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Adaptive CFAR Tests for Detection and Recognition of Target Signals in Radar Clutter

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Abstract
In this paper, adaptive CFAR tests are described which allow one to classify radar clutter into one of several major categories, including bird, weather, and target classes. These tests do not require the arbitrary selection of priors as in the Bayesian classifier. The decision rule of the recognition techniques is in the form of associating the p-dimensional vector of observations on the object with one of the m specific classes. When there is the possibility that the object does not belong to any of the m classes, then this object is to be classified as belonging to one of the m classes or to class $m+1$ whose distribution is unspecified. The tests are invariant to intensity changes in the clutter background and achieve a fixed probability of a false alarm. The results obtained in this paper agree with the simulation results, which confirm the validity of the theoretical predictions of performance of the suggested adaptive CFAR tests.
Keywords Radar clutter, target signal, detection, recognition, adaptive CFAR tests

1 Introduction

Modern air traffic control radar systems rely heavily on automatic target detection and tracking to maximize air traffic safety. Moving target indicator and moving target detector algorithms achieve good target detection performance through the suppression of most or all forms of radar clutter. Unfortunately, real-time information on airborne hazards to aircraft, such as birds and storm systems, is also suppressed. The ability to classify clutter and hence identify these hazards can thus contribute significantly to air traffic safety.

The process of classification can be formalized as follows. The unprocessed radar data are passed through a feature extractor, which transforms the available data samples into a set of separable features. These features are derived from the reflection coefficients computed using the multisegment version of Burgs formula[1]. The aforementioned coefficients (that contain all spectral information, including the mean Doppler shift) are then transformed and grouped to satisfy the requirements for multivariate Gaussian behaviour. Only information that is different from class to class is maintained, and in such a form that a reliable decision, based on a discriminant function derived from the above features, may be made.

The classification problem consists in the following. There are m classes (populations), the elements (objects) of which are characterized by p measurements (features). Next, suppose that we are investigating a certain object on the basis of the corresponding p measurements. We postulate that this object can be regarded as a random drawing from one of the m populations but we do not know from which one. We suppose that m samples are available, each sample being drawn from a different class. The elements of these samples are realizations of p-dimensional normal random variables with unknown parameters. After a sample of p-dimensional vectors of observations on the object is drawn from a class known a priori to be one of the above set of m classes, the problem is to infer from which class the sample has been drawn. The decision rule should be in the form of associating the sample of observations on the object with one of the m samples and declaring that the object has come from the same class as the sample with which it is associated. When there is the possibility that the object does not belong to any of the m classes, then this object is to be classified as belonging to one of the m classes or to class $m + 1$ whose distribution is unspecified.

Stehwien and Haykin [2] solved the problem of statistical classification of radar clutter in a Bayesian framework. In this paper, the problem is treated in a non-Bayesian setting. A classification technique is described which allows one to classify radar clutter into one of several major categories, including bird, weather, and target classes. This technique is based on applying the theory of generalized maximum likelihood ratio testing for composite hypotheses. The unknown parameters are then estimated using maximum likelihood estimators. This approach does not require the arbitrary selection of priors as in the Bayesian classifier. Yet the generalized likelihood ratio test (GLRT) is widely preferred because of its nice asymptotic (large sample size) properties such as consistency, unbiasedness, and constant false alarm rate (CFAR). It is also called the uniformly most powerful invariant (UMPI) test since it exhibits the UMP property among the class of tests that are invariant to a natural set of transformations. The asymptotic performance of the GLRT becomes equivalent to the test with perfectly known parameters. The main feature of the proposed classification technique is the class elimination rule. When certain conditions are met, the decision is taken to eliminate specific class from further considerations, and the classification process is continued with a reduced number of classes. The class elimination rule is based on the generalized likelihood ratio.

The outline of the paper is as follows. A problem of signal detection in clutter is considered in Section 2. Section 3 is devoted to a problem of target signal recognition.

2 Signal detection in clutter

The problem of detecting the unknown deterministic signal s in the presence of a clutter process, which is incompletely specified, can be viewed as a binary hypothesis-testing problem. The decision is based on a sample of observation vectors $x_i = (x_{i1}, ..., x_{ip})', i = 1(1)n$, each of which is composed of clutter $w_i = (w_{i1}, ..., w_{ip})'$ under the hypothesisH_0 and a signal $s = (s_1, ..., s_{\mathrm{p}})'$ added to clutter w_{i} under the alternativeH_1, where $n > p$. The two hypotheses that the detector must distinguish are given by

$$H_0 : \mathbf{X} = \mathbf{W}(clutteralone) \tag{1}$$

$$H_1 : \mathbf{X} = \mathbf{W} + \mathbf{c}\mathbf{s}^{'}(signalpresent) \tag{2}$$

Where

$$\mathbf{X} = (\mathbf{x}_1, ..., \mathbf{x}_n)^{'} \tag{3}$$

$$\mathbf{W} = (\mathbf{w}_1, ..., \mathbf{w}_n)^{'} \tag{4}$$

are $n > p$ random matrices, and

$$\mathbf{c} = (1, ..., 1)^{'} \tag{5}$$

is a column vector of n units. It is assumed that $w_i, i = 1(1)n$, are independent and normally distributed with common mean 0 and covariance matrix (positive definite) $\mathbf{Q}$, i.e.

$$\mathbf{w}_i \sim N_p(\mathbf{0}, \mathbf{Q}), \forall i = 1(1)n. \tag{6}$$

Thus, for fixed n, the problem is to construct a test, which consists of testing the null hypothesis

$$H_0 : \mathbf{x}_i \sim N_p\left(\mathbf{0}, \mathbf{Q}\right), \forall i = 1(1)n. \tag{7}$$

versus the alternative

$$H_1 : \mathbf{x}_i \sim N_p\left(\mathbf{s}, \mathbf{Q}\right), \forall i = 1(1)n. \tag{8}$$

where the parameters $\mathbf{Q}$ and $\mathbf{s}$ are unknown.

Remark 1. Characterization of the multivariate normality is given by the following theorem.

Theorem 1 (Characterization of the multivariate normality). Let $x_i, i = 1(1)n$, be n independent p-multivariate random variables ($n \geq p+2$) with common mean bfa and covariance matrix (positive definite) bfQ. Let $z_k, k = p+2, , n$, be defined by

$$z_k = \frac{k-(p+1)}{p}\frac{k-1}{k}(x_k - \bar{x}_{k-1})' S_{k-1}^{-1}\left(x_k - \bar{x}_{k-1}\right)$$

$$= \frac{k-(p+1)}{p}\left(\frac{|S_k|}{|S_{k-1}|}-1\right), k = p+2,...,n, \tag{9}$$

where

$$\bar{x}_{k-1} = \sum_{i=1}^{k-1} x_i/(k-1), \tag{10}$$

$$S_{k-1} = \sum_{i=1}^{k-1} (x_i - \bar{x}_{k-1})(x_i - \bar{x}_{k-1})', \tag{11}$$

then the $\mathbf{x}_i$ $(i = 1, \ldots, n)$ are $N_p(\mathbf{a}, \mathbf{Q})$ if and only if $z_{p+2}, ..., z_n$ are independently distributed according to the central$F$ distribution with p and $1, 2, ...n-(p+1)$ degrees of freedom, respectively.

Proof. The proof is similar to that of the characterization theorems [3-4] and so it is omitted here.

2.1 Goodness-of-fit testing for the multivariate normality

The results of Theorem 1 can be used to obtain test for the hypothesis of the form H_0 : $\mathbf{x}_i$ follows $N_p(\mathbf{a}, \mathbf{Q})$ versus H_a : $\mathbf{x}_i$ does not follow $N_p(\mathbf{a}, \mathbf{Q}), \forall i = 1(1)n$ The general strategy is to apply the probability integral transforms [5] of $z_k, \forall k = p+2\,(1)\,n$, to obtain a set of i.i.d. $U(0,1)$ random variables under H_0. Under H_a this set of random variables will, in general, not be i.i.d. $U(0,1)$. Any statistic, which measures a distance from uniformity in the transformed sample (say, a Kolmogorov-Smirnov statistic), can be used as a test statistic.

2.2 GMLR statistic and its distribution

One of the possible statistics for testing H_0 versus H_1 is given by the generalized maximum likelihood ratio (GMLR)

$$GMLR = \max_{\theta\in\Theta_1} L_{H_1}(X;\theta) \Big/ \max_{\theta\in\Theta_0} L_{H_0}(X;\theta), \tag{12}$$

where $\mathbf{q} = (\mathbf{s}, \mathbf{Q}), \mathbf{Q}_0 = \{(\mathbf{s}, \mathbf{Q}) : \mathbf{s} = \mathbf{0}, \mathbf{Q} \in Q_p\}, \mathbf{Q}_1 = \mathbf{Q} - \mathbf{Q}_0, \mathbf{Q} = \{(\mathbf{s}, \mathbf{Q}) : \mathbf{s} \in \mathbf{R}^p, \mathbf{Q} \in Q_p\}, Q_p$ denotes the set of $p \times p$ positive definite matrices. Under H_0, the joint likelihood for $\mathbf{X}$ based on (7) is

$$L_{H_0}(X;\theta) = (2\pi)^{-np/2}|Q|^{-n/2}\exp\left(-\sum_{i=1}^{n} x'_i Q^{-1} x_i/2\right)., \tag{13}$$

Under H_1, the joint likelihood for $\mathbf{X}$ based on (8) is

$$L_{H_1}(X;\theta) = (2\pi)^{-np/2}|Q|^{-n/2}\exp\left(-\sum_{i=1}^{n} (x_i - s)'Q^{-1}(x_i - s)/2\right). \tag{14}$$

It can be shown that

$$GMLR{=}\left|\hat{Q}_0\right|^{n/2}\left|\hat{Q}_1\right|^{-n/2}, \tag{15}$$

where

$$\hat{Q}_0{=}X'X/n, \tag{16}$$

$$\hat{Q}_1{=}\ (X'-\hat{s}c')(X'-\hat{s}c')'/n, \tag{17}$$

and

$$\hat{s}{=}X'c/n \tag{18}$$

are the well-known maximum likelihood estimators of the unknown parameters $\mathbf{Q}$ and $\mathbf{s}$ under the hypotheses H_0 and H_1, respectively. It can be shown, after some algebra, that (15) is equivalent finally to the statistic

$$y = T_1' T_2^{-1} T_1/n, \tag{19}$$

where $\mathbf{T}_1 = \mathbf{X}'\mathbf{c}, \mathbf{T}_2 = \mathbf{X}'\mathbf{X}$.It is known that$(\mathbf{T}_1, \mathbf{T}_2)$ is a complete sufficient statistic for the parameter $\mathbf{q} = (\mathbf{s}, \mathbf{Q})$ Thus, the problem has been reduced to consideration of the sufficient statistic $(\mathbf{T}_1, \mathbf{T}_2)$. It can be shown that under H_0 , the result (19) is a $\mathbf{Q}$-free statistic y which has the property that its distribution does not depend on the actual covariance matrix $\mathbf{Q}$. This is given by the following theorem.

Theorem 2 (PDF of the GMLR statistic y). Under H_0, the statistic y is subject to a noncentral beta-distribution with the probability density function (PDF)

$$f_{H_1}(y;n,q) = \left[\mathrm{B}\left(\tfrac{p}{2}, \tfrac{n-p}{2}\right)\right]^{-1} y^{\left(\frac{p}{2}\right)-1}(1-y)^{\left(\frac{n-p}{2}\right)-1}$$
$$\times e^{-q/2}{}_1F_1\left(\frac{n}{2};\frac{p}{2};\frac{qy}{2}\right), 0 < y < 1, \tag{20}$$

where $F_1(a;b;x)$ is the confluent hypergeometric function [6],

$$q = n\left(s'Q^{-1}s\right) \tag{21}$$

is a noncentrality parameter representing the generalized signal-to-noise ratio (GSNR). Under H_0, when $q = 0$, (20) reduces to a standard beta-function density of the form

$$f_{H_0}(y;n) = \left[\mathrm{B}\left(\frac{p}{2}, \frac{n-p}{2}\right)\right]^{-1} y^{\left(\frac{p}{2}\right)-1}(1-y)^{\left(\frac{n-p}{2}\right)-1}, 0 < y < 1. \tag{22}$$

Proof. The proof is given by Nechval [7] and so it is omitted here.

It is clear that the statistic y is equivalent to the statistic

$$v = \left[(n-p)/p\right]y/(1-y) = \left[n(n-p)/p\right]\left(\hat{s}'\left[\hat{G}_1\right]^{-1}\hat{s}\right), \tag{23}$$

where

$$\hat{G}_1 = n\hat{Q}_1 = (X' - \hat{s}c')(X' - \hat{s}c')' = \sum_{i=1}^{n} (x_i - \hat{s})(x_i - \hat{s})'. \tag{24}$$

Here the following theorem clearly holds.

Theorem 3 (PDF of the GMLR statistic v). Under H_1, the statistic v is subject to a noncentral F-distribution with p and $n-p$ degrees of freedom, the probability density function of which is

$$f_{H_1}(v; n, q) = \left[\mathrm{B}\left(\frac{p}{2}, \frac{n-p}{2}\right)\right]^{-1} \frac{\left(\frac{p}{n-p}\right)^{p/2} v^{p/2-1}}{\left(1 + \frac{p}{n-p}v\right)^{n/2}}$$

$$\times\, e^{-q/2} {}_1F_1\left(\frac{n}{2}; \frac{p}{2}; \frac{q}{2}\left(\frac{p}{n-p}v\left(1 + \frac{p}{n-p}v\right)^{-1}\right)\right), 0 < v < \infty, \tag{25}$$

where q is a noncentrality parameter given by (21). Under H_0, when $q = 0$, (25) reduces to a standard F-distribution with p and $n - p$ degrees of freedom,

$$f_{H_0}(v; n) = \left[\mathrm{B}\left(\tfrac{p}{2}, \tfrac{n-p}{2}\right)\right]^{-1} \frac{\left(\frac{p}{n-p}\right)^{p/2} v^{p/2-1}}{\left(1 + \frac{p}{n-p}v\right)^{n/2}}, 0 < v < \infty, \tag{26}$$

Proof. The proof follows by applying Theorem 2 and being straightforward it is omitted.

2.3 Test for signal detection based on the GMLR statistic

The test of H_0 versus H_1, based on the GMLR statistic v, is given by

$$v \begin{cases} > h, \ then \ \ H_1 \ \ (signalpresent), \\ \leq h, \ then \ \ H_0 \ \ (clutteralone), \end{cases} \tag{27}$$

and can be written in the form of a decision rule u(v) over $\{v : v \in (0, \infty)\}$,

$$u(v) = \begin{cases} 1, \ v > h \quad (H_1), \\ 0, \ v \leq h \quad (H_0), \end{cases} \tag{28}$$

where $h > 0$ is a threshold of the test which is uniquely determined for a prescribed level of significance α so that

$$\sup_{\theta \in \Theta_0} E_\theta \{u(v)\} = \alpha. \tag{29}$$

For fixed n, in terms of the probability density function (26), tables of the central F-distribution permit one to choose h to achieve the desired test size (false alarm probability P_{FA}),

$$P_{\text{FA}} = \alpha = \int_{h}^{\infty} f_{H_0}(v;n)dv. \tag{30}$$

Furthermore, once h is chosen, tables of the noncentral F-distribution permit one to evaluate, in terms of the probability density function (25), the power (detection probability P_D) of the test,

$$P_{\text{D}} = \gamma = \int_{h}^{\infty} f_{H_1}(v;n,q)dv. \tag{31}$$

The probability of a miss is given by

$$\beta = 1 - \gamma. \tag{32}$$

It follows from (30) that the GMLR test is invariant to intensity changes in the clutter background and achieves a fixed probability of a false alarm, i.e. the resulting analyses indicate that the test has the property of a constant false alarm rate (CFAR). Also, no learning process is necessary in order to achieve the CFAR. Thus, operating in accordance to the local clutter situation, the test is adaptive.

When the parameter $\mathbf{q} = (\mathbf{s}, \mathbf{Q})$ is unknown, it is well known that no the uniformly most powerful (UMP) test exists for testing H_0 versus H_1 [8]. However, some hypothesis testing problems that do not admit UMP decision rules (tests) nevertheless exhibit certain natural invariance properties [8-9]. These properties suggest restricting attention to a limited class of decision rules, viz., the invariant decision rules. It is then sometimes possible to derive decision rules that are UMP within this limited class. In this sense, invariance is a concept of fundamental importance in hypothesis testing. The following theorem shows that the test (27) is UMPI for a natural group of transformations on the space of observations.

Theorem 4 (UMPI test). For testing the hypothesis $H_0(1)$ versus the alternative $H_1(2)$, the CFAR test given by (27) is uniformly most powerful invariant (UMPI).

Proof. The proof is similar to that of Nechval [10] and so it is omitted here.

A robustness property of the v-test can be studied in the following set-up. Let $\mathbf{X} = (\mathbf{x}_1, ..., \mathbf{x}_n)'$ be an $n \times p$ random matrix with a PDF φ,let C_{np} be the class of PDF s on $\mathbf{R}^{np}$ with respect to Lebesque measure $d\mathbf{X}$, and let H be the set of nonincreasing convex functions from $[0, \infty)$ into $[0, \infty)$. We assume $n \geq p + 1$.

For $\mathbf{s} \in \mathbf{R}^p$ and $\mathbf{Q} \in Q_p$ define a broad class of PDFs on $\mathbf{R}^{np}$ as follows

$$C_{np}(s,Q) = \left\{ \begin{array}{c} f \in C_{np} : f(X;s,Q) \\ = |Q|^{-n/2}\eta\left(\sum_{i=1}^{n}(x_i - s)'Q^{-1}(x_i - s)\right), \eta \in \mathrm{H} \end{array} \right\} \tag{33}$$

In this model, it can be considered the testing problem

$$H_0 : \phi \in C_{np}(0,Q), Q \in \mathrm{Q}_p \tag{34}$$

versus

$$H_1 : \phi \in C_{np}(s,Q), s \neq 0, Q \in \mathrm{Q}_p \tag{35}$$

and shown that v-test is UMPI. Clearly if $(\mathbf{x}_1, ..., \mathbf{x}_n)$ is a random sample of $\mathbf{x}_i \sim N_p(\mathbf{s},\mathbf{Q}), i = 1(1)n$, or $\mathbf{X} \sim N_{np}(\mathbf{cs}', \mathbf{I}_n \otimes \mathbf{Q})$, the PDF φ of $\mathbf{X}$ belongs to $C_{np}(\mathbf{s},\mathbf{Q})$. Further if $f(\mathbf{X};\mathbf{s},\mathbf{Q}))$ belongs to $C_{np}(\mathbf{s},\mathbf{Q})$, then

$$g_*(X;s,Q) = \int_0^\infty f(X;s,rQ)dG_*(r) \tag{36}$$

also belongs to $C_{np}(\mathbf{s},\mathbf{Q})$ where G_* is a distribution function on $(0,\infty)$, and so $C_{np}(\mathbf{s},\mathbf{Q})$ contains the (np-dimensional) multivariate t-distribution, the multivariate Cauchy distribution, the contaminated normal distribution, etc. [10-12]. Here the following theorem holds.

Theorem 5 (Robustness property). For problem (34)-(35), the CFAR v-test is UMPI and the null distribution of v is F-distribution with d.f.s p and $n-p$, i.e., the CFAR test is still UMPI in a broad class of distributions given by (33), and the null distribution under any member of the class is the same as that under normality.

Proof. The proof is similar to that of Nechval [10] and so it is omitted here.

2.4 Risk minimization

For fixed n, in terms of the above probability density functions in (25) and (26), the probability of making the first type of wrong decision (false alarm probability) is found by

$$\alpha(h;n) = \int_h^\infty f_{H_0}(v;n)dv \tag{37}$$

and the probability of making the second type of wrong decision (the probability of a miss) by

$$\beta(h;n,q) = \int_0^h f_{H_1}(v;n,q)dv. \tag{38}$$

Any value of $\mathbf{s}$ will result in a value for q that is greater than zero. As the value of $\mathbf{s}$ increases, the value of q will also increase. A good detector is certainty expected to minimize α and β in some manner. For example, the Neyman-Pearson criterion defines optimality to be that of maximizing $1-\beta$ subject to the constraint that $\alpha \leq \alpha_0$, where α_0 is a fixed constant between zero and unity. For this criterion, the optimum threshold can be found from (30). In general the structure of an optimum detector depends on the signal (or the signal-to-noise ratio).

Let us assume that a noncentrality parameter q representing the generalized signal-to-noise ratio (GSNR) is given. If we let w_α and w_β be the unit weight (cost) of the probability of making the first type of wrong decision (α) and the probability of making the second type of wrong decision (β), respectively, then the optimal threshold of test, h^*, can be found by solving the following optimization problem (with respect to h):

Minimize

$$R(h;n,q) = w_\alpha \alpha(h;n) + w_\beta \beta(h;n,q) \tag{39}$$

subject to

$$h \in (0,1), \tag{40}$$

where $R(h;n,q)$ is a risk representing the weighted sum of the false alarm risk and the miss risk. It can be shown that h^* satisfies the equation

$$w_\alpha f_{H_0}(h^*;n) = w_\beta f_{H_1}(h^*;n,q). \tag{41}$$

Generally, the miss risk is more important that the false alarm risk, so that $w_a \leq w_b$.

If the sample size of observations, n is not bounded above, then the optimal value n^* of n can be found as

$$n^* = \inf n : \left(\begin{array}{l} \alpha(h^*;n)+\beta(h^*;n,q) \leq \vartheta, \\ h^*= \arg \min\limits_{h\in(0,1)} R(h;n,q) \end{array} \right), \tag{42}$$

where ϑ is a preassigned value of the sum of the false alarm risk and the miss risk.

3 Target signal recongnition

Suppose that the hypothesis H_0: (clutter alone) is rejected. Then the target (signal in clutter) classification problem using the target identity information consists in the following. Let the target signal belong to one of m classes and each class has equal a priori probability. There is available a sample of radar measurements of size n from each class. The elements of the sample from the jth class are realizations of p-dimensional random variables $\mathbf{s}_i(j) \sim N_p(\mathbf{s}(j), \mathbf{Q}(j)), i = 1(1)n,$

with unknown parameters $\mathbf{s}(j)$ and $Q(j)$ for each $j \in \{1, \ldots, m\}$. We are investigating a detected target signal on the basis of the corresponding sample of size n of p-dimensional radar measurements $\mathbf{r}_i = (r_{i1}, ..., r_{ip})', i = 1(1)n$, where $r_i \sim N_p(\mathbf{s}, \mathbf{Q})$. We postulate that this target signal can be regarded as a random drawing from one of the m classes but we do not know from which one. The problem is to classify a detected target signal as belonging to one of the m specified classes. When there is the possibility that a target signal does not belong to any of the m above classes, it is desirable to recognize this case.

Let $\mathbf{r}_i$ and $\mathbf{s}_i(j)$ be the ith observation of the target and jth class variable, respectively. It is assumed that all observation vectors,$\mathbf{r}_i = (r_{i1}, ..., r_{ip})', \mathbf{s}_i(j) = (s_{i1}(j), ..., s_{ip}(j))', i = 1(1)n$, are independent of each other, where n is a number of paired observations. Let $\mathbf{x}_i(j) = \mathbf{r}_i - \mathbf{s}_i(j), i = 1(1)n$, be paired comparisons leading to a series of vector differences. Thus, for classification of a detected target signal as belonging to the jth class, it can be obtained and used a sample of n independent observation vectors $\mathbf{X}(j) = (\mathbf{x}_1(j), ..., \mathbf{x}_n(j)), j \in \{1, \ldots, m\}$. It is assumed that under $H_0(j)$,$\mathbf{x}_i(j) \sim N_p(\mathbf{0}, \mathbf{Q} + \mathbf{Q}(j)), \forall i = 1(1)n$, where $\mathbf{Q} + \mathbf{Q}(j)$ is a positive definite covariance matrix. Under $H_1(j)$,
$\mathbf{x}_i(j) \sim N_p(\mathbf{a}(j), \mathbf{Q} + \mathbf{Q}(j))$,$\forall i = 1(1)n$, where $\mathbf{a}(j) = (a_1(j), ..., a_p(j))' \neq (0, ..., 0)'$ is a mean vector. For fixed n, the problem is to construct a test which consists of testing the null hypothesis $H_0(j) : \mathbf{x}_i(j) \sim N_p(\mathbf{0}, \mathbf{Q} + \mathbf{Q}(j)), \forall i = 1(1)n$, versus the alternative $H_1(j) : \mathbf{x}_i(j) \sim N_p(\mathbf{a}(j), \mathbf{Q} + \mathbf{Q}(j)), \forall i = 1(1)n$, where the parameters $\mathbf{a}(j), \mathbf{Q}$ and $\mathbf{Q}(j)$ are unknown. The CFAR test of $H_0(j)$ versus $H_1(j)$ $(j \in \{1, ..., m\})$ is based on the statistic given by (23),

$$v(j) = [n(n-p)/p] \left(\hat{a}'(j) \left[\hat{G}_1(j) \right]^{-1} \hat{a}(j) \right), \tag{43}$$

where

$$\begin{aligned} \hat{G}_1(j) &= (X'(j) - \hat{a}(j)c')(X'(j) - \hat{a}(j)c')' \\ &= \sum_{i=1}^{n} (x_i(j) - \hat{a}(j))(x_i(j) - \hat{a}(j))'. \end{aligned} \tag{44}$$

The test of H_0 versus H_1, based on the GMLR statistic $v(j)$, is given by

$$v(j) \begin{cases} > h(j), \ then \ H_1(j) \ (targetdoesnotbelongtoclassj), \\ \leq h(j), \ then \ H_0(j) \ (targetbelongstoclassj), \end{cases} \tag{45}$$

where $h(j) > 0$ is a threshold of the test which is uniquely determined for a prescribed level of significance $\alpha(j)$ so that

$$\sup_{\theta(j) \in \Theta_0(j)} E_{\theta(j)} \{u(v(j))\} = \alpha(j), \tag{46}$$

where $\theta\left(j\right)=\left(\mathbf{a}\left(j\right),\mathbf{Q}+\mathbf{Q}\left(j\right)\right),\Theta_0\left(j\right)=\left\{\left(\mathbf{a}\left(j\right),\mathbf{Q}+\mathbf{Q}\left(j\right)\right):\mathbf{a}\left(j\right)=\mathbf{0},\left(\mathbf{Q}+\mathbf{Q}\left(j\right)\right)\in Q_p\right\}$,

$$u(v(j))=\begin{cases}1, & v(j)>h(j)\quad (H_1(j)),\\ 0, & v(j)\le h(j)\quad (H_0(j)).\end{cases} \tag{47}$$

Thus, if $v(j)>h(j)$ then the jth target class is eliminated from further consideration.

If $(m-1)$ target classes are so eliminated, then the remaining class (say, kth) is the one to which a detected target signal being classified belongs.

If all the target classes are eliminated from further consideration, we decide that a detected target signal belongs to the $(m+1)$th class whose distribution is unspecified.

If the set of target classes not yet eliminated has more than one element, then we declare that a detected target signal belongs to the class j^* if

$$j*=\arg\max_{j\in D}\left(h(j)-v(j)\right), \tag{48}$$

where D is the set of target classes not yet eliminated by the above test.

Now consider the situation in which a detected target signal $\mathbf{s}$ is related to the true target signal of the jth class, $\mathbf{s}(j)$, by

$$\mathbf{s}=u\mathbf{s}(j)=u(\mathbf{s}_1(j),...\mathbf{s}_p(j))',j\in\{1,...,m\} \tag{49}$$

where ν is a scalar amplitude parameter. It is assumed that the target signal vectors $s(j),j=1(1)m$, are known. The generalized maximum likelihood ratio statistics for this recognition problem are given by

$$\max_{\upsilon}\left\{\max_{Q}L_{H_1(j)}(X;\upsilon,Q)\right\}\Big/\max_{Q}L_{H_0(j)}(X;Q), \tag{50}$$

where

$$L_{H_0(j)}(X;Q)=(2\pi)^{-np/2}|Q|^{-n/2}\exp\left(-\sum_{i=1}^{n}x'_iQ^{-1}x_i/2\right), \tag{51}$$

$$L_{H_1(j)}(X;\upsilon,Q)=(2\pi)^{-np/2}|Q|^{-n/2}\exp\left(-\sum_{i=1}^{n}(x_i-\upsilon s(j))'Q^{-1}(x_i-\upsilon s(j))/2\right) \tag{52}$$

are the likelihood functions under $H_0(j)$ and $H_1(j),j\in\{1,\ldots,m\}$, respectively, and

$$\begin{aligned}&\max_{\upsilon}\{\max_{Q}L_{H_1(j)}(X;\upsilon,Q)\}\\&=\max_{\upsilon}\frac{1}{(2\pi)^{np/2}\left|\widehat{Q}_1(j)\right|^{n/2}}\exp\left(-\frac{np}{2}\right),\end{aligned} \tag{53}$$

$$\max_{Q} L_{H_0(j)}(X;Q) = \frac{1}{(2\pi)^{np/2}\left|\widehat{Q}_0\right|^{n/2}} \exp\left(-\frac{np}{2}\right). \tag{54}$$

The well-known maximum likelihood estimates (MLEs) of the unknown covariance matrix $\mathbf{Q}$ under the respective hypotheses, $H_0(j)$ and $H_1(j)$, are given by

$$\widehat{Q}_0 = \frac{1}{n}\sum_{i=1}^{n} x_i {x'}_i = \frac{1}{n}XX', \tag{55}$$

$$\widehat{Q}_1(j) = \frac{1}{n}\sum_{i=1}^{n} (x_i - \upsilon s(j))(x_i - \upsilon s(j))'$$

$$= \frac{1}{n}(X - \upsilon s(j)c')(X - \upsilon s(j)c')'. \tag{56}$$

After several algebraic manipulations, (28) reduces to the following clutter-adaptive test of detection of the jth target signal, $j \in \{1, \ldots, m\}$:

$$\begin{cases} > h(j), & then \quad H_1(j), \\ \leq h(j), & then \quad H_0(j), \end{cases} \tag{57}$$

where

$$z(j) = \frac{\left[s'(j)(XX')^{-1}Xc\right]^2}{[s'(j)(XX')^{-1}s(j)][1 - c'X'(XX')^{-1}Xc]}, \tag{58}$$

$h(j) > 0$ is a threshold of the test which is uniquely determined for a prescribed level of significance $\alpha(j)$ so that the probability of a false alarm is equal to $\alpha(j)$.

Theorem 6 (PDF of the GMLR statistic $v(j)$). The probability density function of $v(j)$ under hypothesis $H_1(j)$ is given as follows:

$$f_{H_1(j)}(v(j); n, q(j)) = \int_0^1 f(v(j);g, n, q(j)) f(g; n) dg, \tag{59}$$

where

$$f(g;n) = \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n-p}{2}\right)\Gamma\left(\frac{p-1}{2}\right)} (1-g)^{\frac{p-3}{2}} g^{\frac{n-p-2}{2}}, \tag{60}$$

for $0 \leq g \leq 1$, and

$$f(v(j); g, n, q(j)) = \frac{\Gamma\left(\frac{n-p+1}{2}\right)\exp\left(-\frac{q(j)g}{2}\right)}{\Gamma\left(\frac{n-p}{2}\right)\Gamma\left(\frac{1}{2}\right)}$$

$$\times[1-v(j)]^{\frac{n-p-2}{2}}[v(j)]^{-\frac{1}{2}}{}_1F_1\left(\frac{n-p+1}{2};\frac{1}{2};\frac{q(j)gv(j)}{2}\right) \tag{61}$$

for $0 < v(j) < 1$. In (61) ${}_1F_1\,(a;b;x)$ is the confluent hypergeometric function, and $q(j)$ is the generalized signal-to-noise ratio (GSNR) defined by

$$q(j) = \mathrm{GSNR} = nv^2 s'(j)Q^{-1}s(j). \tag{62}$$

Under hypothesis $H_0(j)$, no signal is present. Thus, if one sets $q(j) = 0$ in (61),

$$f_{H_0(j)}(v(j);n) = \frac{\Gamma\left(\frac{n-p+1}{2}\right)}{\Gamma\left(\frac{n-p}{2}\right)\Gamma\left(\frac{1}{2}\right)}[1-v(j)]^{\frac{n-p-2}{2}}[v(j)]^{-\frac{1}{2}}, 0 < v(j) < 1. \tag{63}$$

Proof. The proof is similar to that of Theorem 2 and so it is omitted here. Finally, in terms of the above probability density functions in (59) and (63) the probability of false alarm is given by

$$P_{\mathrm{FA}}(j) = \int_{h(j)}^{1} f_{H_0(j)}(v(j);n)dv(j) \tag{64}$$

and the probability of detection of the jth target signal is

$$P_{\mathrm{D}}(j) = \int_{h(j)}^{1} f_{H_1(j)}(v(j);n,q(j))dv(j). \tag{65}$$

Thus, if $v(j) < h(j)$ then the jth target class is eliminated from further consideration.

If $(m-1)$ target classes are so eliminated, then the remaining class (say, kth) is the one to which a detected target being classified belongs.

If all the target classes are eliminated from further consideration, we decide that we deal with a clutter alone.

If the set of target classes not yet eliminated has more than one element, then we declare that a detected target belongs to the class j^* if

$$j^* = \arg\max_{j\in D}(v(j) - h(j)), \tag{66}$$

where D is the set of target classes not yet eliminated by the above test.

4 Conclusion

The main idea of this paper is to find a test statistic whose distribution, under the null hypothesis, does not depend on unknown (nuisance) parameters. This allows one to eliminate the unknown parameters of noisy processes, which are changing with time and position, from the problem.

The authors hope that this work will stimulate further investigation using the approach on specific applications to see whether obtained results with it are feasible for realistic applications.

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