounded disturbance with unknown model affects the overhead crane, (*ii*) does not require prior knowledge of system real/nominal parameters and disturbance or their bounds, (*iii*) does not use SMC approach and free from complexity of the backstepping-based control laws. Conducted numerical experiments fully validated all theoretical results.

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A. THEOREM 3.1 PROOF

Proof
Proof is divided into two steps. The first one is to show asymptotic convergence of parametric error $\tilde{\theta}(t) = \theta - \hat{\theta}(t)$ to zero. In the second one we prove convergence of the tracking error $\tilde{\xi}(t)$ to zero.

Step 1. The differential equation for the parametric error is written as follows:

$$\dot{\tilde{\theta}}(t) = \gamma\Delta(t)\left(\Delta(t)\hat{\theta}(t) - \mathcal{Y}(t)\right) = -\gamma\Delta^2(t)\tilde{\theta}(t) - \gamma\Delta(t)\mathcal{W}(t). \tag{A.37}$$

Following Theorem 2 from [6], in case $\mathcal{W} \in L_2$ and $\Delta \notin L_2$ we have $\lim\limits_{t\to\infty}\tilde{\theta}(t) = 0$, $\tilde{\theta}(t) \in L_\infty$.

Step 2. The control law (3.31) is substituted into (3.19) to obtain:

$$\begin{gathered}\dot{\tilde{\xi}}(t) = A_K\tilde{\xi}(t) - b_0\left(K^{\mathrm{T}}\xi(t) + e_1^{\mathrm{T}}Kx_{1d}\right) + \\ +b_0 g(\xi)\left[\frac{\cos(x_3)}{\phi^{\mathrm{T}}(t)\theta}\left[\frac{\phi^{\mathrm{T}}(t)\hat{\theta}(t)}{\cos(x_3)}\tau_b(t) + \tilde{\tau}_d(t) + \Omega^{\mathrm{T}}(t)\tilde{\theta}(t)\right] + F(\xi)\right] = \\ = A_K\tilde{\xi}(t) + b_0\frac{g(\xi)\cos(x_3)}{\phi^{\mathrm{T}}(t)\theta}\left[-\frac{\phi^{\mathrm{T}}(t)\tilde{\theta}(t)}{\cos(x_3)}\tau_b(t) + \tilde{\tau}_d(t) + \Omega^{\mathrm{T}}(t)\tilde{\theta}(t)\right]\end{gathered} \tag{A.38}$$

where $\tilde{\tau}_d(t) = \tau_d(t) - \tau_{df}(t)$.

Then error equations can be written in the following cascade form:

$$\begin{aligned}
&\dot{\tilde{\xi}}\left(t\right)=F_1\left(\tilde{\xi},\tilde{\theta},\tilde{\tau}_d,t\right),\\
&\dot{\tilde{\theta}}\left(t\right)=F_2\left(\tilde{\theta},t\right),\\
&\dot{\tilde{\tau}}_d\left(t\right)=F_3\left(\tilde{\tau}_d,t\right),
\end{aligned}\tag{A.39}$$

where

$$\begin{aligned}
&F_1\left(\tilde{\xi},\tilde{\theta},\tilde{\tau}_d,t\right):=A_K\tilde{\xi}\left(t\right)+\\
&+b_0\tfrac{g(\xi)\cos(x_3)}{\phi^{\mathrm{T}}(t)\theta}\left[-\tfrac{\phi^{\mathrm{T}}(t)\tilde{\theta}(t)}{\cos(x_3)}\left(\tfrac{1}{g\left(\tilde{\xi}+\xi_{ref}\right)}\left(K^{\mathrm{T}}\tilde{\xi}\left(t\right)+K^{\mathrm{T}}\xi_{ref}\left(t\right)+e_1^{\mathrm{T}}Kx_{1d}\right)-F\left(\tilde{\xi}+\xi_{ref}\right)\right)+\right.\\
&\left.+\tilde{\tau}_d\left(t\right)+\Omega^{\mathrm{T}}\left(t\right)\tilde{\theta}\left(t\right)\right],\\
&F_2\left(\tilde{\theta},t\right):=-\gamma\Delta^2\left(t\right)\tilde{\theta}\left(t\right)-\gamma\Delta\left(t\right)\mathcal{W}\left(t\right),\\
&F_3\left(\tilde{\tau}_d,t\right):=-k\tilde{\tau}_d+\dot{\tau}_d\left(t\right).
\end{aligned}$$

According to step 1, when the premises of the theorem under consideration are met, we have $\lim\limits_{t\to\infty}\tilde{\theta}\left(t\right)=0$, and since $\dot{\tau}_d\left(t\right)\in L_p\cap L_\infty$, $1\leqslant p<\infty$, then, owing to Lemma 2.19 from [15, p. 83], it is also true that $\lim\limits_{t\to\infty}\tilde{\tau}_d\left(t\right)=0$. Then, as $\sup\limits_{t\geqslant t_0}\sup\limits_{\left\|\tilde{\theta}\right\|\leqslant\tilde{\theta}_{\max}}\left\|\nabla_{\tilde{\theta}}F_2\left(t,\tilde{\theta}\right)\right\|<\infty$ (because of normalisation in (3.29)), following Theorem 3.2 from [17], it is concluded that the error $\tilde{\xi}\left(t\right)$ converges asymptotically to zero, which was to be proved.

□

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