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A State Space Filtering-Based Approach for Price Prediction

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Abstract:
We present a method of the forecasting and the data filtering of a linear dynamic system based on the dimension reduction of the space of unobservable states. The method relies on the singular value decomposition of the Hankel matrix. The decomposition is used to calculate unknown parameters of the model. The elements of the singular value decomposition are separated into blocks enabling to estimate the initial state and the system matrices and predict the system dynamics and the data filtering by identifying exponential trends and periods of seasonal fluctuations.
To illustrate the quality of fitting and the determined periods of an oscillatory system with trends and the white noise, we conducted numerical simulations of such systems. The parameter estimates were obtained with high precision. Then, daily electricity price data from the NordPool system from 2016 to 2020 were used to generate in-sample and out-of-sample forecasts.
The advantages of the proposed method include the ability to handle ill-conditioned matrices and to determine the periods of oscillatory systems. This is significant due to the presence of seasonality in many economic indicators. In the analyzed daily electricity price data, the method identified the presence of biweekly and monthly seasonality.

Keywords: state space method, principal components, time series, singular value decomposition, filter, electricity price forecasting

1. INTRODUCTION

The representation of systems in the state space is a common way to describe the dynamics of linear systems. State space models constitute one of the classes of time series models that are successfully applied in management of technical systems and solving system identification problems, as well as in the fields of filtering and forecasting in economics and finance. Descriptions of these models in economic and financial contexts can be found in [13, 15, 17]. These works also discuss parameter estimation methods for such models, including maximum likelihood and Bayesian methods.

One of the first approaches in this paradigm is the renowned Kalman filter, see, e.g., [12]. Among the earliest applications of state space methods and the Kalman filter, particularly in economics and finance, notable works include [5, 8].

In this work, in addition to filtering, the problem of estimating the hyperparameters of the state space model is considered. In such situations, making forecasts includes preliminary estimation of unknown hyperparameters. In this regard, we rely on and develop the methodology described in [2, 16].

The model used in this work is as follows: there is a linear evolution of hidden states, where observed variables are obtained by projecting from the space of hidden unobservable

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variables. Algebraically, this concept is expressed as a system of two vector linear equations, where one difference equation describes the change in the hidden state x_t over time t, while the other relates the observed parameters y_t and the unobservable parameters x_t. Random factors influence the system, causing the proposed dynamics not to be fully reproduced. Random deviations are modeled as mutually independent random variables.

To identify the system, it is necessary to estimate the dimension of the vector x_t of hidden variables, the initial state x_1, and the corresponding representation matrices in state space. The dimension of the hidden state space is a hyperparameter that is either externally specified or determined by cross-validation methods. The approach based on the singular value decomposition of the Hankel matrix is used to calculate the unknown parameters of the model [2, 3, 10, 16].

This study focuses on forecasting daily electricity prices from 2016 to 2020, and additionally proposes a data filtering method. On a sample of 1170 observations, it was found that the short-term out-of-sample forecast is comparable to traditional econometric models and to the well-known Holt–Winters' filtering method based on the mean absolute error indicator even in a stationary segment of the time series.

In addition to modeling on real data and comparing the forecast quality of the proposed method with well-known models, we present several numerical simulations. These simulations demonstrate the specific characteristics of data behavior that the proposed state space model can identify.

2. BASICS

In general, an autoregressive system can be represented in the state space as follows:

$$\begin{aligned} x_{t+1} &= Ax_t + \xi_t, \\ y_t &= Cx_t + \eta_t, \end{aligned}$$

where $t \in \mathbf{N}$, x_t is an unobservable $k \times 1$ state vector, x_1 is deterministic (but unobservable), y_t is an observable $l \times 1$ vector, $(\xi_t, \eta_t)_{t \in \mathbf{N}}$ is an unobservable white noise, and A and C are state space matrices. It is easily seen that the expected value

$$\mathbf{E} y_{t+1} = CA^t x_1.$$

For the sake of simplicity, we set $\xi_t = 0$ and $\eta_t = 0$ for all t. In such a case,

$$y_{t+1} = CA^t x_1. \tag{2.1}$$

Having a sample of size T the Hankel matrix H can be written as follows for an odd-sized sample:

$$H = \begin{bmatrix} y_1 & y_2 & \cdots & y_n \\ y_2 & y_3 & \cdots & y_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_n & y_{n+1} & \cdots & y_{2n-1} \end{bmatrix} = \Gamma_{1:n}\Omega_{1:n} \tag{2.2}$$

and for an even-sized sample:

$$H = \begin{bmatrix} y_1 & y_2 & \cdots & y_{n+1} \\ y_2 & y_3 & \cdots & y_{n+2} \\ \vdots & \vdots & \ddots & \vdots \\ y_n & y_{n+1} & \cdots & y_{2n\cdot} \end{bmatrix} = \Gamma_{1:n}\Omega_{1:(n+1)}, \tag{2.3}$$

where $n = \left[\frac{T}{2}\right]$. By virtue of formula (2.1) the Hankel matrix is the product of the observability matrix

$$\Gamma_{1:n} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

and the controllability matrix

$$\Omega_{1:n} = [x_1 \quad Ax_1 \quad \dots \quad A^{n-1}x_1]$$

for an odd-sized sample, or

$$\Omega_{1:(n+1)} = [x_1 \quad Ax_1 \quad \dots \quad A^n x_1]$$

for an even-sized sample.

Therefore, if we find a decomposition of the Hankel matrix as a product of two matrices as in (2.2) or (2.3), we can estimate the matrices of the system:

$$\hat{C} = \hat{\Gamma}_{1:1}, \quad \hat{A} = \left(\hat{\Gamma}_{1:(n-1)}\right)^{+} \hat{\Gamma}_{2:n}, \quad \hat{x}_1 = \hat{\Omega}_{1:1}, \tag{2.4}$$

where $\left(\hat{\Gamma}_{1:(n-1)}\right)^{+}$ is the Moore–Penrose pseudoinverse of the matrix $\hat{\Gamma}_{1:(n-1)}$ excluding the last block, and the matrix $\hat{\Gamma}_{2:n}$ does not include the first block. The hat sign indicates that these are estimates of matrices Γ and Ω that will be suggested now. As in [16], we apply the singular value decomposition of the observed matrix $\hat{H} = \hat{U}\hat{S}\hat{V}'$, where $\hat{S}$ is a diagonal matrix of singular values arranged in decreasing order, and $\hat{U}$ and $\hat{V}$ are orthogonal matrices. We consider an odd T since the case for even T is similar. These matrices can be split into parts corresponding to, as we assume, the signal and noise:

$$\hat{U} = \left[\hat{U}_{1:k} \quad \hat{U}_{(k+1):n} \right], \quad \hat{V} = \left[\hat{V}_{1:k} \quad \hat{V}_{(k+1):n} \right], \quad \hat{S} = \begin{bmatrix} \hat{S}_{1:k} & 0 \\ 0 & \hat{S}_{(k+1):n} \end{bmatrix},$$

where $\hat{U}_{1:k}$ and $\hat{V}_{1:k}$ are the matrices consisting of first k columns of matrices $\hat{U}$ and $\hat{V}$ correspondingly, while $\hat{S}_{1:k}$ is the square $k \times k$ matrix having in its diagonal first k singular values of $\hat{H}$. The estimates for the observability and controllability matrices can be found as follows:

$$\hat{\Gamma}_{1:n} = \hat{U}_{1:k}\hat{S}_{1:k}^{1/2}, \qquad \hat{\Omega}_{1:n} = \hat{S}_{1:k}^{1/2}\hat{V}_{1:k}'.$$

Once the estimates $\hat{C}$, $\hat{A}$ and $\hat{x}_1$ are obtained from (2.4), formula (2.1) can be used for forecasting.

It should be noted that matrices C, A and vector x_1 are determined, in general, up to a non-degenerate linear transformation with matrix P: $\tilde{C} = CP^{-1}, \tilde{A} = PAP^{-1}, \tilde{x}_1 = Px_1$. But, for forecasting and filtering, it doesn't matter.

However, in practice, formula (2.1) can lead to significant computational errors, as the matrix A is multiplied by itself many times. To find a more accurate estimate, let us express (2.1) as $y_t = CA^{n-1}A^{t-T}A^{n-1}x_1 = \Gamma_{n:n}A^{t-T}\Omega_{n:n}$, considering odd T. A suggested estimate for $t \geq T$ is the following

$$\hat{y}_t = \hat{\Gamma}_{n:n}\hat{A}^{t-T}\hat{\Omega}_{n:n}.$$

In addition to the forecasting task, we propose a possible solution to the data filtering problem. Let's consider the case for odd T, as for even T the calculations are similar. It is noted that for $t \leq T$

$$y_t = \Gamma_{k:k}\Omega_{(t-k+1):(t-k+1)},$$

where $1 \leq k \leq t$. The proposed estimate is set as an average value of estimates with different parameters k in the following way

$$\hat{y}_t = \frac{1}{\min(t,n) - \max(0, t+1-n)} \sum_{k=\max(0,t+1-n)}^{\min(t,n)} \hat{\Gamma}_{k:k}\hat{\Omega}_{(t-k+1):(t-k+1)}.$$

In order to select the dimension k of the state vector (State Space Dimension, SSD), one may apply any cross-validation method, e.g. the Mean Absolute Error (MAE) of the one-day-ahead (or n-days-ahead) forecast in a rolling-sample scheme.

3. MODELING

3.1. Simulation modeling

Let us consider three model examples that illustrate the method's performance on noisy data with some oscillations.

1) Oscillatory system with two periods

$$y_t = \sin(\pi t/15) + \sin(\pi t/20) + \varepsilon_t/2,$$

where (ε_t) are independent identically distributed random variables with $\varepsilon_t \sim \mathcal{N}(0,1)$. A grid with $t = 0, \dots, 299$ of length 300 was taken, and a trajectory was simulated on it.

To estimate the oscillation period, the method described in [2, 3] was applied. According to this method, the spectrum of the matrix $\log(A)$ is found. We assume that matrix A does not have multiple eigenvalues. Under this condition, oscillation with angular frequency ω will correspond to a pair of complex conjugate numbers $z = \alpha + i\omega$ and $\bar{z} = \alpha - i\omega$, where the oscillation period is given by π/ω.

In the first example, the estimated periods of oscillation were 30.49 and 39.93, while the original periods in the system before noise were 30 and 40 (see Fig. 3.1). In the two following examples, the estimated oscillation periods were the same as in the first one, up to rounding accuracy (see Fig. 3.2 and 3.3).

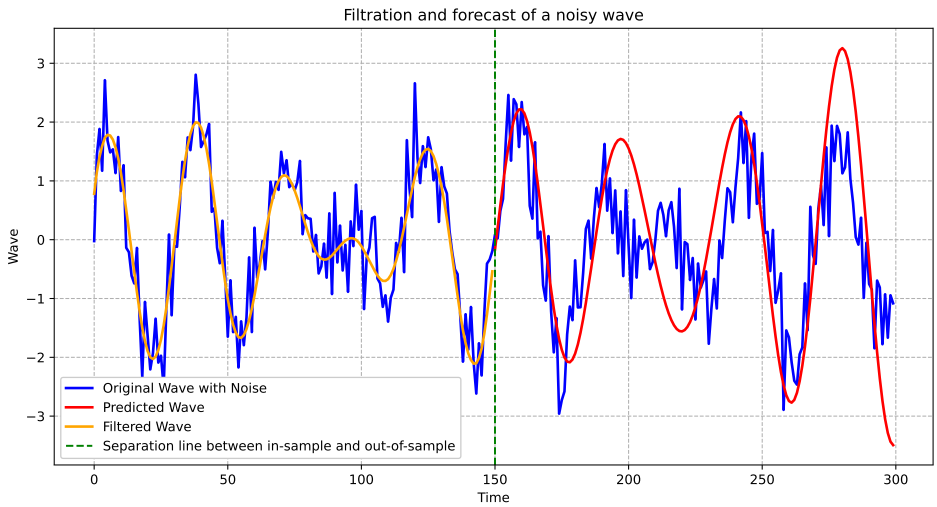

Fig. 3.1. Filtration and forecast of a noisy wave.

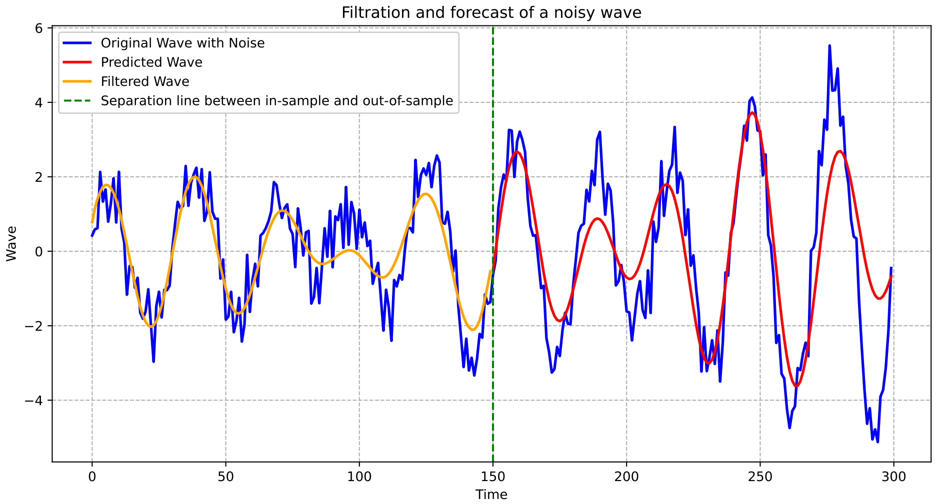

Fig. 3.2. Filtration and forecast of a noisy wave with exponents.

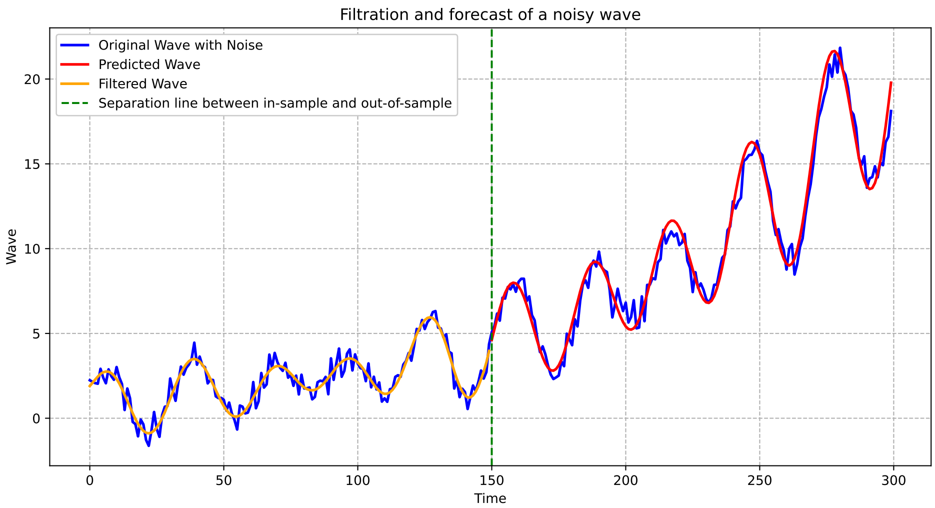

Fig. 3.3. Filtration and forecast of a noisy wave with exponential trend.

2) Oscillatory system with variable amplitude

$$y_t = e^{0.005t}\sin(\pi t/15) + e^{-0.001t}\sin(\pi t/20) + \varepsilon_t/2.$$

3) System with oscillations and exponential trend

$$y_t = e^{0.01t} + e^{0.005t}\sin(\pi t/15) + e^{-0.001t}\sin(\pi t/20) + \varepsilon_t/2.$$

3.2. Electricity price modeling

For modeling based on real data, we will consider electricity prices. The electricity market is of interest due to the presence of high volatility, multiple seasonality and calendar effects. In [11], it is shown that hourly electricity prices exhibit intraday, weekly, and monthly periodicities, as well as variable volatility.

We use daily NordPool system data prices† from 2016 to 2020, as considered in the paper [9], because it serves as the unconstrained market clearing reference price for the European Nordic region. Daily prices were transformed into $/kWh.

The sample consists of 1170 observations from January 01, 2016, to June 26, 2020. For out-of-sample forecasting, a rolling window of $22 * 6$ days (number of working days in a month multiplied by the number of months) is used (see Fig. 3.4).

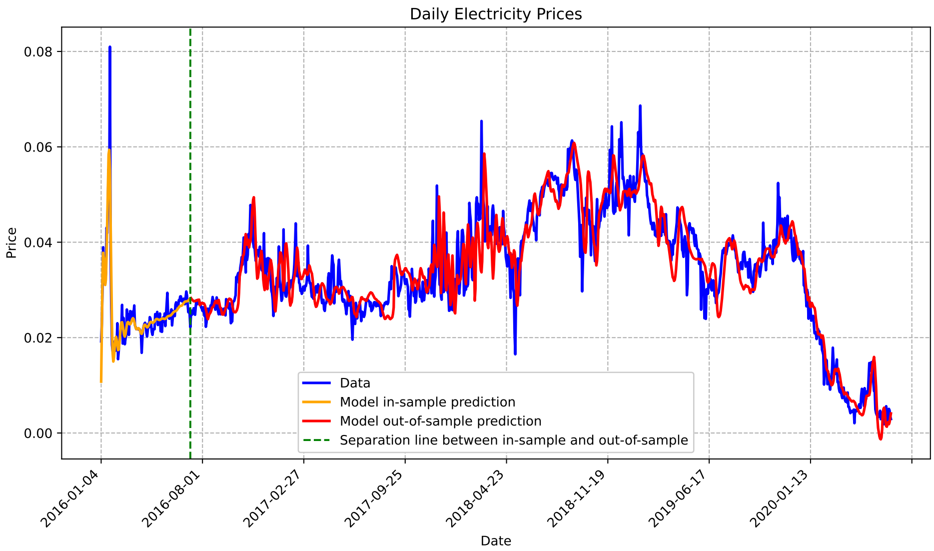

Fig. 3.4. Electricity price with the model filtration and 1-day-ahead forecasts with a 6 month floating window. The estimated periods on the interval 2016-01-01 to 2016-06-30 are 11.22 and 22.34.

To determine the state space dimension k, we used the mean absolute error for 1, 5, 22 days in a rolling-sample scheme with an interval size for modeling of 226 days (half a year). From the graph, it can be observed that the optimal dimensions for forecasting are $k = 5$ for daily and weekly predictions (see Fig. 3.6 and 3.7). It should be noted that for large dimensions k, out-of-sample errors increase, although in-sample errors decrease.

A comprehensive review of literature on electricity price forecasting (EPF) methods and their comparison from 2012 to 2022 is presented in the article [14]. Notably, during relatively stable times, when conditions for building traditional econometric models are met, such methods work well and demonstrate high predictive characteristics. Even considering high volatility, the combined ARMA-type model and GARCH-type model are still applied [4, 6, 11].

To make a comparison with ARMA-type models, for which we selected a stationary series from January 01, 2016, to June 30, 2016. The results of corresponding tests are provided in table 3.1.

Table 3.1. Stationarity tests results

Test	**Statistic**	**p-value**
ADF	-5.3567	4.17×10^{-6}
KPSS	0.2103	> 0.1
DF-GLS	-4.6647	5.38×10^{-6}

†The Nord Pool exchange has free data https://www.nordpoolgroup.com/en/Market-data1/Dayahead/Area-Prices/SYS1/Daily/?view=table

Besides, an alternative and well-established data filtering method with seasonality, Holt–Winters exponential smoothing, was considered.

Fig. 3.5 shows the mean absolute error dependencies for our method for different values of k, as well as for the ARMA(1,1) model, which proved to be the best model of this class on the training set, and the ETS model using Holt–Winters' method. One-day-ahead, weekly, and monthly forecasts were made using a rolling-window scheme for all models.

Over an extended period, the markets often exhibit a complex nonlinear structure as well as natural non-stationarity. An attempt to forecast the price for the next six months is shown in Fig. 3.5. It can be seen that our presented model captures the trend, while the ARMA model and the Holt–Winters' method, even though it gives more accurate in-sample forecast, degenerate into a horizontal line. The fact that these models struggle to predict the behavior of such data has long been known, so researchers seek approaches to improve their quality. A survey on subspace methods for estimating linear dynamic models can be found in [1].

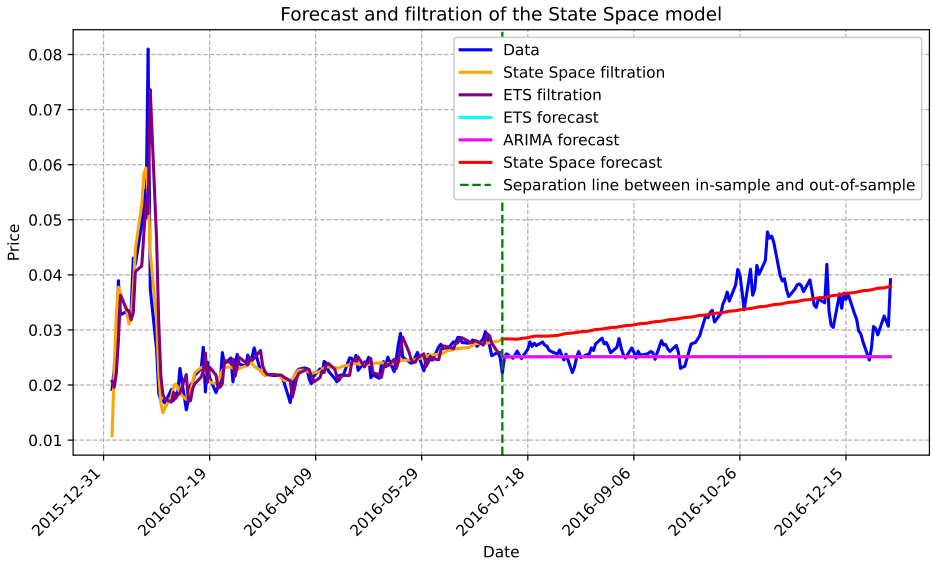

Fig. 3.5. State-space filtration and forecast and comparison with ARIMA forecast and Holt–Winters ETS filtration. The periods calculated by the State Space model are 11.22 and 22.34 days.

Fig. 3.6, 3.7 and 3.8 show how the MAE of considered method changes depending on the dimension of the state space k. For the short-term forecasts, the optimal value of k is 5, whereas for monthly forecasts, it is equal to 1.

The implementation of our forecasting and filtering method in the Python programming language can be found at the following link [7].

4. CONCLUSION

Methods of forecasting based on the state space representation of a system have a wide range of applications for estimating various dynamic systems. However, in different research contexts and on different types of data, choosing the appropriate method is a non-trivial task, considering the inherent advantages and disadvantages of each method. Our proposed forecasting method and an improved filtering method allow working with ill-conditioned system matrices. The ability to identify periods of fluctuations is also of interest for understanding the behavior of cyclical indicators, particularly economic ones.

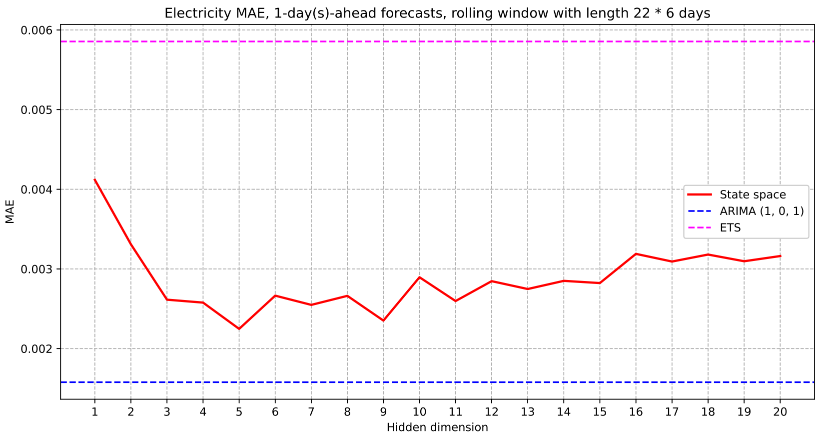

Fig. 3.6. 1-day-ahead MAE over 2016 (6 month window).

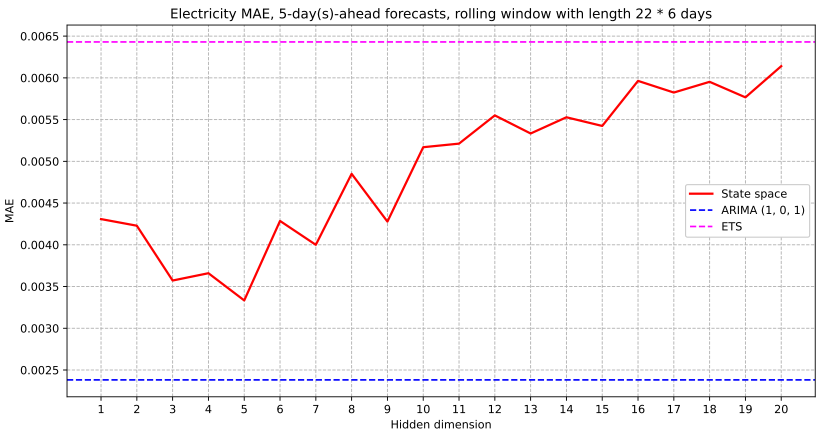

Fig. 3.7. 5-days-ahead MAE over 2016 (6 month window).

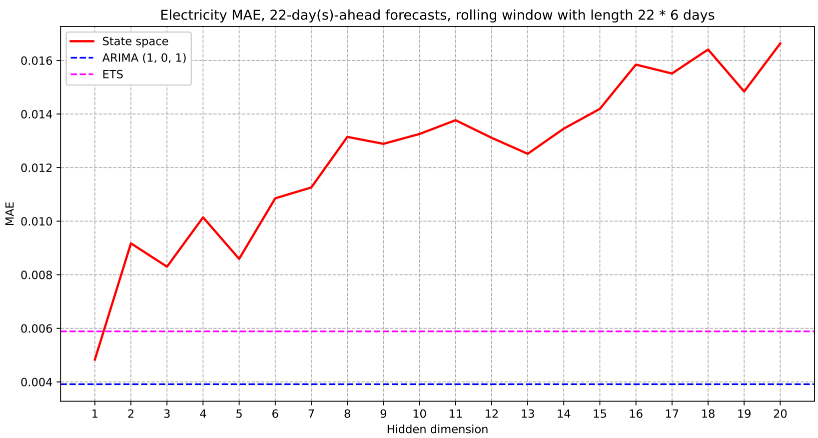

Fig. 3.8. 22-days-ahead MAE over 2016 (6 month window).

To illustrate the method, simulation modeling was conducted on typical types of oscillatory systems, and the forecast quality was evaluated with different horizons for electricity prices.

During the comparison of model forecasts on electricity price data, it was found that the forecasts of the proposed method yield a comparable mean absolute error to the forecasts of traditional econometric models such as ARMA on stationary series and a well-known seasonal data filtering method like Holt–Winters' method. The mean absolute error was calculated for forecasts with a rolling window of 6 months.

It is worth noting that the method performs well in handling multiple seasonality and exponential trends inherent in data with such a complex structure as electricity prices. Additionally, oscillation periods were calculated, and they turned out to be 11 and 22 days, corresponding to two-week and monthly seasonality.

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