Teaching Chaos with a Pendulum to Greek Secondary School Students

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Abstract

In this paper we report on how a commercial "Chaos pendulum system" was used to teach aspects of chaos theory to Greek upper secondary school students. The principal aim of this project was to investigate to what extent students can develop an understanding of the chaotic behaviour using representations in the phase space.

The didactical methodology used was that of the "Physics Workshop" where students follow a discovery type laboratory course with detailed lab worksheets and are guided to the study of chaotic motion starting from observations of the oscillation of a simple physical pendulum.

Interesting conclusions concerning students' ability in using the technology associated with the microcomputer based laboratory (MBL), the development of a qualitative understanding of the chaotic behaviour using the representation in the phase space and in interpreting graphs can be drawn from the analysis of their recorded answers.

1 Why teaching Chaos?

The study of non-linear systems has become a major area of research in science as well as in philosophy. This research has led to a shift in the emphasis concerning fundamental scientific C philosophical concepts such as 'Determinism', 'Predictability', 'Causality', and 'Chance'.

There are systems, the chaotic pendulum is one of them, which are very 'sensitive' to small changes in the initial conditions when the process is running. Subtle changes of the initial parameters lead to a large deviation in the path followed so that detailed prediction of the system's behavior is not possible. Despite the irregular behavior of the system in the representation of the movement in the phase space a characteristic structure emerges indicating a kind of order.

A number of non-linear systems develop complex self-organizing structures. Self-organizing structures as well as structures that appears in the phase space could be described by the model of fractals. In this sense, chaos, self-organisation and fractals are intimately linked.

From an educational point of view this new domain has an important contribution to scientific literacy by developing a more adequate worldview and a more relevant view concerning the nature of science[1] . More specifically, teaching non-linearity to students makes them understand that the mechanistic worldview has been challenged, the physical world is not governed by simple deterministic laws leading to complete predictability, that causality and chance are an integral part of physical reality.

A considerable amount of research work in science education has been done on issues of non-linear systems in the last years[1-4] . Nevertheless there is a deficiency in empirical investigations concerning the possibility to teach and learn of aspects of the chaos theory using representations in the phase space. The principal aim of this research is to investigate to what extent students can develop an understanding of the chaotic behavior using representations in the phase space. For this task our approach adopts the MBL methodology, which allows a direct connection between the real experiment and the abstract graphical representation, based on the 'Workshop Physics' project using the PASCO chaotic pendulum system.

In the following paragraphs a short description of the MBL Methodology, the 'Workshop Physics' project and the PASCO chaotic pendulum system is given.

2 Microcomputer Based Laboratory C (MBL) Methodology

The development of the laboratories using MBL-tools has been inspired by the pedagogical approaches applied in 'RealTime Physics'[5] and 'Tools for Scientific Thinking'[6] and by research in Physics Education[7-9] .

For some decades sensors interfaced with a computer have been used in most physics research laboratories. An attachment of a sensor to a computer creates a very powerful system for collection, analysis and display of experimental data.

Today several systems, specially developed for schools and undergraduate courses, are commercially available. In a Microcomputer Based Laboratory, students do real experiments using different sensors connected to a computer via an interface. Experimental data from real-world experiment are immediately presented on the computer screen in a graphical format and can immediately be analysed. One of the main educational advantages of using MBL is the real-time display of experimental results and graphs thus facilitating direct connection between the real experiment and the abstract representation. Because data are quickly taken and displayed, students can easily examine the consequences of a large number of changes in experimental conditions during a short period of time. Brasell (1987)[10] has shown that even a very short delay in the presentation of experimental data is detrimental for student comprehension.

3 The 'Workshop Physics' course

The 'Workshop Physics' course has developed since 1986 at Dickinson College by P. Laws (2004) and is based on MBL methodology. The major objective of Workshop Physics courses is to help students understand the basis of knowledge in physics as an interplay between observations, experiments, definitions and mathematical description. The main idea of the course is to engage students in laboratory activities without any previous lecturing. In this discovery type laboratory course the students gain real-world experience using hands-on experiments. Students following "Workshop Physics" courses learn to use computer tools to collect data and to develop mathematical models of phenomena.

The Workshop Physics curriculum is centred on a workbook-style Activity Guide, that consists of twenty-eight units[11] . These units are based on activities that include predictions, qualitative observations, explanations, equation derivations, mathematical model building, quantitative experiments and problem solving.

Students in our sample class did activities selected from Unit 15 on "Oscillations, Determinism and Chaos". This unit includes a series of observations and experiments on a physical pendulum system that becomes increasingly complex and eventually appears to undergo chaotic behaviour.

The major advantage of the Workshop Physics for us is the 'step by step' approach used in Unit 15 where, starting from observations of the motion of the physical pendulum, students can gradually built up their knowledge up to a point of understanding chaotic behaviour.

4 The Chaotic Pendulum System

The Chaotic pendulum system by PASCO is one of the chaotic pendulums that are commercially available. A detailed comparison of the chaotic pendulums available has been published[12].

The PASCO Chaos pendulum can be used in two different configurations: a horizontal and a vertical. Both configurations have been analysed and modelled in the relevant literature. We have used the horizontal configuration, shown in Fig.1, a brief description of which follows in the next paragraphs.

The oscillator consists of an aluminum disk connected to two springs. A mass attached on the edge of the aluminum disk makes the oscillator non-linear. The frequency of the sinusoidal driver can be varied to investigate the progression from predictable motion to chaotic motion. Magnetic damping can be also adjusted to change the character of the chaotic motion. The angular position and velocity of the disk are recorded as a function of time using a Rotary Motion Sensor connected to a computer via an interface. Data are recorded and displayed

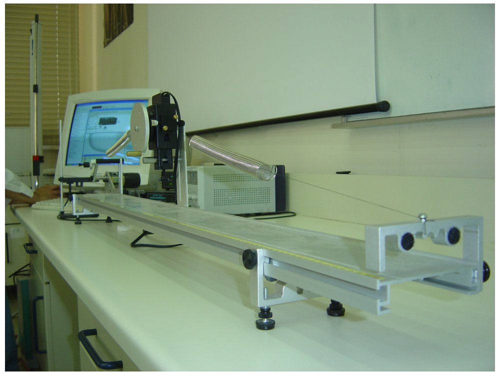


Fig.1 The PASCO chaos pendulum with the horizontal configuration

using the DATA STUDIO software.

The chaotic behaviour of the driven non-linear pendulum is explored by graphing its motion in phase space and by making a Poincare plot. These plots are compared to the motion of the pendulum when it is not chaotic.

A real time phase space diagram is made by graphing the angular velocity versus the displacement angle of the oscillation, while the Poincare plot is also graphed in real time and can be superimposed on the phase plot. This is achieved by recording the point on the phase plot once every cycle of the driver arm as the driver arm blocks a photogate.

The variables of the experiment (driving frequency, displacement amplitude and magnetic damping) can be varied so as to cause the regular motion to become chaotic.Students, using the DATA STUDIO software, can view the following graphs:

1. Angular Displacement (θ) vs. time (t)
2. Angular velocity (ω) vs. time (t)
3. Phase Space: Angular Velocity (ω) vs. Angular Displacement (θ)
4. Poincare Plot: Angular Velocity (ω) vs. Angular Displacement (θ) plotted only once per period of the driving force.

The phase space and the Poincare plot are particularly useful for recognizing chaotic oscillations. When the motion is chaotic, the graphs do not repeat.

5 Didactical Activities

Our sample class consists of six gifted final year secondary school students, with high scores in maths and physics, selected from a private school based in Athens and worked in two teams of three persons during three lab sessions of three hours each (a total of nine hours of lab instruction) in ASEL where the apparatus is based.

The students had not any previous knowledge either of the concepts or the graphical representations (eg Phase Space, Poincare mapping) associated with chaotic motion and they were not lectured on them before commencing on the lab course.

In accordance with the Greek upper secondary school science curriculum the students had a fairly good knowledge of simple harmonic motion, rotational dynamics and electromagnetism and so it was rather easy for them to understand the operation of the dc motor driver, the rotational sensor and the interface during a demonstration lab period conducted by the teacher.

The guidelines of the activities selected from Unit 15 on "Oscillations, Determinism and Chaos" of 'Workshop Physics' were modified to meet the tasks and the aim of the present study.

The students had to make observations, discuss the phenomena, give explanations and draw graphics, following the instructions in a worksheet designed by our group to meet the needs of the present research.

The performance of the students, both individually and as a team, has been evaluated from the questions completed in the laboratory worksheets and from the discussions among them and with the teacher.

During the lab period the teacher was giving the necessary information concerning the operation of the devices, analyzing the instructions for doing the experiment when necessary, and intervening only when the discussion among students had reached a dead end.

Below a detailed description of the procedure followed is given highlighting the 'step by step' approach and the students' performance in every step.

Activity One: Observation of the Oscillation of the Physical Pendulum.

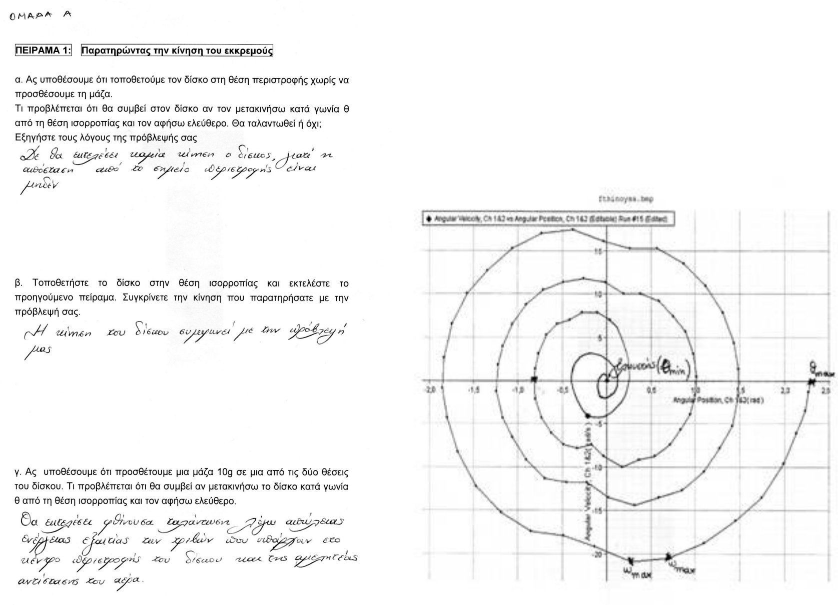
In the first activity students made a series of observations, at different oscillation angles, of the motion of the aluminium disk mounted on the low friction bearing with and without various quantities of mass attached to its edge. This disk was part of the physical pendulum system. The students keep a record of their observations and discuss about the oscillation of the physical pendulum (Fig.2).

Activity Two: Time Series and Phase Space Plot of the Physical Pendulum.

Next the rotary motion sensor was attached to a Universal Laboratory Interface. The students using the DATA STUDIO software could create a plot of the angular displacement of the pendulum as a function of time in real time (time series plot) as well as a plot of the phase space (angular velocity as a function of angular displacement - phase plot).

The students observe the time series graph, the phase plot graph and the experiment in real time.

It has to be emphasised that the students had no difficulty in understanding


ΟΜΑΔΑ Α

ΠΕΙΡΑΜΑ 1: Παρατηρώντας την κίνηση του εκκρεμούς

α. Ας υποθέσουμε ότι τοποθετούμε τον δίσκο στη θέση περιστροφής χωρίς να προσθέσουμε τη μάζα.
Τι προβλέπεται ότι θα συμβεί στον δίσκο αν τον μετακινήσω κατά γωνία θ από τη θέση ισορροπίας και τον αφήσω ελεύθερο. Θα ταλαντωθεί ή όχι;
Εξηγήστε τους λόγους της πρόβλεψής σας

Δε θα εκτελέσει καμία κίνηση ο δίσκος, γιατί η απόσταση από το σημείο περιστροφής είναι μηδέν

β. Τοποθετήστε το δίσκο στην θέση ισορροπίας και εκτελέστε το προηγούμενο πείραμα. Συγκρίνετε την κίνηση που παρατηρήσατε με την πρόβλεψή σας.

Η κίνηση του δίσκου συμφωνεί με την πρόβλεψή μας

γ. Ας υποθέσουμε ότι προσθέτουμε μια μάζα 10g σε μια από τις δύο θέσεις του δίσκου. Τι προβλέπεται ότι θα συμβεί αν μετακινήσω το δίσκο κατά γωνία θ από τη θέση ισορροπίας και τον αφήσω ελεύθερο.

Θα εκτελέσει φθίνουσα ταλάντωση λόγω απώλειας ενέργειας εξαιτίας των τριβών που υπάρχουν στο κέντρο περιστροφής του δίσκου και της αμελητέας αντίστασης του αέρα.

Fig.2 Observation of the oscillation of the physical pendulum

the phase space graph. They started from the graph of the time series, which is very familiar to them and they can easily connect it with the motion of the disk (experiment in the real world). Then they correspond specific points of the time series graph to the graph of the phase space (see Fig.3) and connect these with the oscillation in the real experiment.

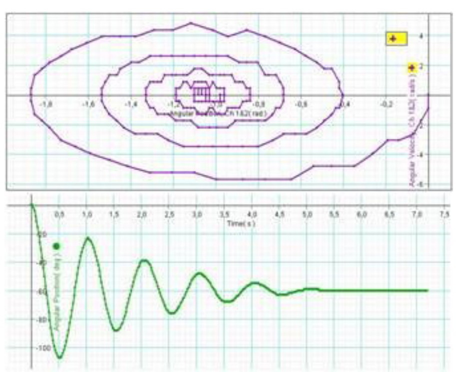


Fig. 3 Time series and phase space plot of the physical pendulum

Activity Three: Determination of the Natural Frequencies of the System. Students were asked to modify the physical pendulum so that a string with the two springs attached and the driver motor are coupled to the aluminium disk with the edge mass in place (see Fig.4).

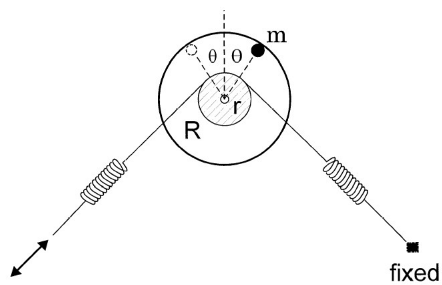


Fig.4 Determination of the natural frequencies of the system

Then with the motor turned off the students explored the natural frequencies of the system with and without the spring coupling and with and without the edge mass in place. The students using DATA STUDIO software create the time series graph $\omega(\mathrm{t})$ and take a direct reading of the natural frequency of the system in the various cases.

Then students activate the driver motor and they set the driver frequency very close to the natural frequencies of the physical pendulum system thus creating a situation "near resonance". The shape of the phase space graph is elliptic (see Fig.5).

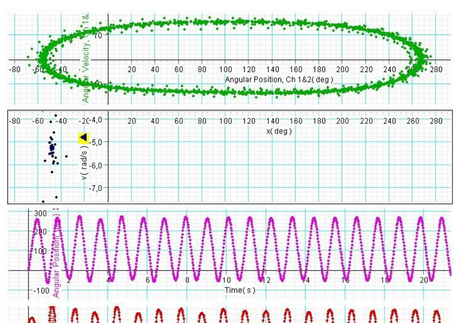


Fig.5 The shape of the phase space graph

"At resonance" the amplitude in the time series graph remains constant (within experimental error) while the shape of the graph in the phase space is a circle. The students had some difficulty in understanding the shape of the phase space graph and the teacher had to talk about the phase difference between $\theta(t)$ and

ω(t) in the corresponding time series graphs at resonance.

At this point the Poincare plot was introduced. A photogate was placed in the driver arm so as the plot can be produced. The students could not understand the physical significance and could not make any connection with the actual experiment despite the efforts of the teacher.

Activity Four: Exploring Conditions for Chaotic Motion-Sensitivity to initial conditions

Students were asked to place the damping magnet fairly close to the metal pendulum disk, activate the driver motion and set the driver frequency in a position that is different than any of the natural frequencies of the system and see if they can achieve a situation in which there is an irregular pattern in the time series graph (angular position vs. time). A typical pattern is shown in Fig.6 for the time series graph and the corresponding phase space plot.

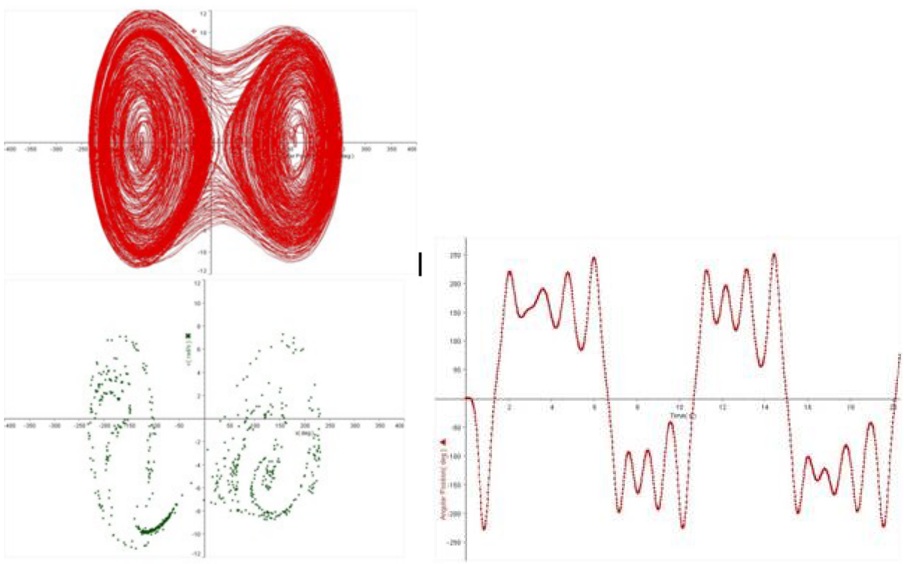


Fig.6 The time series graph and the corresponding phase space

An important observation in this sample activity was to have each participant attempt to repeat a pattern like that shown in Fig.6 by setting up the same initial conditions for the pendulum. All attempts failed after only a few seconds of run time. The students realize that this system is so sensitive to the initial values of angular position and angular velocity that it is impossible to recreate the initial conditions accurately enough to repeat a pattern on a graph.

In this activity students initially show serious difficulties to explain the final form of the graph in the phase space and connect it directly with the motion of the device. Nevertheless, they managed to give a qualitative explanation of the shape of the phase space graph by connecting it to the time series graph and then connect the motion of the pendulum with the time series graph.

More specifically, students connect the random number of peaks on the waveform in the time series graph with the number of oscillations of the pendulum. In the next step, they relate the random number of peaks on the waveform in the time series graph with the irregular way of shaping the graph in the phase space and by extension to the motion of the pendulum.

Concerning the Poincar plot, students can make use of the experimental apparatus and produce the plot, they can use the shape of the plot as a criterion to identify chaotic motion but they cannot relate it to phase space plot and/or understand the physical meaning of it despite the intervention of the teacher.

In order to clarify if and to what extent students had develop an understanding of the phenomena discussed during this laboratory period, the same students have been asked six months later to repeat the course. In this second phase, the main emphasis was on students' predictions about the process of the experiment and the kind of graphs expected. After the experiment was carried out students had to compare their predictions and their graphs with the graphs produced by the software based on the date collected from the apparatus and discuss the possible agreement or differences.

The results of this second phase showed that students were able to make predictions about the behaviour of the system which are very successful, and their graphs in most cases are similar to the graphs drawn by the software. Their explanations were also very satisfactory. In the following Fig.7 and 8, the graphs drawn by the two groups of students are presented concerning their prediction about phase space plots for the oscillation of the physical pendulum and chaotic motion.

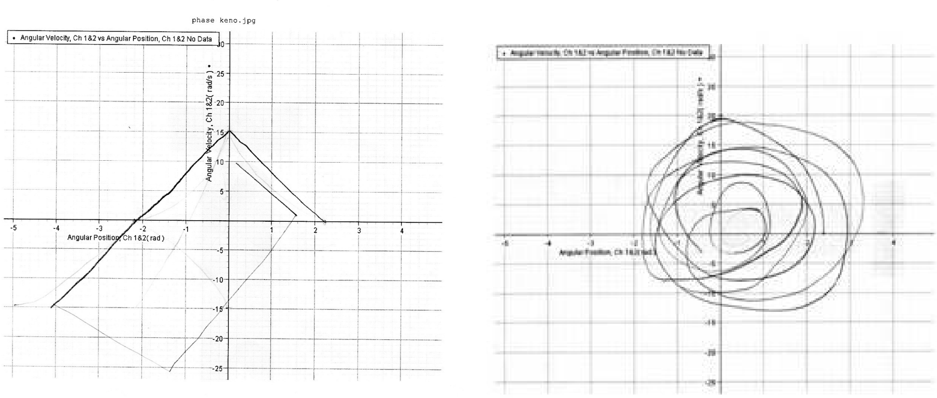


Fig.7 Phase space plots for the oscillation of the physical pendulum and chaotic-motion by group A

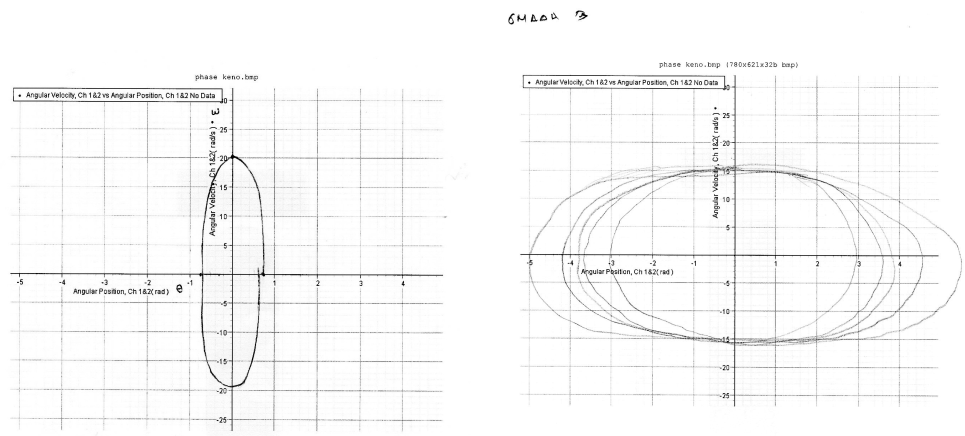


Fig.8 Phase space plots for the oscillation of the physical pendulum and chaotic-motion by group B

6 Discussion and Conclusions

The results presented here are based mainly on students' worksheets and the discussion, the teacher had with the students during and after the lab period. In sum, we can say, that despite the difficulties students have, they can go through these activities and develop up to a point a qualitative understanding of the chaotic behavior using the representation in the phase space.

The 'step by step' approach, ie starting from observing the motion of the simple physical pendulum and ending with the study of the complex motion of the chaotic pendulum system, in combination with the simultaneous presence of the real experiment and the corresponding graphs facilitate students' understanding.

The students had the opportunity to discuss the graphs of simple motions, which are up to a point familiar to them, and connect them with the real experiments. They connect the time series graph with the phase space graph and by extension with the motion of the pendulum. Based on the gained knowledge they could make also connections between the real experiment and the graphs in the case of the chaotic motion and give up to a point explanations of the observed complex behavior and the corresponding graphs. Interventions of the teacher are necessary and concern mainly the clarification of the graphs in the phase space.

This study gave insights about difficulties and opportunities of teaching a new field of physics using the MBL methodology. The results presented here are stem from a preliminary explorative study. Thus they need further elaboration and should be tested in a broader sample. A further investigation based on the results presented here is intended. This work also contributes in the field of pendulum studies[13] by revealing aspects of the educational dynamic of the pendulum. Apart from using the simple pendulum in cross disciplinary teaching giving students an understanding of the way that science has developed in relation to other forms of cultural life, the physical pendulum can be utilised to promote knowledge about the nature of science where causality and chance are characteristics of an integrated scientific worldview.

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