Adv Syst Sci Appl 2024; **03**:31–42
Published online at https://ijassa.ipu.ru.

The Problem of Dynamic Regulation of Train Formation on a Marshalling Yard

Elena Musatova[1*], Alexander Lazarev[1], Alexander Osminin[2], Alexey Baushev[3]

[1] *V. A. Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia*
[2] *Volga State Transport University, Samara, Russia*
[3] *Emperor Alexander I St. Petersburg State Transport University, Saint Petersburg, Russia*

Abstract: The problem of dynamic freight trains formation at a railway station is considered. This problem is part of a two-step strategy to create a dynamic train formation plan. It is assumed that technical routes of all cars and the predicted arrival time of new groups of cars at the station are known. The problem is to determine the set of cars and the departure time of each train formed at this station, taking into account the availability of locomotives and restrictions on the length of trains. The goal is to minimise the total waiting time for cars at the station. Mathematical models for different versions of the problem are presented as an integer and a mixed integer linear programming problems. Results of computational experiments allow to estimate the influence of the amount of information about arriving trains to the station on the value of the objective function.

Keywords: train formation, integer linear programming, daily planning.

1. INTRODUCTION

The problem of developing a train formation plan (TFP) plays a key role in improving the quality of freight rail transportation. In different countries there are various approaches to the organization of freight rail transport. There are two main widely used operational strategies for freight rail transportation [1]. In the first case, the priority is the scheduled plan. This means that the departure of trains is carried out exactly according to a predetermined schedule, regardless of the actual number of cars that need to be transported on a certain day. Such plans require an accurate forecast of the number of cars. In the USA this type of plan is used in rail transportation. In the second case, priority is given to the tonnage plan, i.e. the departure of a train is carried out immediately after the station has the necessary number of cars to form a train in a given direction. Russia, China and many other countries use such freight transportation strategy.

In accordance with these two approaches, the formulation of optimization problems corresponding to them also differs. In general, the problem of calculating the train formation plan refers to the problems of network routing [2]. From the point of view of the mathematical model structure, two areas of research can be distinguished: the construction of linear and nonlinear models. At the same time, most modern research is devoted to linear models of the TFP (see, for example, [1, 3, 4]), that is, problems of integer linear programming and mixed integer linear programming. Examples of nonlinear models can be, for example, the problems considered in [5, 6].

*Corresponding author: nekolyap@mail.ru

As a rule, the problem of train formation is a large-scale problem of designing a flow network and routing with millions, and even billions of decision variables. The size and mathematical complexity of this problem do not allow us to solve it accurately with the help of any commercial software [7]. In this regard, most of the studies in the field of train formation and the distribution of car traffic are devoted to the development of fast heuristic algorithms (see, for example, [3,4]). In addition, the decomposition of the general problem into a number of successively solved problems is used. The following subproblems of railway planning are distinguished [8]:
1) the problem of combining cars into blocks [9]. Each block is characterized by a departure station and a destination station, while the cars included in it may have other stations of formation and destination;
2) the problem of constructing routes and train schedules [2];
3) the problem of assigning blocks to trains [10].

However, the direct use of foreign experience in solving the TFP problem for Russian Railways is impossible due to differences in the organization of the freight transportation process. The existing methodology for creating the TFP was developed to solve the problems of a planned economy and does not meet the conditions of the modern market economy. As a result, the actual planning is reduced to a daily adjustment of the TFP, based on the experience of process engineers and decision makers. The fast operation of planning algorithms is extremely important in this problem due to the fact that the moments and volumes of incoming trains to a marshalling yard differ from year to year and from day to day.

The formulation of the TFP problem in Russian Railways is determined by a specific procedure for constructing the formation plan. So there are two plans for the trains formation: a basic plan and an adaptive one. The basic TFP is built a year in advance and determines for each station destinations of trains formed at it. The problem of the adaptive formation plan is to determine *technical routes* for cars, i.e. sequences of stations through which the cars must go to destination stations, as well as determining which car is assigned to which train. Our work deals with the adaptive TFP problem. Building an adaptive formation plan is a high-dimensional NP-hard problem. Moreover, various local changes and inconsistencies that arise in practice do not allow solving the problem for the entire railway network at the same time. An approach is proposed that focuses on the possibility of implementing the developed algorithms for constructing an approximate solution of the TFP problem in real railway practice.

The adaptive TFP problem is divided into two subproblems: the problem of technical routing and the problem of dynamic regulation. This article is devoted to the problem of dynamic regulation, where it is necessary for a single station to determine a content (set of cars) and a time of departure of each train formed at this station. Our goal is to build mathematical models of dynamic formation of trains at the station and trace the influence of the length of the planning horizon on the solution to the problem.

The article is organised as follows. Section 2 includes a description of the problem, notation and basic terms. In Section 3 a mathematical statement of the problem is presented. In Section 4 we present results of computational experiment using solver CPLEX for problems of small dimensions. Results of solving high-dimensional problems are given in Section 5. In Section 5 we consider a special case of the problem that is easier to solve. Section 7 contains conclusions and suggestions.

2. PROBLEM DESCRIPTION

A train formation station (or marshalling yard) is considered. Let $K = \{1, \ldots, n\}$ be a set of destinations of trains departing from the station. This means that it is possible to form n types of trains with different destination stations. As a planning horizon, we choose the time interval $[t_0, t_{max}]$, the length of which depends on the available information about arrival of freight trains at the station. In practice, planning can be carried out a day ahead.

$T = \{t_1, \dots, t_m\}$ specifies moments of trains arrival to the station in the considered planning horizon. At the initial moment of time, n storages corresponding to n destinations already contain groups of cars, called blocks. In addition, there are r_0 locomotives at the station. These are the locomotives that delivered the blocks to the station before the initial point in time, but were not yet used for new train departures.

At time moments t_i, $i \in I = \{1, \dots, m\}$, trains consisting of blocks arrive at the station. Arrived blocks of cars, for which this station is not the final one, are distributed among the storages in accordance with their destinations, thereby increasing the number of cars in the corresponding potential trains. Each arrived locomotive can be used to form a new train.

Let G be a set of all blocks considered on the given planning horizon, including the blocks located in the storages. Each block $g \in G$ has the following characteristics:

- τ_g is the time moment of arrival at the station (if a block g arrived at the station before the initial moment, then we set $\tau_g = t_0 = 0$);
- k_g is the destination of g;
- l_g is the number of cars in the group.

Each block is an indivisible group of cars, i.e. all cars of the block must leave the station in one train. In Section 5 we will additionally consider another version of the statement where the division of blocks is allowed. Let G_k be the set of blocks of destination k and $L_{\min}$, $L_{\max}$ be respectively the minimum and the maximum numbers of cars included in one train formed at the given station, i.e. the departure of a train can only take place if the number of cars in it satisfies the upper and lower limits. At each time moment from set T it is necessary to make a decision whether to sent trains to some destinations. Also we have to determine which blocks will be included in these trains. We assume that the time required to form a train is a constant equal to β. The goal function is the total number of car-hours spent at the station for the period under consideration.

3. MATHEMATICAL STATEMENT

Let's number all the locomotives that are at the station at the zero moment of time plus all the locomotives that will arrive in the considered time period, from 1 to $r_0 + m$ in the order of their arrival at the station. Let $J = \{1, \dots, r_0 + m\}$ be the set of all locomotives. Denote by φ_j the moment of arrival of the locomotive j at the station. If locomotive j arrived at the station before the beginning of the time period under consideration, then we set $\varphi_j = 0$. Let's introduce the following decision variables:

- binary variable y_{jik}, $j \in J$, $i \in I$, $k \in K$, equals to 1 if locomotive j leaves the station as part of a new train at time $t_i + \beta$ and has a destination k, and 0 otherwise.
- binary variable x_{gij}, $g \in G$, $i \in I$, $j \in J$, is equal to 1 if block g leaves the station at time $t_i + \beta$ with locomotive j, and 0 otherwise.
- binary variable z_g, $g \in G$, takes the value 1 if block g does not leave the station in the given planning horizon, and 0 otherwise.

We list the main constraints. Each block leaves the station once, or does not leave it at all in the current planning period:

$$\sum_{i \in I} \sum_{j \in J} x_{gij} + z_g = 1 \quad \forall g \in G. \tag{3.1}$$

Each locomotive can be used at most once:

$$\sum_{i \in I} \sum_{k \in K} y_{jik} \leq 1 \quad \forall j \in J. \tag{3.2}$$

Each block leaves the station no earlier than it arrives there:

$$x_{gij} = 0 \ \ \forall g \in G, \ \ \forall j \in J, \ \ \forall i \in I : \ \tau_g > t_i \tag{3.3}$$

and no earlier than its locomotive arrives there:

$$x_{gij} = 0 \ \ \forall g \in G, \ \ \forall j \in J, \ \ \forall i \in I : \ \varphi_j > t_i. \tag{3.4}$$

Each locomotive can depart as part of a new train not earlier than its arrival at the station:

$$y_{jik} = 0 \ \ \forall j \in J, \ \ \forall i \in I : \ \varphi_j > t_i, \ \ \forall k \in K. \tag{3.5}$$

There is a restriction on the number of cars in the trains being formed:

$$L_{\min} y_{jik} \le \sum_{g \in G: k_g = k} l_g x_{gij} \le L_{\max} y_{jik} \ \ \forall k \in K, \ \ \forall i \in I, \ \ \forall j \in J. \tag{3.6}$$

Finally, we add the condition of binary variables:

$$x_{gij}, y_{jik}, z_g \in \{0, 1\} \quad \forall g \in G, \ \ \forall i \in I, \ \ \forall j \in J, \ \ \forall k \in K. \tag{3.7}$$

The problem of minimizing the total car-hours at the station in the given planning horizon is considered. The total number of car-hours for blocks that will be sent in the planning period is determined as

$$\sum_{g \in G} l_g \sum_{i \in I} \sum_{j \in J} (t_i + \beta) x_{gij} - \sum_{g \in G} \tau_g l_g \sum_{i \in I} \sum_{j \in J} x_{gij}.$$

For cars remaining at the station by the end of the period under consideration, the total waiting time is equal to

$$t_{\max} \sum_{g \in G} l_g z_g - \sum_{g \in G} \tau_g l_g z_g.$$

The sum of these quantities can be written as

$$\sum_{g \in G} l_g \sum_{i \in I} \sum_{j \in J} (t_i + \beta) x_{gij} + t_{\max} \sum_{g \in G} l_g z_g - \sum_{g \in G} \tau_g l_g (\sum_{i \in I} \sum_{j \in J} x_{gij} + z_g).$$

Due to the constraint (3.1), we get that the objective function of the problem can be written as

$$\sum_{g \in G} l_g \sum_{i \in I} \sum_{j \in J} (t_i + \beta) x_{gij} + t_{\max} \sum_{g \in G} l_g z_g - \sum_{g \in G} \tau_g l_g \to \min. \tag{3.8}$$

We denote problem (3.1)-(3.8) by (P_1). It can be seen that the number of variables is equal to $|J||I||K| + |J||I||G| + |G|$, where $|\cdot|$ is the number of elements in the corresponding set. The number of constraints is a value of the same order.

Theorem 3.1:
Problem (P_1) is strongly NP-hard.

Proof
We consider the multiple subset sum problem with identical capacities (MSSP-I) which is strongly NP-hard [11]. In this problem we are given a set of values $\overline{w}_j$, $j \in \{1, \dots, \overline{g}\}$, and we are looking for $\overline{m}$ subsets of this values with the maximal total sum provided that the sum in each subset does not exceed a capacity $\overline{c}$.

Problem MSSP-I is a special case of problem (P_1) where the number of train destinations $n = 1$, all $m = \overline{m}$ trains arrive at zero time moment, $|G| = \overline{g}$, $l_j = \overline{w}_j$, $j \in \{1, \dots, \overline{g}\}$,

$L_{\min} = 0$, $L_{\max} = \bar{c}$ and the storage already contains groups of cars. As all locomotives are available at zero time moment, in this special case we just have to redistribute the cars by m trains, taking into account the trains in the storage, to maximise the number of cars sent from the station (or minimise the number of cars remaining at the station). This reduction of the problem MSSP-I to the special case of problem (3.1)-(3.8) proves the theorem. □

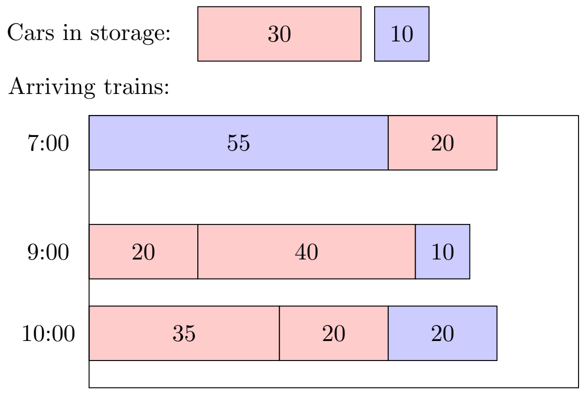

Fig. 3.1. Initial data

In practice, a train is formed as soon as the required number $l_{\min} >= L_{min}$ of cars has accumulated at the station for some destination and a locomotive is available for this departure. We will call this strategy the *current approach to planning* (CAP). Let's consider example 3.1 showing that CAP may be non-optimal.

Example 3.1:
Let trains of two destinations be formed at the station. Suppose that $l_{\min} = L_{min} = 65$, $L_{max} = 75$, $\beta = 0$, the planning horizon is 24 hours. During the period under review, 3 trains arrive at the station at 7:00, 9:00 and 10:00, respectively. Fig. 3.1 shows data on the size of blocks in each arriving train and on blocks located in the storages at the initial time moment. Blocks of different destinations have different colours.

Using CAP strategy we form a train at 7:00 (see fig. 3.2). But it turns out to be more profitable to wait for the arrival of the second train, then send two trains at once (see fig. 3.3). Indeed, in the case of CAP the first train contains fewer cars and block of 10 cars remains unshipped until the end of the planning period. The total car-hours at the station for CAP is equal to 3035, while the optimal solution gives us 3015 car-hours.

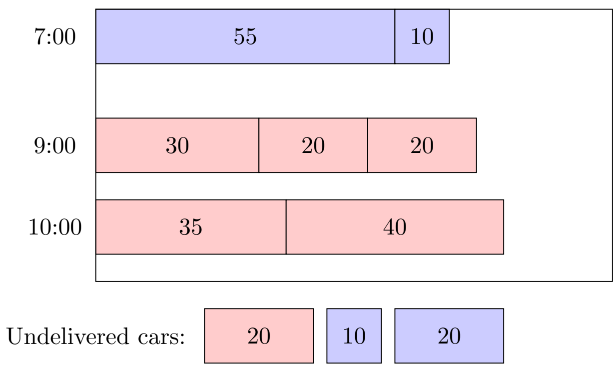

Fig. 3.2. CAP strategy

The purpose of this article is to evaluate the effect of implementing a planning method that uses more information about trains arriving at the station than is done in practice. In fact,

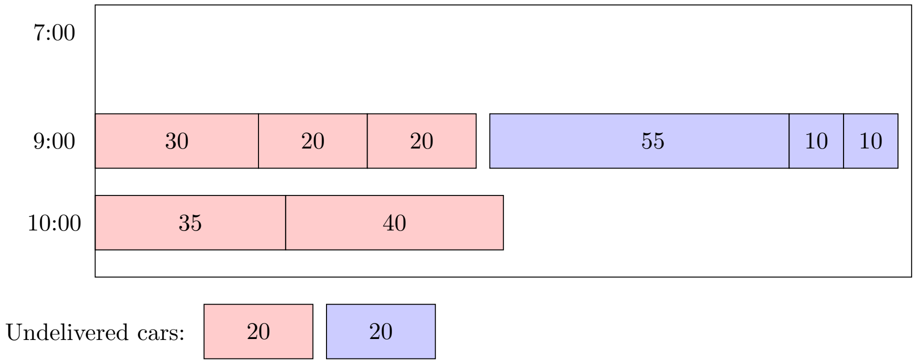

Fig. 3.3. Optimal strategy

if we use information about only one moment of train arrival, we apply CAP: if it is possible to form trains at this time moment, then a train with the maximum possible number of cars is formed. Applying this strategy at every moment of T, we get some feasible solution to the problem (P_1). The task arises of choosing a suitable planning horizon that would improve the goal function compared to the CAP, but would not require too much time to solve the problem.

4. SOLVING PROBLEMS OF SMALL DIMENSIONS

In this section we analyse and compare the behaviour of CAP and an exact method of solving problem (P_1) using small-dimensional test instances. In this experiment, we vary values of parameters m and n to determine how the gain from using the exact method changes.

During computational experiments, CAP was compared with the optimization solver IBM ILOG CPLEX 22.1.0.0. [12]. In CAP, to make a decision at each point in time about which train should be sent, we also used CPLEX to solve the problem (P_1) with $|T| = 1$.

The planning horizon is 24 hours, $l_{\min} = L_{min} = 61$, $L_{max} = 75$, $\beta = 0$. The datasets generated and analysed during the current study are available from the corresponding author on reasonable request. The running time of CAP and the CPLEX solver was limited to 10 min., because, in practice, the decision-making system should work quickly. In Table 4.1 the results of solving the randomly generated problems are given. The following notations are used in the table: "m" is the number of incoming trains, "n" is the number of train destinations generated at the given station, "CAP" is the value of the objective function obtained when applying CAP, "$CPLEX$" is the value of the objective function obtained as a result of CPLEX operation for 10 minutes, "GAP_{CPLEX}" is the gap between the best integer solution and the best bound in CPLEX after 10 minutes of work (by default, the gap equal to 0.01% is set as a stop criterion in CPLEX), "Dif" is the percentage of improvement in the value of the objective function when using CPLEX for problem (P_1). The calculations were performed on a personal computer (Intel Core i7-7700K, 4.2 GHz, 32.0 GB).

As can be seen from the table, using the exact solution method gives a win in all cases, with the exception of instances of very small dimensions. It is also worth noting that the winning percentage varies depending on number of train destinations generated at the given station. In problems with $m < 30$, the smaller the value of n, the greater the winning percentage. This can be explained by the fact that the fewer the number of train destinations, the more likely it is that new cars of given destination will arrive in the near future, and therefore it is more profitable to wait for these cars.

It can also be noted that starting from $m = 25$ CPLEX has not found a guaranteed exact solution in many cases. As the dimension of the problem increases, the GAP will increase. So when running the test example at $m = 140$, $n = 70$ after 10 minutes of CPLEX operation,

Table 4.1. Results of comparing CAP and the exact method on small-dimensional instances.

m	n	CAP	$CPLEX$	GAP_{CPLEX}	Dif
5	2	5903	5865	0,01%	0,66%
5	3	6140	6140	0,01%	0,00%
5	4	6288	6288	0,01%	0,00%
10	2	11146	10978	0,01%	1,50%
10	5	11584	11565	0,01%	0,16%
10	8	14421	14400	0,01%	0,15%
15	5	14514	14167	0,01%	2,39%
15	7	17614	17394	0,01%	1,25%
15	10	22869	22738	0,01%	0,57%
20	5	17861	17693	0,01%	0,94%
20	10	20341	20242	0,01%	0,49%
20	15	31260	31201	0,01%	0,19%
25	5	20882	19753	1,40%	5,41%
25	10	26423	25442	2,36%	3,71%
25	15	30402	29590	0,45%	2,67%
25	20	35303	34870	0,01%	1,23%
30	5	26646	26375	3,31%	1,02%
30	10	34299	33836	0,43%	1,35%
30	15	35486	35243	0,01%	0,69%
30	20	40439	40431	0,01%	0,02%
40	5	34350	34185	2,37%	0,48%
40	10	40586	40566	2,89%	0,05%
40	20	47524	47467	1,42%	0,12%
40	30	53888	53780	0,11%	0,20%

we got $GAP = 95.9\%$ and the value of the objective function which is significantly worse than when using CAP. That is why the next section discusses reducing the planning horizon, i.e. considering fewer arriving trains for solving high-dimensional problems.

5. SOLVING HIGH-DIMENSIONAL PROBLEMS

As we noted above, the exact solution method is not applicable for problems with a large number of arriving trains on the planning horizon. However, in practice, some marshalling yards of Russian railways (for example, Sverdlovsk) operate with up to 8000 cars, i.e. with more than 130 arriving freight trains per day and form new trains in almost 70 destinations. One of the approaches to planning at stations of this size is to use less information about incoming trains, that is, to reduce the planning horizon of the problem.

In our experiment, we examined randomly generated high-dimensional problems. To solve them, we first used CAP. Next, we used the following approach. Let ν be the number of train arrival moments that we take into account when planning. The problem (P_1) was solved for $T = \{t_1, \dots, t_\nu\}$. If in the received schedule some blocks and locomotives were sent, they were removed from the input data, and then the problem was solved with $T = \{t_{\nu+1}, \dots, t_{2\nu}\}$ and so on. Since the size of ν strongly affects the complexity and time of solving the problem, we have considered those options for ν that allow us to solve the general problem (P_1) in less than 10 minutes. As a result, the values $2, 3, 4, 5, 6$ were considered for ν. Note that $\nu = 1$ coincides with CAP. The test results are shown in the table 5.2. The cells of the table show the values of the objective function at different values of ν. The best values of the objective function for each example are highlighted in gray. Dif here shows the maximum improvement in the value of the objective function relative to the value obtained by CAP.

We can see from the table that it cannot be argued that increasing the planning horizon necessarily improves the final value of the target function per day. For example, the algorithm with $\nu = 5$ shows a worse result than with $\nu = 4$. The largest number of minimum values of the objective function is obtained at $\nu = 6$. However, in many other cases, the value of the function at $\nu = 6$ is worse than when using CAP. Note also that the spread of the values of the objective function when using 6 variants does not exceed 1%, except the instance with $m = 50$, $n = 10$.

Thus, it can be argued that in high-dimensional problems the exact method of solving the problem (P_1), which takes into account all trains arriving per day, is not applicable, since it requires too much time. Besides, considering additional 1, ...,5 trains arriving at the station does not give a gain in the value of the objective function by more than 1%, while an increase in the number of trains under consideration does not necessarily lead to a decrease in car-hours.

The question remains how far the solution obtained by CAP is from the optimal one. A partial answer to this question will be given in the next section, where a special case of the problem will be considered.

Table 5.2. Solving high-dimensional instances.

m	n	CAP	$\nu = 2$	$\nu = 3$	$\nu = 4$	$\nu = 5$	$\nu = 6$	Dif
50	10	52215	52478	50686	50865	51193	50060	4,13%
60	10	60947	61236	60932	60817	60928	61146	0,21%
70	10	63682	63787	63911	63609	63858	64071	0,11%
80	10	74505	74526	74265	74503	74442	73932	0,77%
90	10	79333	79063	78925	79434	79369	78792	0,68%
100	10	87419	87482	88023	87700	87684	88006	0,00%
110	10	97006	96985	97080	96575	96715	96572	0,45%
120	10	108222	108289	108266	108153	108344	108508	0,06%
130	10	121505	121650	121368	121266	121665	121521	0,20%
140	10	126714	127093	126214	126998	126683	126627	0,39%
50	20	51584	51609	51600	52009	51699	52041	0,00%
60	20	63448	63377	63351	63222	63548	63298	0,36%
70	20	74696	74687	74612	74013	74351	74656	0,91%
80	20	80400	80427	80372	80643	80381	80357	0,05%
90	20	85539	85464	85570	85704	85805	85591	0,09%
100	20	99386	99264	99420	99389	99356	99431	0,12%
110	20	110324	110354	110265	110235	110215	110418	0,10%
120	20	113795	113817	113510	113595	113677	113382	0,36%
130	20	120042	119951	120382	120157	120180	120192	0,08%
140	20	126933	127063	127258	127576	127277	126875	0,05%
50	25	55104	55087	54850	55138	55094	55096	0,46%
60	30	73218	73232	73170	73202	73188	73217	0,07%
70	35	83956	83907	83775	83895	83793	83788	0,22%
80	40	93114	93011	93026	92813	93295	92868	0,32%
90	45	105207	105079	105125	105038	105086	104977	0,22%
100	50	120615	120638	120565	120419	120436	120465	0,16%
110	55	130184	129782	129778	129948	129917	129558	0,48%
120	60	152286	152290	152265	152207	152250	152005	0,18%
130	65	153508	153507	153688	153961	153359	153190	0,21%
140	70	169040	169138	169043	169215	168975	168997	0,04%

6. SPECIAL CASE OF THE PROBLEM

We will also consider a simplified formulation of the problem, where blocks can be separated, i.e. cars of the same block can be sent by different trains. Obviously, the possibility of splitting blocks increases the bureaucratic difficulties of tracking cars, but reduces the load of stations and the number of car-hours.

Denote by α_{ik}, $i \in I \cup \{0\}$, $k \in K$, the number of cars that arrive at the station at time moment t_i for further shipment in the direction k, i.e.

$$\alpha_{ik} = \sum_{g \in G_k:\, \tau_g = t_i} l_g, \quad i \in I \cup \{0\}, \quad k \in K.$$

We introduce the following decision variables:

- integer variable $\tilde{y}_{ik}$, $i \in I, k \in K$, equals to the number of trains formed at time moment $t_i + \beta$ for destination k.
- integer variable $\tilde{x}_{ik}$, $i \in I$, $k \in K$, is equal to the number of cars departing from the station at time moment $t_i + \beta$ to destination k.

The constraints of the problem take the following form. At any given time moment, we cannot send more cars than are left at the station for a given destination:

$$\tilde{x}_{ik} \leq \sum_{j=0}^{i} \alpha_{jk} - \sum_{j=1}^{i-1} \tilde{x}_{jk} \;\; \forall i \in I, \;\; \forall k \in K. \tag{6.9}$$

We cannot use more locomotives than we have at the station:

$$\sum_{k \in K} \tilde{y}_{ik} \leq r_0 + i - \sum_{k \in K} \sum_{j=1}^{i-1} \tilde{y}_{jk} \;\; \forall i \in I. \tag{6.10}$$

The restriction on the number of cars in the trains can be written as follows:

$$L_{\min} \tilde{y}_{ik} \leq \tilde{x}_{ik} \leq L_{\max} \tilde{y}_{ik} \;\; \forall i \in I, \;\; \forall k \in K. \tag{6.11}$$

The integer condition is

$$\tilde{x}_{ik}, \tilde{y}_{ik} \in N_0 \;\; \forall i \in I, \;\; \forall k \in K, \tag{6.12}$$

where N_0 is the set of non-negative integers. The objective of the problem is to minimise the following function:

$$\sum_{i \in I} \sum_{k \in K} \tilde{x}_{ik}(t_i + \beta) + (\sum_{i=0}^{m} \sum_{k \in K} \alpha_{ik} - \sum_{i \in I} \sum_{k \in K} \tilde{x}_{ik}) t_{\max} \to \min. \tag{6.13}$$

We denote problem (6.9)-(6.13) by (P_2). Unlike problem (P_1) with binary variables, problem (P_2) has integer variables, but the number of variables is equal to $2|I||K|$, that is significantly fewer than in (P_1). It should be noted that the optimal value of the objective function of the problem (P_2) is a lower bound for the problem (P_1). In addition, the integer condition (6.12) can be replaced with a more lenient condition

$$\tilde{x}_{ik} \geq 0, \;\; \tilde{y}_{ik} \in N_0 \;\; \forall i \in I, \;\; \forall k \in K. \tag{6.14}$$

Indeed, denote by (P_3) the problem (6.9)-(6.11), (6.14), (6.13), that is problem (P_2) without requirements for the integers of variables $\tilde{x}_{ik}$, $i \in I$, $k \in K$. We can use (P_2) instead of(P_3) due to the following theorem:

Theorem 6.1:
In any optimal solution of problem (P_3) variables $\tilde{x}_{ik}$, $i \in I$, $k \in K$, take integer values.

Proof
Let's assume the opposite. Consider an optimal solution $\tilde{x}^*_{ik}, \tilde{y}^*_{ik}$, $i \in I, k \in K$. Choose an arbitrary $\bar{k} \in K$ for which there exists $i \in I$ such that $\tilde{x}^*_{i\bar{k}}$ is fractional. Denote by $\bar{I}(\bar{k}) \subset I$ the set of all such indexes i. If this set consist of one index, i.e. $\bar{I}(\bar{k}) = \{\bar{i}\}$, then we change $\tilde{x}^*_{\bar{i}\bar{k}}$ by $[\tilde{x}^*_{\bar{i}\bar{k}}] + 1$, where $[x]$ is the integer part of the number x. It is easy to see that constraints (6.9) and (6.11) are fulfilled for the new value of $\tilde{x}^*_{\bar{i}\bar{k}}$. At the same time, the value of the goal function is strictly better than the previous one.

Now suppose that $w = |\bar{I}| > 1$. Let's arrange all indexes of $|\bar{I}|$ in increasing order:

$$\bar{I} = \{i_1, i_2, \ldots, i_w\}, \;\; i_1 < i_2 < \ldots < i_w.$$

Let $\varepsilon_1, \ldots, \varepsilon_w$ be fractional parts of the corresponding optimal values of the variables $\tilde{x}^*_{i_1\bar{k}}, \ldots, \tilde{x}^*_{i_w\bar{k}}$.

If $\varepsilon_2 + \ldots + \varepsilon_m \leq 1 - \varepsilon_1$ we can change $\tilde{x}^*_{i_1\bar{k}}$ by $[\tilde{x}^*_{i_1\bar{k}}] + 1$ and $\tilde{x}^*_{i_j\bar{k}}$ — by $[\tilde{x}^*_{i_j\bar{k}}]$ for $j \in \{2, \ldots, j_w\}$ without violating constraints (6.9), (6.11) and with strict improvement of the value of the goal function. In the case of $\varepsilon_2 + \ldots + \varepsilon_m > 1 - \varepsilon_1$ we denote by $\bar{j}$ the maximal value of j for which $\varepsilon_2 + \ldots + \varepsilon_j \leq 1 - \varepsilon_1$. Then we can change $\tilde{x}^*_{i_1\bar{k}}$ by $[\tilde{x}^*_{i_1\bar{k}}] + 1$ and $\tilde{x}^*_{i_j\bar{k}}$ — by $[\tilde{x}^*_{i_j\bar{k}}]$ for $j \in \{2, \ldots, \bar{j}\}$ by improving the value of the goal function. Applying the same approach for the remaining indexes $i_{\bar{j}+1}, \ldots, i_w$ the required number of times we will get solution with integer values of $\tilde{x}^*_{i\bar{k}}$ for all $i \in I$. Repeating this procedure for all $k \in K$ for which there are fractional values of $\tilde{x}^*_{ik}$, we will get a feasible solution with integer $\tilde{x}^*_{ik}$, $i \in I$, $k \in K$, and better goal function value, which proves the theorem. □

It is worth noting that the CAP technique is also simplified in the case when it is possible to divide blocks. At each step, there is no need to solve a combinatorial optimization problem of the Knapsack type to make up the train with the largest number of cars. It is enough to choose the destination for which the maximum number of cars is located at the station.

We used the same test data for testing the model (P_3) as for (P_1), but it was assumed that the blocks could be divided. The results are presented in table 6.3. The same notation was used as in table 4.1, except for a new column called "$Gap_{1,2}$", in which we have a percentage decrease in the value of the objective function obtained using CPLEX for problem (P_3) compared to the best found solution to the problem (P_1) (see table 5.2). Values $Gap_{1,2}$ and GAP_{CPLEX} together allow us to estimate the error of solving the problem (P_1). So, for the considered test problems, it can be argued that the error of the solutions found for (P_1) is guaranteed not to be very large. At the same time, it can be seen from the table that the exact method of solving the problem (P_3) allows us to find a solution with acceptable accuracy for high-dimensional problems in 10 minutes.

7. CONCLUSION

In the paper, we proposed and investigated two models of dynamic train formation at a marshalling yard with divisible and indivisible train blocks, respectively. For the first model, computational experiments have shown that in problems of small dimension (a small number of incoming trains per day and a small number of train destinations), it is possible to find an exact solution to the problem of minimizing car-hours at the station per day in an acceptable time. This is impossible for high-dimensional problems. However at the same time, the trains formation without taking into account the trains coming later in such problems gives an acceptable result relative to the found lower bounds. Moreover, increasing information about

Table 6.3. Solving the special case.

m	n	CAP	$CPLEX$	GAP_{CPLEX}	Dif	$Gap_{1,2}$
50	10	50272	49046	0,01%	2,44%	2,03%
60	10	60490	59586	0,01%	1,49%	2,02%
70	10	63422	62210	0,01%	1,91%	2,20%
80	10	73466	72858	0,49%	0,83%	1,45%
90	10	78303	77259	0,29%	1,33%	1,95%
100	10	86858	85753	0,27%	1,27%	1,91%
110	10	96408	95057	0,47%	1,40%	1,57%
120	10	107656	106307	0,61%	1,25%	1,71%
130	10	120075	118958	0,55%	0,93%	1,90%
140	10	125843	124201	0,37%	1,30%	1,59%
50	20	50619	49591	0,01%	2,03%	3,86%
60	20	61268	60517	0,01%	1,23%	4,28%
70	20	73438	72211	0,01%	1,67%	2,44%
80	20	79151	78185	0,15%	1,22%	2,70%
90	20	83164	81306	0,32%	2,23%	4,86%
100	20	97922	96880	0,36%	1,06%	2,40%
110	20	107770	106505	0,53%	1,17%	3,37%
120	20	112236	110087	0,66%	1,91%	2,91%
130	20	118159	116098	0,71%	1,74%	3,21%
140	20	124708	122654	1,22%	1,65%	3,33%
50	25	52875	52308	0,01%	1,07%	4,63%
60	30	72030	71487	0,01%	0,75%	2,30%
70	35	83002	82538	0,01%	0,56%	1,48%
80	40	90429	89742	0,01%	0,76%	3,31%
90	45	102158	101367	0,01%	0,77%	3,44%
100	50	118017	117136	0,01%	0,75%	2,73%
110	55	125432	123572	0,01%	1,48%	4,62%
120	60	146170	143839	0,17%	1,59%	5,37%
130	65	148858	147924	0,06%	0,63%	3,44%
140	70	164399	163193	0,03%	0,73%	3,42%

incoming cars of 2-5 trains in advance does not give a tangible benefit. If we talk about the model for the case of divisible cars blocks, then this problem can be solved with high accuracy for any dimensions that have a practical application.

REFERENCES

1. Yaghini, M., Mirghavami, M., & Zare Andaryan, A. (2021) A mathematical formulation and an lp-based neighborhood search matheuristic solution method for the integrated train blocking and shipment path problem. *Networks*, **78**, 523–542.
2. Cordeau, J.-F., Toth, P., & Vigo, D. (1998) A survey of optimization models for train routing and scheduling. *Transportation science*, **32**, 380–404.
3. Yaghini, M., Momeni, M., & Sarmadi, M. (2014) Solving train formation problem using simulated annealing algorithm in a simplex framework. *Journal of Advanced Transportation*, **48**, 402–416.
4. Zhang, X., Kang, L., Liu, W., & Yao, Y. (2016) Solving train formation plan problem in freight transportation corridors. In *2016 International Conference on Engineering Science and Management*, 102–104, Atlantis Press.

5. Lin, B., Zhao, Y., Lin, R., & Liu, C. (2021) Integrating traffic routing optimization and train formation plan using simulated annealing algorithm. *Applied Mathematical Modelling*, **93**, 811–830.
6. Zhao, Y. & Lin, B. (2019) The multi-shipment train formation optimization problem along the ordered rail stations based on collection delay. *IEEE Access*, **7**, 75935–75948.
7. Ahuja, R. K., Jha, K. C., & Liu, J. (2007) Solving real-life railroad blocking problems. *Interfaces*, **37**, 404–419.
8. Xiao, J. & Lin, B. (2016) Comprehensive optimization of the one-block and two-block train formation plan. *Journal of Rail Transport Planning & Management*, **6**, 218–236.
9. Barnhart, C., Jin, H., & Vance, P. H. (2000) Railroad blocking: A network design application. *Operations research*, **48**, 603–614.
10. Xiao, J., Pachl, J., Lin, B., & Wang, J. (2018) Solving the block-to-train assignment problem using the heuristic approach based on the genetic algorithm and tabu search. *Transportation Research Part B: Methodological*, **108**, 148–171.
11. Kellerer, H., Pferschy, U., & Pisinger, D. (2004) *Knapsack Problems*. Springer-Verlag, Berlin, Heidelberg.
12. (2024, December 06), Ibm ilog cplex optimization studio. Available https://www.ibm.com/products/ilog-cplex-optimization-studio.

