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A New Approach on the Modelling and Analysis Stability of a Class of Fractional-Order Quasi-Polynomial Systems

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Abstract: Stabilization and observation for nonlinear fractional derivative systems remain open problems in automatic due to the fractional nature and nonlinearity of these systems. The present paper studies global stability by the return output for fractional systems. First, we give some definitions of fractional calculus and the quasi-polynomial (QP) and Lotka-volterra (LV) systems. Then, we analyze their stabilities as well as linear (LMI) and bilinear (BMI) matrix inequalities. In order to solve the controller design problem. The goal of this paper is to investigate the global and local stability of a dynamic fractional order system using the quasi-polynomial and LV representation. Then, we use the LMI to study the stabilization of this fractional system.

Keywords: quasi-polynomial systems, fractional derivative, LMI, BMI, stability

1. INTRODUCTION

In the last decades, the analysis problem of control and stability of dynamic systems has attracted the attention of researchers [10, 20]. These studies are applied in several fields in reality such as chemistry, technology and physical systems including electrical networks, economic systems and electrical systems [2].

Stabilization and observation for nonlinear fractional derivative systems remain open problems in automatic due to the fractional nature and nonlinearity of these systems. Indeed, stabilization is one of the major concerns of both researchers and engineers. There are many methods of stabilizing linear systems; they are generally based on pole placement techniques or the minimization of a quadratic criterion and lead to feedback, the implementation of which requires the use of observers when the condition is partially measured. Lyapunov's first method makes it possible to use these results for the control of nonlinear systems from their linearizations and the use observers.

Many dynamical systems are represented by a non-integer order dynamic model, usually based on the notion of differentiation or integration of the non-integer order. Studying the stability of fractional order systems [12–17] is more difficult than for their counterparts (integer order systems). On the one hand, fractional systems are regarded as memory systems, in particular for the consideration of initial conditions, and on the other hand they

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have a much more complex dynamic.

Lotka-Volterra and quasi-polynomial are models proven to be well adapted to canonical forms generally applicable to models of ordinary non-linear ODE differential equations, since a large class of these models can be written in these types [5]. Furthermore, the approach of global and local stability analysis of Generalized LV models are abundant [3, 4, 11].

LV systems are generally used in some scientific fields, such as population biology. The form and properties of the generalized and classical LV algebraic forms were treated by Hernandez-Bermejo and Fairen in [19].

The goal of this paper is to investigate the global and local stability of a dynamic fractional order system using the quasi-polynomial and LV representation. Then, we use the LMI to study the stabilization of this fractional system.

2. BACKGROUND MATERIAL AND PRELIMINARY RESULTS

Throughout this paper, $M_{p,m}(\mathbb{R})$ denotes the set of real matrices with p rows and m columns, $M_m(\mathbb{R})$ the set of square real matrices of size m, $diag(\omega_1, \ldots, \omega_m)$ represent the diagonal matrix which contains the elements $\omega_1, \ldots, \omega_m$ and B_{jk} is the element of the j-th row and the k-th column of a matrix B.

In this section, we use the new easy definition of the fractional derivative (see [7]). This new definition is a natural extension of the usual derivative, and it verifies the first four properties of the fractional derivative in the sense of Riemann-Liouville and in the sense of Caputo (see [7]). Also, this definition coincides with the known fractional derivatives on the polynomials.

Definition 2.1:
Let $h : [0, \infty) \longrightarrow \mathbb{R}$, $u > 0$ and $\alpha \in (0, 1)$. We define the conformable fractional derivative of h of order α by :

$$\mathcal{T}^{\alpha}(h)(u) = \lim_{\epsilon \to 0} \frac{h(u + \epsilon u^{1-\alpha}) - h(u)}{\epsilon}. \tag{2.1}$$

Furthermore, if h is α-differentiable (the conformable fractional derivative $\mathcal{T}^{\alpha}(h)(u)$ exist) in some $(0, u_1)$, $u_1 > 0$ and $\lim_{u \to 0^+} \mathcal{T}^{\alpha}(u)$ exists, then clearly $\mathcal{T}^{\alpha}(u)(0) = \lim_{u \to 0^+} \mathcal{T}^{\alpha} h(u)$.

Theorem 2.1:
Let $\alpha \in [0, 1[$ and the functions $y_1, \ldots, y_p$ are α-differentiable at a point $u > 0$. Then

(1) $\mathcal{T}^{\alpha}(k_1 y_1 + k_2 y_2) = k_1 \mathcal{T}^{\alpha}(y_1) + k_2 \mathcal{T}^{\alpha}(y_2)$ *for all* $k_1, k_2 \in \mathbb{R}$.
(2) $\mathcal{T}^{\alpha}(u) = 0$, *for all* $u \in \mathbb{R}$.
(3) $\mathcal{T}^{\alpha}(y_1 y_2) = y_1 \mathcal{T}^{\alpha}(y_2) + y_2 \mathcal{T}^{\alpha}(y_1)$.
(4) $\mathcal{T}^{\alpha}\left(\frac{y_1}{y_2}\right) = \frac{y_2 \mathcal{T}^{\alpha}(y_1) - y_1 \mathcal{T}^{\alpha}(y_2)}{y_2^2}$.
(5) *If y is differentiable, then* $\mathcal{T}^{\alpha}(y) = u^{1-\alpha} \frac{dy(u)}{du}$.
(6) $\mathcal{T}^{\alpha}(y^m) = m y^{m-1} \mathcal{T}^{\alpha}(y)$ *for all* $m \in \mathbb{N}^*$.
(7) $\mathcal{T}^{\alpha}(\prod_{k=1}^{p} y_k) = \sum_{k=1}^{p} \mathcal{T}^{\alpha}(y_k) \prod_{\substack{i=1 \\ i \neq k}}^{p} y_i$ *for all* $m \in \mathbb{N}^*$.

Proof
Properties (1) through (5) have already been approved by Khalil et al. in [7].

To prove property (6). Let $u>0$ and $m\in\mathbb{N}^*$, we have

$$\begin{aligned}
\mathcal{T}^\alpha(y^m)(u) &= \lim_{\epsilon\to 0}\tfrac{y^m(u+\epsilon u^{1-\alpha})-y^m(u)}{\epsilon},\\
&= \lim_{\epsilon\to 0}\left(\tfrac{y(u+\epsilon u^{1-\alpha})-y(u)}{\epsilon}\right)\left(\lim_{\epsilon\to 0}\sum_{k=0}^{m-1} y^k(u+\epsilon u^{1-\alpha})y^{m-k-1}(u)\right),\\
&= \mathcal{T}^\alpha(y)(u)\sum_{k=0}^{m-1} y^k(u)y^{m-k-1}(u),\\
&= \mathcal{T}^\alpha(y)(u)\sum_{k=0}^{m-1} y^{m-1}(u),\\
&= my^{m-1}(u)\mathcal{T}^\alpha(y)(u).
\end{aligned}$$

Now, for the last property, we use recurrence reasoning. For all $u>0$, we suppose that y_{p+1} is α-differentiable and $\mathcal{T}^\alpha(\prod_{k=1}^{p} y_k)=\sum_{k=1}^{p}\mathcal{T}^\alpha(y_k)\prod_{\substack{i=1\\ i\neq k}}^{p} y_i$.

We have

$$\begin{aligned}
\mathcal{T}^\alpha(\prod_{k=1}^{p+1} y_k) &= \mathcal{T}^\alpha((\prod_{k=1}^{p} y_k)y_{n+1}),\\
&= y_{n+1}\mathcal{T}^\alpha(\prod_{k=1}^{p} y_k)+(\prod_{k=1}^{p} y_k)\mathcal{T}^\alpha(y_{n+1}),\\
&= (\sum_{k=1}^{p}\mathcal{T}^\alpha(y_k)\prod_{\substack{i=1\\ i\neq k}}^{p} y_i)y_{n+1}+(\prod_{k=1}^{p} y_k)\mathcal{T}^\alpha(y_{n+1}),\\
&= \sum_{k=1}^{p}\mathcal{T}^\alpha(y_k)\prod_{\substack{i=1\\ i\neq k}}^{p+1} y_i+(\prod_{k=1}^{p} y_k)\mathcal{T}^\alpha(y_{n+1}),\\
&= \sum_{k=1}^{p+1}\mathcal{T}^\alpha(y_k)\prod_{\substack{i=1\\ i\neq k}}^{p+1} y_i
\end{aligned}$$

According to the reasoning by recurrence we proved the relation (7). □

3. LOTKA-VOLTERRA AND QUASI-POLYNOMIAL MODELS

We will define some notions concerning the QP and LV fractional order systems as well as their stability analysis and that of LMI and BMI.

QP models are systems of FDEs in this form :

$$\mathcal{T}^\alpha(y_i)=y_i\left(L_i+\sum_{j=1}^{m} M_{ij}\prod_{k=1}^{p} y_k^{B_{jk}}\right),\ i=1,\ldots,p. \tag{3.2}$$

where $\mathcal{T}^\alpha$ is defined in Definition 2.1, $y=[y_1,\ldots,y_p]^T\in\mathbb{R}_+^p$, $L=[L_1,\ldots,L_p]^T\in\mathbb{R}^p$, $M\in M_{p,m}(\mathbb{R})$ and $B\in M_{m,p}(\mathbb{R})$. without the loss of generality, we can assume that $rank(B)=p$ and $m\geq p$ (see [5]).

Considering $z_j = \prod_{k=1}^{p} y_k^{B_{jk}}$ and using the 6th and 7th properties of Theorem 2.1. If $rank(B) = p$, then the fractional differential system FDEs (3.2) becomes in the LV form :

$$\mathcal{T}^{\alpha}(z_j) = z_j \left(N_j + \sum_{i=1}^{m} A_{ji} z_i \right), \ j = 1, \ldots, m \tag{3.3}$$

where $A = BM$ and $N = BL$.

3.1. *Input-affine QP system models*

We consider the input-affine nonlinear system fractional order model following :

$$\begin{aligned} \mathcal{T}^{\alpha}(y) &= \phi(y) + \sum_{l=1}^{r} \varphi_l(y) \vartheta_l, \\ \theta &= \psi(y). \end{aligned} \tag{3.4}$$

where y, ϑ and θ are the state vector, the input vector and the output vector, respectively. The function ϕ, φ_l for $l = 1, \ldots, r$ and ψ are in QP-form. Then the input-affine QP-system with p-inputs is :

$$\mathcal{T}^{\alpha}(y_i) = y_i \left(L_0 + \sum_{j=1}^{m} M_{0_{ij}} \prod_{k=1}^{p} y_k^{B_{jk}} \right) + \sum_{l=1}^{r} y_i \left(L_{l_i} + \sum_{j=1}^{m} M_{l_{ij}} \prod_{k=1}^{p} y_k^{B_{jk}} \right) \vartheta_l, \ i = 1, \ldots, p. \tag{3.5}$$

where $M_0, M_l \in M_{p,m}(\mathbb{R})$, $B \in M_{m,p}(\mathbb{R})$ and $L_0, L_l \in M_{p,1}(\mathbb{R})$ for $l = 1, \ldots, r$. The related input-affine LV model in this form :

$$\mathcal{T}^{\alpha}(z_j) = z_j \left(N_{0_j} + \sum_{k=1}^{m} A_{0_{jk}} z_k \right) + \sum_{l=1}^{r} z_j \left(N_{l_j} + \sum_{k=1}^{m} A_{l_{jk}} z_k \right) \vartheta_l, \ j = 1, \ldots, m \tag{3.6}$$

where $N_0 = BL_0 \in M_{m,1}(\mathbb{R}),\ N_l = BL_l \in M_{m,1}(\mathbb{R}),\ A_0 = BM_0 \in M_m(\mathbb{R})$ and $A_l = BM_l \in M_m(\mathbb{R})$ for $l = 1, \ldots, r$.

3.2. *Transforming non-QP FDEs models into QP-form*

The non-linear FDEs is writing in this form :

$$\begin{aligned} \mathcal{T}^{\alpha}(y_i) &= \sum_{i_{s1}, \ldots, i_{sn}, j_s} a_{i_{s1}, \ldots, i_{sn}, j_s} y_1^{i_{s1}} \ldots y_n^{i_{sn}} g(\bar{y})^{j_s}, \\ y_i(0) &= y_{i0}, \qquad i = 1, \ldots, p \end{aligned} \tag{3.7}$$

where $g(\bar{y})$ is not written in this form : $\prod_{k=1}^{p} y_k^{C_{jk}}$, $j = 1, \ldots, m$ with $C \in M_{m,p}(\mathbb{R})$.

In addition,

$$\frac{\mathcal{T}^{\alpha} g}{\partial y_i} = \sum_{e_{s1}, \ldots, e_{sn}, e_s} b_{e_{s1}, \ldots, e_{sn}, e_s} y_1^{e_{s1}} \ldots y_n^{e_{sn}} g(\bar{y})^{e_s}. \tag{3.8}$$

We introduce a new auxiliary variable

$$\eta = f^q \prod_{k=1}^{p} y_k^{p_k}, q \neq 1. \tag{3.9}$$

Then, equations (3.7) can write in QP-form :

$$\mathcal{T}^\alpha(y_i) = y_i \left(\sum_{i_{s1},\ldots,i_{sn},j_s} a_{i_{s1},\ldots,i_{s1},j_s} \eta^{j_s/q} \prod_{k=1}^{p} y_k^{i_{sk}-\delta_{sk}-j s p_k/q} \right), \ i = 1,\ldots,n \tag{3.10}$$

where $\delta_{sk} = 1$ if $s = k$ and 0 otherwise. Additionally, a new QP FDEs appears for the new variable η :

$$\mathcal{T}^\alpha \eta = \eta \sum_{s=1}^{p} \left(p_s \mathcal{T}^\alpha(y_s) y_s^{-1} + \sum_{\substack{i_{s\beta} \quad ,j_s \\ e_{s\beta},e_s}} a_{i_{s\alpha},j_s} b_{e_{s\beta},e_s} q \eta^{(e_s+j_s-1)/q} \prod_{k=1}^{p} y_k^{i_{sk}+e_{sk}+(1-e_s-j_s)p_k/q} \right). \tag{3.11}$$

The choose of p and q can be in (3.9) in different approach in order to facilitate the calculations. The simplest choice is $p_s = 0, \ s = 1,\ldots,n$ and $q = 1$. So, if the initial values of the variables introduced according to (3.9) is fixed, at that moment the dynamics of the integrated system will be equivalent to the original non-QP system shown in (3.9). Hence, it is evident that the original system (3.9) is stable whenever the systems (3.10)-(3.11) are stable.

3.3. QP models of process systems

Nonlinear process system models with localized parameters are divided into two forms in terms of their representation in the form of QP. The systems operate ϕ from the input affine spatial state model (3.4) and they are not always in the form of QP. Consequently, the integration of these models in the described QP-form crucial practice.
The characteristics of the input function φ_i of the input-affine space-state model (3.4). Frequently, φ_i is a simple homogeneous linear function of the corresponding state variable y_i : $\varphi_i(y) = const.y_i$ implies $A_l = 0$ in (3.2) and $M_l = 0$ in (3.6).
If $\varphi_i = const$, this form is not accepted when an equation of state in QP-form comes from the incorporation of variables.

3.4. Application

New, we consider the following non-QP model (see [9]) :

$$\begin{array}{rcl} \mathcal{T}^\alpha(X) &=& r_1 X\left(1-\frac{X}{K}\right) - \sigma_1 X + \sigma_2 Y - uX^2 - \frac{\beta XS}{\alpha'+X} - q_1 E_1 X - n_1 XY - \gamma XI, \\ \mathcal{T}^\alpha(Y) &=& (r_2-\sigma_2)Y + \sigma_1 X - vY^2 - n_2 XY, \\ \mathcal{T}^\alpha(S) &=& \frac{\sigma\beta XS}{\alpha'+X} - \delta SI - \mu S - q_2 E_2 S, \\ \mathcal{T}^\alpha(I) &=& \delta SI + \sigma\gamma XI - q_3 E_3 I - \eta I. \end{array} \tag{3.12}$$

Where X, Y, I and S represent biomass densities of the unreserved area, reserved area, infected and susceptible predator, respectively. For the harvesting in the unreserved area, E_1, E_2 and E_3 denote the effort applied the susceptible and infected predator populations,

respectively. r_1, r_2 the fish population growth rates in the reserved and unserved areas, respectively. q_1 and q_2 represents the coefficient of catchability in the unreserved area and the predator species. σ_1 and σ_2 represent the migration rates between the two zones. μ and η represent the rate of death of susceptible and infected predators. n_1 and n_2 are the coefficients of competition. γ the strength of intra-specific between X and I. δ the coefficient of disease transmission. β the rate of search of the prey toward susceptible predators. α' constant of saturation while susceptible predators S attack the prey X. σ is the rate of conversion of susceptible predator I due to prey X. uX^2 and vY^2 represents the reduction terms, in the unreserved area and reserved area respectively.
To transform the model (3.12) into the QP form, we introduce a new variable $Z = \frac{1}{\alpha'+X}$, we get the following QP-model :

$$\begin{cases} \mathcal{T}^\alpha(X) &= X\left(r_1 - \sigma_1 - q_1E_1 + \sigma_2V_2 - (u + \frac{r_1}{K})X - \beta V_1 - n_1Y - \gamma I\right), \\ \mathcal{T}^\alpha(Y) &= Y\left(r_2 - \sigma_2 + \sigma_1V_3 - vY - n_2X\right), \\ \mathcal{T}^\alpha(S) &= S\left(\sigma\beta V_4 - \delta I - \mu - q_2E_2\right), \\ \mathcal{T}^\alpha(I) &= I\left(\delta S + \sigma\gamma X - q_3E_3 - \eta\right), \\ \mathcal{T}^\alpha(Z) &= Z\left(-V_4(r_1 - \sigma_1 - q_1E_1) + (\frac{r_1}{K} + u)V_5 + n_1V_6 + \beta V_7 - \sigma_2V_8 + \gamma V_9\right). \end{cases} \tag{3.13}$$

The 14 quasi-monomials of the QP system model given are : X, Y, Z, I, S, $V_1 = SZ$, $V_2 = X^{-1}Y$, $V_3 = XY^{-1}$, $V_4 = XZ$, $V_5 = X^2Z$, $V_6 = XYZ$, $V_7 = XZ^2S$, $V_8 = YZ$ and $V_9 = XZI$. Then, the QP-model (3.13) is transfomed in the Lotka-Volterra form :

$$\begin{cases} \mathcal{T}^\alpha(X) &= X\left(r_1 - \sigma_1 - q_1E_1 + \sigma_2V_2 - (u + \frac{r_1}{K})X - \beta V_1 - n_1Y - \gamma I\right), \\ \mathcal{T}^\alpha(Y) &= Y\left(r_2 - \sigma_2 + \sigma_1V_3 - vY - n_2X\right), \\ \mathcal{T}^\alpha(S) &= S\left(\sigma\beta V_4 - \delta I - \mu - q_2E_2\right), \\ \mathcal{T}^\alpha(I) &= I\left(\delta S + \sigma\gamma X - q_3E_3 - \eta\right), \\ \mathcal{T}^\alpha(Z) &= Z\left(-V_4(r_1 - \sigma_1 - q_1E_1) + (\frac{r_1}{K} + u)V_5 + n_1V_6 + \beta V_7 - \sigma_2V_8 + \gamma V_9\right), \\ \mathcal{T}^\alpha(V_1) &= V_1\left((\sigma\beta + r_1 - \sigma_1 - q_1E_1)V_4 + (\frac{r_1}{K} + u)V_5 + n_1V_6 + \beta V_7 - \sigma_2V_8 + \gamma V_9 + \sigma\gamma X - \eta - q_3E_3\right), \\ \mathcal{T}^\alpha(V_2) &= V_2\left(r_2 - \sigma_2 - (r_1 - \sigma_1 - q_1E_1) + (\frac{r_1}{K} + u - n_2)X + (n_1 - v)Y + \beta V_1 - \sigma_2V_2 - \gamma I\right), \\ \mathcal{T}^\alpha(V_3) &= -V_3\left(r_2 - \sigma_2 - (r_1 - \sigma_1 - q_1E_1) + (\frac{r_1}{K} + u - n_2)X + (n_1 - v)Y + \beta V_1 - \sigma_2V_2 - \gamma I\right), \\ \mathcal{T}^\alpha(V_4) &= V_4\left(\frac{\mathcal{T}^\alpha(X)}{X} + \frac{\mathcal{T}^\alpha(Z)}{Z}\right), \\ \mathcal{T}^\alpha(V_5) &= V_5\left(\frac{\mathcal{T}^\alpha(X)}{X} + 2\frac{\mathcal{T}^\alpha(Z)}{Z}\right), \\ \mathcal{T}^\alpha(V_6) &= V_6\left(\frac{\mathcal{T}^\alpha(X)}{X} + \frac{\mathcal{T}^\alpha(Z)}{Z} + \frac{\mathcal{T}^\alpha(Y)}{Y}\right), \\ \mathcal{T}^\alpha(V_7) &= V_7\left(\frac{\mathcal{T}^\alpha(X)}{X} + 2\frac{\mathcal{T}^\alpha(Z)}{Z} + \frac{\mathcal{T}^\alpha(S)}{S}\right), \\ \mathcal{T}^\alpha(V_8) &= V_8\left(\frac{\mathcal{T}^\alpha(Z)}{Z} + \frac{\mathcal{T}^\alpha(Y)}{Y}\right), \\ \mathcal{T}^\alpha(V_9) &= V_9\left(\frac{\mathcal{T}^\alpha(X)}{X} + \frac{\mathcal{T}^\alpha(Z)}{Z} + \frac{\mathcal{T}^\alpha(I)}{I}\right). \end{cases} \tag{3.14}$$

4. STABILITY OF QP SYSTEMS FRACTIONAL ORDER

According on the basic concepts of LV and QP systems gave in section 3, we will analysis of the stability of QP systems fractional order and a proposed numerical algorithm to solve the case of non-strict LMI.
The analysis of stability is carried out around an equilibrium point $y^* = (y_1^*, y_2^*, \ldots, y_p^*)$. The solution of the equilibrium point is determined from (3.2) as follows :

$$0 = y_i\left(L_i + \sum_{j=1}^{m} M_{ij} \prod_{k=1}^{p} y_k^{B_{jk}}\right), i = 1, \ldots p, \text{ and } m \geq p \tag{4.15}$$

Suppose that y^* is a positive equilibrium point of (3.2), so $z^* = (z_1^*, z_2^*, \ldots, z_m^*)$ is a positive equilibrium in the case LV systems (3.6). The Lyapunov function associated with LV-system in the following:

$$\begin{aligned} F(z) &= \sum_{i=1}^{m} \omega_i \left(z_i - z_i^* - z_i^* \ln(\tfrac{z_i}{z_i^*}) \right) \\ & \quad \omega_i > 0, \ i = 1, \ldots, m \end{aligned} \tag{4.16}$$

where z^* is correspondent point of equilibrium y^* of system (3.2). The fractional derivate of Lyapunov function is defined by :

$$\mathcal{T}^\alpha F(z) = F'(z).\mathcal{T}^\alpha(z) = \frac{1}{2}(z - z^*)(\mathcal{W}A + A^T\mathcal{W})(z - z^*), \tag{4.17}$$

where $\mathcal{W} = diag(\omega_1, \ldots, \omega_m)$ and A is the invariant characterizing the LV-form (3.6). Consequently, the decreasing nature of Lyapunov's function is equivalent to a feasibility problem of the following LMI :

$$\begin{aligned} \mathcal{W}A + A^T\mathcal{W} &\leq 0 \\ \mathcal{W} &> 0, \end{aligned} \tag{4.18}$$

The possibilities of finding a function of Lyapunov that shows the global asymptotic stability of a QP system can be raised with the help of temporal reparametrization (See [3]). If the model (3.6) are ordered so that the first p lines of B are linearly independent, then $\omega_i > 0$ for $i = 1, \ldots, p$ and $c_j = 0$ for $j = p + 1, \ldots, m$ ensure the global stability.
The global stability of (3.6) is studied and demonstrated in [3, 4] with the Lyapunov function (4.16), which gives the boundedness of the solutions and the global stability of (3.2). It can be noted that the global stability is limited to the positive orthant only for LV and QP models, which is compatible with the nature of the variables.
The global stability of the equilibrium points (3.2) as a function of Lyapunov (4.16) are not related on the value of the vector L as will as the equilibrium points are in the positive orthant. This makes it possible to reach the equilibrium point of the closed-loop system (CLS) when designing the stabilization controller [18].
By using temporal reparametrization [18], we increase the possibilities to find a function of Lyapunov which demonstrates the global asymptotic stability of a QP system.

5. THE CONTROLLER DESIGN PROBLEM

Now, we will study the global feedback stabilization of (3.4) where the state feedback control is on the QP form of the CLS. Also, we will study its global stability using LMI if the return parameters are known and fixed. Otherwise, a feedback design problem will be defined that globally stabilizes the CLS.

The global stabilization of the feedback design problem for QP systems is modeled on (5.19). Let QP entries:

$$\varphi_l = \sum_{i=1}^{s} k_{il} q_i, \quad l = 1, \ldots, r \tag{5.19}$$

where the quasi-monomial functions $q_i = q_i(y_1, \ldots, y_p)$ for $i = 1, \ldots, s$ of the state variables of (3.7) and k_{il} are constants gain. Moreover, the CLS will also be a QP system with the following matrices :

$$\begin{aligned} \bar{M} &= M_0 + \sum_{l=1}^{r}\sum_{i=1}^{s} k_{il} M_{il}, \qquad \bar{B} \\ \bar{L} &= L_0 + \sum_{l=1}^{r}\sum_{i=1}^{s} k_{il} L_{il}. \end{aligned} \tag{5.20}$$

The q_i in the CLS, as well as the matrix $\bar{B}$ can vary considerably depending on the choice of the feedback structure. Hence, in the closed-loop LV, the matrix of the coefficients $\bar{A}$ is expressed in this form :

$$\bar{A} = \bar{B}.\bar{M} = A_0 + \sum_{l=1}^{r}\sum_{i=1}^{s} k_{il} A_{il}. \tag{5.21}$$

Thereafter, the analysis of the global stability of the CLS with an unspecified gain feedback k_{il} gives the next BMI :

$$\bar{A}^T \mathcal{W} + \mathcal{W}\bar{A} = A_0^T \mathcal{W} + \mathcal{W} A_0 + \sum_{l=1}^{r}\sum_{i=1}^{s} k_{il}(A_{il}^T \mathcal{W} + \mathcal{W} A_{il}) \leq 0. \tag{5.22}$$

where the feedback gain parameters is k_{il} and the positive coefficients of the Lyapunov function is c_j, $j = 1, \ldots, m$. Then, there exists globally stabilizing feedback whenever the BMI above is attainable.

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