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Determining SST Aerodynamic Configuration and Power Plant Parameters under Epistemic Uncertainty

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Abstract: The paper considers the problem of determining parameters of the aerodynamic configuration and power plant of an advanced supersonic passenger transport (SST) at the stage of preliminary aerodynamic design under epistemic uncertainty associated with incomplete information about the initial data. Optimization models and algorithms based on them are proposed that operate with the designed SST initial parameters generated by experts on the basis of empirical prediction. Such parameters are proposed to be generated within Liu's uncertainty theory as uncertain quantities expressed by uncertainty distribution functions. The use of uncertainty theory will make it possible to formalize and perform aerodynamic design process by replacing the functions that depend on uncertain quantities with their numerical characteristics. Such numerical characteristics are effectively interpreted by the decision maker, since they have analogues in probability theory – expected value, quantile, variance. The use of uncertainty theory in solving optimization problems under uncertainty provides low computational costs compared to the theory of probability. The paper discusses the use of numerical methods in the proposed algorithms, since, additionally, it is required to solve the black box function optimization problem. This is due to the lack of simple analytical relations between the SST requirements and the SST aerodynamic configuration and power plant parameters. The adequacy of the developed algorithms is demonstrated by the aerodynamic predictions presented by the Pareto fronts of the objective functions, which allow choosing trade-off design solutions.

Keywords: supersonic passenger transport, aerodynamic design, parametric uncertainty, multiobjective optimization.

1. INTRODUCTION

To determine the parameters of the advanced SST aerodynamic configuration and power plant at the stage of preliminary aerodynamic design is an important task, the successful solution of which largely influences the efficiency of its operation, life cycle cost and competitiveness. A characteristic feature of the early stage of the advanced SST design is considered to be the problem of insufficiency or lack of aerodynamic requirements and initial data. Therefore, the use of classical (deterministic) calculation models may be inefficient. To generate the initial data of the preliminary aerodynamic design, experts are involved, operating with subjective estimates of parameter values. Therefore, in the SST preliminary aerodynamic design, it is advisable to use mathematical models that make it possible to carry out calculations under parameter uncertainty (indeterminacy).

Parametric uncertainty is related to the problem of obtaining the exact values of the parameters that are required when using classical deterministic mathematical models. In this case, there are two types of uncertainty reflecting the parameter non-determination - aleatory (objective) and epistemic (subjective). Within the problem of determining SST parameters (design solutions), aleatory uncertainty occurs when the information about stochastic (random)

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parameters is contained in statistical data, and the parameters are modeled by random variables with the probability distributions determined on the basis of these data (statistical modeling). Epistemic uncertainty arises when statistical data are insufficient or not available, and the information about the initial parameters of the SST aerodynamic design is generated by experts.

When solving aerodynamic design problems under uncertainty, it is known to use probability theory [1, 2], interval methods [3, 4], possibility theory [5, 6], fuzzy set theory [7, 8], uncertainty theory [9, 10]. A high interest in applying the theories designed to perform "smart"/"soft" computing is usually due to the lack of statistical data. In papers related to solving design problems under uncertainty, with regard to possible constraints, the optimization and verification of the objective function properties, being considered as numerical characteristics of the distribution of non-deterministic model values, are performed. This is explained by the fact that, depending on the theory chosen to represent the aircraft parameters, the result of calculating a function that depends on non-deterministic quantities is a random variable [11], an interval [12], a fuzzy number [13], a possibility value [14], an uncertain quantity [15], which cannot be directly used in optimization calculations. Therefore, optimization models are developed, in which the functions that depend on non-deterministic parameters are replaced by their numerical characteristics.

The main goals of developing optimization models, in which objective functions and constraints are replaced by their numerical characteristics [16], are to provide:

• the least sensitivity of objective functions to possible changes in the optimized and input parameters, i.e. objective robustness;

• feasibility of solutions for possible changes in the optimized and input parameters of constraints, i.e. feasibility robustness.

The tasks of preliminary aerodynamic design are complicated by the need to solve the black box function optimization problem that occurs when the structure of the objective functions/constraints is hidden or complex.

The technique of determining the advanced SST aerodynamic configuration and power plant parameters under uncertainty considered in the paper is based on the deterministic approach to developing the aerodynamic configuration of a supersonic cruise aircraft [17], which includes iterative procedures for numerically solving a system of nonlinear equations. This does not allow using optimization models with analytical relations.

Traditional methods of solving such problems under uncertainty are based on the use of probability theory. In the general case, the calculation of objective functions and the verification of constraints that depend on a set of random parameters are associated with the calculation of multidimensional integrals. For example, in papers devoted to solving reliability-based design optimization (RBDO) problems [18], in which the main attention is paid to the fulfillment of constraints under parametric uncertainty - ensuring robust feasibility, many methods are proposed for calculating a multidimensional integral that determines the probability of fulfilling the constraints dependent on random parameters. The use of statistical modeling methods (Monte Carlo method) and its modifications, which reduce the required number of calculations to obtain the probability of fulfilling stochastic constraints, does not provide sufficient computational efficiency in many applied problems [19, 20]. In [21], as an alternative approach to solving this problem, it is proposed to calculate the multidimensional integral by numerical methods, but it is noted that, in practice, the possibilities of numerical integration are limited by a relatively small number of measurements (5-6 parameters). A lot of research is devoted to the FORM (first-order reliability method) [22] and SORM (second-order reliability method) [23] approximation methods, which allow reducing computational costs when calculating the probability of fulfilling stochastic constraints. The SORM usually provides higher accuracy than FORM, since the first and second order derivatives are used for approximation.

When solving problems of preliminary aerodynamic design, it is proposed to use the theory of uncertainty [15] to build optimization models. This approach allows one to determine the parameters of the SST aerodynamic configuration and power plant, when the information about non-deterministic parameters (uncertain parameters) of the objective functions and constraints is generated by experts. In the theory of uncertainty, based on a strictly described axiomatics, the measure of uncertainty $M(\bullet)$ is introduced as the expert's confidence level that the event • will occur. The uncertain parameters are given by uncertainty distribution functions $\Phi_\xi(x) = M\{\xi \le x\}$ (Φ_ξ^{-1} is the inverse function), where x is the value of the uncertain variable ξ. Appendix provides the fundamental concepts of uncertainty theory used in the paper. It will be shown below that uncertainty theory can be effectively applied in aerodynamic design optimization involving black box function optimization and at the same time provide low computational costs. An example of solving the preliminary aerodynamic design problem is given – the aerodynamic configuration and power plant parametrization, which provides the implementation of the required SST performance with the initial data of non-deterministic nature.

2. METHODS AND ALGORITHMS

The problem of the SST aerodynamic configuration and power plant parametrization under uncertainty is proposed to be formalized and solved using the flow chart shown in fig. 1.

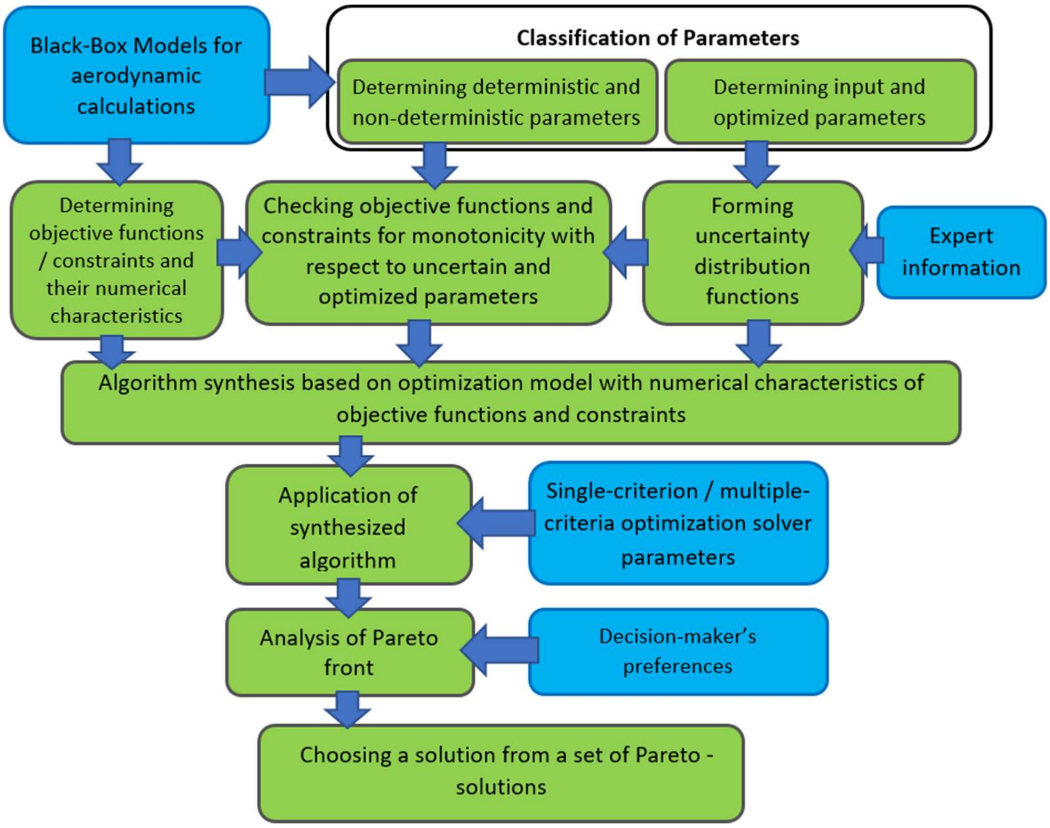

Fig. 1. Flow chart of the aerodynamic design problem formalization and solution

Based on input parameters of the Black-Box Model, sets of parameters are formed that are input and optimized for the optimization model with numerical characteristics of objective functions and constraints. The parameters within these sets are divided into deterministic and non-deterministic. For non-deterministic parameters, experts form the uncertainty distribution functions. As a result of determining the objective functions and their numerical characteristics, optimization models are generated that reflect the technical requirements for the designed object. These models can be generally represented as follows [9]:

$$\begin{cases} \min\limits_{\bar{x}'}/\max\{d_1[f_1(\bar{x}', \bar{\xi})],\ldots,d_m[f_m(\bar{x}', \bar{\xi})]\}, \\ M(g_j(\bar{x}', \bar{\xi})\leq 0)\geq \alpha_{g_j}, j=1, \ldots, p, \\ M((x_z+\delta_z)\geq a_z)\geq \alpha_{a_z}, \\ M((x_z+\delta_z)\leq b_z)\geq \alpha_{b_z}, \\ z=1, \ldots, k. \end{cases}$$

where d_i is the set of numerical characteristics of the objective function $f_i(\bar{x}', \bar{\xi})$, $i=1, \ldots, m$; $g_j(\bar{x}', \bar{\xi})$ is the constraint function; $\bar{x}'=(x_1', \ldots, x_k')$ is the vector of optimized deterministic and uncertain HST (high speed transport) parameters, $x_1'=x_1+\delta_1, \ldots, x_k'=x_k+\delta_k$; $\bar{\xi}$ is the vector of input uncertain parameters; m, p is the number of objective functions and constraints; $M(\bullet)$ is the measure of uncertainty (level of confidence) of the event occurrence •; α_{g_j} is the level of the uncertainty measure (level of confidence) given by the decision maker to fulfill the j-th constraint, $0\leq\alpha_{g_j}\leq 1$, a_z and b_z are the bounds of the optimized parameters; α_{a_z} and α_{b_z} are the levels of the measure of uncertainty used to control the bounds by the optimized uncertain parameter x_z', $0\leq\alpha_{a_z}\leq 1$, $0\leq\alpha_{b_z}\leq 1$.

The generated optimization models are used to synthesize an algorithm that allows one to get a Pareto front to analyze and choose trade-off design solutions. The flow chart of this algorithm is shown in fig. 2.

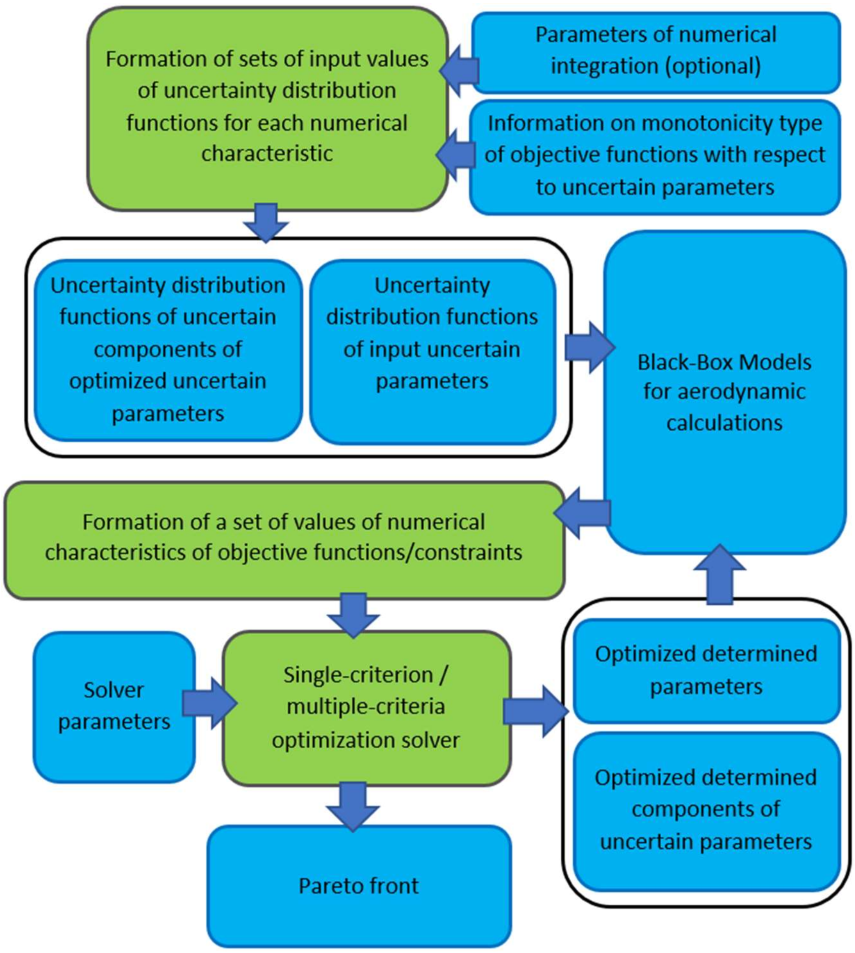

Fig. 2. Generalized flow chart of the synthesized algorithm for solving the aerodynamic optimization problem

To test the method for solving the preliminary aerodynamic design problems under uncertainty, the deterministic approach to determining parameters of the aerodynamic configuration and power plant of a supersonic aircraft [26] was chosen as the basis, the implementation of which is used in the paper as a Black-Box Model.

The method is based on a combination of the Breguet parameter analytical optimization $M_{cr}\cdot(L/D)_{cr}/C_e$, where L – lift, D – drag, M_{cr} is the cruise Mach number, $(L/D)_{cr}$ is the lift-to-drag ratio at cruise conditions, C_e is the cruise specific fuel consumption coefficient, and the numerical solution of the aerodynamic inverse problem – the transition from the flight performance to the aerodynamic configuration and power plant parameters.

The Breguet parameter optimization is carried out under the assumptions adopted in the supersonic aircraft aerodynamic calculation:

- the number of M_{cr} is assumed to be constant, given in the requirements,
- with an increase in cruise altitude $H_{cr}\geq 11$ km the thrust decreases proportionally to air density, i.e. $P(H_{cr})/P(11\text{ km.}) = \rho(H_{cr})/\rho(11\text{ km.})$, at a constant coefficient of specific fuel consumption $C_e(H_{cr}) = C_e(11\text{ km})$,
- the aerodynamic polar is symmetrical quadratic and is expressed as $C_D = C_{D0} + A_{20}C_L^2$, where C_{D0} is the drag coefficient at zero lift, A_{20} is the drag-due-to-lift factor, C_L is the lift coefficient,
- the calculation corresponds to the conditions of horizontal flight at a constant speed corresponding to the number of M_{cr}:

$$\begin{cases} Y_a = G_{des} \\ X_a = P, \end{cases}$$

where X_a - the drag force, P - the projection of the thrust of the power plant on the velocity axis $0X_a$, Y_a - the lift force, G_{des} - the design weight of the aircraft,

- drag coefficient at zero lift and specified M_{cr} is constant when changing the cruise altitude,
- the dependence of the specific fuel consumption coefficient on thrust is assumed to be linear and is expressed as $C_e(P) = C_{e_1} + C_e^P\,(P - P_1)$, where C_{e_1} is the specific fuel consumption coefficient at a given value of thrust P_1, C_e^P is the derivative of the thrust dependence of $C_e(P)$.

Accounting for the conditions of steady horizontal flight makes it is possible to obtain the dependence of $(L/D)/C_e$ on thrust normalized to an altitude of H=11 km:

$$\frac{(L/D)}{C_e} = \frac{G_{des}}{P_{11\text{ km}}\,\rho(H)/\rho(C_{e_1} + C_e^P(P_{11\text{ km}} - P_1))}.$$

Solution of the system of equations, expressing the conditions of steady horizontal flight:

$$\begin{cases} P(H) = \dfrac{\rho(H)(aM_{cr})^2 S}{2}(C_{D0} + A_{20}C_L^2), \\ G_{des} = \dfrac{\rho(H)(aM_{cr})^2 S}{2} C_L, \end{cases}$$

where a is the speed of sound, allows one to obtain the relation between the flight altitude and the available thrust, normalized to H = 11 km (engine operation mode):

Solving the equation $((L/D)/C_e)^P = 0$ with respect to P, it is easy to obtain expression for the optimal thrust corresponding to an altitude of 11 km, which ensures the maximum of the parameter $(L/D)/C_e$:

$$P_{optim} = \frac{4X_0 + P^* + \sqrt{(4X_0 - 2P^*)^2 - 3P^{*2}}}{6}$$

where $P^* = P_1 - \frac{C_{e_1}}{C_e^P}$.

Substituting this formula into the formula for $((L/D)/C_e)_{cr\ max}$, we obtain dependence for the optimal value of $(L/D)/C_e$ in the supersonic cruise flight mode:

$$\left(\frac{(L/D)}{C_e}\right)_{cr\ max} \approx \frac{\sqrt{C_L^{\alpha} q S (P_{optim}\text{-}X_0)}}{P_{optim}(c_{e_{max}} + C_e^P (P_{optim}\text{-}P_1))}$$

The wing area S is determined by solving a system of nonlinear equations relating the flight range, takeoff thrust-to-weight ratio, and maximum subsonic lift-to-drag ratio in accordance with the procedure presented in [26].

The uncertain parameters used in this procedure are:

- $G_{T_{des}}$ - consumption in fractions of takeoff weight during descent,
- $G_{T_{alt}}$ - fuel consumption in fractions of take-off weight during acceleration-climb,
- $G_{f_{load}}$ - functional load weight (equipment, crew, passengers),
- k_{pp} - weight characteristic of the power plant,
- C_f - equivalent friction coefficient corresponding to the wetted area,
- q_1, q_2 - coefficients in the relation of the specific structural weight (weight of 1 m^2 of the structure) to the aircraft volume,
- L_{cl_des} - climb + descent length.

As a result of numerical optimization, the Pareto front is determined in the plane of $M{\cdot}K/C_e$ and $K_{max\ M<1}$, where K means (L/D), fig. 3.

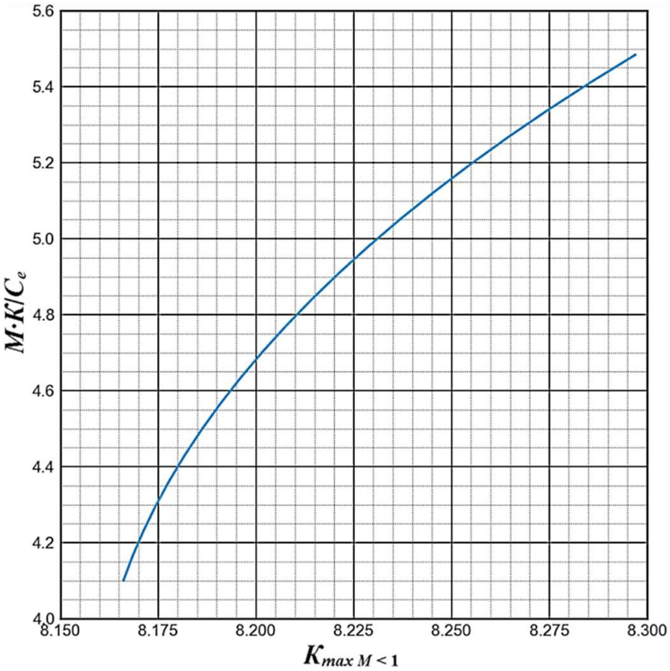

Fig. 3. Pareto front for $M{\cdot}K/C_e$ and $K_{max\ M<1}$

Within the flow charts in Fig. 1 and 2, optimization Models 1-2 are proposed to solve the presented problem of the SST preliminary design under uncertainty.

Model1. Critical values K_{max_cr} and $Br\_par=M{\cdot}K/C_e$.

$$\min_{\bar{x}'} \{E[K_{max\_cr}(\bar{x}', \bar{\xi})]\}, \max_{\bar{x}'} \{E[Br\_par(\bar{x}', \bar{\xi})]\},$$

Model 2. Critical values K_{max_cr} and $Br\_par=M{\cdot}K/C_e$

$$\min_{\bar{x}'} \{\inf_{\alpha}[K_{max\_cr}(\bar{x}', \bar{\xi})]\}, \max_{\bar{x}'} \{\sup_{\alpha}[\mathrm{Br\_par}(\bar{x}', \bar{\xi})]\},$$

In these optimization models, $\bar{x}' = (F, \gamma_v)$ is the vector of optimized parameters, F is the configuration factor, γ_v is the aircraft density, $\bar{\xi} = G_{T_{des}}, G_{T_{alt}}, G_{f_{load}}, k_{pp}, C_f, q_1, q_2, L_{cl\_des})$ is the vector of input uncertain parameters.

The remaining input parameters of the Black-Box Models are considered deterministic and therefore are not specified in the objective functions.

Using the expression for the expected value in Appendix (Theorem 1b), the trapezoid method, and the information about the monotonicity type of the objective functions with respect to uncertain parameters, we obtain expressions for approximate calculations $\mathrm{E}[K_{max\_cr}(\bar{x}', \bar{\xi})]$ and $\mathrm{E}[Br\_par(\bar{x}', \bar{\xi})]$.

$$\begin{aligned}\mathrm{E}\left[\mathrm{K}_{\max\_cr}(\bar{x}', \bar{\xi})\right] = \frac{1}{2n}\sum_{i=0}^{n-1} [K_{max\_cr}(F, \gamma_v, \Phi^{-1}_{G_{T_{des}}}\left(\frac{i}{n}\right), \Phi^{-1}_{G_{T_{alt}}}\left(\frac{i}{n}\right), \Phi^{-1}_{G_{f_{load}}}\left(\frac{i}{n}\right),\\ \Phi^{-1}_{k_{pp}}\left(\frac{i}{n}\right), \Phi^{-1}_{C_f}\left(1-\frac{i}{n}\right), \Phi^{-1}_{q_1}\left(\frac{i}{n}\right), \Phi^{-1}_{q_2}\left(\frac{i}{n}\right), \Phi^{-1}_{L_{cl\_des}}\left(1-\frac{i}{n}\right))+\\ + K_{max\_cr}(F, \gamma_v, \Phi^{-1}_{G_{T_{des}}}\left(\frac{i+1}{n}\right), \Phi^{-1}_{G_{T_{alt}}}\left(\frac{i+1}{n}\right), \Phi^{-1}_{G_{f_{load}}}\left(\frac{i+1}{n}\right), \Phi^{-1}_{k_{pp}}\left(\frac{i+1}{n}\right),\\ \Phi^{-1}_{C_f}\left(1-\frac{i+1}{n}\right), \Phi^{-1}_{q_1}\left(\frac{i+1}{n}\right), \Phi^{-1}_{q_2}\left(\frac{i+1}{n}\right),\\ \Phi^{-1}_{L_{cl\_des}}\left(1-\frac{i+1}{n}\right)],\end{aligned}$$

$$\begin{aligned}\mathrm{E}\left[Br\_par(\bar{x}', \bar{\xi})\right] = \frac{1}{2n}\sum_{i=0}^{n-1} [Br\_par(F, \gamma_v, \Phi^{-1}_{G_{T_{des}}}\left(1-\frac{i}{n}\right), \Phi^{-1}_{G_{T_{alt}}}\left(1-\frac{i}{n}\right), \Phi^{-1}_{G_{f_{load}}}\left(1-\frac{i}{n}\right),\\ \Phi^{-1}_{k_{pp}}\left(1-\frac{i}{n}\right), \Phi^{-1}_{C_f}\left(1-\frac{i}{n}\right), \Phi^{-1}_{q_1}\left(1-\frac{i}{n}\right), \Phi^{-1}_{q_2}\left(1-\frac{i}{n}\right), \Phi^{-1}_{L_{cl\_des}}\left(\frac{i}{n}\right)+\\ +Br\_par(F, \gamma_v, \Phi^{-1}_{G_{T_{des}}}\left(1-\frac{i+1}{n}\right), \Phi^{-1}_{G_{T_{alt}}}\left(1-\frac{i+1}{n}\right), \Phi^{-1}_{G_{f_{load}}}\left(1-\frac{i+1}{n}\right), \Phi^{-1}_{k_{pp}}\left(1-\frac{i+1}{n}\right),\\ \Phi^{-1}_{C_f}\left(1-\frac{i+1}{n}\right), \Phi^{-1}_{q_1}\left(1-\frac{i+1}{n}\right), \Phi^{-1}_{q_2}\left(1-\frac{i+1}{n}\right), \Phi^{-1}_{L_{cl\_des}}\left(\frac{i+1}{n}\right)],\end{aligned}$$

where n is the specified number of parts of the unit interval decomposition.

The need for approximate calculations is due to the impossibility of exact calculation of a definite integral, since $K_{max\_cr}(\bar{x}', \bar{\xi})$ and $Br\_par(\bar{x}', \bar{\xi})$ are defined by Black-Box Models. Approximate calculations can significantly reduce the overall computational performance of solving preliminary design problems. Optimization problems, especially multicriteria ones, require the execution of many similar calculation procedures, which determines the high dependence of computational costs on the method of calculating the numerical characteristics of objective functions and constraints. However, uncertainty theory requires significantly fewer objective function calculations using Black-Box Models than probability theory.

Using the expressions for critical values in Appendix (Definition 2 and Theorem 1a) and information about the types of monotonicity of objective functions with respect to uncertain parameters, we obtain expressions for $\inf_\alpha[K_{max\_cr}(\bar{x}', \bar{\xi})]$ and $\sup_\alpha[Br\_par(\bar{x}', \bar{\xi})]$.

$$\begin{aligned}\inf\nolimits_\alpha[K_{max\_cr}(\bar{x}', \bar{\xi})] = K_{max\_cr}(F, \gamma_v, \Phi^{-1}_{G_{T_{des}}}(\alpha), \Phi^{-1}_{G_{T_{alt}}}(\alpha), \Phi^{-1}_{G_{f_{load}}}(\alpha),\\ \Phi^{-1}_{k_{cy}}(\alpha), \Phi^{-1}_{C_f}(1-\alpha), \Phi^{-1}_{q_1}(\alpha), \Phi^{-1}_{q_2}(\alpha), \Phi^{-1}_{L_{cl\_des}}(1-\alpha)),\\ \sup\nolimits_\alpha[Br\_par(\bar{x}', \bar{\xi})] = Br\_par(F, \gamma_v, \Phi^{-1}_{G_{T_{des}}}(\alpha), \Phi^{-1}_{G_{T_{alt}}}(\alpha), \Phi^{-1}_{G_{f_{load}}}(\alpha),\\ \Phi^{-1}_{k_{cy}}(\alpha), \Phi^{-1}_{C_f}(\alpha), \Phi^{-1}_{q_1}(\alpha), \Phi^{-1}_{q_2}(\alpha), \Phi^{-1}_{L_{cl\_des}}(1-\alpha).\end{aligned}$$

where α is the level of confidence.

It can be seen that the application of Optimization Models 1, 2 requires much less calculation of objective functions using Black-Box Models compared to their stochastic analogues.

The next section presents the results of optimization calculations obtained using the proposed optimization models.

3. RESULTS

The parameter uncertainty distribution functions in Optimization Models 1 and 2 were set by experts with piecewise linear functions (6):

$$\Phi_{(\bullet)}(x)=\begin{cases} 0, \text{ if } x\leq x_1, \\ \alpha_i+\dfrac{(\alpha_{i+1}-\alpha_i)(x-x_{\text{i}})}{x_{i+1}-x_i}, \text{ if } x_i\leq x\leq x_{i+1},\ 1\leq i\leq n, \\ 1, \text{ if } x>x_n\ . \end{cases}$$

where $\alpha_1, \alpha_2,\ldots, \alpha_{p_{(\bullet)}}$ are the levels of confidence in that $(\bullet)\leq x_1$, $(\bullet)\leq x_2$, $(\bullet)\leq x_{p_{(\bullet)}}$, x_1 , x_2 , …, $x_{p_{(\bullet)}}$ are the deterministic values of the uncertain parameter $(\bullet)$ considered by an expert, $x_1 < x_2 < \ldots < x_{p_{(\bullet)}}$, $0 \leq \alpha_1 \leq \alpha_2 \leq\ldots\leq \alpha_{p_{(\bullet)}} \leq 1$.

The use of functions of this type makes it possible to avoid a common error associated with setting parameters by an expert, which, according to physics, can change only in a closed interval, with normal distribution functions having a domain of [-∞; ∞]. In addition, the use of piecewise linear functions simplifies numerical integration when calculating the expected value.

An example of the uncertainty distribution function is shown in Fig. 4.

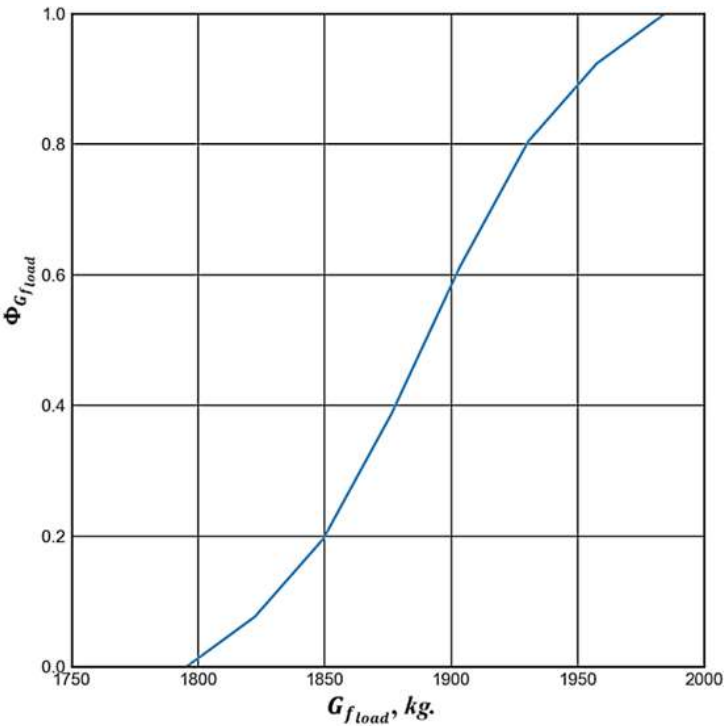

Fig. 4. Uncertainty distribution function for the functional load weight

As a result of optimization calculations according to Models 1-2, Pareto fronts were obtained, which reflect trade-off design solutions on determining the SST aerodynamic configuration and power plant parameters under epistemic uncertainty. Figures 5 and 6 show the Pareto fronts for Models 1 and 2. For comparison, each graph shows the Pareto front, which was obtained by solving the problem under consideration in a deterministic formulation.

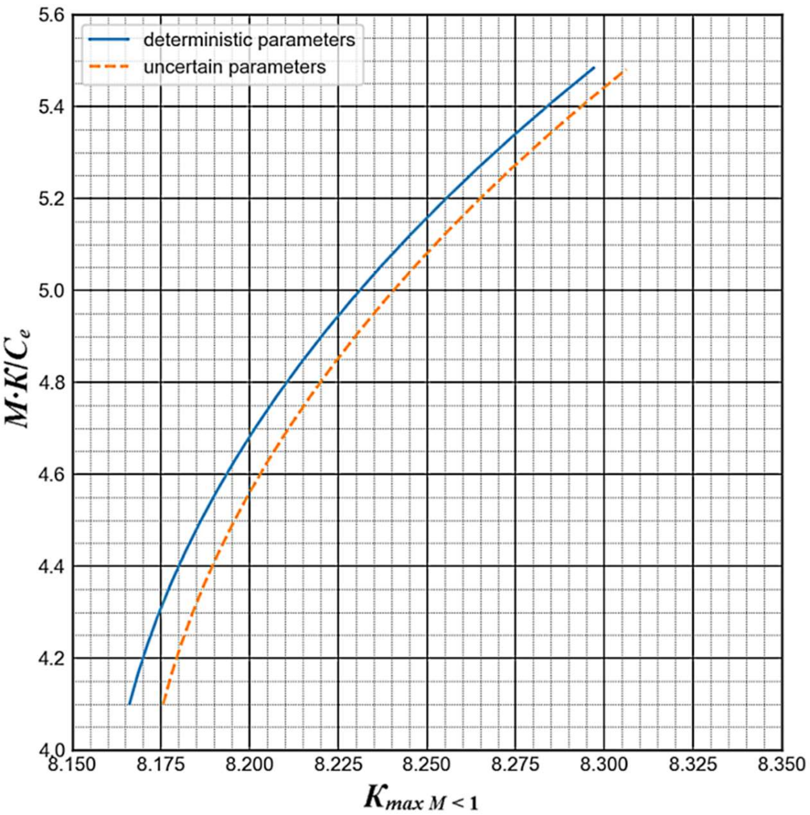

Fig. 5. Optimization Model 1 calculation results

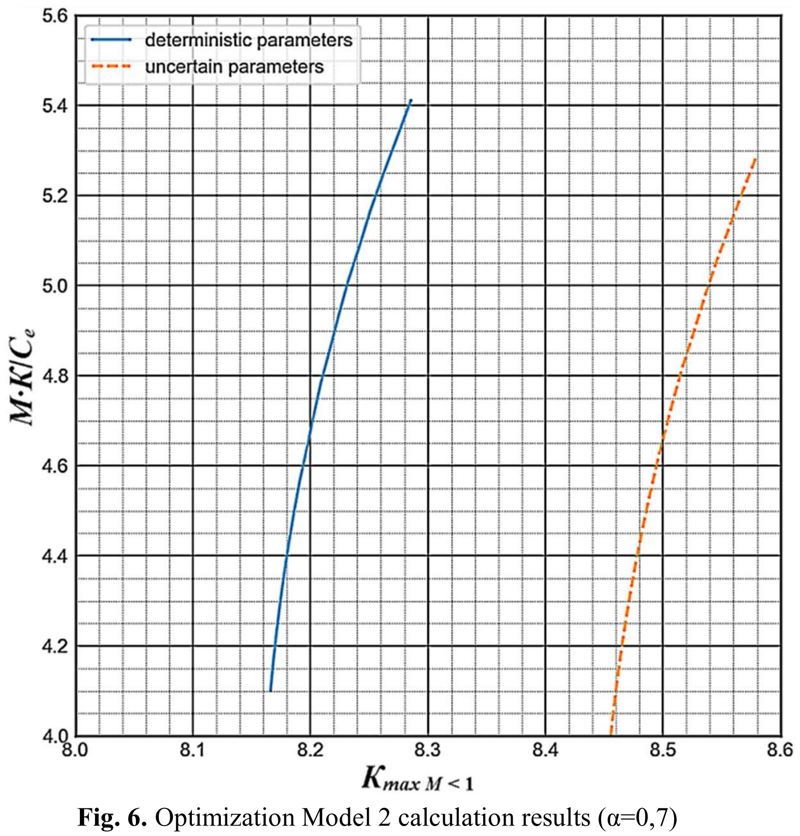

Fig. 6. Optimization Model 2 calculation results (α=0,7)

Figure 4 shows that taking into account the uncertainty of the SST parameters using the expected values in Optimization Model 1 leads to the Pareto front shift to the “worst” range of objective function values (the required growth in the maximum subsonic lift-to-drag ratio to increase the confidence level complicates the aerodynamic design). Model 1 provides optimality in the mean, so the function forms of the objective function uncertainty distribution do not matter when choosing design solutions using this Pareto front. This problem is solved in Model 2. The use of critical values makes it possible to obtain design solutions with a given level of confidence (robustness) and leads to a more pronounced Pareto front shift to the range of the "worst" objective function values (Fig. 5).

Thus, to ensure the reliability of design solutions, it is preferable to use Optimization Model 2 with critical values of objective functions.

4. CONCLUSION

The paper considers the problem of determining the parameters of the aerodynamic configuration and power plant of a supersonic passenger aircraft under epistemic uncertainty at the preliminary design stage, when some of the initial parameters are set by experts and are non-deterministic. Non-determination is the reason for the possibility of implementing parameter values that can lead to a significant deviation of the SST performance from the performance predictions and violation of critical constraints. The paper presents multicri teria optimization models based on uncertainty theory, which provide high computational efficiency when using numerical methods for calculating the numerical characteristics of functions that depend on uncertain quantities. It is shown that the use of the proposed optimization models, which take into account the non-determination of the parameters set by experts, leads to the design solutions that differ from the solutions found using deterministic models. The Pareto fronts obtained as a result of optimization calculations for various confidence levels allow a design engineer to determine the aerodynamic configuration and power plant parameters that provide a rational objective function value trade-off.

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APPENDIX. UNCERTAINTY THEORY: FUNDAMENTAL CONCEPTS

This section presents the main fundamental concepts of the theory of uncertainty [15], which are used to create a theoretical basis for the parametric synthesis of the HST design solutions under epistemic uncertainty.

Definition 1 [15]:

The uncertainty distribution function of the uncertain variable ξ is the function $\Phi: R \to [0, 1]$, defined as:

$$\Phi(x)= M\{\xi \le x\},$$

where $M\{\bullet\}$ is the measure of the uncertainty of event occurrence •, defined in [23] as the expert's confidence level that this event will occur.

Theorem 1 [15]:

Let the function $f(\xi_1, \xi_2, ..., \xi_n)$ be continuous, strictly increasing in $\xi_1, \xi_2, ..., \xi_m$ and strictly decreasing in $\xi_{m+1}, \xi_{m+2}, \dots, \xi_n$. Then if $\xi_1, \xi_2, ..., \xi_n$ are independent uncertain variables with uncertainty distribution functions $\Phi_1, \Phi_2, ..., \Phi_n$, respectively, then:

a) $\xi = f(\xi_1, \xi_2, ..., \xi_n)$ is an uncertain variable with the inverse uncertainty:

$$\Psi^{-1}(\xi)=f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), ..., \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), ..., \Phi_n^{-1}(1-\alpha)),$$

where Φ^{-1} is the inverse uncertainty distribution.

b) Expected value:

$$E[\xi]=\int_0^1 (f(\Phi_1^{-1}(\alpha),\Phi_2^{-1}(\alpha), ..., \Phi_m^{-1}(\alpha),\Phi_{m+1}^{-1}(1-\alpha), ... \Phi_n^{-1}(1-\alpha))d\alpha.$$

c) Variance:

$$V[\xi] = \int_0^1 (f(\Phi_1^{-1}(\alpha),\Phi_2^{-1}(\alpha), \dots, \Phi_m^{-1}(\alpha),\Phi_{m+1}^{-1}(1-\alpha), \dots \Phi_n^{-1}(1-\alpha))-E[\xi])^2 d\alpha.$$

d) For any $\alpha \in [0, 1]$: $M\{f(\xi_1, \xi_2, ..., \xi_n) < 0\} > \alpha$ is equivalent to

$$f(\Phi_1^{-1}(\alpha),\Phi_2^{-1}(\alpha), ... ,\Phi_m^{-1}(\alpha),\Phi_{m+1}^{-1}(1-\alpha), ..., \Phi_n^{-1}(1-\alpha))\le 0.$$

Definition 2 [15]:

The quantiles of the uncertain value ξ (critical values) are:

$$\sup_\alpha[\xi] = \sup\{r \mid M\{\xi \ge r\} \ge \alpha\}, \alpha \in [0, 1],$$
$$\inf_\alpha[\xi] = \inf\{r \mid M\{\xi \le r\} \ge \alpha\}, \alpha \in [0, 1].$$