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Evaluating the Efficacy of the Sliding Mode Algorithm for the Active Suspension System

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Abstract: The active suspension system is installed in the car to improve the smoothness and comfortable ride while driving. In this paper, a quarter-dynamic model is utilized to describe the car's vibrations. The sliding mode control algorithm is proposed to control the operation of the suspension system. The problem's input signal is the excitation from the road surface, while the output signal includes values related to the acceleration and displacement of the vehicle body. In addition, the interaction between the wheel and the road surface, which is expressed through the change of dynamic force, is also considered. The numerical simulation is performed with a sinusoidal bump excitation signal. According to this result, the car body displacement was greatly reduced when using an active suspension controlled by the sliding mode method. The fluctuation of the unsprung mass is insignificant, so it does not affect the car's smoothness. This helps increase wheel stability and holding. The effect that the sliding mode algorithm brings to this study is positive.

Keywords: Sliding Mode Control; Vehicle Vibration; Vehicle Dynamic; MATLAB Simulink

1. INTRODUCTION

In recent years, car vibration has become an important and researched topic. There are many causes of car vibration [1], such as wind and external forces. Besides, the engine and transmission system operation also cause the car to oscillate [2]. However, these factors only affect a small part of the vehicle's vibration. The main cause of car vibration is the stimulus from the road surface [3]. In fact, the road surface is not flat, so it causes the wheels to oscillate. The vibrations from the wheels are transmitted to the car body through the suspension system, which causes the body to vibrate.

Vibrations in the car make passengers feel tired and uncomfort [4]. In addition, the goods' quality and the car's life may be reduced if the oscillation occurs continuously over a long period. The suspension system on a car plays an important role in ensuring the smoothness of the vehicle [5]. The suspension system quenches much of the car's vibrations, while the rest depends on the tires and other factors. Traditional suspension consists of only three main components: the lever arm, the spring, and the damper. Some other components, such as the rubber, stabilizer bar, multi-link bar, etc., can also be considered components of the suspension system [6]. Only springs and dampers are considered for studies related to vehicle vibration and ride comfort [7], meanwhile remaining components, such as the lever arm, stabilizer bar, etc., can be mentioned in the studies of vehicle oscillation and stability [8]. A suspension system using springs and dampers with constant stiffness (K = const, C = const) is called passive suspension. Today, many modern suspension systems have been used to improve the car's smoothness when traveling. Modern suspension systems can either change the spring stiffness (by using pneumatic springs) [9] or change the damping stiffness (by using electromagnetic dampers) [10]. The stiffness of these elements can be changed dynamically to improve the car's smoothness. Although suspension stiffness can be changed based on using the semi-active suspension (damper control) or air suspension (air spring

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control), the system response has not yet reached the ideal state. Therefore, the solution of retrofitting a hydraulic actuator to the suspension system is a perfect fit. The hydraulic actuator can apply force to the vehicle body to suppress unwanted vibrations. The suspension using a separate hydraulic actuator is known as the active suspension [11].

Many papers related to active suspension control have been published in recent years. In [12], Khodadadi and Ghadiri introduced the PID algorithm (Proportional Integral Derivative) to control the suspension system. This algorithm has three coefficients: K_P, K_I, and K_D for amplification, integration, and differential stages, respectively. These coefficients are tuned by a fuzzy algorithm that includes three states: input, output, and processing. The PID algorithm is quite simple and widely used in industry; however, it is only suitable for systems with one input and one output. If the system has a lot of output and/or input, it is necessary to use a different algorithm instead. In [13], Nguyen et al. applied the LQR (Linear Quadratic Regulator) algorithm to control the quarter suspension model. According to Nguyen et al., determining the controller's coefficients involves four specific steps. The determination of these values is based on the point of view of selecting the minimum RMS (Root Mean Square) value of the outputs. An additional filter is necessary if the system is affected by interference signals from the outside. Usually, the Gaussian filter can be combined with the LQ algorithm to become the LQG controller [14].

The above control algorithms are perfectly suitable for linear oscillation systems. However, the actual oscillation of the car is nonlinear, so more complex control algorithms need to be applied. In [15], Qin et al. proposed using RAISC (Road Adaptive Intelligent Suspension Control) for the quarter dynamics model. According to Qin et al., the structure of the RAISC controller is divided into two phases: offline and online. The offline phase assumes the task of determining the optimal values of the controller based on the PSO (Particle Swarm Optimization) method, while the online phase performs the task of generating control signals for the system. In [16], Huang et al. proposed a new adaptive robust control law for the suspension system. This rule can satisfy the uncertainty conditions of the nonlinear system. This controller is synthesized from three component signals. The first signal is received from the system's output, while the other two must undergo filtering and adaptive tuning. For complex nonlinear algorithms, it is common to design indirect rather than direct controllers [16, 17]. Once the dynamic model of the actuator (direct control) is considered, the algorithm becomes much more complex. Therefore, the model of the actuator should be linearized to simplify the control problem. In [18], Nguyen used a quarter model with 5 state variables (the fifth variable is the influence of the actuator). The actuator coefficients are described in the equation (26), and they depend on many factors such as the piston cross-sectional area, fluid pressure, hydraulic cylinder flow, fluid viscosity, fluid leaks, etc. [19]. In [20], Nguyen and Nguyen proposed using the OSMC solution (Optimal Sliding Mode Control) for an active suspension system. The model parameters are optimally selected by the loop algorithm. Besides, there are many other intelligent control algorithms utilized to control the suspension system, such as type-2 fuzzy [21], fuzzy-PI [22], type-2 fractional order fuzzy [23], fuzzy SMC [24], neural network [25], etc. Overall, the efficiency of these algorithms is positive. However, the controller design process is quite complicated.

The author proposes using the SMC method for a quarter suspension model in this study. Because the actuator is considered, the controller designed in this study is a direct controller. Previous studies often only considered the car's smoothness when designing the controller, but ignored the car's stability. In this paper, both issues related to smoothness and stability are considered. The paper's content is divided into 4 sections, including an introduction section, a model section, a simulation section, and a conclusion section. Specific content corresponding to each section is presented in the below sections.

2. MODEL

2.1. Sliding mode control theory

Considering a nonlinear control object with y(t) output and u(t) input signals. The n^{th} derivative of the output signal is a function that depends on the component derivation signals and input signals.

$$y^{(n)}(t)=f\left(y,\dot{y},\ddot{y},...,y^{(n-1)}\right)+u(t). \qquad (1)$$

Assume that the function *f(.)* is undefined and bounded, i.e.:

$$\| f(.)\| < \delta < \infty. \qquad (2)$$

Let x_i be the state variables of the model:

$$\begin{aligned} x_1 &= y \\ x_2 &= \dot{y} \\ &... \\ x_n &= y^{(n-1)} \end{aligned}. \qquad (3)$$

The model of the object is returned in the form of a system of state equations as follows:

$$\begin{cases} \dot{x}_i = x_{i+1} \\ \dot{x} = f(x)+u(t) \end{cases} \quad i=\overline{1,n-1} \qquad (4)$$

where $x=\begin{bmatrix} x_1 & x_2 & ... & x_n \end{bmatrix}^T$.

Assuming that the set point signal is zero, that is, *h(t)* = 0 (Figure 1). If the nonlinear object (4) is restricted by condition (3), there is always a signal response controller that does not depend on the function *f(.)* of the model. So, the control signal can be rewritten as follows:

$$u(t)=(k+\delta)sgn(s). \qquad (5)$$

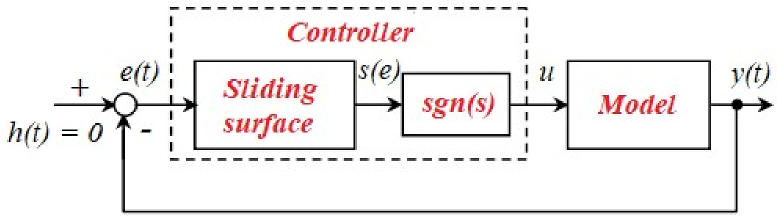

Fig. 1. System control.

Here:
s(e): sliding surface (when s(e) = 0),
e(t): error signal,

$$s(e)=b_0e(t)+b_1\dot{e}(t)+b_2\ddot{e}(t)+...+b_{n-2}e^{(n-2)}(t)+e^{(n-1)}(t), \qquad (6)$$

$$e(t)=h(t)-y(t)=-y(t). \qquad (7)$$

The b_i coefficients of the slip surface need to be chosen appropriately, so that (8) is a Hurwitz polynomial. Once this condition is met, the state variables tend to zero after a specific time (9).

$$p(\gamma)=b_0+b_1\gamma^1+b_2\gamma^2+...+b_{n-2}\gamma^{n-2}+\gamma^{n-1}. \qquad (8)$$

$$\lim_{t\to\infty} x(t)=0. \qquad (9)$$

Because:

$$\begin{cases} e=-x_1 \\ x_i=-e^{(i-1)} \end{cases}. \qquad (10)$$

Therefore, the equation (9) is able to write in the following form:

$$\lim_{T<t\to\infty} e^{(i)} = 0 \tag{11}$$

where T is a finite time point.

If the equation $s(e) = 0$ has coefficients b_i satisfying the condition of the Hurwitz polynomial (8), the sliding surface $s(e)$ tends towards zero, i.e.:

$$s(e)\dot{s} < 0. \tag{12}$$

The equation (12) is called the sliding condition of the controller. From (4), (6), and (12), we have:

$$\dot{s}(e) = \sum_{i=0}^{n-1} b_i e^{(i+1)} = -\sum_{i=0}^{n-2} b_i x_{i+2} - \dot{x}_n = -\sum_{i=0}^{n-2} b_i x_{i+2} - f(x) - u(t). \tag{13}$$

The value of (13) is positive if $s(e)$ is less than zero, and vice versa if $s(e)$ is greater than zero. The control signal $u(t)$ can be rewritten by combining (2) and (13) as follows:

$$\begin{cases} u(t) < -\sum_{i=0}^{n-2} b_i x_{i+2} - \delta; s(e) < 0 \\ u(t) > \sum_{i=0}^{n-2} b_i x_{i+2} + \delta; s(e) > 0 \end{cases}. \tag{14}$$

The control signal $u(t)$, which is determined by the equation (14), does not depend on (4). Therefore, it is considered a robust controller. If condition (2) is not satisfied, it is necessary to define an upper limit of the function $f(.)$, i.e.:

$$|f(x)| < |g(x)|, \forall x. \tag{15}$$

Then condition (13) becomes:

$$\begin{cases} u(t) < -\sum_{i=0}^{n-2} b_i x_{i+2} - g(\underline{x}); s(e) < 0 \\ u(t) > \sum_{i=0}^{n-2} b_i x_{i+2} + g(\underline{x}); s(e) > 0 \end{cases}. \tag{16}$$

2.2. Suspension control model

Considering the active suspension model (Figure 2). The model consists of m_1 (the sprung mass) and m_2 (the unsprung mass). The spring is described by its K stiffness, and the damper is described by its C coefficient. The damping component of the tire is ignored, so only the elastic characteristic of the tire (K_T) is considered. A hydraulic actuator is considered when using an active suspension. The force that is produced by the actuator is considered a state variable of the system.

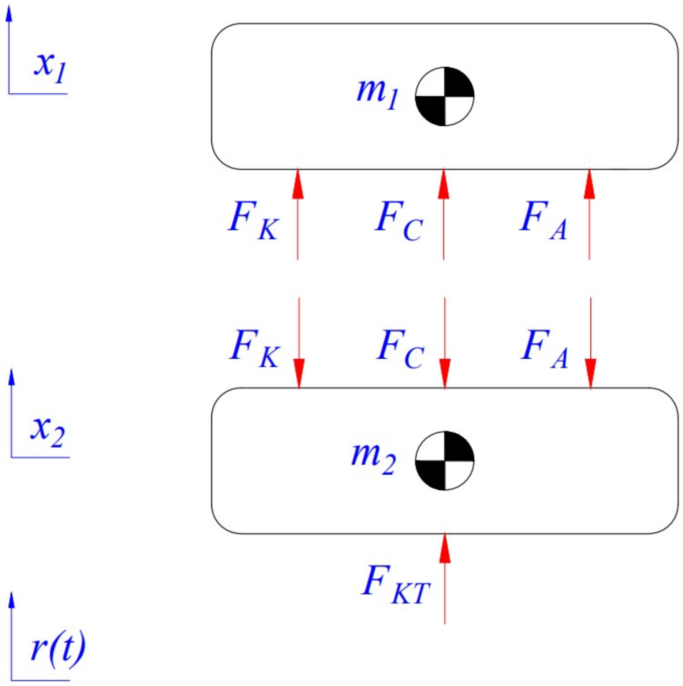

Fig. 2. A quarter dynamic model.

Applying D'Alembert's principle, the car's oscillation is described by equations (17) and (18), respectively, with two degrees of freedom, x_1 and x_2.

$$F_K + F_C + F_A = F_i^{m_1}, \tag{17}$$

$$F_{KT} - F_K - F_C - F_A = F_i^{m_2}. \tag{18}$$

Where:
The inertia force:

$$F_i^{m_1} = m_1\ddot{x}_1, \tag{19}$$

$$F_i^{m_2} = m_2\ddot{x}_2. \tag{20}$$

The elastic force of spring and tire:

$$F_K = K\left(x_2 - x_1\right), \tag{21}$$

$$F_{KT} = K_T\left(r\left(t\right) - x_2\right). \tag{22}$$

The shock absorber resistance force:

$$F_C = C\left(\dot{x}_2 - \dot{x}_1\right). \tag{23}$$

Combining equations from (17) to (23), we get:

$$K\left(x_2 - x_1\right) + C\left(\dot{x}_2 - \dot{x}_1\right) + x_3 = m_1\ddot{x}_1, \tag{24}$$

$$K_T\left(r\left(t\right) - x_2\right) - K\left(x_2 - x_1\right) - C\left(\dot{x}_2 - \dot{x}_1\right) - x_3 = m_2\ddot{x}_2. \tag{25}$$

Where:
r(t): excitation signal from road surface,
x_3: acting force of the actuator, $x_3 = F_A$,
According to [26], the state variable x_5 can be approximated by the equation (26).

$$\dot{x}_3 = \lambda_1 u\left(t\right) - \lambda_2 x_3 + \lambda_3\left(\dot{x}_2 - \dot{x}_1\right). \tag{26}$$

With:
u(t): the output voltage signal of the controller,
λ_i: actuator's coefficients.
Let's:

$$x_4 = \dot{x}_1, \tag{27}$$

$$x_5 = \dot{x}_2. \tag{28}$$

Taking the derivative of the variables x_4 and x_5, we get:

$$\dot{x}_4 = \frac{1}{m_1}\left(-Kx_1 + Kx_2 + x_3 - Cx_4 + Cx_5\right), \quad (29)$$

$$\dot{x}_5 = \frac{1}{m_2}\left(Kx_1 - \left(K + K_T\right)x_2 - x_3 + Cx_4 - Cx_5\right). \quad (30)$$

The output signal of the simulation problem is the sprung mass displacement.

$$y\left(t\right) = x_1. \quad (31)$$

The model's higher-order derivative signal has the form of the equation (32) (n = 5):

$$y^{(5)} = \frac{K_T}{\chi m_1 m_2} \times \begin{bmatrix} KC\left(\frac{1}{m_1} + \frac{1}{m_2}\right)x_1 + C\left(-\frac{K}{m_1} - \frac{K + K_T}{m_2}\right)x_2 \\ + \left(-\frac{C}{m_1} - \frac{C}{m_2} - \lambda_2\right)x_3 + \left(\frac{C^2}{m_1} + \frac{C^2}{m_2} - K - \lambda_3\right)x_4 \\ + \left(-\frac{C^2}{m_1} - \frac{C^2}{m_2} + \left(K + K_T\right) + \lambda_3\right)x_5 \end{bmatrix} + \frac{K_T \lambda_1}{\chi m_1 m_2} u\left(t\right). \quad (32)$$

The sliding surface is an important component of the SM controller. According to (6), the sliding surface is written in the following form:

$$s\left(e\right) = \gamma_0 e + \gamma_1 \dot{e} + \gamma_2 \ddot{e} + \gamma_3 e^{(3)} + \gamma_4 e^{(4)}. \quad (33)$$

The control signal is written as a function depending on the sliding surface of the controller.

$$u\left(t\right) = c\left(y_s^{(5)} - \sum_{i=1}^{5} a_i x_i + \sum_{i=1}^{4} \gamma_i e^{(i)}\left(t\right) + Rsgn\left(\sum_{i=0}^{4} \gamma_i e^{(i)}\left(t\right)\right)\right). \quad (34)$$

Where: a_i, γ_i, c, and R are coefficients.

After the control algorithm has been established, the simulation will be performed.

3. SIMULATION

In this work, the numerical simulation method is used to investigate car vibration. The simulation process is done in the MATLAB/Simulink environment. A sinusoidal bump excitation is utilized (Figure 3). This is a common type of excitation signal that is frequently used in many simulation problems related to suspension control. The bump from the road surface is the input to the oscillation problem, while the output of the problem includes changes in displacement, acceleration, dynamic force, etc. These results are represented in the time domain corresponding to the simulation time. The maximum and RMS values of the results obtained are used for comparison and evaluation.

The following mathematical function represents the excitation signal from the road:

$$h\left(t\right) = h_0 sin\left(2\pi ft + \varphi\right). \quad (35)$$

There are three scenarios covered when simulating, including:

+) A car using traditional suspension system (None)

+) A car using linear control suspension (PID)

+) A car using nonlinear control suspension (SM)

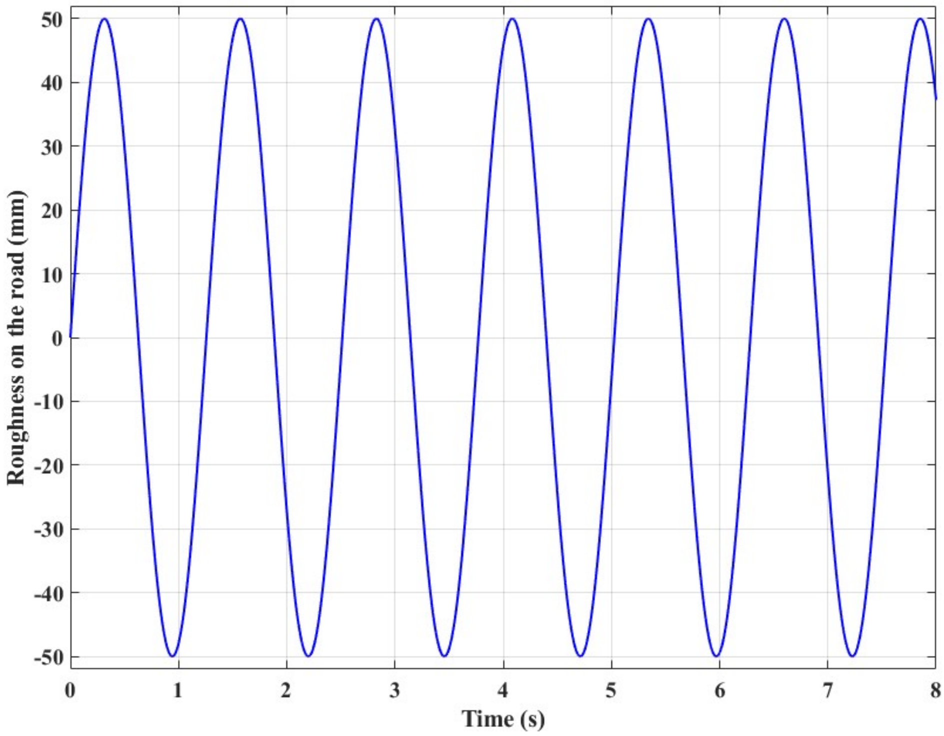

Fig. 3. Bump on the road.

The technical parameters of the simulation vehicle are shown in Table 1.

Table 1. The simulation vehicle parameters.

Symbol	Unit	Value	Description
K	N/m	40500	Coefficient spring
K_T	N/m	174000	Coefficient tire
C	Ns/m	3150	Coefficient damper
m_2	kg	36	Unsprung mass
m_1	kg	410	Sprung mass

The road excitation signal used is sinusoidal. This is a low-frequency cyclic variable signal that is often used in problems related to control in suspension systems.

The change in the vehicle body displacement over time is shown in Figure 4. The vehicle body oscillation is greatest when using a traditional suspension system without control. In contrast, car vibrations are smaller if an active suspension system is used. The vertical displacement of the sprung mass is 70.83 (mm), 23.36 (mm), and 10.03 (mm) respectively, corresponding to three investigated situations: None, PID, and SM. The RMS of values related to vehicle vibrations is also commonly used to assess car smoothness. In this case, the RMS value of displacement corresponding to the situations is: 45.81 (mm) – None, 16.54 (mm) – PID, and 7.07 (mm) – SM.

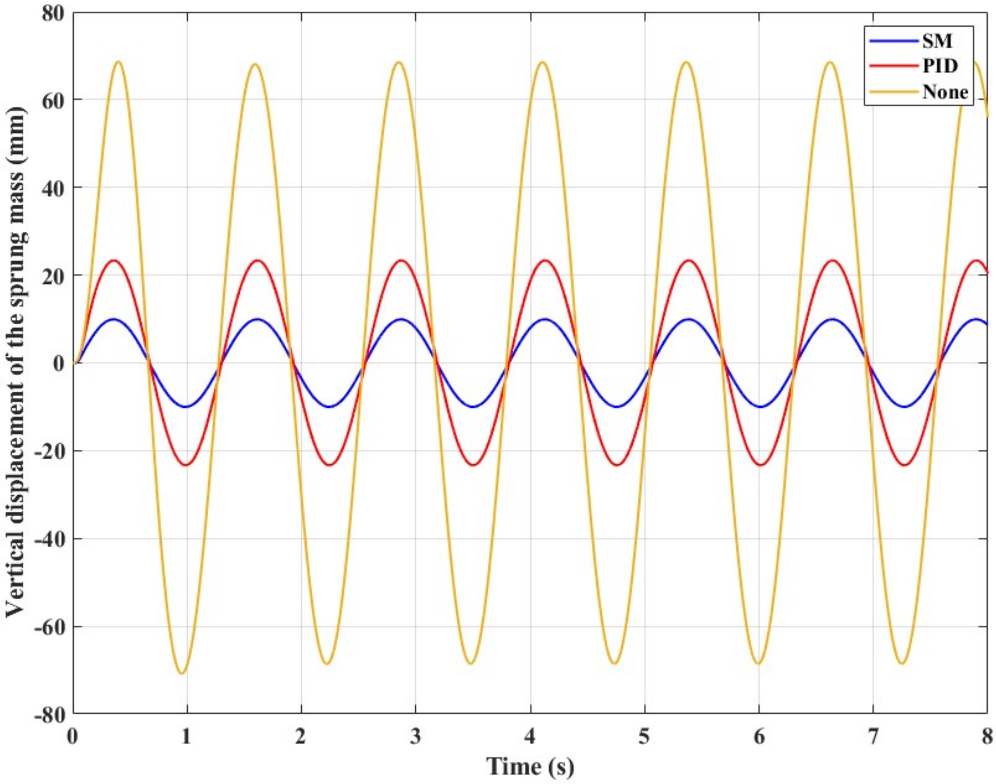

Fig. 4. Vertical displacement (sprung mass).

When the displacement of the vehicle body changes, the value of the sprung mass acceleration will change accordingly. The change in vehicle body acceleration is shown in Figure 5. In the first stage, the change in acceleration is the largest; these values can be up to 2.52 (m/s^2), 1.82 (m/s^2), and 1.08 (m/s^2), corresponding to the three investigated cases. This value changes cyclically in later stages and reaches a more stable state. Because this is a continuous periodic oscillation, the RMS indicator should be used. The result of the acceleration value when calculating according to the RMS criterion is 1.28 (m/s^2) – None, 0.44 (m/s^2) – PID, and 0.19 (m/s^2) – SM, respectively. According to this result, the car body's RMS acceleration when using the active suspension system directed by the SM method is only 14.84% compared to the car using the traditional suspension system. Meanwhile, the RMS value of acceleration when the PID algorithm is used is 34.38% compared to the first situation. This leads to the conclusion that the SM control algorithm enhances the stability and smoothness of the sprung mass.

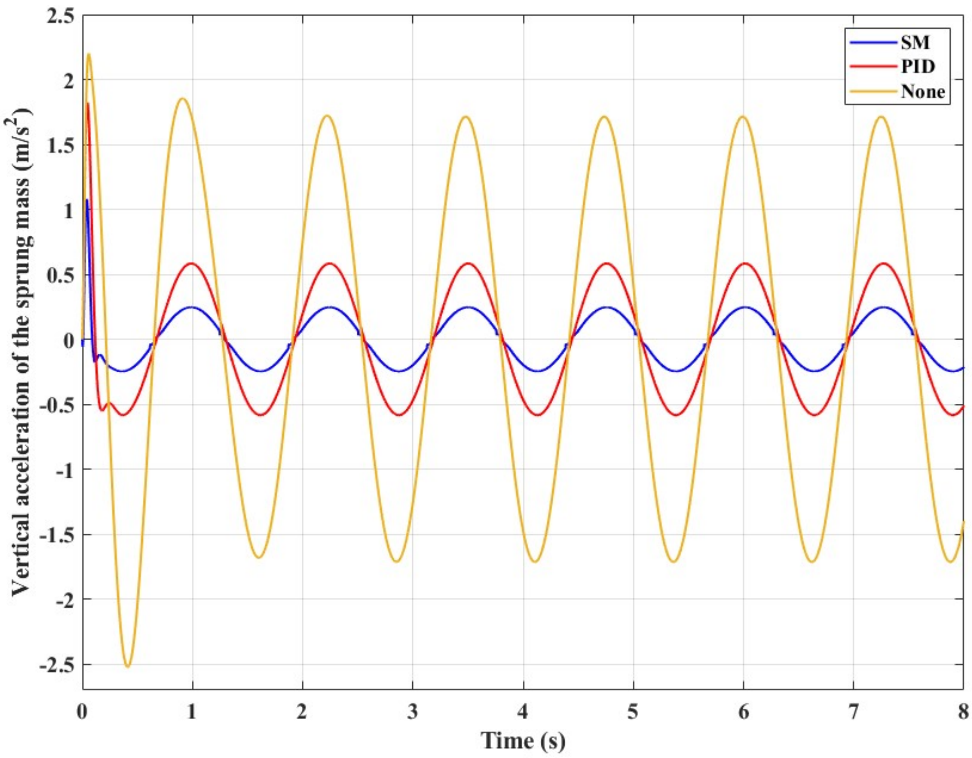

Fig. 5. Vertical acceleration (sprung mass).

The vehicle body oscillation can be further improved using the active suspension system with the SM mentioned above algorithm. However, it is still necessary to investigate the oscillations of the unsprung mass. Besides, the change in the dynamic load of the wheel is of great significance to assessing the car's stability when moving.

Figure 6 shows the change in the unsprung mass's vertical displacement during the simulation period. This value tends to be asymptotic to the excitation signal from the road (Figure 3). The difference in values in all three situations is very small, namely 54.84 (mm), 51.57 (mm), and 50.82 (mm), which correspond to the maximum values; 38.51 (mm), 36.68 (mm), and 36.15 (mm), which correspond to the RMS values.

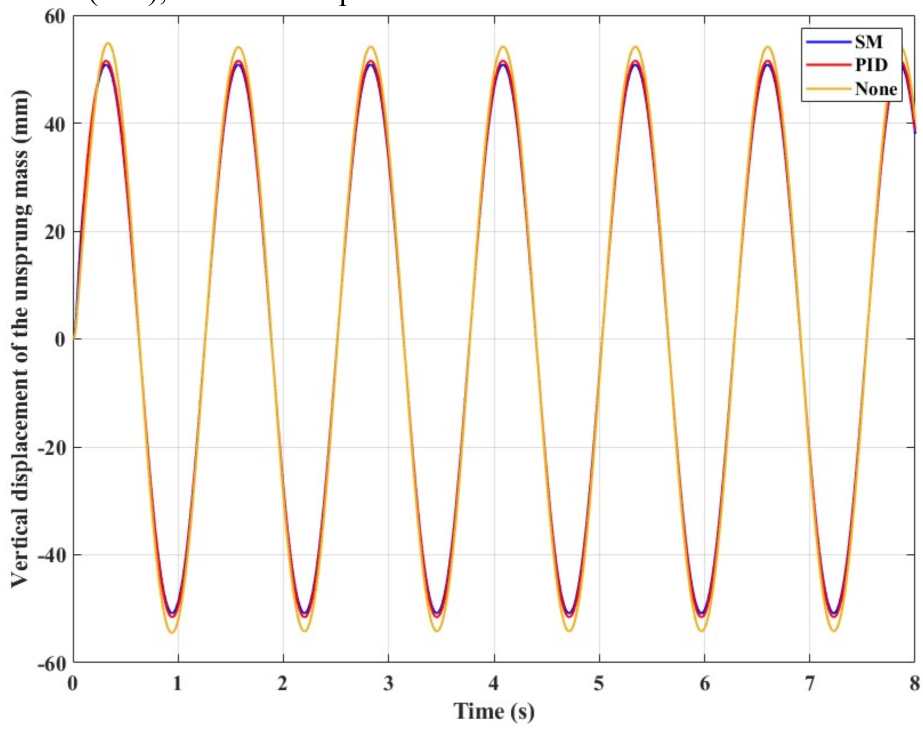

Fig. 6. Vertical displacement (unsprung mass).

For the acceleration of the unsprung mass, the difference exists in the first period (Figure 7). The acceleration in the SM case is larger than the other two; however, this only happens for a very small amount of time. In the subsequent phases of the oscillation, the difference between the values is very small. For the car using the active suspension controlled by the SM solution, "chattering" occurred when calculating the acceleration of the unsprung mass. The unsprung mass acceleration signal vibrates at the phase transition's beginning. This vibration is not large, so it does not affect the smoothness and stability of a car.

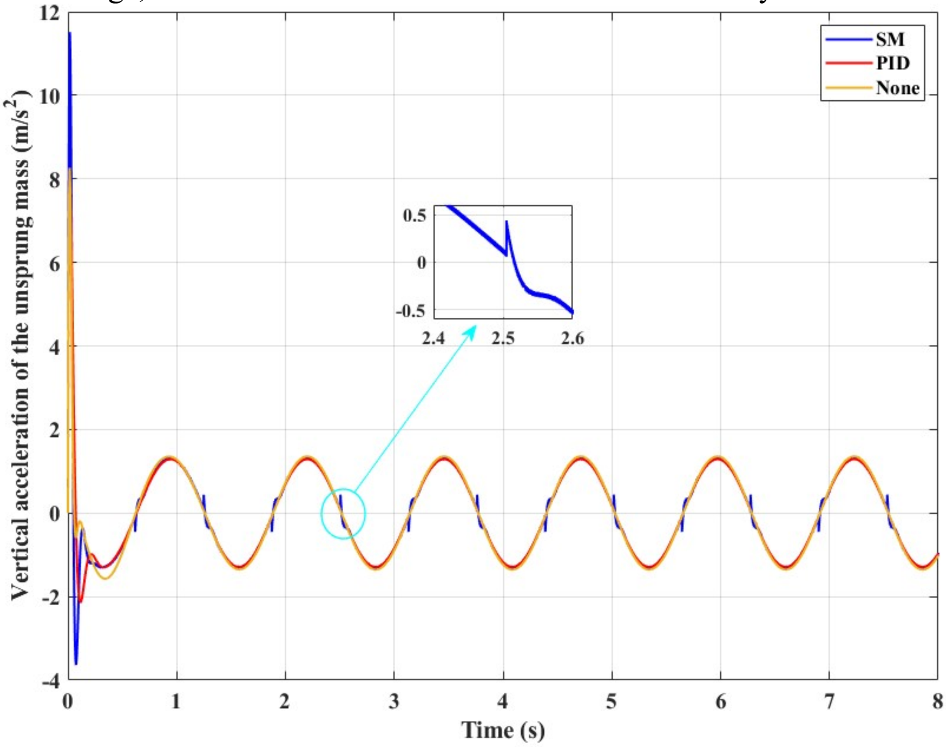

Fig. 7. Vertical acceleration (unsprung mass).

In order to improve the ride comfort of the automobile, the actuator of the active suspension system needs to generate actuating forces that affect both the sprung and the unsprung mass. If the impact force is too large, the change in dynamic load will also be larger. This increases the risk of wheels separating from the road and can lead to vehicle instability. Therefore, the change of dynamic load at the wheel is one of the very important factors to help evaluate the automobile's stability. This change is shown in Figure 8.

As a result of Figure 8, the value of the dynamic force changes continuously based on the excitation signal from the road. The biggest change belongs to the situation where cars only use conventional suspension systems. The dynamic load on wheels was reduced from 4286.97 (N) to 3381.09 (N). Meanwhile, the minimum dynamic load values of the other two situations are 3384.33 (N) and 3596.45 (N), respectively, for PID and SM situations. The RMS value of the dynamic load did not change too much, reaching 4331.92 (N), 4295.78 (N), and 4290.47 (N), respectively.

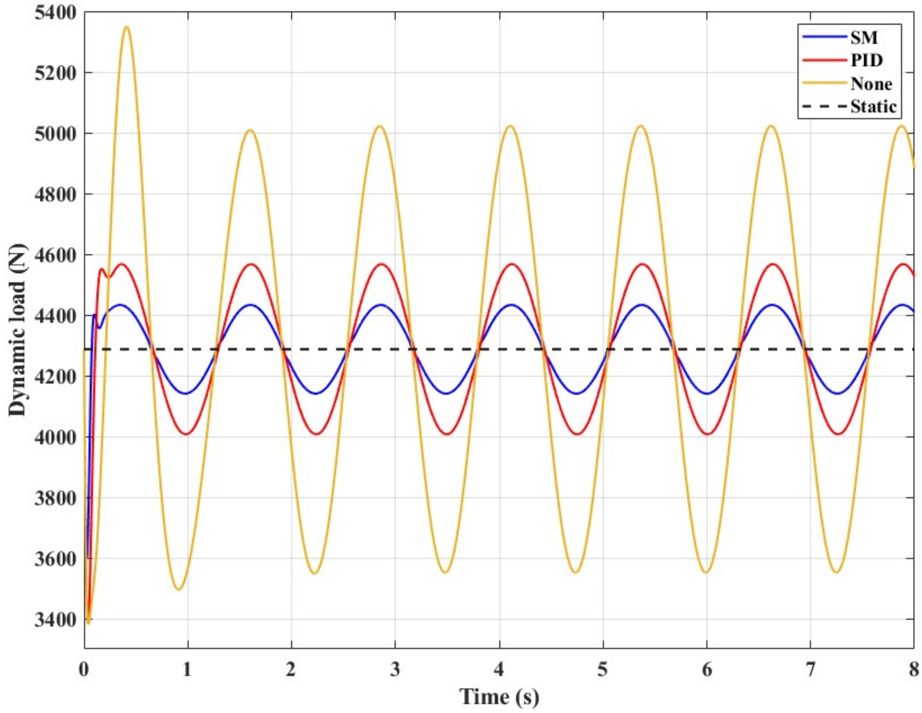

Fig. 8. Dynamic load.

The change in the dynamic load is expressed as a percentage, as shown in Figure 9. This graph depicts the change in Figure 8 as a percentage to all three scenarios. During the first oscillation phase, the "None" situation variation can be as close as 25%. In subsequent oscillation phases, the value of "None" does not exceed 17.1%, while the values of "PID" and "SM" are only about 6.8% and 3.3%, respectively. This result has shown that the actuation force of the actuator when a car uses the active suspension system does not harm the vehicle's stability.

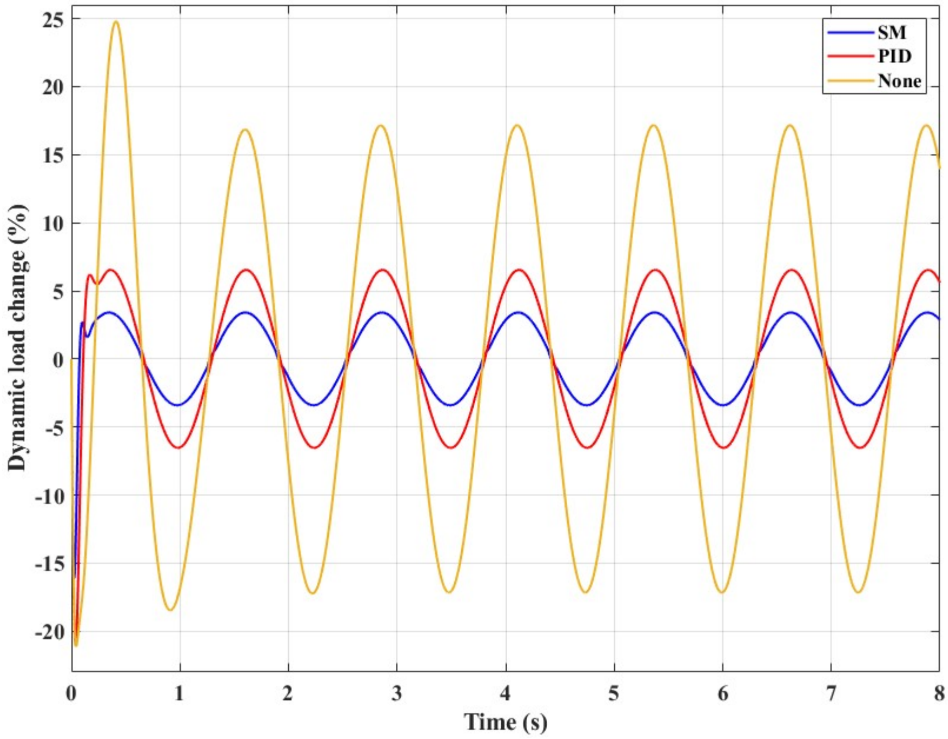

Fig. 9. Dynamic load change.

4. CONCLUSION

Irritation of the road is the main cause of car vibration. The suspension is utilized to quench these oscillations. The active suspension system with hydraulic actuators is more efficient than the conventional one. As a result, the car's smoothness and stability have improved. In this study, the author proposes to use the SMC solution to control the active suspension system in cars. A quarter-dynamic model that incorporates hydraulic actuators is used to describe vehicle vibrations. The numerical simulation is conducted in the MATLAB/Simulink environment in the time domain.

According to the research findings, the car body displacement and acceleration values are strongly reduced when utilizing the active suspension system directed by the SM algorithm. In addition, the oscillation of the unsprung mass is not large (equivalent to vehicles using passive suspension). So, the comfort and smoothness of a car are improved. Besides, the change in dynamic load at the wheel when using active suspension is smaller than traditional suspension. This helps to enhance the vehicle's stability when oscillating.

The SM algorithm can help the active suspension operate more steadily; however, the phenomenon of "chattering" still exists. In some cases, "chattering" can have a negative effect on the smoothness of the car (usually with high-frequency nonlinear oscillations). In the near future, studies on reducing the "chattering" phenomenon when applying the SM algorithm to the suspension system will be carried out.

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