

Modeling of Consensus in Bilateral Trade Negotiations

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Abstract: Any trade transaction is preceded by negotiations between at least two parties, under which a sale and purchase are made. This work studies the factors that influence the time it takes for the parties to reach mutual agreement in the presence of coalitions. De Groot's model of the process of achieving consensus based on regular Markov chains is presented in the work. It is built on the assumption that participants in the negotiation process exchange opinions and can influence the point of view of other members of the negotiating parties during the discussion. The main advantage of such a model is the ability to identify and study the key characteristics of the negotiating parties. The obtained modeling results helped to formulate practical recommendations for making an agreed-upon decision within the framework of trade negotiations. Analysis of the current model of face-to-face negotiations between the two parties creates softer conditions for the composition of the group, and the negotiations themselves contribute to the development of new ways to take into account the interests of both sides.

Keywords: trade negotiations, consensus, coalitions, regular Markov chains, number of negotiations.

1. INTRODUCTION

1.1. Historical Aspect of the Development of Trade Negotiations and Their Role in Modern Economic Development

Trade negotiations are one of the oldest types of human activity aimed at solving problems when organizing cooperation or resolving conflict situations in the trade process. They have the same ancient history as conflicts and wars and were used in practice long before the formation of legal procedures.

The first information about the existence of trade relations can be found in ancient times. Already three and a half millennia BC, the first king of Assyria-Babylonia from the Sumerian dynasty established trade relations with the north and south from his capital, Sirtella. The prosperity of today's civilization owes much to international and national trade.

Any trade transaction is preceded by negotiations between at least two parties, under which a sale and purchase are made. There are many historical examples of such negotiations, from antiquity to the present day.

The first attempts to describe the negotiation process are associated with the name Francois de Callières (1645–1717), a French diplomat and advisor to French King Louis XIV. Callières created the concept of diplomacy based on the absolute priority of negotiations between equal partners over force and violence, manifested in its various forms in interstate relations. He defined the goals, objectives, forms, and methods of the negotiation process and

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other aspects of professional diplomacy [6]. Although his book was devoted to diplomatic negotiations, it can rightfully (with some reservations) be attributed to the trade sphere.

The heart of a trade deal is negotiation. Modern researchers of the negotiation process focus on such aspects of this activity as communications [21, 26], strategy and tactics of negotiations [27], fair equilibrium prices [24, 28], the best choice from alternatives [5], etc.

At the same time, as a rule, the dynamics of the negotiation process and the issues of time to reach agreement between the parties (consensus) are on the periphery of the research.

The trade negotiation process is not continuous. As a rule, it consists of several rounds (stages). Their number, in fact, determines the time it takes to reach consensus. It should be emphasized that the time of the negotiation process until the parties reach an agreement can be considered an integral indicator characterizing the effectiveness of negotiations. An increase in the time of the negotiation process may indicate poor selection of negotiators, a lack of flexibility in negotiations, excessive ambition, etc. Thus, the analysis of the negotiation process in this regard is a relevant task that meets the ideas of globalization.

During the first round of negotiations, the parties get to know each other and determine their starting positions, which need to be brought closer together during subsequent rounds.

Further, we will assume that two parties participate in the negotiation process. This assumption does not in any way detract from the value of the study, since most trade negotiations are bilateral.

As noted above, the main function of negotiations is to reach an agreement, without which a trade deal cannot take place. Joint activity with the other party focused mainly on finding a mutually acceptable solution is the most important characteristic of the negotiation process and, in this sense, is opposed to the unilateral actions of the participants. It should be noted that many researchers generally view negotiations as a process of finding a joint solution to a problem with the other side.

In order to emphasize the importance of this point, R. Fisher and W. Urey introduced the special term “BATNA”, which is an abbreviation for the English “Best Alternative to Negotiated Agreement”, as one of the main elements of the negotiation process [10]. Negotiations between the two parties will not take place if at least one of the participants has BATNA, i.e., an alternative that he considers to be more advantageous than the negotiated solution.

If, when discussing the initial position, it turns out that the parties’ proposals do not have any common ground – in other words, there is no common negotiation space – then such negotiations are unlikely to end in agreement [30]. Moreover, if, for some reason, for example, due to an incorrect assessment of the situation by one of the parties, the negotiations end in mutual agreement, in the future they will be perceived as unfair or dishonest.

Therefore, the authors of the present study proceed from the fact that there is a negotiation space (points of intersection of the parties’ proposals) that makes it possible to achieve a convergence of positions during the negotiation process.

It should be emphasized that the presence of negotiation space and the absence of BATNA are necessary conditions for achieving consensus but are clearly insufficient. Even the absence of BATNA does not guarantee the achievement of agreement in trade negotiations, which may be due, for example, to the psychophysical characteristics of the specialists participating in the discussion, the complexity of the subject of discussion, and other factors. Thus, during the negotiation process, two coalitions are formed (according to the number of parties), which leads to a deadlock and a conflict between the parties.

In the theory of conflictology, there are works devoted to the search for consensus in coalitions [14, 16, 19, 20]. These works examined the issues of resolving conflicts within specific cases, which does not allow us to draw general conclusions related to the achievement of agreement by the parties during negotiations.

The purpose of this work is to study the factors that influence the time it takes for the parties to reach mutual agreement (consensus) in the presence of coalitions in bilateral trade negotiations.

1.2. The Role of Consensus as a Method of Agreement in Trade Negotiations

An insoluble situation with coalitions can be changed in different ways: to come up with some compromise from one side or from both coalitions (parties), or to invite an external expert to moderate negotiations. In the first case, the main idea of “removing” the conflict and achieving consensus is associated with the proposal to relax demands (i.e., making a concession), which one party to the negotiations makes to the other, and in the second case, the main idea is associated with the streamlining of the negotiation process by the moderator (an external expert). If there is no search for a compromise between the coalitions (parties) or the absence of external intervention, negotiations may drag on for years and consensus will not be achieved.

Consensus research raises many questions related to ensuring the reproducibility of research results. An analysis by a collaboration of evidence-based psychologists found that out of 100 original experimental studies in the field of social psychology, only some groups were able to reproduce no more than 39 experiments [1]. Thus, it is not possible to organize socio-psychological studies of the nature of consensus for large groups.

The complexity of formalizing the negotiation procedure itself leads to the search for fundamentally new approaches to solving the problem of finding consensus. The phenomenon of reaching consensus can be analyzed on the basis of a modeling methodology that is not limited by the number of group participants. The relevance of building such models is related to the need to study factors that influence the time to reach consensus in trade negotiations in order to increase their efficiency and reduce time costs.

De Groot's model of the process of achieving consensus based on regular Markov chains presented in the work showed its efficiency in studies of general consensus in a group [2–4, 31]. The model is built on the assumption that discussion participants exchange opinions and can influence the opinions of other group members during the discussion. The main advantages of this model are the ability to identify and study the key characteristics of the participants in the negotiation process and develop practical recommendations to avoid delaying the process of making an agreed-upon decision. Recently, this model has been applied in various domains such as network automation management [7], negotiation processes [23], and social media management [8].

2. METHODS AND MATERIALS

2.1. Description of the Theoretical Consensus Building Model Based on Regular Markov Chains

Let us describe a model of consensus in the process of negotiation based on regular Markov chains [3].

1. Let some question concerning the parameter Θ (not necessarily real) in a group be discussed. It is assumed that the value of the parameter Θ itself is not known/not fully known. Each of the negotiators holds his initial opinion about Θ .

Thus,

n – the number of members of the group participating in the discussion;

$S(0) = (S_{01}, S_{02}, \dots, S_{0n})$ – the vector of the initial opinions of these members, where S_{0i} is the initial opinion of the i -th member, $i = 1, \dots, n$. For example, in the structure of Russian exports to China a significant share is occupied by pulp and paper products (paper, cardboard). Thus, in the process of trade negotiations, the quality indicators of these products, prices, delivery times and other characteristics related to the import of

goods are discussed. Therefore, the vector $S(0)$ describes the initial values of any characteristic along which the sides of negotiations must agree.

Note that the opinion S_{0i} of each member ($i = 1, \dots, n$) can vary depending on that member's degree of confidence in the opinions of other, as well as the degree of self-confidence. Group members exchange their opinions about the vector $S(0)$ during a group meeting. Therefore, each of them can either change their opinion or keep it the same at any stage of the negotiation.

- Let's set the probability of trust of the i -th participant to the opinion of the j -th participant by a strict inequality $0 < p_{ij} < 1$ ($i = 1, \dots, n; j = 1, \dots, n, i \neq j$). This is because consensus often requires flexibility in decision-making as well as active participation. If $i = j$, let p_{ii} be the probability that the i -th member trusts himself, and we also suppose $0 < p_{ii} < 1$. Also, these values can be explained as levels of authoritarianism: the higher the p_{ii} value, the higher the authoritarianism of the group member. The issues related to authoritarianism are described quite well in the literature, which allows us to introduce an appropriate scale (authoritarianism – a socio-psychological characteristic of a person reflecting his/her desire to maximally subordinate partners in interaction and communication to his/her influence). This is important for practical guidance on negotiation processes.

Let's form a quadratic confidence matrix consisting of probabilities $P = (p_{ij})$ ($i = 1, \dots, n; j = 1, \dots, n$). The resulting matrix P is stochastic (the condition $\forall i \in \overline{1, n} [\sum_{j=1}^n p_{ij} = 1]$ is satisfied).

- The vector of opinions of the members of the negotiation process at the first stage can be calculated by the formula

$$S^T(1) = P \cdot S^T(0) = (S_{11}, S_{12}, \dots, S_{1n})^T,$$

where $S^T(\cdot)$ is a column vector of dimension $n \times 1$. And after the k -th step, we get the vector

$$S^T(k) = (S_{k1}, S_{k2}, \dots, S_{kn})^T = P \cdot S^T(k-1) = P^k \cdot S^T(0).$$

- The iterative process stops at the m -th step when all rows of the matrix P^m become equal with a given accuracy. In this case, we consider that group cohesion has been achieved, i.e. consensus has been reached. Mathematically, this means that the matrix P has reached the final matrix F after being raised to degree m . Since the final matrix F does not change in subsequent iterations, the vector of negotiators' opinions $S^T(m) = P^m \cdot S^T(0) = (S_{m1}, S_{m2}, \dots, S_{mn})^T$. Thus, after m iterations a consensus emerges.

Comment. In Markov chain theory, a necessary and sufficient condition for the convergence of the matrix P to the final matrix F any vector of initial opinions is the regularity of the matrix P [13]. That is, for the regularity of the matrix it is sufficient for the sums of probabilities on the rows of the matrix P to be equal to 1, with the strict inequality $0 < p_{ij} < 1$ being satisfied for any probabilities p_{ij} [17] (Theorem 4.1.2).

2.2. Description of the Theoretical Consensus Building Model in Two Coalitions as a Special Case of the Consensus Model

Let us consider the theoretical model of consensus using the example of having two coalitions for a group of five people. For such a case, the confidence matrix P can have, for example, the following form:

$$P = \begin{pmatrix} \boxed{\begin{matrix} 0.7 & 0.3 \\ 0.5 & 0.5 \end{matrix}} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} & \boxed{\begin{matrix} 0.5 & 0.5 & 0 \\ 0.1 & 0.7 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{matrix}} \end{pmatrix}.$$

Thus, we consider two coalitions, the first involving two negotiators and the second involving three negotiators.

In the coalition model, the regularity condition of the trust matrix P is violated, so in this situation consensus cannot be reached for any $n \geq 2$ [13]. In other words, the condition $0 < p_{ij} < 1$ for any rows is violated. Matrices of this type and their corresponding Markov chains are decomposable.

If a compromise solution is reached, it will allow to get rid of the existing coalitions. Such solutions can be different. For example, it is possible to make a concession by members of one coalition to the second coalition. Mathematically, this corresponds to redistributing the i -negotiator's probabilities of trust p_{ij} for the members of his coalition by the probabilities p_{ij} for the members of the competing coalition. The probabilities p_{ij} are redistributed through additional reasoning in the deliberation process. A model of such concessions is described in the work of the social psychologist Homans [15]. The article considers the scheme of functioning of small groups on the basis of utility exchange. We can talk about two variants of concessions:

- unilateral concessions, where members of one coalition redistribute their probabilities of trust to members of the other coalition;
- bilateral concessions, where members of one coalition redistribute their confidence probabilities to members of the second coalition and, conversely.

We will find the time it takes to reach consensus in the presence of unilateral concessions in different types of coalitions. Unilateral concessions in the negotiation process have been examined in a number of papers, such as [20]. However, they did not analyze the factors affecting the time until the final decision is reached.

2.3. Elements of Experiment Planning for the Analysis of the Theoretical Model of Coalitions

It is known that as the size of a group increases, interactions between its members become less frequent and formal, which leads to a weakening sense of belonging, and this is an obstacle to the formation of group cohesion [18]. Based on this premise, it is obvious that the group of negotiators should not be too large so that direct interaction between its members can be ensured. Let us take this fact into account further when setting up the experiment.

For the general case of consensus without coalitions, it has been revealed in a number of papers that the time of consensus depends primarily on the authoritarian number of group members and their size [2–4]. There are many combinations of size, concessions, and the authoritarianism of negotiators. In order to control this variability, experimental planning is applied [10].

1) Let's build a full-scale experiment with an orthogonal plane. In such an experiment, all combinations of factor level are realized, which allows us to independently assess the influence of each factor and each pair interaction [10].

2) In order to draw stable conclusions about the average number of conversations, the experimental plan will include the simulation of 100 parallel experiments per line [9]. Each series of parallel experiments that correspond to one line of the plan includes information about the reaction. In addition, parallel experiments protect us from errors and allow us to evaluate reproducibility errors.

3) The homogeneity of experiment variances is tested by the Cochran G -Test at 0.05 [10]:

$$G(v-1, l-1) = \frac{\hat{\sigma}_{max}^2}{\hat{\sigma}_1^2 + \hat{\sigma}_2^2 + \dots + \hat{\sigma}_l^2}$$

where $\hat{\sigma}_{(\cdot)}^2$ – experimental (\cdot)-line variation, v – number of parallel experiments in one line. If the obtained $G_{estimated}$ is less than the $G_{critical}$, then we can recognize the homogeneity of the variances of parallel experiments at a given level of significance. In this case, we can proceed to building a model. Cochran test for checking the homogeneity of variances works for normally distributed variables. For each line of the plan we checked the

consistency of the obtained data with the normal distribution by Pearson's criterion (the significance level for the test is assumed to be 0.05).

4) Regression analysis of the experimental results was performed in the context of *Excel-16* and *Statistica-15* with a significance level of $\alpha = 5$ percent.

3. CONSENSUS MODEL IN TWO MONO-AUTHORITARIAN COALITIONS

3.1. Description of the Experiment with Unilateral Concession and its Theoretical Results

Let each coalition consist of members (negotiators) whose authoritarian levels are close to each other, i.e., their corresponding probabilities vary within ± 0.005 of some predetermined probability p . In this case, we will assume that negotiations are conducted by mono-authoritarian coalitions.

Assumptions and definitions

Build a model of unilateral concessions when there are two mono-authoritarian coalitions in a group of 20 negotiators. Other works have shown that such a number of negotiators in a group is optimal in terms of the number of rounds of negotiations, all other things being equal [31].

The first stage. The response choice.

The response will be the average number of rounds m before consensus is reached, i.e. the average time to reach consensus. From now on, we'll count the number of rounds.

The second stage. The factors of choice.

After a preliminary analysis, the following factors were selected:

$X0$ – coalition impact index I – is the quotient of the first coalition to the second coalition;

$X1$ – average authoritarianism of the first coalition negotiators;

$X2$ – average authoritarianism of the second coalition negotiators;

$X3$ – unilateral concession that members of the first coalition make to the second coalition.

The third stage. The factor-levels choice.

Preliminary analysis showed that the constructed mathematical model is nonlinear. Since we do not assume to go out of the framework of quadratic polynomial it was not assumed that two or three factor levels are sufficient in such models. The choice of these factor levels was made after research. After preliminary analyses, the levels were selected (Table 3.1).

Table 3.1. Factors and levels of the first experiment factors

factor levels	factors			
	$X0$	$X1$	$X2$	$X3$
lower «-1»	0.25	0.005	0.005	10
medium «0»	1	0.05	0.05	50
upper «+1»	4	0.895	0.895	75

Let us describe the levels of each factor.

Factor $X0$. Level “-1” corresponds to two coalitions with the number of members $n_1 = 4$ and $n_2 = 16$ ($I = 1/4$), level “0” corresponds to two coalitions with the number of members $n_1 = n_2 = 10$ ($I = 1$) and level “+ 1” corresponds to two coalitions with the number of members $n_1 = 16$ and $n_2 = 4$ ($I = 4$).

Factor $X1$. Level “-1” corresponds to the authoritarianism of the members of the first coalition that makes a concession, from 0 to 0.01 (with an average value of 0.005), level “0” corresponds to authoritarianism from 0.045 to 0.055 (with an average value of 0.05), level “+ 1” corresponds to the authoritarianism from 0.89 to 0.90 (with an average value of 0.895).

Factor $X2$. Level “-1” corresponds to the authoritarianism of the members of the second coalition that accept a concession from the first coalition, from 0 to 0.01 (with an average

value of 0.005), level “0” corresponds to authoritarianism from 0.045 to 0.055 (with an average value of 0.05), level “+1” corresponds to the authoritarianism from 0.89 to 0.90 (with an average value of 0.895).

Consider *the remarks for the factors X1 and X2.*

Level «-1» is chosen so as to ensure consideration of the group of negotiators with low authoritarianism. For a group of negotiators with low authoritarianism, we will have that

$$\forall i = \overline{1, n} \left[p_{ii} < \frac{1}{n} \right].$$

Based on this estimate, for groups of 20 negotiators, an upper boundary is provided $p_{ii} < 0.005$. Authoritarianism varied in the range of ± 0.005 from the average level. If $p_{ii} > \frac{1}{n}$, the group ceases to be a low authoritarian group because its members can trust themselves more than others in the group [3]. In a group with low authoritarian members, we can get the following situation for some $i = \overline{1, n}$:

$$\forall j = \overline{1, n}; j \neq i \left[p_{ii} < p_{ij} \right],$$

i.e., there is a negotiator whose level of confidence in his opinion is less than the level of confidence in any other.

Level «0» is chosen to ensure that a group of negotiators with an upper bound on authoritarianism is considered $p_{ii} = 0.005$. Authoritarianism was varied in the range of ± 0.005 from the average level.

Level «+1» level is chosen to ensure that the group of negotiators with high levels of authoritarianism is taken into account:

$$\forall i = \overline{1, n} \exists j = \overline{1, n}; \left[p_{ii} > p_{ij} \right], \\ j \neq i$$

i.e., there is at least one member whose level of confidence in his opinion exceeds the level of confidence in any other member of the group. Note that for a group with authoritarianisms $p_{ii} = 1$ we obtain a decomposable matrix

$$\begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \end{pmatrix}.$$

It is shown in Markov chain theory that this matrix does not converge to the final matrix [17]. Therefore, the level «+1» is chosen close to the $p_{ii} = 1$. Authoritarianism varied in the range of $\pm 0,005$ from the average level.

Factor X3. Level “-1” corresponds to a concession of Y = 10 percent, level “0” corresponds to a concession of Y = 50 percent, and level “+1” corresponds to a concession of Y = 75 percent.

The fourth stage. The planning matrix choice.

The orthogonal design of the full-factor experiment included 81 lines and 4 factors in total. Preliminary analysis showed that the model would not be linear, so we will include pairwise interactions.

The fifth stage. The experiment.

For each factor level, the modelling of elements p_{ii} of matrix P was carried out using a uniform distribution law. It was taken into account that the sum of probabilities in each row should equal 1, i.e. that the matrix P should be stochastic. Next, all probabilities in the rows of matrix P for the first coalition were recalculated in proportion to the amount of concession it made to the second coalition. The probabilities of the second coalition accepting the concession remained the same. For each line, the average m is calculated. Then, for all lines, the homogeneity of dispersion is checked by Cochran's G-test. The homogeneity of variances was confirmed at a level of only 15 percent. Let us analyze the variability of each experimental line at the end of the chapter.

The following regression model was obtained:

$$\hat{m} = 36.7 + 20.0 \cdot X2 - 15.5 \cdot X3 + 10.9 \cdot X2 \cdot X3 \quad (3.1)$$

The plan matrix is orthogonal, which allows us to interpret the model coefficients straightly. The model (1) is adequate according to Fisher's criterion, the determination shown is high enough, which also demonstrates the good quality of the model ($R^2 = 0.723$, $F(3, 77) = 66.853$, $p_{value} = 2.2 \cdot 10^{-21} < 0.05$). Fig. 3.1 shows the spline of the model along the experimental points derived from the factors X_2 и X_3 included into the models. It confirms the results of the preliminary studies, which showed that the model is nonlinear with respect to the included factors X_2 and X_3 . A joint increase in factors X_2 and X_3 leads to an increase in the value of m (Fig. 3.1). Let us carry out a theoretical analysis of the resulting model (3.1) and further give its interpretation.

The sixth stage. Results.

Result 1.

Using the step-by-step inclusion of variables, confirmed the inclusion gave the following results:

- a statistically significant relationship between the output variable m and the regressors X_2 , X_3 and X_2X_3 , while the strongest one was revealed with the regressor X_2 ;
- a statistically insignificant relationship between the output variable m and the regressors X_0 , X_1 and all remaining pairwise interactions.

The structure of the final matrix in each such case is remarkable: the rows corresponding to the first coalition consist of zeros, as shown in the example from section 2.2.

Also, based on the results of the experiment, it is important to note one non-random fact of the model's behavior, consistent with the studies for the general group without coalitions [17]: if $X_2 = -1$, then for fixed values of X_3 the average number of rounds m will be slightly greater than for the level $X_2 = 0$, as shown in Fig. 3.1.

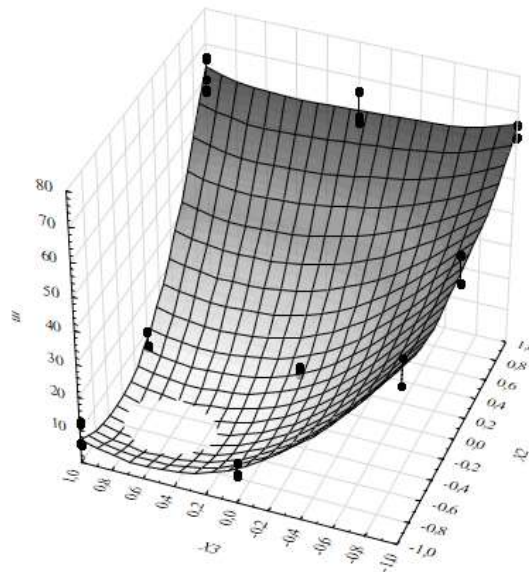


Fig. 3.1. Spline based on experimental data (X_2 – authoritarianism of the coalition accepting the concession, X_3 – concession, m – average number of rounds)

Result 2.

The regression graph (3.1) resulting from the experiment is presented in Fig. 3.2. Let us fix each factor in the experiment in turn and analyze the dependencies (Fig. 3.3, Fig. 3.4).

1. When X_2 is fixed, we obtain the following section equations (Fig. 3.3):

$$X_2 = -1, \hat{m} = 16,7 - 26,4 \cdot X_3,$$

$$X_2 = 0, \hat{m} = 36,7 - 15,5 \cdot X_3,$$

$$X_2 = 1, \hat{m} = 56,7 - 4,6 \cdot X_3.$$

2. When X_3 is fixed, we obtain the following section equations (Fig. 3.4):

$$X_3 = -1, \hat{m} = 52,2 + 9,1 \cdot X_2,$$

$$X_3 = 0, \hat{m} = 36,7 + 20,0 \cdot X_2,$$

$$X3 = 1, \hat{m} = 21.2 + 30.9 \cdot X2.$$

When fixing $X2$:

- the largest slope angle is given by the first equation for $X2 = -1$, which records high sensitivity to changes in factor $X3$: the decrease of $X3$ by 1 unit implies an average increase of m response for 26–27 rounds;
- the smallest slope angle is given by the last equation for $X2 = 1$, which indicates low sensitivity to changes in factor $X3$: the increase of $X3$ by 1 unit implies an average decrease of m response for only 4–5 rounds.
- When fixing $X3$:
- the largest slope angle is given by the last equation for $X3 = 1$, which records high sensitivity to changes in factor $X2$; the increase of $X2$ by 1 unit implies an average increase of m for 30–31 rounds;
- the smallest slope angle is given by the last equation for $X3 = -1$, which records low sensitivity to changes in the factor $X2$; the increase of $X2$ by 1 unit implies an average increase of m for only 9–10 rounds.

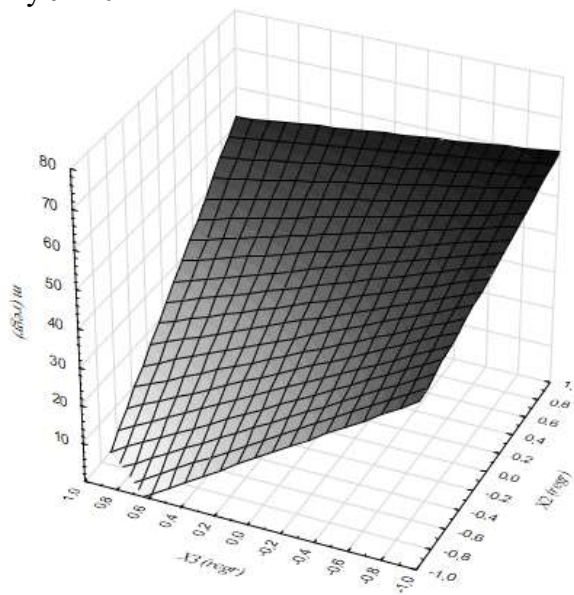


Fig. 3.2. Regression model based on experimental data ($X2$ – authoritarianism of the coalition accepting the concession, $X3$ – concession, m – average number of rounds)

Result 3.

In Figures 3.3 and 3.4 and in the sections of the regression model, it can be clearly seen that the greatest slope is reached when $X3$ is fixed, which indicates the sensitivity of the model, particularly to changes in factor $X2$. It is also clearly evident that the effect of factor $X3$ becomes less as it tends to $X2 = 1$ (the sections in Fig. 3.4 converge as $X2 \rightarrow 1$, the slope angle at $X2 = 1$ in the sections displayed in Fig. 3.3 is the smallest among the others).

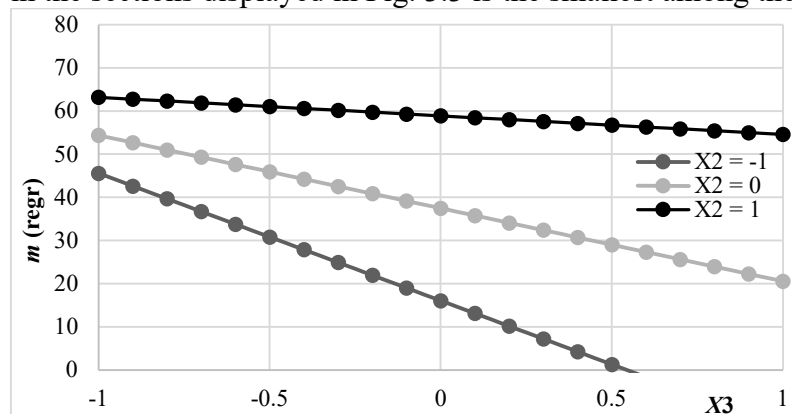


Fig. 3.3. Sections of the regression model when factor $X2$ is fixed

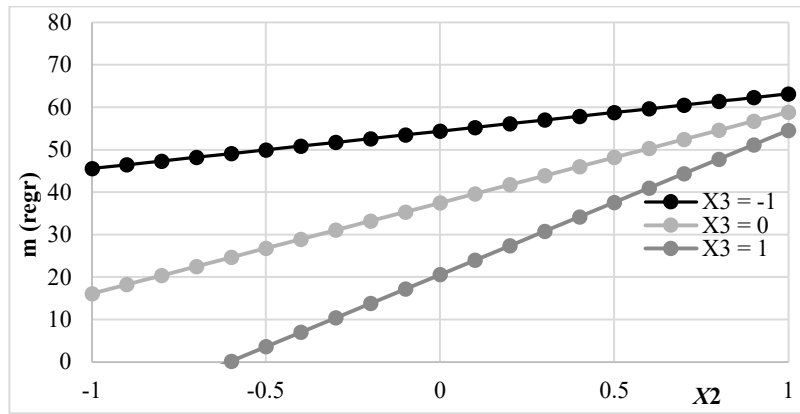


Fig. 3.3. Sections of the regression model when factor X3 is fixed

Result 4.

The highest values of $m = 55-66$ according to experimental data are observed at $X2 = 1$, which is confirmed by the model within the experimental region:

$$X2 = 1, \hat{m} = 56.7 - 4.6 \cdot X3;$$

for different $X3 \in [-1, 1]$, the values according to the model will be $\hat{m} = 56.7 \pm 4.6$. The model also shows high values for $X2 = 0$ when $X3 = -1$, which is also evident from the experimental data.

Note that outside the experimental area, for any values of $X3$ when factor $X2$ increases (i.e., as the authoritarianism of the group members accepting the concession tends to 1), we can obtain an arbitrarily large number of rounds since, as noted earlier, we can arrive to decomposable matrix.

Result 5.

The smallest value $m = 5-13$ according to experimental data is observed at $X3 = 1$ (Table 3.2) and at $X3 = 0$, if $X2 \neq 1$, which is confirmed by the model within the experimental area:

$$X3 = 1, \hat{m} = 21.2 + 30.9 \cdot X2;$$

level of values $m = 5-13$ in Fig. 3.3 is achieved only for $X2 = 0$ and $X2 = -1$ at a high value of $X3$, in Fig. 3.4 is achieved only for $X3 = 0$ and $X3 = 1$ at low values of $X2$.

3.2. Variability in the Number of Negotiations in the Experiment and Particular Cases of the Model

The experiment examined the effect of various factors on the average number of negotiation rounds conducted before consensus is reached. It was found that there is different variability in the number of negotiations under different conditions. Variability allows us to understand the limits of variation in the number of negotiations in coalitions, as there is no complete homogeneity within a coalition. Therefore, it characterizes the constancy of the number of steps taken to reach consensus.

Let us compile a table where we display the main range of variances together with the corresponding levels of factors (Table 3.2).

Result 6.

From the analysis of Table 3.2 it is clear that, on average, the tests are subject to the greatest variation when factor $X0 = 1$ (moreover, when factor $X2 = 1$, then the level of factor $X3$ is not important).

The smallest variation is most often observed in the tests when:

- a) $X0 = -1$,
- b) $X0 = 1$ when $X2 \neq 1$ и $X3 \neq 1$,
- c) $X0 = 0$ when $X2 \neq 1$.

Note that in the case of $X0 = -1$ there are no tests with high or medium variation, and that the factor $X1$ does not affect the variation of the tests.

An additional analysis of variance showed that factor X_0 has a statistically significant effect on the variation (Fisher's criterion: $F_{\text{estimated}} = 33.602$, $F_{\text{critical}} = F_{0,05}(2, 78) = 3.114$ and $F_{\text{estimated}} > F_{\text{critical}}$) [9].

Let us consider special cases of the model for each level of factor X_0 .

The obtained regression partial dependencies under conditions of homogeneous variances (at a significance level of 5 percent) have the form:

$$X_0 = -1. \hat{m} = 37.5 + 21.4 \cdot X_2 - 16.9 \cdot X_3 + 12.6 \cdot X_2 \cdot X_3.$$

$$X_0 = 0. \hat{m} = 35.6 + 22.6 \cdot X_2 - 14.0 \cdot X_3 + 10.4 \cdot X_2 \cdot X_3.$$

$$X_0 = 1. \hat{m} = 37.0 + 16.1 \cdot X_2 - 15.7 \cdot X_3 + 9.7 \cdot X_2 \cdot X_3.$$

For the level $X_0 = 0$ at $X_2 = X_3 = 0$, the response value \hat{m} is the smallest among all other levels of X_0 , which is consistent with the simulation results and is essential in interpretation (see section 3.3).

In general, the coefficients in the equations are close and reflect the same behavior of the dependent variable. Therefore, to obtain general conclusions when interpreting the model, you can use the general model (3.1) obtained in section 3.1.

Table 3.2. Ranges of variances of parallel tests of the experiment

variation levels	factors				variation
	X_0	X_1	X_2	X_3	
lower	-1	any	any	any	0-2
	0	any	0; -1	any	
	1	any	0	0; -1	
medium	0	any	-1	any	2-10
	1	any	0	1	
upper	1	any	1	any	10-32
			-1	1	

3.3. Experiment with a Bilateral Concession

With a bilateral concession, it is expected that the negotiation process should move faster. But how fast? And will the process always be faster? In order to find out, we will conduct a shortened experiment with two equal coalitions with the same level of authoritarianism of members. Let us briefly describe it.

Factors X_1 , X_2 . Level “-1” corresponds to the authoritarianism of coalition members from 0 to 0.01 (with an average value of 0.005), level “0” corresponds to authoritarianism from 0.045 to 0.055 (with an average value of 0.05), level “+1” corresponds to the authoritarianism of coalition members from 0.89 to 0.90 (with an average value of 0.895). Consider *the remarks for the factors X_1 and X_2 in 3.2.*

Factor X_3 . Level “-1” corresponds to a bilateral concession of $Y = 10$ percent, level “0” corresponds to $Y = 50$ percent and level “+1” corresponds to $Y = 75$ percent.

The full-factor experiment orthogonal plan included 9 lines and 2 factors in total. For each level, the modeling of elements p_{ii} of matrix P was carried out using the uniform law of distribution under given conditions so that the sum of probabilities within each row equals 1. Next, all probabilities in the rows of matrix P for both coalitions (sides) were recalculated in proportion to the concessions they make for each other. For each line, the average m is calculated.

Result 7.

It is possible to identify cases not only when we observe a decrease in the number of rounds but also cases where their number may increase (highlighted in bold in Table 3). These cases correspond to the factor levels: $X_1, X_2 = -1$ or $X_1, X_2 = 0$ and $X_3 = 1$. The largest decrease in the average number of negotiation rounds m is observed for the factor

levels: $X1, X2 = 1$ and $X3 = 0$ or $X3 = 1$. Note that a bilateral small concession ($X3 = -1$ or $X3 = 0$) led in all cases to a decrease in m .

Table 3.3. Results of a full factorial experiment for mono-authoritarian coalitions with bilateral concessions

$X1, X2$	$X3$	$X1$	$X2$	$X3$
-1	-1	47.6	22.8	52
-1	0	8.1	4.0	50
-1	1	5.9	7.3	rise
0	-1	47.5	22.8	52
0	0	8	4.0	50
0	1	5.6	7.2	rise
1	-1	66.1	33.3	50
1	0	65.6	9.0	86
1	1	65.8	7.0	89

The modeling revealed that variability m decreased on average by 2 times compared to the variability with unilateral concession.

This shortened experiment served the purpose of identifying possible cases of reduction/increase in m . The experiment does not include the analysis of coalitions with different authoritarian levels of negotiators and different numbers of group members with mutual concessions. This requires further detailed research in accordance with the stated objectives.

3.4. Interpretation of the Model of Mono-Authoritarian Coalitions

Interpretation of results 1 and 3.

The most significant factor that influences the number of rounds of negotiations to reach consensus is the authoritarianism of the coalition accepting the concession. If the coalition accepting the concession consists only of non-authoritarian members, then the average number of rounds will be slightly higher than in the case when the members of this coalition have authoritarian levels close to $1/n$. Since the coalition accepting the concession ultimately forms a consensus decision, it is very important that this group includes specialists who are ready to listen to others and at the same time have their own opinion; that is, they are not absolutely non-authoritarian or absolutely authoritarian. It was revealed that with average values of concession and average authoritarianism of the coalition accepting the concession, the smallest number of agreements will be in the case when the coalitions are equal in number. This corresponds to general recommendations for negotiations [27].

The influence of the authoritarianism of the coalition that moves towards rapprochement, on the total number of rounds of negotiations is statistically insignificant. The size of the coalition also does not have a significant effect on the value of m .

Interpretation of result 2.

A high sensitivity to the size of the concession was revealed for a coalition with non-authoritarian members that accepts this concession: as the concession increases, the number of rounds quickly decreases. It was revealed that with an increase in concessions from 10 percent to 75 percent, the number of rounds can be reduced by 5 or more times.

Low sensitivity to the size of the concession was revealed for the authoritarian coalition accepting this concession: with an increase in the concession, the number of rounds is reduced slightly.

Thus, the size of the concession plays a significant role in reducing the number of rounds only for non-authoritarian coalitions that accept the concession.

Interpretation of result 4.

The highest value of the number m was found for authoritarian coalitions accepting a concession. The size of this authoritarian coalition does not significantly affect the average number of rounds in this case. The negative role of authoritarianism among the group members was also noted in the general case, i.e., during negotiations within one group without the presence of coalitions [31]. Note that by replacing the authoritarian members of the coalition accepting the concession with non-authoritarian members, it is possible to reduce the negotiation time for a 50 percent concession by 4 or more times, and for a 75 percent concession by 10 or more times.

Interpretation of result 5.

The smallest value of the number of rounds of the negotiation process within the experiment was revealed at the highest value of the concession, but only if it is accepted by a non-authoritarian coalition (all other things being equal). This finding is consistent with Result 2: for an authoritarian coalition accepting a concession, the size of the concession has little effect on the number of negotiation rounds.

Interpretation of result 6.

The greatest variation in average negotiation time occurs when a large coalition makes any significant concession to an authoritarian small coalition.

The smallest variation in average negotiation time is observed when:

- a small coalition makes any concession;
- a larger coalition makes a concession to a coalition that consists of non-authoritarian members (increasing concessions in this case entails an increase in the variation in the average number of negotiation rounds).

Thus, in the first case, we can unexpectedly observe both a small and a large number of rounds of the negotiation process; in the second case, low variability in the number m is observed. The variation in the number m is not affected by the authoritarianism of the coalition that makes the concession.

Interpretation of result 7.

With a bilateral concession, both cases of reduction in the number m and cases of its increase were identified.

The average number of negotiation rounds increases when coalitions consist of members with low or close to $1/n$ authoritarianism and make a large concession. Thus, the transfer of responsibility for the decision made from one side to the other leads to an increase in the time of the negotiation process.

The average number of rounds of the negotiation process is reduced to the maximum when authoritarian coalitions decide to make a mutual concession.

Note that a small bilateral concession leads to a reduction in the negotiation process.

4. INTERPRETATION OF THE RESULTS FOR THE TRADE NEGOTIATION PROCESS. FINDINGS

The obtained modeling results make it possible to develop general recommendations for the negotiation process of the two parties to avoid delay in the negotiation process. In what follows, we will call a coalition (side) authoritarian if all its members have high authoritarianism ($p_{ii} \geq 0.9$), and non-authoritarian if its members have low authoritarianism ($p_{ii} < 1/n$). The values of authoritarianism p_{ii} can be obtained by expert assessment of the components developed by F. Fiedler and later tested in many laboratory experiments [12]. Let us consider the main recommendations arising from the present study.

1. Without a concession in the negotiations, the parties (if there are contradictions) will not be able to reach a consensus. At least one of the parties must make a concession.
2. It is advisable (other things being equal) for the parties to be equal in number.
3. Negotiators on both sides should not all be completely non-authoritarian or authoritarian.

4. If condition 3 is met, then, from the point of view of reducing negotiation time, it is better if both parties make a concession.

If condition 1 is not met, then reaching a consensus is possible only by replacing the authoritarian members of one side with specialists who are ready to make a concession. Note that resolving the conflict by replacing negotiators in order to eliminate coalitions does not always lead to the expected consensus [29]. A striking example of this situation is the conflict in Panama during the construction of a hydroelectric power station between two coalitions: indigenous peoples and the company Generadora del Istmo S. A. (GENISA). The replacement of members of one of the coalitions did not lead to the development of a consensus decision. Therefore, a concession serves as a primary factor in developing a solution during negotiations.

Next, we assume that condition 1 is satisfied, but the following condition is not fulfilled:

- a) condition 2. Achieving consensus will be faster if one of the possibilities is implemented:
- any concession is made by the minority side, and the other side will not be absolutely authoritarian,
 - a larger coalition makes a concession, while the party accepting the concession consists of non-authoritarian members (increasing concessions in this case entails an increase in the variation in the average number of negotiation rounds).
- b) condition 3. Reaching consensus can be achieved by replacing absolutely authoritarian members or absolutely non-authoritarian members of one side with members with average authoritarianism.

Analysis of the current model of face-to-face negotiations between the two parties creates softer conditions for the composition of the group, and the negotiations themselves contribute to the development of new ways to take into account the interests of both sides.

5. CONCLUSION

Issues of achieving consensus, which is based, as a rule, on the possibility and ability of negotiators to compromise, currently have been poorly studied; the complexity of formalizing the negotiation procedure itself leads to the search for fundamentally new approaches to constructing mathematical models. The relevance of developing such models for studying the interaction between experts in groups and the factors influencing the time to reach consensus is determined by the possibility of delaying the negotiation process.

The model considered in the work is built on the assumption that participants in the negotiation process exchange opinions and can influence the point of view of other members of the negotiating parties during the discussion. The main advantage of such a model is the ability to identify and study the key characteristics of the negotiating parties. It helped to formulate practical recommendations for making an agreed-upon decision within the framework of trade negotiations.

Despite these advantages, the considered model does not reveal the problem of correlating consensus and truth, when the deliberate formation of a group of “ignoramuses” leads to a consensus that is far from the truth [25, 22]. Therefore, the problem of approaching the truth of a collective decision deserves special attention and analysis of the experience of using consensus procedures and is beyond the scope of the current study.

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