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Savage's Solution to the Problem of Three-Currency Deposit Diversification: Program Tools and Modeling Results

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Abstract: This paper presents the development of computing tools for finding the optimal structures of multi-currency deposits in terms of guaranteed risk under uncertain exchange rates. The approach utilizes Savage's minimax regret concept to calculate risk and guaranteed risk functions explicitly, assuming only the limits of possible changes in uncertain parameters are known. The Excel environment implements the algorithm for calculating the optimal solution that minimizes income loss due to incomplete information. Computational experiments analyzed the dependence of the optimal guaranteed risk on problem parameters, such as interest rates of currencies and boundaries of uncertain factors. The results can be used to analyze financial management problems in conditions of incomplete information.

Keywords: optimization, incomplete information, minimax regret solution, deposit diversification, Excel

1. INTRODUCTION

The search of the optimal structure of a multi-currency deposit is complicated by the uncertainty of future values of economic parameters, in particular, the unknown values of the exchange rates of the currencies used. A typical situation is the so-called substantial uncertainty, in which there are not only no exact values of the uncertain parameters, but also no statistical characteristics. The Decision Maker (DM) knows only the limits of the possible values of these parameters. This leads to the emergence of risk, understood as a deviation of the result obtained from the desired or expected result.

One approach to measuring and minimizing risk is Savage's concept of minimax regret. According to this , risk is interpreted as a loss from the ignorance of uncertain factors. The principle of the best-guaranteed outcome [2] is usually applied to the risk function to compute the smallest guaranteed risk. This is the approach used below in the problem of optimal deposit diversification across three currencies and risk is understood according to Savage or regret risk [10] . The point of view used is that of the DM who allows risk and seeks to minimize it.

The concept of Savage optimality [10] and other approaches to problems with uncertain parameters were considered in publications [5–7]. The paper [14] investigated the solution of Savage's minimax regret in the problem of deposit diversification for two types of currencies. The article [8] is devoted to finding a solution with optimal guaranteed risk in a similar problem with three currencies. In it, an explicit form of the optimal solution is obtained and a constructive algorithm to compute it is proposed. The current paper continues this research.

The proposed algorithm is implemented in Excel. An economic interpretation of the initial and derived parameters of the problem is given. Debugging and experimental calculations are carried out. The case of equal guaranteed yield of all three currencies (at minimum future rates) is investigated analytically. Numerical modeling of the dependence of the minimum guaranteed risk and the structure of the optimal deposit on the parameters of the problem, such as the upper bounds of the future values of exchange rates and interest rates, is carried out.

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2. STATEMENT OF THE PROBLEM

At the beginning of the deposit period, the DM allocates one ruble among three types of deposits, buying dollars in the amount of x_d rubles and euros in the amount of x_e rubles at the initial rates K_d and K_e respectively, leaving $1 - x_d - x_e$ rubles in a ruble deposit. At the end of the deposit period, the accrued interest (at the known interest rates d_d and d_e) in dollars and euros are converted into rubles at exchange rates unknown at the beginning of the deposit period: $y_d \in [a_d, b_d]$, $y_e \in [a_e, b_e]$. Here a_d and b_d (a_e and b_e) are the lower and upper limits of possible changes in the final dollar (euro) exchange rates. In accordance with these rules, the final income in ruble terms can be presented in the form:

$$f(x,y) = (1+r)(1 - x_d - x_e) + x_d \frac{1+d_d}{K_d} y_d + x_e \frac{1+d_e}{K_e} y_e. \tag{2.1}$$

It is desirable to maximize this indicator by choosing a strategy $x = (x_d, x_e) \in X = \{x_d + x_e \leq 1, x_i \geq 0\ (i = d, e)\}$ and the DM should take into account the possibility of realizing any uncertainty $y = (y_d, y_e) \in Y = [a_d, b_d] \times [a_e, b_e]$.

Thus, the mathematical model of the diversification problem is represented by an ordered triplet $\Gamma = < X, Y, f(x,y) >$ where $f(x,y)$ is the income function of the depositor (DM), and the set X of DM's strategies x and the set Y of uncertainties y have the form:

$$X = \{x = (x_d, x_e) | x_d + x_e \leq 1, x_i \geq 0\ (i = d, e)\}, \tag{2.2}$$

$$Y = \{y = (y_d, y_e) |\ y_i \in [a_i, b_i]\ (i = d, e)\}. \tag{2.3}$$

The problem Γ is a single-criteria decision-making problem under uncertainty: maximize a linear function of x with uncertain coefficients on a polyhedron (triangle) X given the uncertain factors. The presence of uncertainty leads to the concept of risk as a possibility of the deviation of some results from their desired or expected values.

In the framework of Wald's maximin approach [13], this problem can be considered as an antagonistic game of DM against "Nature". Its solution is a "cautious" maximin strategy obtained according to the principle of the best-guaranteed result. Note that in the considered problem it has a rather trivial form: it is necessary to calculate the profitability of each type of currency at the most unfavorable possible combination of final exchange rates $y = (a_d, a_e)$ and invest all funds in the currency for which this indicator is maximized. If the maximum is reached on several currencies, the whole ruble amount is distributed among them in an arbitrary way. This result is true for any number of currencies [14].

As we know, the Wald criterion is very pessimistic and focuses on the worst case. It does not allow for risk and often gives extremely underestimated results. On the other hand, many DMs accept risk and need tools to minimize it. One of these is the principle of minimax regret (Savage's criterion). It also uses the notion of the best-guaranteed outcome, but applies not to the original quality indicator $f(x,y)$ but to the so-called risk (regret) function.

3. SAVAGE'S PRINCIPLE OF MINIMAX REGRET

Let $f(x,y)$ be the income, X be the set of DM strategies, and Y be the set of uncertainties. Then $f\ (z,y)$ is the best possible income if there is an uncertainty y. However, the DM does not know in advance, what the value y of the uncertain parameter will be. The difference

$$\Phi(x,y) = \max_{z \in X} f\ (z,y) - f(x,y) \tag{3.1}$$

is called the Savage risk (regret) function. It represents the loss due to uncertainty which is the difference between the best outcome that could have been obtained under known y and the actual outcome of some strategy x. This risk depends on both strategy x and uncertainty y.

To minimize risk, the DM can use the concept of the best-guaranteed outcome (Wald's principle [13]).

These considerations lead to the following definition (Savage's minimax regret principle [10]).

Definition 3.1:

Let us call the solution x^r the risk-guaranteed solution (RGS) of the problem Γ, if

$$\Phi^r = \max_{y\in Y} \Phi(x^r, y) = \min_{x\in X} \max_{y\in Y} \Phi(x, y), \tag{3.2}$$

where the risk (loss) function $\Phi(x,y)$ is defined in (3.1).

For the sake of brevity, we also refer to the decision x^r and risk Φ^r as the optimal decision and the optimal risk, respectively.

According to Definition 3.1, the construction of a RGS contains four steps.

Step 1: Calculate $f[y] = \max_{z\in X} f(z, y)$ for each uncertainty $y \in Y$.

Step 2: Calculate the risk function $f[y] - f(x, y)$.

Step 3. Compute the internal maximum in (3.2) which is the guaranteed risk $\max_{y\in Y} \Phi(x, y) = \max_{y\in Y}(f([y] - f(x, y)) = \Phi[x] \geq \Phi(x, y)$ for each strategy $x \in X$.

Step 4. Compute the external minimum in (3.2) which is the best-guaranteed risk $\Phi^r = \min_{x\in X} \Phi[x] = \Phi[x^r]$.

4. CONSTRUCTING A RISK-GUARANTEED SOLUTION

The explicit form of the risk function for the diversification problem of a three-currency deposit was obtained in [8]:

$$\Phi(x,y) = f[y] - f(x,y) = \\ = \begin{cases} \Phi_1(x,y) = (1+r) - f(x,y) = \\ [(1+r) - \xi_d]x_d + [(1+r) - \xi_e y_e]x_e, y \in Y_1, \\ \Phi_2(x,y) = \xi_d y_d - f(x,y) = \\ (1-x_d)[\xi_d y_d - (1+r)] + [1+r-\xi_e y_e]x_e, y \in Y_2, \\ \Phi_3(x,y) = \xi_e y_e - f(x,y) = \\ [1+r-\xi_d y_d]x_d + [\xi_e y_e - (1+r)](1-x_e), y \in Y_3, \end{cases} \tag{4.1}$$

where, for the sake of brevity, we used the notations $\xi_d = \frac{1+d_d}{K_d}, \xi_e = \frac{1+d_e}{K_e}$.

The sets Y_1, Y_2, Y_3 are subsets of the set Y defined by additional linear inequations:

$$Y_1 = \{y \in Y, 1+r \geq \xi_d y_d, \quad 1+r \geq \xi_e y_e\}, \tag{4.2}$$

$$Y_2 = \{y \in Y, \xi_d y_d \geq 1+r, \quad \xi_d y_d \geq \xi_e y_e\}, \tag{4.3}$$

$$Y_3 = \{y \in Y, \xi_e y_e \geq 1+r, \quad \xi_e y_e \geq \xi_d y_d\}. \tag{4.4}$$

It is obvious that $Y_1 \cup Y_2 \cup Y_3 = Y$. Functions $\Phi_i(x,y)$ $(i = 1,2,3)$ are bilinear functions of the variables x and y. Therefore, the risk function $\Phi(x,y)$ for any fixed strategy x is a piecewise linear function of uncertainty y. The sets Y_1, Y_2, Y_3 are regions (polygons) of linearity on y of the risk function, i.e. the risk function $\Phi(x,y)$ coincides with the linear (by x) function $\Phi_i(x,y)$, $y \in Y_i$. In this case, it is said that the risk function $\Phi(x,y)$ is defined at the point (x,y) by the function $\Phi_i(x,y)$.

For each strategy $x \in X$ the guaranteed risk $\Phi[x]$ was calculated [8]:

$$\Phi[x] = \max\{\alpha_d x_d + \alpha_e x_e, \quad \beta_d(1-x_d) + \alpha_e x_e, \quad \beta_e(1-x_e) + \alpha_d x_d\}, \quad x \in X, \tag{4.5}$$

where the following notations are used:

$$\alpha_d = [(1+r) - \xi_d a_d], \ \alpha_e = [(1+r) - \xi_e a_e], \tag{4.6}$$

$$\beta_d = [\xi_d b_d - (1+r)], \ \beta_e = [\xi_e b_e - (1+r)]. \tag{4.7}$$

The secondary (derivative) coefficients $\alpha_d, \alpha_e, \beta_d, \beta_e$ depend on the above-mentioned initial parameters. The substantive interpretation for them is given below (see Section 6).

The best (smallest) guaranteed risk $\Phi^r = \min_{x\in X} \Phi[x] = \Phi[x^r]$ is computed in [8] by considering the problem $\min_{x\in X} \Phi[x]$ separately on the boundary and on the interior of the set of the depositor's strategies X (triangle A(1;0)O(0;0)B(0;1)) followed by the selection of the best solution:

$$\min_{x\in X}\Phi[x] = \min\{\min_{x\in[0,A]}\Phi[x], \min_{x\in[0,B]}\Phi[x], \min_{x\in[A,B]}\Phi[x], \min_{x\in\text{intX}}\Phi[x]\}. \#(4.8)$$

The results obtained can be restated in economic terms in the following form.

Proposition 4.1:

If the optimal guaranteed risk is achieved by a combination of dollar and ruble components, then the solution of the form

$$x^{OA} = (x_d^{OA}, 0), 0 < x_d^{OA} < 1)$$

wher $x_d^{OA} = \begin{cases} \frac{\beta_d-\beta_e}{\alpha_d+\beta_d}, & if \quad \beta_e \geq 0, \\ \frac{\beta_d}{\alpha_d+\beta_d}, & if \quad \beta_e \leq 0, \end{cases}$ *is optimal.*

The last two formulas can be combined into a single expression:

$$x_d^{OA} = \frac{\beta_d - \beta_e(sign\beta_e + 1)/2}{\alpha_d + \beta_d}.$$

Proposition 4.2:

If the optimal guaranteed risk is achieved by a combination of euros and rubles, the solution of the form

$$x^{OB} = (0, x_e^{OB}), 0 < x_e^{OB} < 1$$

where $x_e^{OB} = \begin{cases} \frac{\beta_e-\beta_d}{\alpha_e+\beta_e}, if \ \beta_d \geq 0, \\ \frac{\beta_e}{\alpha_e+\beta_e}, if \ \beta_d \leq 0, \end{cases}$ *is optimal.*

Here also both formulas can be combined into a single expression:

$$x_e^{OB} = \frac{\beta_e - \beta_d(sign(\beta_d) + 1)/2}{\alpha_e + \beta_e}.$$

Proposition 4.3:

If the optimal guaranteed risk is achieved by a combination of dollars and euros, then some solution from the set:

$$\{x^{12} = (x_e^{12}, 1 - x_e^{12}), x^{13} = (x_e^{13}, 1 - x_e^{13}), \ x^{23} = (x_e^{23}, 1 - \ x_e^{23})\} \cap X$$

where $x_e^{12} = \frac{\alpha_d}{\alpha_d+\beta_d}$, $x_e^{13} = \frac{\beta_e}{\alpha_e+\beta_e}$, $x_e^{23} = \frac{\alpha_d+\beta_e}{\alpha_e+\alpha_d+\beta_e+\beta_d}$ *is optimal.*

We speak about full diversification if the guaranteed risk function $\Phi[x]$ reaches the minimum value Φ^* on the set X at some interior point X and for all points of the boundary frX $\Phi[x] > \Phi^*$.

This condition means that the optimal diversification necessarily uses all three currencies. In the following, we restrict ourselves to this condition, since other possible situations are covered by Propositions 4.1–4.3.

Proposition 4.4:

If the optimal guaranteed risk is achieved only when all three currencies are used, then the optimal solution is the following:

$$x^* = (x_d^*, x_e^*) = (\frac{\beta_d}{\alpha_d + \beta_d}, \frac{\beta_e}{\alpha_e + \beta_e})$$

provided that $0 < x_d, x_e < 1$ *and* $x_d + x_e < 1$.

Any point of a local minimum of a convex function on a convex set is simultaneously a point of global (on this set) minimum. Hence, if the point of internal minimum x^* exists and $x^* = (x_d^*, x_e^*) \in intX$, it will be the RGS. Whether it exists or not depends on the relationship between the parameters of the problem under consideration. Very often, the RGS is on the boundary of the set of admissible solutions. This means that the optimal diversification contains only two (or even one) of the three currencies.

The list of all possible candidates for the optimal solution specified in Propositions 4.1–4.4 should be accomplished by three single-currency variants: (0;0) – only rubles, (1;0) – only dollars, (0;1) – only euros. As a result, we obtain an exhaustive set of nine solutions, at least one of which will be optimal.

5. ALGORITHM FOR CONSTRUCTING A RISK-GUARANTEED SOLUTION

Combining the results of Propositions 4.1–4.4 leads to the following scheme [8] for computing the risk-guaranteed diversification strategy and the smallest guaranteed risk.

Step 1. Set numerical values of the interest rates r, d_d, d_e and the current exchange rates K_d, K_e; set numerical values of the boundaries of possible changes in dollar and euro exchange rates a_d, b_d and a_e, b_e , respectively. Any available information can be used, e.g. expert estimates and forecasts, DM's own assumptions, etc.

Step 2. Calculate the secondary parameters α_d, α_e, β_d, β_e from (4.6-4.7).

Step 3**.** Compute the nine candidate points for the optimal solution. The corresponding formulas for these points are presented in Table 1 below**.**

Step 4: Remove points that do not belong to the set X.

Step 5: Calculate the values of the guaranteed risk for the remaining points, select the solution with the best (minimum) guaranteed risk.

Table 1. Candidates for solutions and verification of their admissibility conditions [8]

N	Candidates for solutions	Formulas	Checking the conditions
1	The point O=(0;0) (rubles)	(0;0)	–
2	The point A=(1;0) (dollars)	(1;0)	–
3	The point B=(0;1) (euro)	(0;1)	–
4	The point $(x_d^{OA}, 0)$ (rubles and dollars)	$x_d^{OA} = \dfrac{\beta_d - \beta_e(sign\beta_e + 1)/2}{\alpha_d + \beta_d}$	$0 < x_d^{OA} < 1$
5	The point $(0, x_e^{OB})$ (rubles and euro)	$x_e^{OB} = \dfrac{\beta_e - \beta_d(sign\beta_d + 1)/2}{\alpha_d + \beta_d}$	$0 < x_d^{OB} < 1$
6	The point $(1 - x_e^{12}, x_e^{12})$ (dollars and euro)	$x_e^{12} = \dfrac{\alpha_d}{\alpha_d + \beta_d}$	$0 < x_e^{12} < 1$
7	The point $(1 - x_e^{13}, x_e^{13})$ (dollars and euro)	$x_e^{13} = \dfrac{\beta_e}{\alpha_e + \beta_e}$	$0 < x_e^{13} < 1$
8	The point $(1 - x_e^{23}, x_e^{23})$ (dollars and euro)	$x_e^{23} = \dfrac{\alpha_d + \beta_e}{\alpha_e + \alpha_d + \beta_e + \beta_d}$	$0 < x_e^{23} < 1$
9	The internal point in X (rubles, dollars and euro)	$(x_d^*, x_e^*) = = (\dfrac{\beta_d}{\alpha_d + \beta_d}, \dfrac{\beta_e}{\alpha_e + \beta_e})$	$0 < x_d, x_e < 1$ $x_d + x_e < 1$

6. ECONOMIC AND GAME INTERPRETATION

Let us explain the economic meaning of the derived parameters of the problem. Recall that r, d_d, d_e are the interest rates on the corresponding deposits (ruble, dollar, euro); K_d and K_e are dollar and euro exchange rates against the ruble, respectively, at which the currency is purchased at the beginning of the year; a_d, a_e are the lower and b_d, b_e are the upper limits of the dollar and euro exchange rates, respectively, at the end of the deposit period. These parameters are assumed to be known to the DM at the beginning of the year. However, the values of uncertain parameters, i.e. the final values of the dollar and euro exchange rates are known only to the precision of the sets: $y_d \in [a_d, b_d]$, $y_e \in [a_e, b_e]$. Income, measured in rubles, for a one ruble allocation $(1 - x_d - x_e, x_d, x_e)$ is determined by the linear function (2.1) with uncertain coefficients.

The secondary coefficients $\alpha_d, \alpha_e, \beta_d, \beta_e$ depend on the above initial parameters and have the following meaningful interpretations. The expression $\alpha_d = \left[(1+r) - \frac{1+d_d}{K_d} a_d\right]$ defines the difference between the yield $(1+r)$ of one ruble in a ruble deposit and the yield when using it to buy dollars at the beginning of the period at the rate of K_d ruble/dollar and selling the total amount $(1+d_d)$ at the minimum rate a_d at the end of the period. The parameter $\alpha_e = \left[(1+r) - \frac{1+d_e}{K_e} a_e\right]$ defines the difference between the yields of the ruble and the euro if the uncertain euro exchange rate takes the minimum possible value.

Coefficient β_d (β_e) sets the difference between the yield of one ruble invested in a dollar (euro) deposit and the yield of a ruble in a ruble deposit at the highest final dollar (euro) exchange rate.

As noted above, the problem of the best-guaranteed income can be regarded as an antagonistic game between DM and Nature. This game has, at the given uncertainty structure, a rather simple solution even in the case of any finite number of currencies [14]. Savage's minimax regret principle is related to the consideration of a more complex structure, the target risk function (3.1), to which Wald's minimax principle is then applied. Thus, the problem of computing optimal risk can also be interpreted as an antagonistic game between DM and Nature-uncertainty with the risk function as the payoff function. However, unlike the aforementioned DM-Nature game with income as a payoff function, this game does not typically have a saddle point [12] in pure strategies. Indeed, under the assumptions made, the maximin and minimax in this game exist but are not equal. The maximin is equal to zero, which is the optimal value of the risk function under the uncertainty known to the DM in advance, i.e. actually without uncertainty. The minimax for a non-negative risk function is non-negative and equals zero only in degenerate cases, when there exists a strategy that is optimal under any uncertainty. Hence the absence of a saddle point [12].

In this paper, the DM's guaranteed strategy was found by direct computing ("by definition") the minimax of the risk function.

In its original formulation, the problem under consideration is close to Markowitz's problem of optimal portfolio structure [3], first formulated in 1952 and since developed in numerous publications. Despite such a long history, it is still the case that "many investment efficiency indicators are based on Markowitz's portfolio theory" [1].

In Markowitz’s theory, the returns of individual portfolio assets are random variables with respect to which mathematical expectations and covariance matrices are assumed to be known [4]. The expected return of the whole portfolio and its variance are considered as two indicators to be optimized – return and risk. From a formal point of view, this is a two-criterion optimization problem [9] with linear (return) and quadratic (risk as variance of return) criteria and simple linear constraints. Various methods of finding Pareto-optimal solutions and ways to construct efficient portfolios have been developed [11]. Most of them use techniques of parameterization, for example, optimization by one criterion while restricting the values of another criterion, to construct the Pareto frontier.

The subject of this paper is close to the topic of optimizing a portfolio with three assets, two of which are risky. However, there are two important differences – in the available information and in risk assessment. For a number of reasons, it is not always possible to obtain satisfactory estimates of the statistical characteristics of asset returns and one has to be satisfied with interval forecasts. The use of dispersion as a measure of risk is computationally convenient, but has well-known disadvantages. Among them is the need for information on the correlation of assets. The use of an alternative risk measure – Savage risk – develops additional directions of portfolio theory and practice.

7. ANALYTICAL SOLUTION IN THE SPECIAL CASE

Consider the special case when $\alpha_d = \alpha_e = 0$. This means that at minimum final values of dollar and euro exchange rates all three currencies give equal returns. Then, from the point of view of the Wald principle, any admissible diversification plan gives the same (and, therefore, optimal) result [14]. This is explained by the fact that this principle takes into account only the worst possible value $y = (y_d, y_e) \in Y$ for each strategy $x = (x_d, x_e) \in X$. Thus, in our case, obviously, for all x will be $y = (a_d, a_e)$. In contrast to Wald's principle, the Savage optimal solution is found taking into account both the lower and upper bounds of uncertainty b_d, b_e and the coefficients β_d, β_e depending on them.

The guaranteed risk function (4.5) takes the form

$$\Phi[x] = \max\{0,\ \beta_d(1 - x_d),\ \beta_e(1 - x_e)\}, \qquad x \in X \tag{7.1}$$

From the condition $\alpha_d = \alpha_e = 0$ and the definition of the coefficients $\alpha_d, \alpha_e, \beta_d, \beta_e$ it follows that $\beta_d > 0, \beta_e > 0$. In addition, $(1 - x_d) \geq 0, (1 - x_e) \geq 0$ for $x \in X$. Therefore,

$$\Phi[x] = \max\{\ \beta_d(1 - x_d), \beta_e(1 - x_e)\}, \qquad x \in X \tag{7.2}$$

Recall that the set of admissible solutions has the form:

$$X = \{x = (x_d, x_e) | x_d + x_e \leq 1, \qquad x_i \geq 0\ (i = d, e)\} \tag{7.3}$$

Obviously, a solution $x = (x_d, x_e)$ such that $x_d + x_e < 1$, cannot be the minimum point of the function (7.2), since a small increase in both variables will result in a decrease in the function. Therefore, the point of minimum of the function $\Phi[x]$ on the set X lies on the segment [A,B] $= \{x_d + x_e = 1, x_d \geq 0, x_e \geq 0\}$, where $x = (1 - x_e, x_e)$. In the particular case $\alpha_d = \alpha_e = 0$ taking into account the conditions $\beta_d > 0, \beta_e > 0$ we obtain from **4.3** as a corollary:

Proposition 7.1:

If $\alpha_d = \alpha_e = 0$ *then the guaranteed risk function* $\Phi[x]$ *reaches the minimum value on the set* X *at some point in the set* $\{(1; 0), (0; 1), x_e^{23} = (\beta_d/(\beta_e + \beta_d), \beta_e/(\beta_e + \beta_d)\}$.

In the derivation of the corollary, it is taken into account that some points from proposition 4.3 are duplicated here and that condition $x_e^{23} \in X$ is fulfilled automatically in this case.

For the final choice of the solution, we compute the value of the guaranteed risk function at the candidate points: $\Phi[(1,0)] = \max\{\ \beta_d(1 - 1), \beta_e(1 - 0)\} = \beta_e$, $\Phi[(0,1)] = \beta_d$, $\Phi[(x_d^{23}, x_e^{23})] = \max\{\ \beta_d\beta_e/(\beta_e + \beta_d), \beta_e\beta_d/(\beta_e + \beta_d)\ \} = \beta_d\beta_e/(\beta_e + \beta_d)$.

The last value is smaller than the values at the other two points. Hence, the RGS in this case is unique and has the form:

$$x^r = \left(\frac{\beta_d}{\beta_e + \beta_d}, \frac{\beta_e}{\beta_e + \beta_d}\right) \tag{7.4}$$

It gives the minimum guaranteed risk $\Phi^r = \Phi[x^r] = \beta_d\beta_e/(\beta_e + \beta_d)$.

In the case of equal ruble profitability of all three currencies (at minimum values of the final exchange rates), the optimal solution in terms of guaranteed risk is to distribute the ruble between the dollar and the euro in proportion to the upper possible values of their final exchange rates. If $\beta_d = \beta_e = \beta$, the optimal solution is $x^r = (0.5; 0.5)$.

In the general case, the analytical search for the optimal solution of the diversification problem is difficult and requires the use of numerical methods and algorithms.

8. COMPUTATIONAL TOOLS AND SOME RESULTS

The suggested method is implemented in the Excel 2016 environment in two variants – user and research . The first is designed for a user with minimal skills of working in a ready-made Excel spreadsheet. The work is reduced to the input of the above-mentioned initial data and the analysis of the results. The corresponding spreadsheet is consistent with Table 1 and contains the calculation formulas for the points, of which at least one is the desired solution.

Table 2 reproduces part of the spreadsheet to input data.

Table 2. Input of initial data. Input cells are highlighted in green

	E	F	G	H	I	J	K	L
9		DATA INPUT				Growth indices (for information)		
10		CURRENCY	RUBLE	DOLLAR	EURO	(1+r) =	1,12	- ruble deposit growth index
11		INTEREST RATES	r	d_d	d_e	DOLLAR	EURO	
12			12,00%	3,00%	3,00%	$(1+d_d)*(a_d/K_d)$	$(1+d_e)*(a_e/K_e)$	- lowest growth indices (at y=a)
13		INITIAL RATES VALUES		K_d	K_e	0,9728	0,9785	
14				90	100	$(1+d_d)*(b_d/K_d)$	$(1+d_e)*(b_e/K_e)$	- highest growth indices (at y=b)
15		FINAL RATES VALUES	1	y_d	y_e	1,2017	1,1845	
16		LOW BOUNDS (a_d,a_e)	1	85	95			
17		UPPER BOUNDS (b_d,b_e)	1	105	115			

Table 3 shows a part of the Excel spreadsheet with calculations corresponding to the given initial parameters from Table 2. Column B contains the text of the formulas from Table 1, columns C and D contain the corresponding calculations. The results are checked by logical operators for the fulfillment of the condition $x \in X$ (descriptions of condition are in the column J). If the condition is not fulfilled, the text "NOT CANDIDATE" is written in the cell of column E, otherwise the result of the calculation of the guaranteed risk function at the corresponding point is given. Then the minimum of these results is calculated. The corresponding cell (or cells in case of non-singularity) is highlighted by means of conditional formatting, and the optimal solution is located to the left of it. According to the values of the target function at the other acceptable candidate points, it is possible to estimate the increase in risk when reducing the number of currencies used**.**

Table 3. Calculating the optimal solution

A	B	C	D	E	F	G	H	I	J	K
	TABLE FOR CHECKS, EXCLUDING UNACCEPTABLE POINTS AND SELECTING THE MINIMUM POINT									
	CALCULATION FORMULAS	xd - dollars	xe - euro	CHECKING RESULT	Φ	Φ1	Φ2	Φ3	CHECKING FOR:	
1	POINT O(0,0) (RUBLES ONLY)	0,000	0,000	0,082	0,082	0,000	0,082	0,064	NOT REQUIRED	
2	POINT A(1,0) (DOLLARS AND RUBLES)	1,000	0,000	0,212	0,212	0,147	0,000	0,212	NOT REQUIRED	
3	POINT B(0,1) (EURO AND RUBLES)	0,000	1,000	0,223	0,223	0,142	0,223	0,000	NOT REQUIRED	
4	$x_d^{OA} = \frac{\beta_d - \beta_e(sign(\beta_e)+1)/2}{\alpha_d + \beta_d}.$	0,075	0,000	0,076	0,076	0,011	0,076	0,076	1>xd>0	
5	$x_e^{OB} = \frac{\beta_e - \beta_d(sign(\beta_d)+1)/2}{\alpha_e + \beta_e}$	0,000	-0,083	NOT CANDIDATE	0,070	-0,012	0,070	0,070	1>xe>0	
6	$x_e^{12} = \frac{\alpha_d}{\alpha_d + \beta_d}$	0,357	0,643	0,144	0,144	0,144	0,144	0,076	xd,xe>0	
7	$x_e^{13} = \frac{\beta_e}{\alpha_e + \beta_e}$	0,687	0,313	0,145	0,145	0,145	0,070	0,145	xd,xe>0	
8	$x_e^{23} = \frac{\alpha_d + \beta_e}{\alpha_e + \alpha_d + \beta_e + \beta_d}$	0,513	0,487	0,144	0,144	0,144	0,109	0,109	xd,xe>0	
9	$x^* = (x_d^*, x_e^*) = (\frac{\beta_d}{\alpha_d + \beta_d}, \frac{\beta_e}{\alpha_e + \beta_e})$	0,357	0,313	0,097	0,097	0,097	0,097	0,097	xd,xe>0,xd+xe<1	
10	OPTIMAL SOLUTION:	0,075	0,000		0,075541667	0,011	0,076	0,076		
	Minimal guaranteed risk Φ*=			0,076						

The research variant contains a number of additional features. For example, instead of using the initial data, it is possible to directly specify the secondary parameters $\alpha_d, \alpha_e, \beta_d, \beta_e$ which also fully determine the solution.

However, the direct assignment of the derived parameters cannot be arbitrary, since their values must be consistent with the original parameters (see formulas (4.6–4.7)). For example, as noted above, if $\alpha_d = \alpha_e = 0$, then the parameters β_d, β_e cannot be negative.

Table 4 corresponds to the case considered above: $\alpha_d = \alpha_e = 0$ at $\beta_d = 0{,}2, \beta_e = 0{,}3$.

Table 4. The case of equal minimal currency returns: $\alpha_d = \alpha_e = 0$; $\beta_d = 0{,}2, \beta_e = 0{,}3$

A	B	C	D	E	F	G	H	I	J	K
	TABLE FOR CHECKS, EXCLUDING UNACCEPTABLE POINTS AND SELECTING THE MINIMUM POINT									
	CALCULATION FORMULAS	xd - dollars	xe - euro	CHECKING RESULT	Φ	Φ1	Φ2	Φ3	CHECKING FOR:	
1	POINT O(0,0) (RUBLES ONLY)	0,000	0,000	0,300	0,300	0,000	0,200	0,300	NOT REQUIRED	
2	POINT A(1,0) (DOLLARS AND RUBLES)	1,000	0,000	0,300	0,300	0,000	0,000	0,300	NOT REQUIRED	
3	POINT B(0,1) (EURO AND RUBLES)	0,000	1,000	0,200	0,200	0,000	0,200	0,000	NOT REQUIRED	
4	$x_d^{0A} = \frac{\beta_d - \beta_e(sign(\beta_e)+1)/2}{\alpha_d + \beta_d}$.	-0,500	0,000	NOT CANDIDATE	0,300	0,000	0,300	0,300	1>xd>0	
5	$x_e^{0B} = \frac{\beta_e - \beta_d(sign(\beta_d)+1)/2}{\alpha_e + \beta_e}$	0,000	0,333	0,200	0,200	0,000	0,200	0,200	1>xe>0	
6	$x_e^{12} = \frac{\alpha_d}{\alpha_d + \beta_d}$	1,000	0,000	NOT CANDIDATE	0,300	0,000	0,000	0,300	xd,xe>0	
7	$x_e^{13} = \frac{\beta_e}{\alpha_e + \beta_e}$	0,000	1,000	NOT CANDIDATE	0,200	0,000	0,200	0,000	xd,xe>0	
8	$x_e^{23} = \frac{\alpha_d + \beta_e}{\alpha_e + \alpha_d + \beta_e + \beta_d}$	0,400	0,600	0,120	0,120	0,000	0,120	0,120	xd,xe>0	
9	$x^* = (x_d^*, x_e^*) = (\frac{\beta_d}{\alpha_d + \beta_d}, \frac{\beta_e}{\alpha_e + \beta_e})$	1,000	1,000	NOT CANDIDATE	0,000	0,000	0,000	0,000	xd,xe>0,xd+xe<1	
10	OPTIMAL SOLUTION:	0,400	0,600		0,12	0,000	0,120	0,120		
			Minimal guaranteed risk Φ*=	0,120						

The software implementation of the algorithm provides means for multivariate calculations, in which one or two parameters are varied. For example, by fixing the parameters α_d, α_e, it is possible to build a table showing the dependence of the optimal result on the varied values β_d, β_e. Alternatively, it is possible to study the optimal risk dependence on the lower bounds of rates α_d, α_e when the values β_d, β_e are fixed . On the other hand, we can construct a table of the dependence of the optimal risk on the values of dollar and euro interest rates. There are also other experiments, for example, studying the dependence of the optimal deposit structure on the interest rate of one of the currencies.

Table 5 shows the results of multivariate calculations of the optimal risk at all combinations of upper bounds of rates b_d from 97 to 117 and b_e from 108 to 128 respectively with step 1. Other parameters are fixed at the values from Table 2.

Table 5. Dependence of optimal risk on upper limits of exchange rates b_d, b_e.
In the upper line are dollar exchange rate values, in the left column – euro

	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117
108	0,000	0,002	0,012	0,021	0,029	0,036	0,042	0,048	0,053	0,057	0,061	0,065	0,068	0,071	0,074	0,077	0,080	0,082	0,084	0,086	0,088
109	0,003	0,003	0,012	0,021	0,029	0,036	0,043	0,048	0,053	0,058	0,062	0,066	0,070	0,073	0,076	0,078	0,081	0,083	0,086	0,088	0,090
110	0,012	0,012	0,013	0,023	0,031	0,039	0,046	0,052	0,057	0,062	0,067	0,071	0,074	0,078	0,081	0,084	0,087	0,089	0,092	0,094	0,096
111	0,020	0,020	0,022	0,024	0,033	0,041	0,049	0,055	0,061	0,066	0,071	0,075	0,079	0,083	0,086	0,089	0,092	0,095	0,097	0,100	0,102
112	0,027	0,027	0,030	0,032	0,035	0,044	0,052	0,058	0,065	0,070	0,075	0,080	0,084	0,088	0,091	0,095	0,098	0,101	0,103	0,106	0,108
113	0,034	0,034	0,037	0,039	0,042	0,046	0,055	0,062	0,068	0,074	0,079	0,084	0,089	0,093	0,097	0,100	0,103	0,106	0,109	0,112	0,114
114	0,039	0,040	0,043	0,046	0,049	0,052	0,057	0,065	0,072	0,078	0,084	0,089	0,093	0,098	0,102	0,105	0,109	0,112	0,115	0,118	0,120
115	0,044	0,045	0,048	0,052	0,056	0,059	0,063	0,068	0,076	0,082	0,088	0,093	0,098	0,103	0,107	0,111	0,114	0,118	0,121	0,124	0,127
116	0,049	0,049	0,053	0,057	0,061	0,065	0,069	0,073	0,079	0,086	0,092	0,098	0,103	0,108	0,112	0,116	0,120	0,124	0,127	0,130	0,133
117	0,053	0,054	0,058	0,062	0,067	0,071	0,075	0,080	0,084	0,090	0,096	0,102	0,108	0,113	0,117	0,122	0,126	0,129	0,133	0,136	0,139
118	0,057	0,058	0,062	0,067	0,071	0,076	0,081	0,085	0,090	0,094	0,101	0,107	0,113	0,118	0,123	0,127	0,131	0,135	0,139	0,142	0,145
119	0,061	0,061	0,066	0,071	0,076	0,081	0,086	0,091	0,095	0,100	0,105	0,111	0,117	0,123	0,128	0,132	0,137	0,141	0,144	0,147	0,149
120	0,064	0,064	0,070	0,075	0,080	0,085	0,090	0,095	0,101	0,106	0,111	0,116	0,122	0,128	0,133	0,138	0,142	0,146	0,148	0,150	0,152
121	0,067	0,067	0,073	0,078	0,084	0,089	0,094	0,100	0,105	0,111	0,116	0,121	0,127	0,133	0,138	0,143	0,146	0,149	0,151	0,153	0,156
122	0,070	0,070	0,076	0,082	0,087	0,093	0,098	0,104	0,110	0,115	0,121	0,126	0,132	0,138	0,143	0,147	0,149	0,152	0,154	0,157	0,159
123	0,072	0,073	0,079	0,085	0,090	0,096	0,102	0,108	0,114	0,120	0,125	0,131	0,137	0,143	0,146	0,149	0,152	0,155	0,157	0,160	0,162
124	0,074	0,075	0,081	0,087	0,093	0,099	0,105	0,111	0,117	0,123	0,129	0,136	0,142	0,146	0,149	0,152	0,155	0,157	0,160	0,163	0,165
125	0,077	0,078	0,084	0,090	0,096	0,102	0,109	0,115	0,121	0,127	0,133	0,140	0,145	0,148	0,151	0,154	0,157	0,160	0,163	0,166	0,168
126	0,079	0,080	0,086	0,092	0,099	0,105	0,112	0,118	0,124	0,131	0,137	0,143	0,147	0,151	0,154	0,157	0,160	0,163	0,166	0,168	0,171
127	0,081	0,082	0,088	0,095	0,101	0,108	0,114	0,121	0,127	0,134	0,140	0,146	0,149	0,153	0,156	0,159	0,162	0,165	0,168	0,171	0,174
128	0,083	0,084	0,090	0,097	0,104	0,110	0,117	0,124	0,130	0,137	0,144	0,148	0,151	0,155	0,158	0,162	0,165	0,168	0,171	0,174	0,176

It is convenient to analyze the obtained data using Excel graphical tools. Thus, Fig. 1 shows graphs of the first 10 rows of Table 5. For the remaining eleven values of b_e the graphs are similar. The graphs for the columns of the table also look similar. As one would expect, the optimal risk is an increasing nonlinear function for each parameter b_d, b_e.

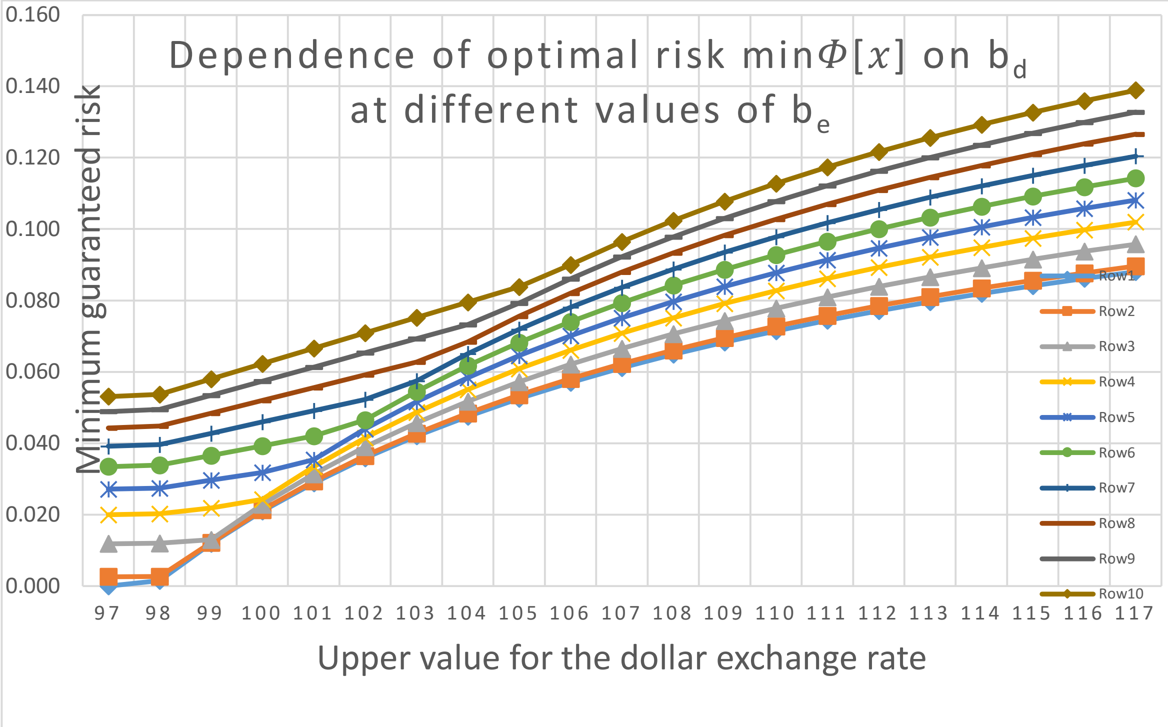

Fig. 1. Graphs of the dependence of the optimal risk on the parameter b_d at different values of the parameter b_e. Row 1: $b_e = 108$, ..., Row 10: $b_e = 117$

Table 6 shows the optimal deposit structures when the interest rate for the ruble changes from 15% to 6% with steps of -0.5%. Other parameters are fixed at the following values: dollar rate – 3%, euro rate – 3%, initial exchange rates – 90 rubles/dollar and 100 rubles/euro, dollar exchange rate limits – from 85 to 105, euro – from 95 to 115. The last line shows the corresponding values of the optimal guaranteed risk. As can be seen, it decreases monotonically with the growth of the ruble interest rate. Fig. 2 gives a visual representation of the dynamics.

Table 6. Dependence of the deposit structure on the ruble interest rate

Ruble Rate	15,0%	14,5%	14,0%	13,5%	13,0%	12,5%	12,0%	11,5%	11,0%	10,5%	10,0%	9,5%	9,0%	8,5%	8,0%	7,5%	7,0%	6,5%	6,0%
Dollar	0,075	0,075	0,075	0,075	0,075	0,075	0,075	0,075	0,075	0,075	0,075	0,075	0,075	0,075	0,513	0,513	0,513	0,513	0,513
Euro	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,487	0,487	0,487	0,487	0,487
Ruble	0,925	0,925	0,925	0,925	0,925	0,925	0,925	0,925	0,925	0,925	0,925	0,925	0,925	0,925	0,000	0,000	0,000	0,000	0,000
Risk	0,048	0,052	0,057	0,062	0,066	0,071	0,076	0,080	0,085	0,089	0,094	0,099	0,103	0,108	0,109	0,109	0,109	0,109	0,109

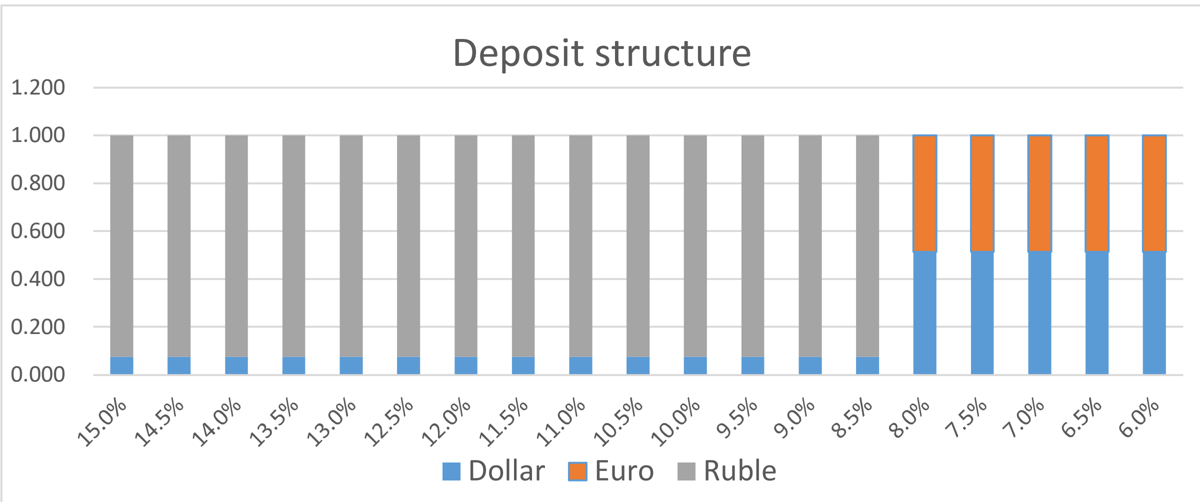

Fig 2. Optimal deposit structure

Table 7 shows the results of calculations of the minimum guaranteed risk for dollar and euro interest rates in the range from 0,25% to 5% with steps of 0.25%. Other parameters are fixed at the following values: initial exchange rates – 90 rubles/dollar and 100 rubles/euro, dollar exchange rate limits – from 85 to 105, euro – from 95 to 115; ruble interest rate is assumed to be 12%.

Table 7. Optimal guaranteed risk at different dollar (top row) and euro (left column) interest rates

	0,25%	0,50%	0,75%	1,00%	1,25%	1,50%	1,75%	2,00%	2,25%	2,50%	2,75%	3,00%	3,25%	3,50%	3,75%	4,00%	4,25%	4,50%	4,75%	5,00%
0,25%	0,046	0,048	0,050	0,052	0,054	0,055	0,057	0,059	0,060	0,062	0,063	0,064	0,066	0,067	0,068	0,069	0,070	0,071	0,072	0,073
0,50%	0,047	0,049	0,051	0,052	0,054	0,056	0,058	0,059	0,061	0,062	0,064	0,065	0,067	0,068	0,069	0,070	0,071	0,072	0,073	0,074
0,75%	0,047	0,049	0,051	0,053	0,055	0,057	0,059	0,060	0,062	0,063	0,065	0,066	0,068	0,069	0,070	0,071	0,072	0,073	0,074	0,075
1,00%	0,048	0,050	0,052	0,054	0,056	0,058	0,059	0,061	0,063	0,064	0,066	0,067	0,069	0,070	0,071	0,072	0,074	0,075	0,076	0,076
1,25%	0,048	0,051	0,053	0,055	0,057	0,059	0,060	0,062	0,064	0,065	0,067	0,068	0,070	0,071	0,072	0,074	0,075	0,076	0,077	0,078
1,50%	0,049	0,051	0,053	0,055	0,057	0,059	0,061	0,063	0,065	0,066	0,068	0,069	0,071	0,072	0,073	0,075	0,076	0,077	0,078	0,079
1,75%	0,050	0,052	0,054	0,056	0,058	0,060	0,062	0,064	0,066	0,067	0,069	0,070	0,072	0,073	0,075	0,076	0,077	0,078	0,079	0,080
2,00%	0,052	0,053	0,055	0,057	0,059	0,061	0,063	0,065	0,067	0,068	0,070	0,071	0,073	0,074	0,076	0,077	0,078	0,079	0,081	0,082
2,25%	0,054	0,055	0,056	0,058	0,060	0,062	0,064	0,066	0,067	0,069	0,071	0,072	0,074	0,075	0,077	0,078	0,079	0,081	0,082	0,083
2,50%	0,056	0,057	0,058	0,059	0,061	0,063	0,065	0,067	0,068	0,070	0,072	0,073	0,075	0,077	0,078	0,079	0,081	0,082	0,083	0,084
2,75%	0,058	0,059	0,060	0,061	0,062	0,063	0,065	0,067	0,069	0,071	0,073	0,075	0,076	0,078	0,079	0,081	0,082	0,083	0,084	0,085
3,00%	0,060	0,061	0,062	0,063	0,063	0,064	0,066	0,068	0,070	0,072	0,074	0,076	0,077	0,079	0,080	0,082	0,083	0,084	0,086	0,087
3,25%	0,062	0,063	0,063	0,064	0,065	0,066	0,067	0,069	0,071	0,073	0,075	0,077	0,078	0,080	0,081	0,083	0,084	0,086	0,087	0,088
3,50%	0,063	0,064	0,065	0,066	0,067	0,068	0,069	0,070	0,072	0,074	0,076	0,078	0,079	0,081	0,083	0,084	0,085	0,087	0,088	0,089
3,75%	0,065	0,066	0,067	0,068	0,069	0,070	0,071	0,072	0,073	0,075	0,077	0,079	0,080	0,082	0,084	0,085	0,087	0,088	0,089	0,091
4,00%	0,066	0,067	0,068	0,070	0,071	0,072	0,073	0,074	0,075	0,076	0,078	0,080	0,081	0,083	0,085	0,086	0,088	0,089	0,091	0,092
4,25%	0,068	0,069	0,070	0,071	0,072	0,073	0,074	0,076	0,077	0,078	0,079	0,081	0,082	0,084	0,086	0,087	0,089	0,091	0,092	0,093
4,50%	0,069	0,070	0,071	0,073	0,074	0,075	0,076	0,077	0,078	0,079	0,081	0,082	0,084	0,085	0,087	0,089	0,090	0,092	0,093	0,095
4,75%	0,070	0,072	0,073	0,074	0,075	0,076	0,078	0,079	0,080	0,081	0,082	0,083	0,085	0,086	0,088	0,090	0,091	0,093	0,094	0,096
5,00%	0,072	0,073	0,074	0,075	0,077	0,078	0,079	0,080	0,081	0,083	0,084	0,085	0,086	0,087	0,089	0,091	0,093	0,094	0,096	0,097

Fig. 3 shows graphs of the dependence of the optimal risk on the dollar interest rate d_d at the first 10 fixed values of the interest rate d_e for euro (the first 10 rows of Table 7). For the remaining 11 values of d_e the graphs are similar. The graphs for the columns of the table also look similar.

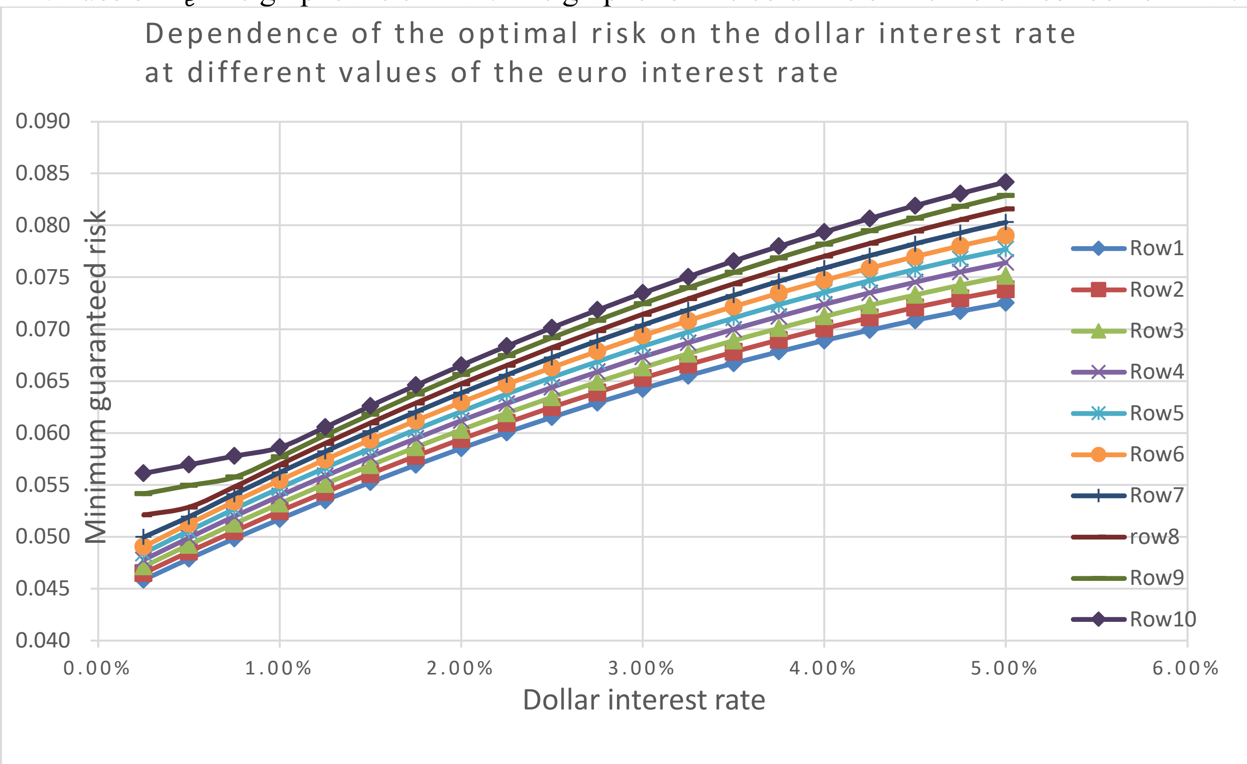

Fig 3. Graphs of the dependence of the optimal risk on the interest rate d_d for dollar at different values of the interest rate d_e for the euro. Row 1: $d_e = 0{,}25\%$, ..., Row 10: $d_e = 2{,}5\%$

The results give a partial idea of the capabilities of the proposed method and its software implementation, which gives an experienced Excel user other possibilities to analyze the problem under consideration by means of, for example, a "What-if" analysis.

CONCLUSION

The paper considers the problem of the optimal allocation of deposits to three of currencies (ruble, dollar, and euro) with uncertain future exchange rates of the dollar and euro. It is assumed that only the limits of possible changes in these uncertain parameters are known. The concept of Savage's minimax regret is used. According to this, the original problem is reduced to a minimax problem with the benefit lost due to uncertainty as the target function (the risk function). The risk function and the guaranteed risk function are found in explicit form. After that, the problem is reduced to finding the minimum point of a piecewise linear function under linear constraints.

Explicit formulas are obtained for nine "representative" candidate points, at least one of which is the optimal solution. The final choice is made by direct comparison of the criterion values at these points.

Computational tools have been developed in the Excel environment to find the structures of a three-currency deposit, optimal in terms of guaranteed risk, under conditions of unknown future exchange rates.

The use of this computing environment has allowed the development of tools that combine ease of use for individuals with minimal spreadsheet skills and extensive data analysis tools for advanced users.

The method is implemented in two variants – user and research. The first one is designed for those who have minimal skills of working in a ready-made Excel spreadsheet. This work is reduced to the input of the initial data and substantive analysis of the automatically obtained results. The second version contains a number of additional features: direct specification of the derived parameters of the profitability of individual currencies, multiple calculations varying one or two parameters of the problem, and a graphical visualization of the results.

Debugging and experimental calculations were performed. In particular, the case of the equal guaranteed profitability of all three currencies (at minimum future exchange rates) was analytically investigated and used for debugging the program. Numerical modeling of the dependence of the minimum guaranteed risk on the maximum possible future values of currency rates was carried out. A monotonic increase in risk for each of these variables was revealed.

The tables of dependence of the optimal deposit structure on the ruble interest rate (with other parameters fixed), as well as the tables of dependence of the optimal risk on the dollar and euro interest rates were constructed.

The results can be used in analyzing problems of financial management under conditions of incomplete information.

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