Adv Syst Sci Appl 2025; **3**:25–39
Published online at https://ijassa.ipu.ru.

A Mathematical Model for Reducing Divorce Cases According to Social Indicators for Morocco: Optimal Control Approach

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Abstract: In this research, we discuss a mathematical model for a person's social status with a focus on marital status. In most societies, the marital status of men and women can be categorized as follows: Unmarried refers to a young man or woman of legal marriageable age who is neither married nor engaged. The second category is engagement, a stage that precedes marriage, often characterized by acquaintance and typically of short duration. The third status is marriage, which is customarily conducted according to the traditions and customs of each religion or culture. The next category is separation, referring to married couples who are experiencing difficulties and are living apart but are not legally divorced. Following that is divorce, which denotes a legal dissolution of marriage. The final status is widowed.
Our research aims to address and reduce the phenomenon of divorce, which has become widespread, with more than 50 percent of marriages reportedly lasting less than a year. We identify two control strategies that help minimize the number of divorced individuals while increasing the number of married individuals. The optimal control problem is formulated and analyzed by applying a discrete version of Pontryagin's Maximum Principle. Numerical simulations are performed to validate the theoretical results.

Keywords: The marital status model, optimal control, Pontryagin's maximum principle.

1. INTRODUCTION

Divorce is one of the social-psychological problems that has become increasingly prevalent in societies worldwide, especially in Arab communities. It results in many negative effects, including family disintegration, the spread of hostility, the emergence of children's adverse behaviors, and their reluctance toward marriage in the future. Additionally, divorce can lead to psychological disorders that may cause behavioral deviations later in life.

Marriage, as we can see now, confronts many challenges. Marriage's sacramental essence is increasingly changing, and divorce is being integrated into the legal system through legislation. The legal measures have now made it possible for an unhappy couple to seek a way out of the dead lock in the wedding lock. This has brought about dynamic changes in the social environment. Divorce, desertion, and separation are frequent occurrences in a modern family, whereas they were a rare phenomenon in traditional society.

Marriages are not always successful, as some end in discord. Divorce is the final indication of a marriage breakdown. It is the legal method of dissolving a marriage. The conjugal relationship is the central bond uniting the family in any society. When this bond is broken, the family is automatically broken. The existence of family groups as a functioning unit depends upon the continuation of many personal relationships. When this bond breaks down, the family organisation breaks down.

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The practice of divorce has existed for centuries. Divorce is described as a court-ordered legal dissolution of a marriage. Divorce can also be defined as the legal separation of two people impacted by a court's judgment or decree, either completely terminating the marriage relationship or suspending its consequences as far as the parties' cohabitation is concerned. Thus, from the two definitions, it may be established that a divorce can be said to be validly granted when it is made by a judgement or decree of a court with the aim of dissolving the marriage completely or suspending its effects, where these effects are the rights and obligations acquired by and imposed on the couple in their cohabitation as a result of the marriage contract. Divorce is the only recognised procedure for annulling a marriage contract.

People who are in a relationship before marriage are one of the compartments we discussed in our mathematical model, a man and a woman who are in a relationship can be considered two mature people in the dating or courtship stage. In some countries and cultures, two people in a relationship mean that they practice family life as a married couple and can have children as long as the marriage contract exists between them. In some other countries, the union of two people or their sexual relations outside of marriage is considered a crime punishable by law. As for the model we are discussing, we consider only two related people who are engaged or in a state of acquaintance before marriage, according to Moroccan law, which criminalizes relationships outside the institution of marriage. The High Commission for Planning (HCP) [11], a national statistical institution, has included Social Indicators for Morocco: 2023 Edition, exciting and indicative digital data on the development of many aspects of personal status issues for Moroccans, especially regarding issues of marriage and divorce. Annually, HCP addresses the state of the population structure (demography) in detail. Since the Family Code came into effect, the number has increased from 236.574 during the year 2004 to a total of 275.978 marriages concluded in 2019. However, there was a significant decline in 2020 until it reached 194.318, and after that there was a slight increase until it reached 251.847 in 2022.

50% of marriages in Morocco end in divorce. The Morocco divorce rate increased from 45.01% in 2017 to 48.83% in 2018, then 50.34% in 2019 to 55.17% in 2020 during the COVID-19 pandemic. The year 2021 marked a slight drop in this rate, which fell to 51.18% and back again in 2022 to 74.95%. In numbers, divorce cases registered in Moroccan courts were 107,136 in 2017, followed by an increase of 7.75% the next year with 115,436 cases. The year 2019 registered 129,417 cases, which dropped to 105,471 in 2020.

In our mathematical system, we discuss a mathematical model of family dynamics based on marital status. People in society are considered to be classified into six classes: virgin S, engagement C, married M, separated P, divorced D, widowed W. Many research themes close to the subject of the individual's status in society have been discussed, for example, law, religion, and statistics [1, 7–9, 18, 21, 24]. M.Lhous in [12] has talked about a mathematical model of monogamous marriage. And in [13], M.Lhous, by using a multi-region discrete-time model, developed a mathematical model of the family status in various areas, as well as the outcome of region interconnectivity on marital status. A.Sakkoum [22] has talked about a discrete mathematical model of Islamic polygamy.

Optimal control theory is widely utilized as accessible and successful tool for managers to create and simulate control schemes, see [2, 23]. We use an optimal control strategy to reduce divorces and increase the number of spouses at a low cost. The first two controls (u_1, u_2) are introduced to decrease the number of divorces using reconciliation, advice, seminars, and classes within the framework of the accompanying relationship to solve the problems for which the spouses want to divorce, explain to adults the positive effects of marriage on the psychological and social equilibrium of the individual and society, and provide clarifications of the psychological problems that divorce entails for children and all members of both families. The second control (u_3, u_4) lies in issuing laws that make divorce not easy. Is chosen for the individuals who started the divorce process. This control is regarded as a time-consuming and expensive legal processes. There are several approaches for computing the

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Fig. 1.1. Divorce rate in Morocco between 2017 and 2022 [11].

best control for a given mathematical model. Pontryagin's maximum principle [19] calculates The optimal control for a system under a specified constraint.

The paper is structured as follows. In Section 2, we explore equations of social status. In Section 3, we create Positivity of Solutions and Invariant Area. In Section 4, free equilibrium and basic reproduction number analysis are conducted, proving the local stability of the free equilibrium point and the global stability. In Section 5, we present some findings on the existence of optimum control and use Pontryagin's maximal principle to analyse control strategies and deduce the essential conditions for optimal control. The numerical simulations are reported in Section 6. Finally, we give conclusions in Section 7.

2. MATHEMATICAL MODEL

In this social system, we create a mathematical model to minimise divorces and separations. The notations and their descriptions are shown below. S(t): number of unmarried persons reach marriageable age. $C(t)$: number of people who are in a relationship before marriage. M(t): number of people who are married. P(t): the number of persons who are separated but not divorced. D(t): the number of divorced individuals. W(t): number of people who are widowed. Λ is the percentage of people who are recruited. who remain unmarried when they reach marriageable age. This individual got into a couple without marriage at a rate of α_1. α_2 is the rate at which individuals in a couple become single. This individual got married at a rate of β_1 and engaged at a rate of β_2. r_1 is the rate of married individuals who become widowed. r_2 is the rate of widows who become married. r_3 represents the proportion of widows in relationships. The unhappy couple moved past their disagreements and remarried at a rate of λ_1. λ_2 rate of broken marriage individuals but not divorced. δ_1 rate of married people getting divorced. δ_2 rate of divorced individuals who become married. ρ_1 is the rate of failed marriages ending in permanent divorce. The rate at which divorced individuals enter the engaged population is ρ_2, while the natural death rate is μ. Furthermore, we assume that members blend in a uniform manner, meaning they have the same degree of interaction, and that factors such as sex, colour, and social standing have no bearing on the likelihood of divorce. Table 1 depicts and describes the model's state variables, while Table 2 provides

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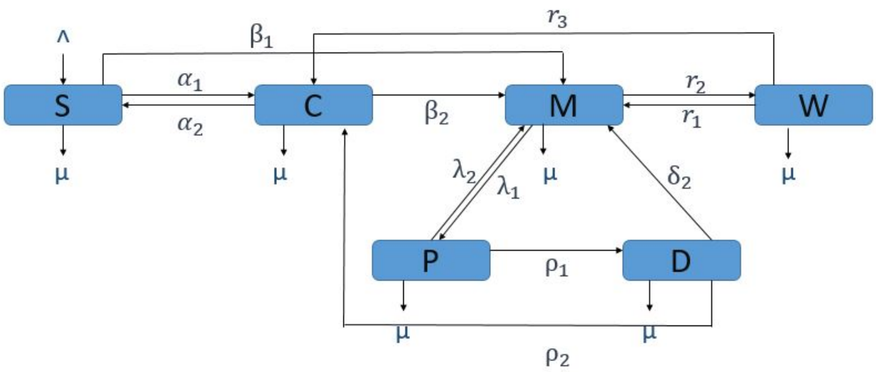

Fig. 2.2. The schematic illustration of the SCMPDW model.

a description of the model's parameters. Figure 1 displays the model's compartmental flow diagram.

In light of the aforementioned presumptions, the differential equation system that governs the model is as follows:

$$\frac{dS}{dt} = \Lambda - (\alpha_1 + \beta_1 + \mu)S + \alpha_2 C \tag{2.1}$$

$$\frac{dC}{dt} = \alpha_1 S - (\alpha_2 + \beta_2 + \mu)C + r_3 W + \rho_2 D \tag{2.2}$$

$$\frac{dM}{dt} = \beta_1 S + \beta_2 C - (r_1 + \mu + \lambda_2 D)M + \delta_2 D + \lambda_1 P + r_2 W \tag{2.3}$$

$$\frac{dP}{dt} = \lambda_2 MD - (\lambda_1 + \rho_1 + \mu)P \tag{2.4}$$

$$\frac{dD}{dt} = \rho_1 P - (\delta_2 + \rho_2 + \mu)D \tag{2.5}$$

$$\frac{dW}{dt} = r_1 M - (r_2 + r_3 + \mu)W \tag{2.6}$$

with positive initial conditions given by

$$S_0 = S(0) \geqslant 0, C_0 = C(0) \geqslant 0, M_0 = M(0) \geqslant 0, P_0 = P(0) \geqslant 0, \\ D_0 = D(0) \geqslant 0 \ \ and \ \ W_0 = W(0) \geqslant 0. \tag{2.7}$$

All parameters of the system are considered to be positive at all times t.

Variable State	Description
$S(t)$	Number of Single adults at time t.
$C(t)$	Number of engaged without marriage adults at time t.
$M(t)$	Number of married adults at time t.
$P(t)$	Number of broken adults at time t.
$D(t)$	Number of divorced individuals at time t.
$W(t)$	Number of widowed at time t.
$N(t)$	Total number of population at time t.

Table 2.1. Model's (2.1)-(2.6) variable states.

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Parameter	Description
Λ	Recruitment percentage of adults at the age of single.
α_1	Average percentage of single adults who got engaged.
α_2	Average percentage of engaged adults who got single.
β_1	Average percentage of single adults who got married.
β_2	Average percentage of engaged adults who got married.
r_1	Average percentage of married adults who become widowed.
r_2	Average percentage of widowed who got married.
r_3	Average percentage of widowed who got engaged.
λ_1	percentage of Broken adults who renew their previous marriage.
λ_2	The contact percentage of divorced individuals with married individuals.
δ_2	percentage of divorced individuals who got married.
ρ_1	percentage of broken adults who got divorced.
ρ_2	percentage of divorced individuals who got engaged.
μ	Natural death percentage of adults.

Table 2.2. Description of model parameters (2.1)-(2.6).

2.1. Positivity of Solutions

A marriage model system must demonstrate that every state variable is always non negative in order to be considered socially realistic.

Theorem 2.1:
For all solutions of system (2.1)-(2.6) with intial conditions (2.7) the functions $S(t)$, $C(t)$, $M(t)$, $P(t)$, $D(t)$, $W(t)$ are nonnegative for every $t \geq 0$.

Proof
We start our investigation of system (2.1)-(2.6) by looking at a few basic model characteristics. Firstly, we will show that any solution of (2.1)-(2.6), initiated from a nonnegative initial condition in $\mathbb{R}^6_+$, will remain nonnegative. Specifically, we can see that

$$\begin{aligned}
\frac{dS}{dt}|_{S=0} &= \Lambda + \alpha_2 C > 0 \text{for all} C \geqslant 0 \\
\frac{dC}{dt}|_{C=0} &= \alpha_1 S + r_3 W + \rho_2 D \geqslant 0 \;\; for \;\; all \;\; S, W, D \geqslant 0 \\
\frac{dM}{dt}|_{M=0} &= \beta_1 S + \beta_2 C + \delta_2 D + \lambda_1 P + r_2 W \geqslant 0 \qquad for \;\; all \;\; S, C, D, P, W \geqslant 0 \\
\frac{dP}{dt}|_{P=0} &= \lambda_2 M D \geqslant 0 \;\; for \;\; all \;\; M, D \geqslant 0 \\
\frac{dD}{dt}|_{D=0} &= \rho_1 P \geqslant 0 \;\; for \;\; all \;\; P \geqslant 0 \\
\frac{dW}{dt}|_{W=0} &= r_1 M \geqslant 0 \;\; for \;\; all \;\; M \geqslant 0
\end{aligned}$$

This proves that $\mathbb{R}^6_+$ is positively invariant with respect to system (2.1)-(2.6), meaning that any solution of (2.1)-(2.6) will remain in $\mathbb{R}^6_+$ for all times. Which proves that any solution of (2.1)-(2.6) is nonnegative. □

2.2. Invariant Region

Theorem 2.2:
For all $t > 0$ solutions of system (2.1)-(2.6) with intial conditions (2.7) are contained in the

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region $\Omega \subset R^6_+$, characterized by

$$\Omega = \{(S(t), C(t), M(t), P(t), D(t), M(t)) \in R^6_+ : N(t) \leq \frac{\Lambda}{\mu}\}. \tag{2.8}$$

Proof
The sum of all equations in the model system yields

$$\frac{dN(t)}{dt} = \frac{dS(t)}{dt} + \frac{dC(t)}{dt} + \frac{dM(t)}{dt} + \frac{dP(t)}{dt} + \frac{dD(t)}{dt} + \frac{dW(t)}{dt}.$$

The change in the overall population is defined as

$$\frac{dN}{dt} = \Lambda - \mu N.$$

The answer of inequation is

$$N(t) = \frac{\Lambda}{\mu} - (\frac{\Lambda}{\mu} - N_0)e^{-\mu t},$$

where $N_0 = N(0)$.
Using the theorem of Birkhoff-Rota [4], we observe that, if $N_0 < \frac{\Lambda}{\mu}$, then $N \longrightarrow \frac{\Lambda}{\mu}$ asymptotically as $t \longrightarrow \infty$ in Ω and the total population size $N \longrightarrow \frac{\Lambda}{\mu}$, which means that $0 \leqslant N \leqslant \frac{\Lambda}{\mu}$ and the solutions are bounded. As a result, all of the model's possible solutions converge in the region Ω [10]. □

3. ANALYSIS OF FREE EQUILIBRIUM, BASIC REPRODUCTION NUMBER R_0

Fundamentally, the divorce–free equilibrium can be obtained by taking the equation of the system with $S = C = M = P = D = M = 0$ into consideration, we arrive at:

$$M_0 = \frac{\Lambda[\beta_1 n_2 + \beta_2 \alpha_1] n_6}{[n_1 n_2 - \alpha_1 \alpha_2][\alpha_1(r_1 + \mu) n_6 + \beta_1 r_1 r_3 - \alpha_1 r_1 r_2 - r_1 r_3]}$$

$$S_0 = \frac{\Lambda}{n_1} + \frac{\alpha_2}{n_1} C_0$$

$$C_0 = \frac{1}{\alpha_1 \beta_2 + \beta_1 n_2}[\alpha_1 r_1 + \alpha_1 \mu + \frac{\beta_1 r_1 r_3}{n_6} - \frac{\alpha_1 r_1 r_3}{n_6}] M_0,$$

$$W_0 = \frac{r_1}{n_6} M_0.$$

where, $n_1 = \alpha_1 + \beta_1 + \mu$, $n_2 = \alpha_2 + \beta_2 + \mu$ and $n_6 = r_2 + r_3 + \mu$.
Therefore, system (2.1)-(2.6) has a unique free equilibrium state:

$$E_0 = (S_0, C_0, M_0, 0, 0, W_0) \tag{3.9}$$

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3.1. Basic Reproduction Number R_0

To calculate the fundamental R_0, we employed the next generation matrix approach of (2.1)-(2.6), where F is the matrix of fresh divorce or separation terms and V is the matrix of transition terms. The matrices F and V are calculated using the coefficients of P and D in the system's fourth and fifth equations. Equations of the model are re-written, beginning with newly infected classes:

$$\frac{dP}{dt} = \lambda_2 MD - (\lambda_1 + \rho_1 + \mu)P \tag{3.10}$$

$$\frac{dD}{dt} = \rho_1 P - (\delta_2 + \rho_2 + \mu)D \tag{3.11}$$

$$F=\begin{bmatrix} 0 & \lambda_2 M_0 \\ 0 & 0 \end{bmatrix} \text{ and } V=\begin{bmatrix} (\lambda_1+\rho_1+\mu) & 0 \\ -\rho_1 & (\delta_2+\rho_2+\mu) \end{bmatrix}$$

R_0 (spectral radius of FV^{-1}) must be taken into consideration. Thus, the reproduction number R_0 can be obtained as follows:

$$R_0 = \rho(FV^{-1}) = \frac{\lambda_2 M_0}{(\lambda_1+\rho_1+\mu)(\delta_2+\rho_2+\mu)} \tag{3.12}$$

3.2. Local Stability of Free Equilibrium Point

Theorem 3.1:
E_0 is locally asymptotically stable if $R_0 < 1$. Whereas, E_0 is unstable if $R_0 > 1$.

Proof
Consider the model system (2.1)-(2.6), at the equilibrium point (E_0), the Jacobian becomes:

$$\begin{pmatrix} X_1 & \alpha_2 & 0 & 0 & 0 & 0 \\ \alpha_1 & X_2 & 0 & 0 & \rho_2 & r_3 \\ \beta_1 & \beta_2 & X_3 & \lambda_1 & \lambda_2 M_0 + \delta_2 & r_2 \\ 0 & 0 & 0 & X_4 & \lambda_2 M_0 & 0 \\ 0 & 0 & 0 & \rho_1 & X_5 & 0 \\ 0 & 0 & r_1 & 0 & 0 & X_6 \end{pmatrix} \tag{3.13}$$

with:

$$X_1 = -(\alpha_1+\beta_1+\mu),\ X_2 = -(\alpha_2+\beta_2+\mu),\ X_3 = -(\alpha_1+\mu),\ X_4 = -(\lambda_1+\rho_1+\mu),$$
$$X_5 = -(\delta_2+\rho_2+\mu) \quad and \quad X_6 = -(r_3+r_2+\mu).$$

Three eigenvalues of (13) are $n_1 = -(\alpha_1+\beta_1+\mu)$, $n_2 = -(\alpha_2+\beta_2+\mu)$, $n_3 = -(\alpha_1+\mu)$, and $n_6 = -(r_3+r_2+\mu)$, while the remaining two eigenvalues are obtained from the 2 by 2 matrix

$$A = \begin{pmatrix} -(\lambda_1+\rho_1+\mu) & \lambda_2 M_0 \\ \rho_1 & -(\delta_2+\rho_2+\mu) \end{pmatrix} \tag{3.14}$$

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If the Routh-Hurwitz [15] condition is met, the eigenvalues of matrix A will be real and negative. Applying the Routh-Hurwitz condition:

- $\star$ $tr(A) < 0$,
- $\star$ $\det(A) > 0$.

$$\begin{aligned} tr(A) &= -(\lambda_1 + \rho_1 + \mu) - (\delta_2 + \rho_2 + \mu) \\ &= -\big((\lambda_1 + \rho_1 + \mu) + (\delta_2 + \rho_2 + \mu)\big). \end{aligned}$$

Therefore, we have $tr(A) < 0$.

$$\begin{aligned} \det(A) &= (\lambda_1 + \rho_1 + \mu)(\delta_2 + \rho_2 + \mu) - \rho_1 \lambda_2 M_0 \\ &= (\lambda_1 + \rho_1 + \mu)(\delta_2 + \rho_2 + \mu)[1 - \frac{\rho_1 \lambda_2 M_0}{(\lambda_1 + \rho_1 + \mu)(\delta_2 + \rho_2 + \mu)}] \\ &= (\lambda_1 + \rho_1 + \mu)(\delta_2 + \rho_2 + \mu)[1 - \rho_1 R_0,] \end{aligned}$$

thus, $\det(A) > 0$ if, $R_0 < \dfrac{1}{\rho_1}$.

Using theorem 2 of (2.1)-(2.6), we deduce that the free equilibrium point is locally asymptotically stable. □

3.3. Global Stability of Free Equilibrium Point

We create a Lyapunov function [14] to demonstrate the free equilibrium point's global stability. We define the following:

$$L = \frac{P}{\lambda_1 + \rho_1 + \mu} + D. \tag{3.15}$$

We will show that $\dfrac{dL}{dt} \leqslant 0$ for all $t \geqslant 0$. We have:

$$\begin{aligned} \frac{dL}{dt} &= \frac{d}{dt}\frac{P}{\lambda_1 + \rho_1 + \mu} + \frac{dD}{dt} \\ &= \frac{\lambda_2 MD - (\lambda_1 + \rho_1 + \mu)P}{\lambda_1 + \rho_1 + \mu} + \big(\rho_1 P - (\delta_2 + \rho_2 + \mu)D\big) \\ &= \big(\frac{\lambda_2 M}{\lambda_1 + \rho_1 + \mu} - (\delta_2 + \rho_2 + \mu)\big)D - (1 - \rho_1)P \\ &= (\delta_2 + \rho_2 + \mu)[\frac{\lambda_2 M}{(\lambda_1 + \rho_1 + \mu)(\delta_2 + \rho_2 + \mu)} - 1]D - (1 - \rho_1)P \\ &= (\delta_2 + \rho_2 + \mu)[R_0 - 1]D - (1 - \rho_1)P. \end{aligned}$$

Thus, if $R_0 < 1$, $\dfrac{dL}{dt}$ is negative. The biggest compact invariant set in Ω is the singleton set E_0. According to LaSalle's invariance principle [3], E_0 is asymptotically stable in Ω.

4. THE OPTIMAL CONTROL PROBLEM

As well, we recognize Morocco, like other countries, suffers until this moment from the phenomenon of divorce. Divorce is found in all age groups, young and old. We suggest a

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strategy to control and decrease the rate of divorces, first u_1 and u_2 to advise and find ways of reconciliation before falling into divorce. The second control u_3 and u_4 lies in issuing laws that make divorce not easy. The controlled model corresponding to (2.1)-(2.6) has the form:

$$\frac{dS}{dt} = \Lambda - (\alpha_1 + \beta_1 + \mu)S + \alpha_2 C \tag{4.16}$$

$$\frac{dC}{dt} = \alpha_1 S - (\alpha_2 + (1+u_1)\beta_2 + \mu)C + r_3 W + \rho_2 D \tag{4.17}$$

$$\begin{aligned}\frac{dM}{dt} &= \beta_1 S + (1+u_1)\beta_2 C - (r_1 + \mu + \lambda_2 D)M \\ &+ (1+u_4)\delta_2 D + (1+u_3)\lambda_1 P + r_2 W\end{aligned} \tag{4.18}$$

$$\frac{dP}{dt} = \lambda_2 MD - ((1+u_3)\lambda_1 + (1-u_2)\rho_1 + \mu)P \tag{4.19}$$

$$\frac{dD}{dt} = (1-u_2)\rho_1 P - ((1+u_4)\delta_2 + \rho_2 + \mu)D \tag{4.20}$$

$$\frac{dW}{dt} = r_1 M - (r_2 + r_3 + \mu)W, \tag{4.21}$$

and the challenge is to minimize the objective functional

$$\begin{aligned}J(u_1,u_2,u_3,u_4) &= AD(T) + BP(T) - CM(T) + \\ &\int_0^T (AD(t) + BP(t) - CM(t) + \frac{I}{2}u_1^2 + \frac{J}{2}u_2^2 + \frac{K}{2}u_3^2 + \frac{L}{2}u_4^2)dt\end{aligned} \tag{4.22}$$

where u_1, u_2, u_3, u_4 are measurable functions satisfying

$$0 \leqslant u_{i,\min} \leqslant u_i \leqslant u_{i,\max} \leqslant 1, \quad 1 \leqslant i \leqslant 4$$

$$J(u_1^*, u_2^*, u_3^*, u_4^*) = \min\{J(u_1,u_2,u_3,u_4)\}. \tag{4.23}$$

4.1. The Optimal Control: Existence

We initial demonstrate the existence of the system's solutions, followed by the presence of optimal control.

Theorem 4.1:
The optimal control problem given by (4.16)-(4.21), (4.23) has a solution $(u_1^*, u_2^*, u_3^*, u_4^*)$.

Proof
We shall employ Fleming and Rishel [6] to demonstrate the existence of the optimal control.

- As a result, the collection of controls and their respective state variables is not zero. We'll utilize a simplified version of the existence result.
- The control space $\mathscr{U}$, is convex and closed by definition.
- $J(u_1, u_2, u_3, u_4)$ is convex in $\mathscr{U}$.
- The main sides of system equations are continuous, limited higher by a sum of bounded control and state, and can be represented as a linear function of u_1, u_2, u_3, and u_4, with coefficients varying with time and state.
- The integrand in the objective functional,

$$A_1 D(t) + A_2 P(t) - A_3 M(t) + \frac{I}{2}u_1^2 + \frac{J}{2}u_2^2 + \frac{K}{2}u_3^2 + \frac{L}{2}u_4^2$$

  is clearly convex on $\mathscr{U}$.

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- It rests to show that there exist constants $\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5 > 0$ and ζ such that

$$A_1D(t) + A_2P(t) - A_3M(t) + \frac{I}{2}u_1^2 + \frac{J}{2}u_2^2 + \frac{K}{2}u_3^2 + \frac{L}{2}u_4^2 \geqslant \zeta_1 + \zeta_2|u_1|^{\zeta} + \zeta_3|u_2|^{\zeta} + \zeta_4|u_3|^{\zeta} + \zeta_5|u_4|^{\zeta},$$

where $\zeta_1 = \inf_{t\in[0,T]}\{A_1D(t) + A_2P(t) - A_3M(t)\}$, $\zeta_2 = I$, $\zeta_3 = J$, $\zeta_4 = K$ and $\zeta_5 = L$ and $\zeta = 2$. The state variables are being bounded $(N(t) \leqslant \frac{\Lambda}{\mu}\ \ and\ \ N(t) = S(t) + C(t) + M(t) + P(t) + D(t) + M(t),\ \ \forall t \geqslant 0)$, then, from Fleming and Rishel [6], we conclude that there exists an optimal control of (4.23).

4.2. The Optimal Control Characterization

At the same time by using Pontryagin's maximum principle [19], we derive the necessary conditions for our optimal control. For this purpose we define the Hamiltonian as

$$H = A_1D(t) + A_2P(t) - A_3M(t)) + \frac{I}{2}u_1^2 + \frac{J}{2}u_2^2 + \frac{K}{2}u_3^2 + \frac{L}{2}u_4^2 + \sum_{i=1}^{5}\varsigma_i f_i(S, C, M, P, D, W).$$

Where f_i denotes the right side of the difference equation for the i^{th} state variable at the time step. □

Theorem 4.2:
Considering optimal controls $u_1^*, u_2^*, u_3^*, u_4^*$ and their respective solutions S, C, M, P, D, and W in the appropriate state system (2.1)-(2.6), there are adjoint variables. ς_1, ς_2, ς_3, ς_4, ς_5, and ς_6 meet

$$\begin{aligned}
\varsigma_1^{'} &= \varsigma_1(\alpha_1 + \beta_1 + \mu) - \varsigma_2\alpha_1 - \varsigma_3\beta_1 \\
\varsigma_2^{'} &= -\varsigma_1\alpha_2 + \varsigma_2(\alpha_2 + (1 + u_1)\beta_2 + \mu) - \varsigma_3(1 + u_1)\beta_2 \\
\varsigma_3^{'} &= A_3 + \varsigma_3[(r_1 + \mu + \lambda_2D] - \varsigma_4\lambda_2D - \varsigma_6r_1 \\
\varsigma_4^{'} &= -A_2 - \varsigma_3(1 + u_3)\lambda_1 + \varsigma_4[(1 + u_3)\lambda_1 + (1 - u_2)\rho_1 + \mu] - \varsigma_5(1 - u_2)\rho_1 \\
\varsigma_5^{'} &= -A - \varsigma_2\rho_2 + \varsigma_3[\lambda_2M - (1 + u_4)\delta_2] - \varsigma_4\lambda_2M + \varsigma_5[(1 + u_4)\delta_2 + \rho_2 + \mu]\} \\
\varsigma_6^{'} &= -\varsigma_2r_3 - \varsigma_3r_2 + \varsigma_6(r_2 + r_3 + \mu).
\end{aligned}$$

With the transversality conditions at time T_f, $\varsigma_1(T_f) = 0$, $\varsigma_2(T_f) = 0$, $\varsigma_3(T_f) = A_3$, $\varsigma_4(T_f) = -A_2$, $\varsigma_5(T_f) = -A_1$, $\varsigma_6(T_f) = 0$. Furthermore, for $t \in [0, T_f]$, the optimal controls u_1^*, u_2^* and v_1^* are given by

$$\begin{aligned}
u_1^* &= \min[1, \max(0, \frac{(\varsigma_2 - \varsigma_3)\beta_2C}{I})], \\
u_2^* &= \min[1, \max(0, \frac{(\varsigma_5 - \varsigma_4)\rho_1P}{J})], \\
u_3^* &= \min[1, \max(0, \frac{(\varsigma_4 - \varsigma_3)\lambda_1P}{K})], \\
u_4^* &= \min[1, \max(0, \frac{(\varsigma_5 - \varsigma_3)\delta_2D}{L})].
\end{aligned}$$

Proof
We derive the necessary conditions for our optimal control. For this purpose, the Hamiltonian is described as below.

$$H = A_1D(t) + A_2P(t) - A_3M(t) + \frac{I}{2}u_1^2 + \frac{J}{2}u_2^2 + \frac{K}{2}u_3^2 + \frac{L}{2}u_4^2 + \sum_{i=1}^{5} \varsigma_i f_i(S, C, M, P, D, W).$$

where

$$\begin{aligned}
f_1(S,C,M,P,D,W) &= \Lambda - (\alpha_1+\beta_1+\mu)S + \alpha_2 C,\\
f_2(S,C,M,P,D,W) &= \alpha_1 S - (\alpha_2 + (1+u_1)\beta_2 + \mu)C + r_3W + \rho_2 D,\\
f_3(S,C,M,P,D,W) &= \beta_1 S + (1+u_1)\beta_2 C - (r_1 + \mu + \lambda_2 D)M\\
&+ (1+u_4)\delta_2 D + (1+u_3)\lambda_1 P + r_2 W,\\
f_4(S,C,M,P,D,W) &= \lambda_2 MD - ((1+u_3)\lambda_1 + (1-u_2)\rho_1 + \mu)P,\\
f_5(S,C,M,P,D,W) &= (1-u_2)\rho_1 P - ((1+u_4)\delta_2 + \rho_2 + \mu)D,\\
f_6(S,C,M,P,D,W) &= r_1 M - (r_2 + r_3 + \mu)W.
\end{aligned}$$

Using Pontryagin's maximum principle [19, 20], the adjoint equations and transversality requirements for $t \in [0, T_f]$ can be derived as follows:

$$\begin{aligned}
\varsigma_1' = -\frac{\partial H}{\partial S} &= -\{-\varsigma_1(\alpha_1+\beta_1+\mu) + \varsigma_2\alpha_1 + \varsigma_3\beta_1\}\\
&= \varsigma_1(\alpha_1+\beta_1+\mu) - \varsigma_2\alpha_1 - \varsigma_3\beta_1\\
\varsigma_2' = -\frac{\partial H}{\partial C} &= -\{\varsigma_1\alpha_2 - \varsigma_2(\alpha_2 + (1+u_1)\beta_2 + \mu) + \varsigma_3(1+u_1)\beta_2\}\\
&= -\varsigma_1\alpha_2 + \varsigma_2(\alpha_2 + (1+u_1)\beta_2 + \mu) - \varsigma_3(1+u_1)\beta_2\\
\varsigma_3' = -\frac{\partial H}{\partial M} &= -\{-A_3 - \varsigma_3[(r_1 + \mu + \lambda_2 D] + \varsigma_4\lambda_2 D + \varsigma_6 r_1\}\\
&= A_3 + \varsigma_3[(r_1 + \mu + \lambda_2 D] - \varsigma_4\lambda_2 D - \varsigma_6 r_1\\
\varsigma_4' = -\frac{\partial H}{\partial P} &= -\{A_2 + \varsigma_3(1+u_3)\lambda_1 - \varsigma_4[(1+u_3)\lambda_1 + (1-u_2)\rho_1 + \mu] + \varsigma_5(1-u_2)\rho_1\}\\
&= -A_2 - \varsigma_3(1+u_3)\lambda_1 + \varsigma_4[(1+u_3)\lambda_1 + (1-u_2)\rho_1 + \mu] - \varsigma_5(1-u_2)\rho_1\\
\varsigma_5' = -\frac{\partial H}{\partial D} &= -\{A_1 + \varsigma_2\rho_2 - \varsigma_3[\lambda_2 M - (1+u_4)\delta_2] + \varsigma_4\lambda_2 M - \varsigma_5[(1+u_4)\delta_2 + \rho_2 + \mu]\}\\
&= -A - \varsigma_2\rho_2 + \varsigma_3[\lambda_2 M - (1+u_4)\delta_2] - \varsigma_4\lambda_2 M + \varsigma_5[(1+u_4)\delta_2 + \rho_2 + \mu]\}\\
\varsigma_6' = -\frac{\partial H}{\partial W} &= -\{\varsigma_2 r_3 + \varsigma_3 r_2 - \varsigma_6(r_2 + r_3 + \mu)\}\\
&= -\varsigma_2 r_3 - \varsigma_3 r_2 + \varsigma_6(r_2 + r_3 + \mu)
\end{aligned}$$

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For $t \in [0, T_f]$, the optimal controls u_1^*, u_2^*, u_3^* and u_4^* can be solved from the optimality condition

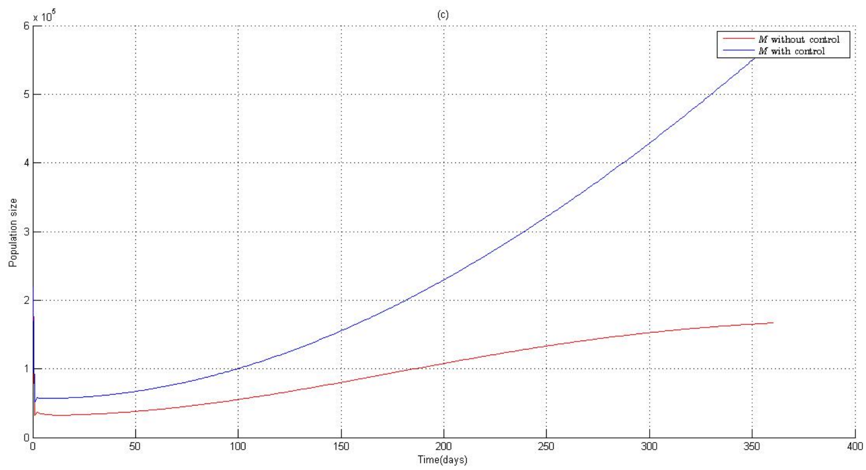
$$\begin{aligned}
\frac{\partial H}{\partial u_1} &= Iu_1 - \beta_2 C\varsigma_2 - \beta_2 C\varsigma_3 = 0\\
\frac{\partial H}{\partial u_2} &= Ju_2 + \rho_1 P\varsigma_4 - \rho_2 P\varsigma_5 = 0\\
\frac{\partial H}{\partial u_3} &= Ku_3 + \lambda_2 P\varsigma_3 - \lambda_2 P\varsigma_4 = 0\\
\frac{\partial H}{\partial u_4} &= Lu_4 + \delta_2 D\varsigma_3 - \delta_2 D\varsigma_5 = 0
\end{aligned}$$

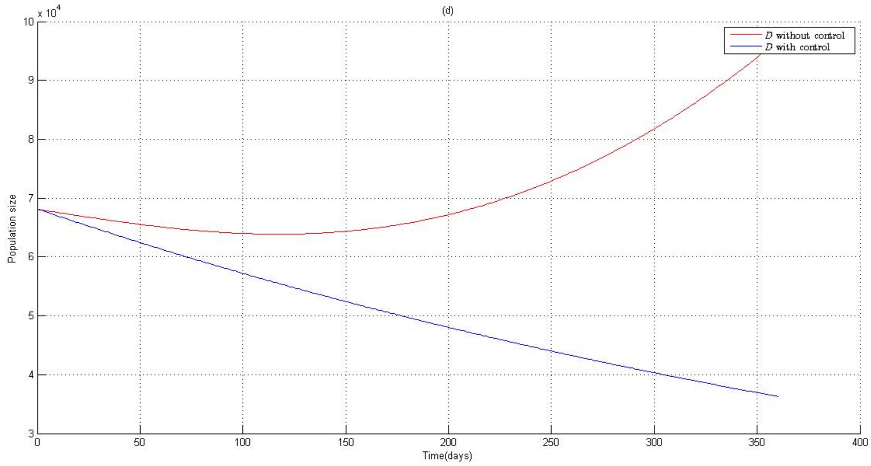
Then, we have $\frac{(\varsigma_2 - \varsigma_3)\beta_2 C}{I}$, $\frac{(\varsigma_5 - \varsigma_4)\rho_1 P}{J}$, $\frac{(\varsigma_4 - \varsigma_3)\lambda_1 P}{K}$ and $\frac{(\varsigma_5 - \varsigma_3)\delta_2 D}{L}$. By the bounds in $\mathscr{U}$ of the controls, we deduce that u_1^*, u_2^*, u_3^* and u_4^* are given the form in the theorem. □

5. NUMERICAL SIMULATION

Our reasons for participating in this investigation was to assess the rate of controls, the number of divorces, and the rate of increase in marriages affect each other. Some numerical simulation results are given in this section mainly To demonstrate the numerical results achieved. The " Social Indicators for Morocco-2023 Edition " document included " exciting and indicative " digital data on the development of many aspects of personal status issues for Moroccans. Especially with regard to issues of marriage, divorce, fertility rates, and childbearing. The values of the model parameters chosen for the simulation are given by document " Social Indicators for Morocco-2023 Edition " [11].

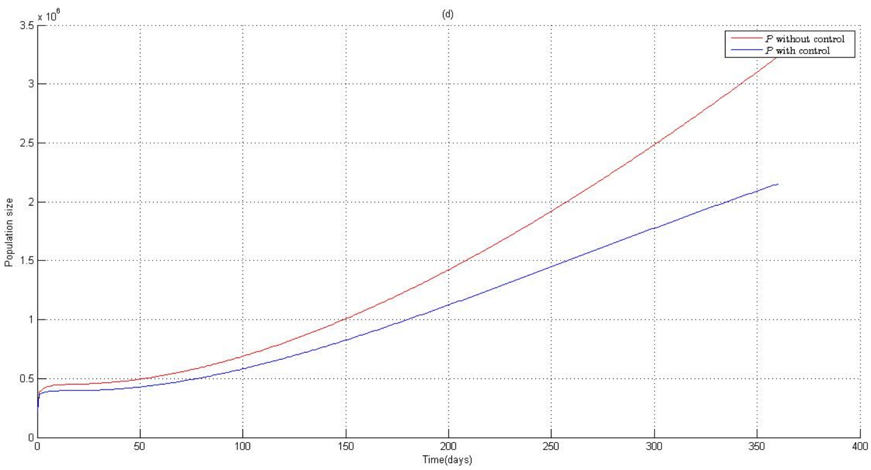
$$\begin{gathered}
S(0) = 184052, C(0) = 153694, M(0) = 258470, P(0) = 183441, D(0) = 68145,\\
W(0) = 81542, \Lambda = 82319, \alpha_1 = 0.0158, \alpha_2 = 0.11, \beta_1 = 0.0003889, \beta_2 = 0.00656,\\
r_1 = 0.364, r_2 = 0.314, r_3 = 0.00684, \lambda_1 = 0.119, \lambda_2 = 0.000025, \delta_2 = 0.0000219,\\
rho_1 = 0.000142, \rho_2 = 0.0000275, \mu = 0.0017.
\end{gathered}$$





The following two figures show developments in the number of marriage contracts and divorces in Morocco in 2022. The development of the number of marriages in Morocco in one year, as we notice that starting from the first day, the development is noticeable day after day. Without controls: On the first day, we launched with more than 200,000 marriage contracts, but more than 50 percent do not last more than a few weeks. After that, we notice a slow increase in the number of marriages, as we moved from 61,228 marriage contracts until we reached 170,349 successful marriage contracts a year later. These results remain much lower than the numbers we started with. As for divorce cases in Morocco, we note that we started with 68,145 on the first day, and with a significant increase, we reached more than 97,000 in one year, and this is an abnormal situation.

With controls: After using the control, we notice a good development from 258,470 marriage contracts on the first day until we reached more than 570,000 marriage contracts per year. After applying the control, it appears that there is an interesting decline in the number of divorces, as the difference was good. We started from 68,145 divorce cases and reached 36,000 divorce cases. This means the continuation of marriage and the decline in divorce.



The following figure represent the changes in the number of separated people, in one year. The changes occurring in this chart show that before implementing the control, we started on the first day with 183441 and after a year had passed, the number of the separated people reached more than 340000. This situation is not reassuring, and this is due to the fact that most of the cases separation reach To divorce, and this is what we reduce from it. After applying the control, we notice that the graph shows an increase in the number of cases separated people but not in such a large way that after a year it reached an average of 210000 cases, and most of them do not reach divorce. This is due to the control that we applied on two levels.

6. CONCLUSION

A mathematical model has been developed as a system to examine the social status of people in all societies. On the one hand, it has been proven that the systems are stable locally and globally without symptoms at the equilibrium point. On the other hand, the two controls are applied by reducing the number of divorce cases as well as increasing the number of marriage cases after we developed controls that gave effectiveness and encouraging results. Finally, numerical simulations were used to evaluate the departments' results, which showed that providing counseling and implementing an awareness campaign, as well as the administrative complexities and severe financial and societal impacts of divorce, can help reduce divorce cases.

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