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Game-Theoretic Models of Quality Management in Organizations under Corruption

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Abstract: We consider game theoretic models of the quality management in an organizational system under corruption. A graph theoretic formalization of the process approach in quality management is proposed. In a static case, a problem of quality management in a two-level organizational system under corruption is formalized as an inverse Stackelberg game with an additional viability condition. It is solved by means of Germeier theorem. In a three-level model we analyze how the Principal can constraint corruption by penalties. The conditions under which the Principal rather supports corruption than constraints it are determined. In a dynamic case, a quality indicator is considered as a state variable which changes in a discrete time due to an equation of dynamics. A two-level model of the type "supervisor-agent" is studied first. It is assumed that a quality requirement is obligatory for the agent but he can weaken this requirement in exchange for a bribe to the supervisor. Then we build and analyze a three-level model by adding the Principal that can charge penalties to other players if a bribe is found. The game theoretic models are investigated numerically by means of simulation modeling.

Keywords: corruption; game theory; quality control; organizational systems; simulation modeling

1. INTRODUCTION

A problem of quality management is very actual for any organization. A well known concept of quality management based on a process approach is used worldwide. There are official ISO 9000 standards that describe and regulate the quality management.

However, employees are active agents whose behavior is strategic. Particularly, the employees can propose bribes to their supervisors if they want to weaken too strong quality requirements, and then a corporative corruption arises.

There is a big stream of the mathematical modeling of corruption. Particularly, some papers are devoted to the corruption in hierarchical organizations. Several important in-sights and recommendations are received.

The authors' concept of corruption modeling is based on two ideas. First, corruption is treated as a feedback on bribe in an organizational control system. Second, a struggle with corruption is successful if some requirements of sustainable development (for example, quality requirements) are satisfied for this system. Therefore, the most adequate model of the corporative corruption is an inverse Stackelberg game with phase constraints. In a static case, an inverse Stackelberg game has a special additional constraint that plays a role of the phase constraint.

For the solution of inverse Stackelberg games we use Germeier theorem. In a static setting it allows for an analytical solution, and in a dynamic setting some algorithms of simulation modeling are used. A different software on Java and Python programming languages is used for numerical calculations.

The contribution of this paper is the following:

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- a problem of quality management in a two-level organizational system under corruption is formalized as a static inverse Stackelberg game with an additional requirement, and its solution is found by means of Germeier theorem;
- in a three-level organizational system a similar model is analyzed analytically and numerically. The conditions under which the Principal supports corruption are determined;
- in a dynamic setting, a problem of quality management in two-level and three-level organizational systems under corruption is formalized as a dynamic inverse Stackelberg game with phase constraints;
- a method of qualitatively representative scenarios in simulation modeling is used for the solution of the mentioned dynamic inverse Stackelberg games.

In Section 2, a literature review on the topic of the paper is provided. In Section 3, we consider static models of quality management under corruption in two-level and three-level organizations. These models are inverse Stackelberg games. In Section 4, we consider dynamic models of the same type, also in two-level and three-level organizational systems. In Section 5, concluding remarks are presented.

2. LITERATURE REVIEW

A concept of quality management was introduced by W. Edwards Deming [22] and developed by many other authors [29,59]. They proposed several principles of quality management, specified their implementation in industrial organizations, and used statistical methods to improve their efficiency. We develop this approach in our concept of sustainable management in organizations [25,52].

Most papers concerned modeling of corruption are based on Gary Becker's idea [11] that struggle with any crime makes sense (is economically rational) if benefits from pre-venting the crime are greater than the respective costs. In application to corruption this idea was developed by Susan Rose-Ackerman [54,55] and many other authors [58]. The main reviews are presented in [3,28]. Political corruption, especially in elections, is considered in [1,45–47]. Cultural aspects of corruption are touched in [30], its psychological aspects are considered in [10]. Other papers are devoted to the corruption in economics and state administration. So called "games of inspection" are described in [21], tax and other audits are studied in [18,19,56]. Influence of competition to corruption is studied in [46,57], compensation of market failures in [22], corruption in natural resources exploitation in [12]. In [20] they analyze a connection between corruption and shadow economics, in [36–38] – a "petty" corruption in licensing. In [28] the authors propose methods of struggle with corruption, in [65,66] they reveal the role of collusion.

Many papers [7–9,17,43,44] are devoted to the corruption in hierarchical organizations. As a rule, all mentioned models use the techniques of static games in normal form or multi-stage games. Essentially less number of papers is dedicated to the dynamic models of corruption based on optimal control models or differential games. Some examples can be found in the manual [27], and papers [14–16,42,68,70]. Thus, in [13] they have shown that an evolution of corruption in economics can result in the «revolution of honesty» due to which the system control vector moves to another state. In the model [16] situations exist in which national economies with the same development parameters are on different corruption levels. The paper [23] is devoted to the cyclicity in political corruption when struggle with corruption interchanges with its silent support on the macroeconomic level. Organization of inspections as an evolutionary game is presented in [33,34], and the same approach is used in [39] for description of the struggle with illegal logging. In [35] they use mean-field games that describe an interaction of a very big number of rational agents, including the case of impact of a special "main" player.

An important domain of description of the strategic behavior of active agents is auction models [4,41]. Two big groups can be differentiated there: models of collusion between the participants of an auction and models of collusion between an auctioneer and some participants. The data of specific studies are presented in [2,30,60], and the normative documents are given in [61-64]. In the last time the attention is attracted by the models of sustainable procurement that consider social and environmental aspects of the auction proposals [53,67].

The authors' concept of modeling corruption is presented in [25,68].

Olsder [49,50] has presented a review of inverse Stackelberg games. An original concept of hierarchical games belongs to Germeier [24]. He proposed a classification of hierarchical games based on a principle of guaranteed payoff of the leader, and proved a very useful theorem about a solution of inverse Stackelberg games in a static setting. In a dynamic setting, this approach was developed by Kononenko [26]. We proposed the respective numerical algorithms for the case of several agents [69]. These algorithms are based on the method of qualitatively representative scenarios in simulation modeling [51] which permits to reduce enumeration essentially.

3. STATIC MODELS

3.1. Quality Management without Corruption

We consider an organizational-economic system as a basic object. This system is a controlled one because its state depends on a production strategy. The system state is described by a set of characteristics that include costs and quality indicators. An economic agent exercises control. Namely, the agent chooses a strategy that maximizes his payoff function (his profit on a period).

So, we receive an optimization problem for the agent in the form:

$$p(q) - r \to \max, (q, r) \in S, \tag{3.1.1}$$

where $q = (q_1, q_2, \dots, q_m)$ is a vector of quality indicators; r – production cost; $p(q)$ – income from the sale of a product with quality indicators q; S – a set of feasible production strategies. A function p is continuous, and S is a compact set.

The quality indicators can reflect absolutely different characteristics of the product. Such indicators can describe consumption quality or characterize production (for example, its environmental impact). Suppose that all indicators take their values on the segment $[0,1]$, and a greater value corresponds to a better quality.

This model is a static one, and the income and cost are calculated for the whole report period. Any solutions are made in the beginning of the period.

An element of the set S is a pair (q, r) that corresponds to such a production strategy which provides a vector of quality indicators q and cost r.

Besides, for any pair of strategies:

$$(q_1, r_1) \in S, (q_2, r_2) \in S, (q_1 > q_2) => (r_1 > r_2);$$

$$(q_1 > q_2) \overset{\text{def}}{=} \forall i \in \{1,2, \dots, m\}(q_{1i} \geq q_{2i}), \exists j: (q_{1j} \geq q_{2j}).$$

We can write $S = PO(f(P))$, where $P = \{(p_1, p_2, \dots, p_n)\}$ is a set of controlled parameters; f– a standard production function; PO – a function that makes the set Pareto-optimal.

Assume that for any element from S we can determine its preimage in P, i.e., such values of the controlled parameters that provide given values r and q. Thus, instead of the solution of the problem on the whole set Pit is sufficient to solve it on the set S and apply to the solution the function f^{-1}. A solution of the problem (3.1.1) can be considered both as a pair(q, r) and the respective set of parameters$(p_1, p_2, \dots, p_n)$.

A viability condition in the static setting means that all essential indicators of the system activity take their values from a given range. For an industrial organization such indicators are, for example, a quality of production, an adequate wage, a satisfaction of some environmental protection requirements, and so on. For the quality indicators q_i these conditions can be written as $0 \le q_i \le 1, i = 1, \dots, m$.

The greater are these indicators, the higher is the production quality. Therefore, a viability condition (an admittance) for an indicator $i$can be written as:

$$q_i \ge a_i, a_i \in [0,1], i = 1, \dots, m. \quad (3.1.2)$$

The value $a_i$determines a minimal admissible value of the respective indicator. If there are no constraints for an indicator then $a_i = 0$.

Thus, we go from the problem (3.1.1) to the problem

$$p(q) - r \to max, (q, r) \in S \cap A, A = [a_1, 1] \times [a_2, 1] \times [a_m, 1] \times [0, \infty). \quad (3.1.3)$$

As viability conditions narrow down a domain S, a maximal payoff found with consideration with these conditions will be less or the same. This fact motivates to neglect a viability condition to increase a payoff.

According to the ISO 9000 international standards, any activity should be considered as a technological process. In any organization these processes have a complex interaction that form a system of processes [22].

Let us consider an industrial system as a tree$D = (X, U)$, directed to the root, where X is a set of processes (vertices), U is a set of flows between these processes (arcs).

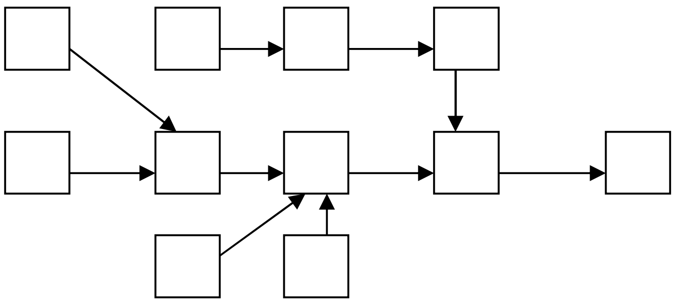


Fig. 3.1.1. An example of a tree-like industrial structure

The root of the tree corresponds to the last production process: its output goes to a consumer. The processes can reflect arbitrary types of activity. Besides a production, it can be an intermediate quality control, a preparation of reports or other documents, a maintenance of a ready production. A production flow from one process to another is characterized by two quantities: a vector of quality indicators q and a production cost r.

An agent's objective is a choice of the optimal set of strategies that provides a maximal payoff. This set distributes the strategies among the processes. In general case a process can have several input flows. Then characteristics of the output flow depend on all these variables. An initial process has no input flows. An output of the last process goes to the consumer, and its characteristics determine the agent's payoff.

This description of an organizational-economic system allows to find a Pareto-optimal set of strategies S step by step, from one process to another. Here we use a dynamic programming principle.

The Java platform was used for writing the application, and the NetBeans 6.9 package served as a development environment. The program builds a set S and finds the strategies that provide a maximal payoff with consideration of viability conditions or without it.

3.2. Two-level System under Corruption

It is not always advantageous for an agent to satisfy the viability conditions. Thus, the Principal who establish these conditions must control their satisfaction. Usually, the Principal is a legislative body, a local control agency or a certification organization. So, the Principal cannot control the viability conditions personally and delegates the control functions to special supervisors.

Consider the following regulation pattern. An organizational-economic system is a controlled object. The first control subject is an agent (a governing body of the organization) who chooses a production strategy. The model is a static one, and the agent knows which vector of quality indicators q and cost r corresponds to each strategy p. Also, the agent knows the viability conditions established by the Principal (the latter does not participate in the model explicitly).

The second control subject is a supervisor who is responsible for the satisfaction of viability conditions. Thus, we receive a two-level hierarchically organized control subsystem "supervisor-agent".

Let us assume first that there is only one quality indicator, and the supervisor has a complete and certain information about its value. If the viability conditions are satisfied then any actions are required, otherwise the supervisor should fix the situation. Besides, the supervisor can charge a penalty to the agent. The penalty value can depend on the difference between normative and actual values of the indicator.

Ideally, the supervisor is an impartial arbitrator who honestly does her job. Her reward is fixed by the Principal in advance, and the possible penalty goes completely to the Principal. However, a real situation may differ.

Suppose that the supervisor can extend the domain of viability by the controlled indicator. Naturally, she does it for a reward (bribe) from the agent.

The check-up procedure for an$i$-th indicator is the following.

1. The Principal reports an admittance range that determines the viability condition. Suppose that an$i$-th component of this vector is equal toa_0.
2. The agent chooses a control strategy. After this, the value of the$i$-th quality indicator is equal toq_T, and the cost is equal tor_T. The viability condition may be satisfied or not.
3. The supervisor checks the value of the$i$-th indicator and exposesq_T.
4. If$q_T < a_0$ then the supervisor reports to the agent a corruption function$a(b)$. This function defines a dependence of the bound of admittance$a$on a bribe value$b$.
5. The agent pays to the supervisor a bribe$b$ and uses a production strategy that provides a new extended admittance condition$q_T \geq a(b)$. In this case the agent has additional cost connected with some production changes and, probably, pays a penalty.

Suppose that the agent does not know when a check-up will be performed and which indicator will be checked. Thus, in a two-level system the corruption concerns only steps 3-5. A mathematical formalization of this situation is a complicated version of the model (3.1.3).

The agent's payoff function$g_2$is his profit, or a difference between an income from new quality indicators and a sum of new production cost, bribe, and penalty. Thus, themodelhastheform

$$g_2(a, b, q, r) = p(q) - r - b - d(q, q_T) \to \max, \quad (3.2.1)$$
$$b \geq 0, (q, r) \in S, q_i \geq a(b), i = 1, \dots, m;$$

where$i$ – index of the controlled indicator; b – bribe value;$a(b)$ – a new bound of admittance by the indicator$i$; q – a new vector of quality indicators; r – a new production cost; S – a compact set of admissible strategies; d –a penalty function for the transfer from an old strategy that provides the quality$q_T$to a new strategy. It includes additional cost and possible penalty for violation of the viability condition. This function is supposed to be given and proportional to the distance between q and q_T, for example, $d(q, q_T) = k\|q - q_T\|, k > 0$.

The control variables are a new production strategy and a bribe. The domain of production strategy is an initial domain of feasible strategies with an additional condition of admittance by the checked i-th indicator. The domain of bribe is a semi-interval $[0, +\infty)$ that is specified by the function $a(b)$.

The supervisor's payoff function also corresponds to her profit. We do not consider a fixed reward, therefore the function coincides with the bribe value:

$$g_1(a,b) = b \to \max, a(b) \in A. \quad (3.2.2)$$

The supervisor chooses $a(b)$. This function must satisfy the following constraints that define the admissible domainA:

$$A: a(0) = a_0, q_T \leq a(b) \leq a_0. \quad (3.2.3)$$

The first condition determines a capture strategy of the supervisor: if there is no bribe then an initial viability condition is satisfied. The second condition means that the supervisor can extend an admittance bound up to the current value of the checked indicator. An additional extension has no reason.

Greater values of the bribe correspond to not less admittance domain. Therefore, $a(b)$is a monotonous non-increasing function.

Thus, we receive the model (3.2.1)–(3.2.3). This is a game theoretic model because the interests of players are different. A greater bribe is advantageous to the supervisor and non-advantageous to the agent, and the payoff function of each player depends on strategies of both players.

This game is a hierarchical one because the supervisor makes the first move when she reports to agent a response function $a(b)$. Given $a(b)$the agent chooses his bribe $b$and control variables p. The supervisor anticipates a best response of the non-benevolent agent and maximizes her guaranteed payoff.

Thus, we receive an inverse Stackelberg game [49,50], or Germeier game of the type Γ_2 [24,26]. However, the agent's payoff function is too complicated.

Let us transform this function. A choice of the production strategy by the agent has no impact to the supervisor's payoff function. In the same time, this choice depends on the admittance bound. Let us introduce the function

$$u(a) = \max_{(q,r)\in S, q_i \geq a} \left(p(q) - r - d(q, q_T)\right). \quad (3.2.4)$$

This function solves a problem of profit maximization for a given value of the admittance bound$a$. The agent can calculate this function for each a without any information about the supervisor's strategy.

As evident from the formula (3.2.4), this function is non-increasing on the segment [0,1] and attains its maximum in the point$q_T$because a further reduction of the quality is non–advantageous to the agent.

Moreover, even small deviations of a from q_T imply some changes in the current production strategy that incurs additional cost. Therefore, $\forall \varepsilon > 0\ \exists \delta > 0: u(q_T) - u(q_T + \varepsilon) > \delta$.

Thus, we can rewrite the model (3.2.2)–(3.2.4) in the form

$$g_1(a,b) = b \to \max_{a(b)}, g_2(a,b) = u(a) - b \to \max_{b}, \quad (3.2.5)$$
$$b \geq 0, q_T \leq a \leq a_0, a(0) = a_0.$$

This model is a Germeier game of the type Γ_2 [64], or an inverse Stackelberg game.

We can generalize the model for the case of several quality indicators. The supervisor's strategy and the function u become multidimensional. However, the idea of solution and the final result are the same, and the supervisor's strategy also includes two important points: the first one corresponds to the current system state, and the second one corresponds to the viability condition.

For solution of the game (3.2.5) we use Germeier theorem [64]. In this case the supervisor's payoff function does not depend directly on the admittance bound. Therefore, any valueof a may serve as her dominant strategy. It is natural to take $a = a_0$ because it corresponds to the law requirements:

$$a_a(b) = a_0, g_1(a_a(b), b) = \max_{a\in[q_T, a_0]} g_1(a,b) = b.$$

The supervisor's punishment strategy consists in the compulsion of the agent to provide the viability condition established by the Principal:

$$a_H(b) = a_0.$$

If the supervisor punishes the agent, then his payoff is equal to $g_2(a_H(b), b) = u(a_0) - b$.

The maximal agent's payoff in the case of his punishment is equal to:

$$L_2 = \max_{b\in[0,\infty)} g_2(a_0, b) = u(a_0).$$

In this case the agent changes a production strategy to provide an initial admittance range for the checked quality indicator. Thus, there is no reason to pay a bribe, and the optimal agent's strategy is

$$E_2 = \{0\}.$$

Let us introduce a set:

$$D_2 = \{(a, b): u(a) - b > u(a_0)\}. \quad (3.2.6)$$

A stepwise form of the bound of this set corresponds to the form of the function $u(a)$. The value $b^*$is a maximal bribe which the agent is agree to pay for $a = q_T$. In this case to pay a bribe is advantageous for the agent. Then, we find

$$K_1 = \sup_{(a,b)\in D_2} g_1(a, b) = b^*. \quad (3.2.7)$$

$$(x_1^\varepsilon, x_2^\varepsilon): g_1(x_1^\varepsilon, x_2^\varepsilon) \geq K_1 - \varepsilon.$$

$$x_1^\varepsilon = q_T, x_2^\varepsilon = b^* - \varepsilon.$$

$$K_2 = \max_{a\in[q_T, a_0]} g_2(a, 0) = 0.$$

We have $K_1 > K_2$, therefore, the ε-optimal supervisor's strategy is

$$a_\varepsilon(b) = \begin{cases} q_T, b = b^* - \varepsilon, \\ a_0, b < b^* - \varepsilon. \end{cases} \quad (3.2.8)$$

It remains to calculateb^*. For this purpose, let us build the setD_2in the coordinates $b, u(a)$. Itisseenthat

$$b^* = u(q_T) - u(a_0). \quad (3.2.9)$$

Thus, the solution of the game is the pair$\left(a_\varepsilon(b), u(q_T) - u(a_0)\right)$, and the respective values of payoff functions are equal to:

$$g_1^* = u(q_T) - u(a_0) - \varepsilon, \quad (3.2.10)$$

$$g_2^* = u(a_0) + \varepsilon.$$

Now consider the supervisor's optimal strategy with incomplete information:

$$a_\varepsilon(b) = \begin{cases} q_T, b = u(q_T) - u(a_0) - \varepsilon, \\ a_0, b < u(q_T) - u(a_0) - \varepsilon. \end{cases} \quad (3.2.11)$$

It has an essential shortage, namely, the supervisor must know the function $u(a)$. As it follows from (3.2.4), this function has a complex structure and depends on three other functions. Meanwhile, a check-up gives to the supervisor a certain information only about an actual value of the quality indicator.

However, the supervisor may not know a complete form of the function$u$. To construct an optimal strategy, it is sufficient to have estimates for two points. Suppose that the supervisor has a sufficient information to evaluate a distribution of the values $u(q_T)$and $u(a_0)$ (denote them by $u_T$andu_0).

$$u_0 \in N(\mu_1, \sigma_1), u_T \epsilon N(\mu_2, \sigma_2).$$

In this case

$$b^* \in N\left(\mu_2 - \mu_1, \sqrt{\sigma_1^2 + \sigma_2^2}\right) = N(\mu, \sigma). \quad (3.2.12)$$

The supervisor chooses a value$b_0$as an estimate for $b^*$and builds a function:

$$a_\varepsilon^0(b) = \begin{cases} q_T, b \geq b_0, \\ a_0, b < b_0. \end{cases} \quad (3.2.13)$$

After that the supervisor reports her strategy to the agent, two variants are possible.

1. A point with coordinates (q_T, b_0)belongs to the domain D. In this case the agent gives a bribeb_0, and the supervisor closes her eyes to the violation. Respectively, the payoffs are equal to:

$$g_1^* = u(q_T) - u(a_0) - b_0,$$
$$g_2^* = b_0.$$

2. A point with coordinates (q_T, b_0) does not belong to the domain D. The agent does not pay a bribe and provide the condition of viability. The supervisor receives nothing:

$$g_1^* = u(q_T) - u(a_0),$$
$$g_2^* = 0.$$

As the supervisor knows a distribution ofb^*, it is possible to calculate the following probability:

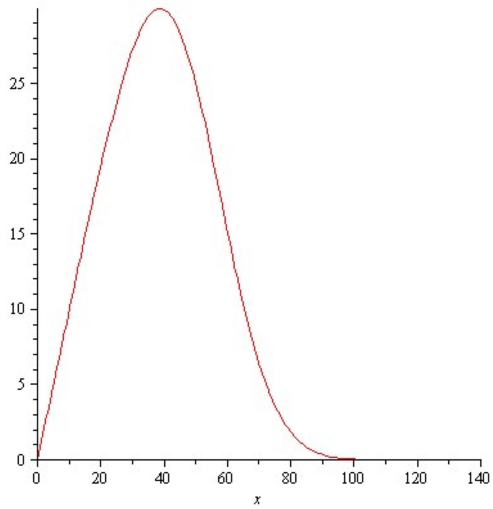
$$P(b^* > b_0) = \frac{1}{\sigma\sqrt{2\pi}} \int_{b_0}^{+\infty} e^{-\frac{(S-\mu)^2}{2\sigma^2}} ds. \quad (3.2.14)$$

In this case we can modify the supervisor's payoff function as follows:

$$f(b_0) = \begin{cases} b_0, b_0 < b^*, \\ 0, b_0 \geq b^*, \end{cases} \quad (3.2.15)$$

$$Ef(b_0) = \frac{b_0}{\sigma\sqrt{2\pi}} \int_{b_0}^{+\infty} e^{-\frac{(S-\mu)^2}{2\sigma^2}} ds \underset{b_0}{\to} \max. \quad (3.2.16)$$

The supervisor maximizes an expectation of her payoff. The problem (3.2.16) was solved numerically by means of the Maple package for different parameters of distribution of b^*. The supervisor's payoff function and its derivative for $b^* \in N(50,15)$are presented in Fig. 3.2.2.



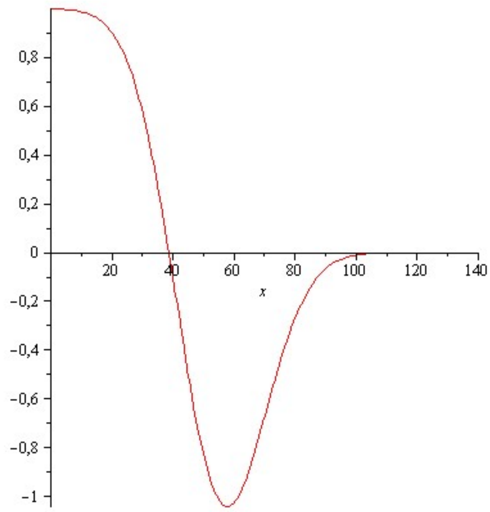


Fig. 3.2.2. An example of the graphs of the supervisor's payoff function
and its derivative for normal distribution

Besides, it is possible to calculate numerically a value of b_0 that is optimal for the supervisor. In the considered example $b_0^* = 38.7$. However, an expectation of the payoff is less than a required bribe: $Ef(b_0^*) = 29.97$.

Using the log-normal distribution as a distribution rule with the same median and variance implied an increasing of the optimal for the supervisor values of bribe and payoff expectation (43.26 and 41.75 respectively) (Fig. 3.2.3).

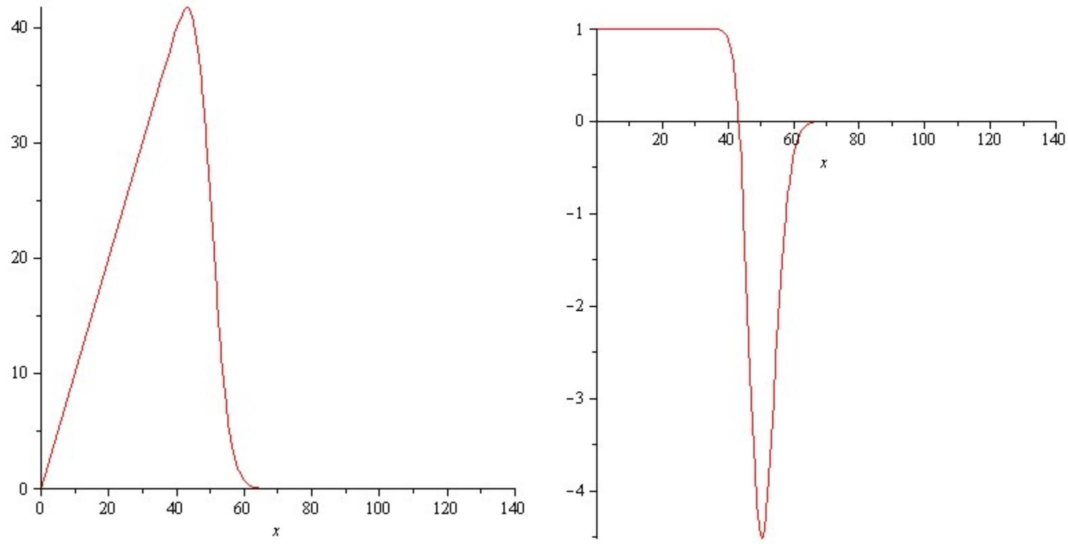


Fig. 3.2.3. An example of the graphs of the supervisor's payoff function and its derivative for log-normal distribution

In the considered two-level model of corruption the Principal does not interfere in the supervisor's activity in any way. Respectively, any corruption interactions remain unpunished. That's why an optimal strategy of the supervisor is a maximal violation of the viability conditions in exchange of the maximal possible bribe from the agent.

It is natural that the Principal is not satisfied and tries to fix the situation. The most evident way to do it is to charge penalties both on a bribe-giver (agent) and on a bribe-taker (supervisor). Let us consider a modified model where the Principal still participates implicitly.

This model is based on the model (3.2.16) where the supervisor does not know an exact value of b^*. The supervisor's payoff function is defined by the formula (3.2.15) and is equal to the bribe or to zero if the agent does not pay a bribe. However, in the latter case the agent may report to the Principal a fact of extortion. Then the Principal charges to the supervisor a penalty as a function of the required bribe and a new admittance bound.

Let us first consider a case when the penalty depends only on a bribe. In this case an admittance bound still does not impact the supervisor's payoff, a punishment strategy and a domain D_2 do not change, and therefore an optimal strategy remains the same. However, the value of $b_0$is found with consideration of a penalty. The supervisor's payoff is equal to:

$$f(b_0) = \begin{cases} b_0, b_0 < b^*, \\ -x(b_0), b_0 \geq b^*. \end{cases} \quad (3.2.17)$$

The objective is to maximize a payoff expectation. This expectation does not depend on b because it is predetermined by supervisor's strategy and a value of b^*:

$$Ef(b_0) = bP(b *> b_0) - x(b_0)\big(1 - P(b *> b_0)\big) \to \max_{b_0}. \quad (3.2.18)$$

A penalty function $x(b_0)$is determined by the Principal and is known to the supervisor. It is established officially and is known in advance. The simplest example is a constant function, i.e., a fixed penalty that does not depend on bribe. Certainly, the penalty may be proportional to the bribe or be a piecewise continuous function. In the latter case the domain segment is subdivided on several intervals, and the values of penalty are different on the intervals.

In general case the Principal tries to choose such penalty function that the supervisor's payoff expectation is negative (for allb_0 or for the values from a domain). In this case the corruption becomes non-advantageous for the supervisor.

Denote by $F_{b^*}(b_0) = P(b^* \le b_0)$ a distribution function of b^*. Then for given b_0 it is necessary for the Principal that an inequality holds:

$$b_0\big(1 - F_{b^*}(b_0)\big) - x(b_0)F_{b^*}(b_0) \le 0, or x(b_0) \ge \frac{b_0\big(1 - F_{b^*}(b_0)\big)}{F_{b^*}(b_0)}. \quad (3.2.19)$$

The main shortage of the model is that the supervisor's payoff depends directly on her information about a real value of b^*. Therefore, if a variance of the respective random variable tends to zero, or the supervisor is able to evaluate quite accurate the agent's losses from correction the viability violations then the penalty increases. In the limit case when the supervisor knows exactly the value of b^*, even a maximal possible penalty is useless because a probability of bribe becomes equal to one.

At last, let us consider a case when the penalty function depends on the admittance bound. To solve the problem in this setting it is necessary to use the following formulation:

$$K_1 = \sup_{(a,b)\in D_2} \Big(bP(b^* > b) - x(a,b)\big(1 - P(b^* > b)\big)\Big). \quad (3.2.20)$$

This solution is less trivial and depends directly on the form of the function $x(a,b)$. In this case the supervisor can decrease an admittance bound not up to the current value of the indicator but to an intermediate value. However, a general trend and the main shortage are the same: if the supervisor is well informed about b^* then no penalty will help to overcome corruption.

3.3. Three-Level System under Corruption

The respective model has the form

$$\begin{aligned} g_0(a,b,c) &= c + z(a) - l_2(a,b,c) \to \min_{c,l_i,\alpha,\beta}, \\ g_1(a,b,c) &= b - l_1(a,b,c) \to \max_{a(b)}, \\ g_2(a,b,c) &= u(a) - b - l_2(a,b,c) \to \max_b. \end{aligned} \quad (3.3.1)$$

Here c is the Principal's anti-corruption cost; $z(a)$ – the Principal's losses from the viability violation (a non-decreasing function); $z(a_0) = 0$; $l_1(a,b,c)$ – a penalty charged to the supervisor; $l_2(a,b,c)$ – a penalty charged to the agent; l_1^*, l_2^* – functions that show which part of a penalty has an economic utility for the Principal. In some cases, penalties may be non-material, for example, a prohibition to hold an office or even an imprisonment. Such penalties punish the briber but have no economic utility for the Principal.

In the absence of corruption the penalty functions are equal to zero, as well as in the absence of the Principal's cost:

$$l_i(a_0, 0, c) = l_i(a,b,0) = 0.$$

In general case, the Principal strategies are $l_1(a,b,c)$, $l_2(a,b,c)$, l_1^*, l_2^* and c. However, in real life the Principal can control only the variable c, and we will stick to this interpretation.

Introduce the following penalty functions:

$$l_i(\mathrm{a,b,c}) = p(\mathrm{a,c}) \cdot s_i(a,b), i \in \{1,2\}. \quad (3.3.2)$$

Here $p(a,c)$ is a probability of detection a fact of corruption. The probability of detection increases with augmentation of anti-corruption cost and is decreasing of an admittance bound a.

Denote $s_i(a,b)$ – a penalty,

$$s_i(a_0, 0) = 0, p(a,0) = 0; \quad (3.3.3)$$

$$\begin{aligned} g_0(a,b,c) &= c + z(a) - l_1(a,b,c) - l_2(a,b,c) \to \min_c, \\ g_1(a,b,c) &= b - p(a,c)s_1(b) \to \max_{a(b)}, \\ g_2(a,b,c) &= u(a) - b - p(a,c)s_2(a,b) \to \max_b, \end{aligned} \quad (3.3.4)$$

An algorithm of solution of the problem (3.3.4) is the following.

1. Given a fixed value of c the supervisor and the agent find a solution of the inverse Stackelberg game (Germeier game Γ_2) – a value of bribe and an admittance bound. This solution depends on the parameter c.
2. The Principal chooses the value of that minimizes its payoff function.

Consider as an example a pure financial penalty. The penalty is equal to the upper bound of the respective intervals. Then the payoff functions have the form:

$$s_1(b) = \begin{cases} 50b, b \in [0,25000] \\ 60b, b \in [25000,150000] \\ 90b, b \in [150000,1000000] \\ 100b, b \in (1000000, +\infty). \end{cases} \quad (3.3.5)$$

$$\tilde{s}_2(b) = \begin{cases} 30b, b \in [0,25000] \\ 40b, b \in [25000,150000] \\ 80b, b \in [150000,1000000] \\ 90b, b \in (1000000, +\infty). \end{cases} \quad (3.3.6)$$

$$s_2(a,b) = \tilde{s}_2(b) - u(a) + u(a_0) \quad (3.3.7)$$

Additional summands in the second function mean that in the case of detection the agent should provide an initial viability condition. These summands are not included in the Principal's payoff function:

$$l_1 * (\mathrm{a},\mathrm{b}) = p(\mathrm{a},\mathrm{c}) \cdot s_1(b); \quad (3.3.8)$$
$$l_2 * (\mathrm{a},\mathrm{b}) = p(\mathrm{a},\mathrm{c}) \cdot \tilde{s}_2(b).$$

Denote:

$$s^*(b) = \begin{cases} 80b, b \in [0,25000] \\ 100b, b \in [25000,150000] \\ 170b, b \in [150000,1000000] \\ 190b, b \in (1000000, +\infty). \end{cases} \quad (3.3.9)$$

The problem takes the form:

$$g_0(a,b,c) = c + z(a) + p(a,c)s*(b) \to \min_c, \quad (3.3.10)$$
$$g_1(a,b,c) = b - p(a,c)s_1(b) \to \max_{a(b)},$$
$$g_2(a,b,c) = u(a) - b - p(a,c)s_2(a,b) \to \max_b.$$

Let us solve the problem (3.3.10) according to the proposed algorithm. Assume that $p(a,c) = p(c)$ because this simplifies the solution of the game $\Gamma_2$essentially. As before, the punishment strategy is: $aH(b) = a_0$. The difference from the model (3.2.5) begins from the definition of the domain D_2.

$$D_2 = \{(a,b): g_2(a,b,c) > u(a_0)\}. \quad (3.3.11)$$
$$u(a) - b - p(\mathrm{c})s_2(a,b) - u(a_0) > 0 \Leftrightarrow$$
$$\big(1 - p(\mathrm{c})\big)\big(u(a) - u(a_0)\big) - b - p(\mathrm{c})\tilde{s}_2(b) > 0 \quad (3.3.12)$$

The domain that corresponds to the inequality (3.3.12) is similar to the domain D_2 from (3.3.6). Let us find a maximal value of b that belongs to the domain:

$$b_i^* = \min\left(\frac{\big(1 - p(\mathrm{c})\big)\big(u(a) - u(a_0)\big)}{1 + p(\mathrm{c})k_i}; e_i\right). \quad (3.3.13)$$

Here k_i is a coefficient at b in the i-th row of the formula (3.3.13), e_i – the right bound of the interval in the i-th row.

$$b^* = \max_{i \in \{1,2,3,4\}} (b_i^*). \quad (3.3.14)$$

It is evident that $b^* > 0$ because each of $b_i^*$is positive. In fact, the supervisor solves the following problem:

$$g_1(b,c) = b - p(c)s_1(b) \to max, \;\; 0 \le b \le b * -\varepsilon, \quad (3.3.15)$$

$$b_0 = \underset{0 \le b < b^* - \varepsilon}{\operatorname{argmax}} \left(b - p(c) s_1(b)\right). \tag{3.3.16}$$

As $s_1(0) = 0$ then $g_1 \ge 0$, and the equality holds when the bribe is equal to zero. Suppose that in this case the supervisor sets an admittance bound equal to a_0 because she has no motivation to change the viability condition. Thus, an ε-optimal supervisor's strategy is the function:

$$a_\varepsilon^0(b) = \begin{cases} q_T, b = b_0, b_0 \ne 0, \\ a_0, otherwise. \end{cases} \tag{3.3.17}$$

The only agent's best response to this function is: $b = b_0$.

Go to the second step of the algorithm. Find such a value of c that minimizes the Principal's payoff function:

$$g_0(a, b, c) = c + z(a) - s*(b) \to \min_c. \tag{3.3.18}$$

Optimal for the game "supervisor-agent" values $a$and b are expressed by c:

$$g_0(a(c), b(c), c) = \begin{cases} c + z(q_T) - p(c)s*(b_0(c)), b_0(c) > 0, \\ c, b_0(c) = 0. \end{cases} \tag{3.3.19}$$

Notice that $b_0(0) > 0$, i.e. $g_0(0) = z(q_T)$. Then, $p(c)$is a non-decreasing function. Suppose that it has a non-decreasing inverse function $c(p)$in the sense that

$$c(p) = \min\{c: p(c) > p\}. \tag{3.3.20}$$

As it is seen from (3.3.13)–(3.3.14), when c increases the value of b^*decreases. The value of $b_0$also decreases because, first, a penalty increases and, second, an interval of taking the maximum decreases. It is simple to check that

$$b_0 = 0 <=> b - p(c)s_1(b) \le 0, \forall b > 0 <=> p(c) \ge \frac{b}{s_1(b)}, \forall b > 0.$$

In turn, the last inequality holds when

$$p(c) \ge \frac{b}{b \cdot \min_{i \in \{1,2,3,4\}} \{\tilde{k}_i\}} = \frac{1}{50}. \tag{3.3.21}$$

Here$\tilde{k}_i$ is a coefficient at b in the i-th row of the supervisor's penalty function.

For big values of $c$the supervisor will also refuse from taking a bribe. However, the Principal's payoff function will increase. Therefore, if c^* is the Principal's optimal strategy then

$$c^* \in \left[0, c\left(\frac{1}{50}\right)\right]. \tag{3.3.22}$$

If a struggle with corruption is the main objective of the Principal then $c^* = c\left(\frac{1}{50}\right)$is its optimal strategy.

It remains to answer the following question: may it be advantageous to the Principal to support a certain level of corruption that minimizes its cost? Otherwise, is there an internal point of maximum on the segment (3.3.22) that does not coincide with its right bound?

In the point $c(1/50)$ the Principal's payoff function has a break. So, let us first estimate $g_0(a(c), b(c), c)$ when $c \to c(1/50) - 0$. Then $a(c) = q_T$. Denote for simplicity $U = u(a) - u(q_T)$.

$$b_0 = \underset{0 \le b < b^* - \varepsilon}{\operatorname{argmax}} (b - p(c)s_1(b)) = min\{b^* - \varepsilon; e_1\};$$

$$b^* = \max_{i \in \{1,2,3,4\}} \min\left(\frac{\left(1 - p(c)\right)\left(u(q_T) - u(a_0)\right)}{1 + p(c)k_i}; e_i\right) =$$

$$= \max_{i\in\{1,2,3,4\}} \min\left(\frac{\frac{49}{50}U}{1+\frac{1}{50}k_i}; e_i\right) =$$

$$= \max\left\{\min\left\{e_1,\frac{49}{80}U\right\}, \min\left\{e_2,\frac{49}{90}U\right\}, \min\left\{e_3,\frac{49}{130}U\right\}, \frac{49}{140}U\right\}.$$

$$b_0 = \min\{b^* - \varepsilon; e_1\} = \min\left\{e_1,\frac{49}{80}U\right\} - \varepsilon.$$

The Principal's payoff is equal to

$$g_0(a,b,c) = \mathrm{c}\left(\frac{1}{50} - 0\right) + \mathrm{z}(q_T) - \frac{80}{50}\cdot\left(\min\left\{e_1,\frac{49}{80}U\right\} - \varepsilon\right).$$

As c is non-decreasing function then in the case when

$$\mathrm{z}(q_T) < \frac{80}{50}\cdot \min\left\{e_1,\frac{49}{80}U\right\},$$

it is advantageous to the Principal to support a non-zero level of corruption. If $z > 40000$ then simple bribes are not advantageous to the Principal.

Let us consider the case

$$p(c) \in \left(0,\frac{1}{100}\right),$$

$$b_0 = \underset{0\le b < b^* - \varepsilon}{\operatorname{argmax}} (\mathrm{b} - p(\mathrm{c})s_1(b) = b^* - \varepsilon.$$

$$b^* = \max_{i\in\{1,2,3,4\}} \min\left(\frac{\big(1-p(c)\big)\big(u(q_T) - u(a_0)\big)}{1+p(c)k_i}; e_i\right) =$$

$$= \max_{i\in\{1,2,3,4\}} \min\left(\frac{\frac{99}{100}U}{1+\frac{1}{100}k_i}; e_i\right) =$$

$$= \max\left\{\min\left\{e_1,\frac{99}{130}U\right\}, \min\left\{e_2,\frac{99}{140}U\right\}, \min\left\{e_3,\frac{99}{180}U\right\}, \frac{99}{190}U\right\}.$$

Though the calculations are not finished it is already clear that

$$\frac{99}{190}U > e_3 <=> U > 1.919.192 => b_0 = \frac{99}{190}U.$$

$$g_0(a,b,c) \le \mathrm{c}\left(\frac{1}{100}\right) + \mathrm{z}(q_T) - \frac{99}{100}\cdot U.$$

Thus, if $U > 1919192$ and $z(q_T) < (99/100)U$ then for the Principal it is more advantageous to support a maximally possible level of corruption than to get rid of it completely. More accurate evaluations of the parameter c require a specific information about z and U, and functions $\mathrm{c}(p)$ or $p(\mathrm{c})$.

An additional penalty in the case of detection encourages corruption. For example, if a function U contains compensation of the Principal's losses equal to $z(q_T)$ then a probability that the Principal will not fight with corruption essentially increases.

Besides, only very large corporations with immense profits can allow themselves such big bribes. Therefore, one of the ways of fighting with a big corruption is a small business development.

4. DYNAMIC MODELS

Let us consider dynamic modifications of the proposed models.

Agent's production function $p(q)$ reflects his income in dependence on the quality indicator q. A part of the income I is invested to the quality support, and the other part is his profit. The agent must provide a viability condition $q \geq a_0$ that defines quality requirements. However, it is a supervisor who monitors the viability condition and can weaken it subject to the function $a(b)$ in exchange to a bribe b from the agent. This process is implemented during a finite period of time T with a discrete step t.
A dynamical problem of control has the following form:

$$G_1 = \sum_{t=1}^{T} g_1^t = \sum_{t=1}^{T} b^t \to \max, \tag{4.1}$$

$$a(b) \in A; A = \{a: a(0) = a_0; 0 \leq a(b) \leq a_0\}; \tag{4.2}$$

$$G_2 = \sum_{t=1}^{T} g_2^t = \sum_{t=1}^{T} (1 - b^t - I^t) p(q^t) \to \max, \tag{4.3}$$

$$b^t \geq 0; I^t \geq 0; b^t + I^t \leq 1; \tag{4.4}$$

$$q^t \geq a(b^t); \tag{4.5}$$

$$q^{t+1} = I^t p(q^t) - \mu q^t, q^0 = q_0; t = 0,1,2, \dots, T. \tag{4.6}$$

Here G_1, G_2 are total payoffs of the supervisor and the agent respectively for the period T; g_1^t, g_2^t – their current payoffs on the step t; $p(q^t)$ – the agent's production function on the step t; b^t –a share of bribe ("kickback") in the agent's payoff on the step t; I^t – a share of investments to the quality support in the agent's payoff on the step t; $a(b^t)$ – the supervisor's corruption function on the step t; a_0 – an initial threshold value of the quality indicator providing the viability; μ –a coefficient of quality decreasing in the absence of investments; q_0 – an initial value of the quality indicator.
A difference game (4.1)–(4.6) with phase constraints (4.5) has the following information structure. At each step of the discrete time $t = 1,2, \dots, T$:
(1) given the function $a(b)$ the agent chooses the control values b^t, I^t;
(2) the values of $a(b^t), p(q^t), q^{t+1}$ are calculated;
(3) if $q^{t+1} \geq a(b^t)$ then $t := t + 1$ else go to step (1).
For identification of the model (4.1)–(4.6) we use the following hypotheses. The supervisor's corruption function describes capture [61] that is reflected by the conditions (4.2). The function $a(b)$ decreases monotonously on the segment $[0,1]$. In the absence of corruption $(b = 0)$ its value coincides with an initial threshold value a_0, and in case of the maximal kickback $(b = 1)$ the supervisor does not check the viability condition at all.
The function $p(q)$ has standard properties of a production function: it increases monotonously in its domain, it is equal to zero if the quality is equal to zero, it has a positive derivative and a non-positive second derivative. So, it is natural to use a power function.
Notice that in the case of corruption the value of quality may become less than a threshold value a_0 that defines viability, and the value of income decreases. However, to attain a value $a(b) < a_0$ of the quality indicator a less share of investments I may be sufficient. Thus, in dependence on the correlation of values of the model parameters and the form of functions $a(b), p(q)$ the agent should decide whether to give a bribe and how big it should be. Respectively, a model analysis allows to expose the conditions that make corruption non-advantageous.
Now add to the model the Principal that can charge penalties on the supervisor and on the agent if their corruption is detected. The quantities of respective penalties $s_1(a), s_2(a)$ are functions of the threshold value of quality a determined subject to corruption $a(b)$. The functions are supposed to be given. However, to catch a briber the Principal should exercise some efforts. The quantity of these efforts z is the Principal's control variable that serves as an argument of two functions: probability of corruption detection $P(z)$ and the respective cost $C(z)$. In the case of corruption $(a(b) < a_0)$ the Principal is charged a conditional

penalty (an estimate of its losses) proportional to the deviation $a_0 - a$ with a large coefficient M.
This dynamical control problem has the following form:

$$G_0 = \sum_{t=1}^{T} g_0^t = \sum_{t=1}^{T} [s_1(a) + s_2(a)]P(z) - C(z) - M(a_0 - a) \to \max, \tag{4.7}$$

$$z \geq 0; \tag{4.8}$$

$$G_1 = \sum_{t=1}^{T} g_1^t = \sum_{t=1}^{T} \left(b^t - s_1(a)P(z)\right) \to \max, \tag{4.9}$$

$$a(b) \in A; A = \{a: a(0) = a_0; 0 \leq a(b) \leq a_0\}; \tag{4.10}$$

$$G_2 = \sum_{t=1}^{T} g_2^t = \sum_{t=1}^{T} [(1 - b^t - I^t)p(q^t) - s_2(a)P(z)] \to \max, \tag{4.11}$$

$$b^t \geq 0; I^t \geq 0; b^t + I^t \leq 1; \tag{4.12}$$

$$q^t \geq a(b^t); \tag{4.13}$$

$$q^{t+1} = I^t p(q^t) - \mu q^t, q^0 = q_0; t = 0,1,2, \dots, T. \tag{4.14}$$

In comparison with the model (4.1)–(4.6) we added the following denotations: G_0 – a total payoff of the Principal for the period T; g_0^t – its current payoff at the step t; $s_1(a), s_2(a)$ – penalty functions; z – a quantity of the Principal efforts in detecting corruption; $C(z)$ – a function of the Principal's respective cost; $P(z)$ – a probability of detection; M – a conditional penalty coefficient.
The information structure of the difference game (4.7)–(4.14) with phase constraints (4.13) remains the same, and the supervisor and the agent know the payoff functions$s_1(a), s_2(a)$and a quantity of the Principal efforts in detecting corruption z.
A function of cost of the corruption detection $C(z)$is increasing and convex, $C(0) = 0$. A function of the corruption detection $P(z)$ is increasing and concave, $P(0) = 0$. Penalty functions of the supervisor and the agent $s_1(a), s_2(a)$, as in the case of the Principal, are linearly proportional to the deviation $a_0 - a$.
An analytical investigation of the dynamical control problems (4.1)–(4.6), (4.7)–(4.14) seems very complicated. Thus, we used a method of qualitatively representative scenarios in simulation modeling [51]. The results are similar to the static case.

5. CONCLUSION

Mathematical modeling of the social-economic systems is a very complicated and ambitious task. It is absolutely evident in the case of organizational corruption where a problem of model identification is especially difficult.
However, there is a big stream of literature on mathematical modeling of corruption. Even simplified mathematical models increase our understanding of this phenomenon and allow for some practical recommendations on its control.
In this paper we build and investigated game theoretic models of quality management under corruption. A quality management is characterized by viability conditions. We considered static and dynamic inverse Stackelberg games in two-level (supervisor-agent) and three-level (Principal-supervisor-agent) organizations. Some results are received analytically using Germeier theorem, and other are obtained by numerical calculations on the base of simulation modeling. In some cases, we revealed conditions under which the corruption is not advantageous for the players.
In the considered models the Principal does not participate in a corruption activity. Moreover, it has anti-corruption cost and charges penalties to the bribers, and ideally these penalties compensate a damage from violation of the viability conditions. However, for

certain values of the parameters the Principal is not economically interested in the complete eradication of corruption because it increases the costs for corruption detection and excludes penalties.

It is possible to substitute financial penalties by other preventive measures, for example, criminal responsibility and imprisonment. From one side, in this case the Principal will not strive for more penalties. From other side, this policy will encourage corruption on the level Principal-supervisor.

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