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The Model of the Production Side of the Russian Economy: the Updated Version

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Abstract: In this paper we propose the new version of the model of the production side of the Russian economy presented in [25]. We add several new features such as more general production function, more tractable human capital variable and the method of COVID-19 pandemic shock description. The new version of the model is capable of reproducing the wide set of main Russian macroeconomic indicators with sufficiently high accuracy even during the COVID-19 pandemic.

Keywords: dynamic optimization, complementary slackness conditions, production of GDP

1. INTRODUCTION AND LITERATURE REVIEW

In [25] we presented the model of production side of the economy, which represents the aggregate description of all economic agents who produce the added value. This aggregated agent, which can be called Producer, constitutes the important part of economy and, together with Consumer, Government etc., forms the basic structure of general equilibrium description of the economy. The model presented in [25] is based on the solution of the optimization problem of the agent who maximizes his discounted profit flow under technologic, demographic and financial constraints in the spirit of the standard microeconomic description of the firm. The model, however, contains a wider set of economic instruments available to Producer – it is connected not only to labor and capital markets, but also to the banking system via loans in both national and foreign currencies. Several heuristic methods such as relaxation of complementary slackness conditions are used to derive the model which is capable of reproducing the Russian macroeconomic variables with sufficiently high precision.

In this paper we add several new features to the model. First, we use more general constant elasticity of substitution (CES) production function instead of Cobb-Douglas production function. CES production function was introduced in [4] and is widely used both in theoretic and applied works dedicated to the description of the production side of economy both on macro- and microeconomic levels (see [20], [21], [9], [14], [10], [22], [2], [7], [5] and the special issue of Journal of Macroeconomics in June 2008 for examples and discussion). The modelling of gross domestic product using the production functions of different kinds (see for example [11], [12], [15], [18]) can be mentioned separately due to the closeness of these works to ours. The Cobb-Douglas production function, which is a special case of CES, is used more commonly than CES due to its analytic tractability. In particular, Cobb-Douglas production functions are ubiquitous in DSGE models (see for example [28], [29], [8]). However, one of the properties of the Cobb-Douglas production function is that it implies the constant shares of labor and capital contributions to production, which is not confirmed on the Russian macroeconomic data. Hence we decided to switch to more sophisticated CES production function.

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Second, in the new version of the model we use the human development index calculated by the United Nations (see [27]) as a measure of human capital. This index is calculated based on such indicators as gross national income per capita, literacy rate, education level. Note that these indicators are essentially out of control of the production side of economy, so this variable is considered as exogenous in the model.

Third, we introduce the mechanism which exogenously restricts the ability to hire the labor force. It us used to explain the dynamics of employment during the COVID-19 pandemic. It is worth mentioning that the impact of COVID-19 pandemic and the related government measures stimulated the great volume of research. Notable examples include [13], [3], [19], [1], [6], [16] among many others. In our model we use the COVID-19 stringency index introduced in [17], which is a numeric indicator of strictness of COVID-related measures, as a factor which puts a pressure on employment rate.

In the rest of the paper we describe the new version of the model and analyze its ability to reproduce the Russian macroeconomic variables.

2. THE MODEL: STATEMENT

As in [25], we consider the model of the whole sector of the economy, which generates the added value, as a single agent. We formulate the optimization problem, solve it and then, using several heuristics, we reduce the solution to the dynamic model.

As in [25], the investments are divided into two parts: investments in the maintenance of fixed assets $Ju(t)$ and investments in building up fixed assets $Jm(t)$.

The formation of production capital (fixed assets) $M(t)$ is described by the following relations:

$$\frac{\mathrm{d}}{\mathrm{d}t}M\left(t\right) = Jm\left(t\right) - \delta_{am}\left(t\right)M\left(t\right), \tag{2.1}$$

$$0 \le Jm\left(t\right). \tag{2.2}$$

where $\delta_{am}\left(t\right)$ is the coefficient of depreciation rate, which is assumed to be constant and equal to 0.00275. The equation (2.1) allows to calculate the series $Jm\left(t\right)$ based on statistics on the level of fixed assets and depreciation. The balance of investments

$$J\left(t\right) = Jm\left(t\right) + Ju\left(t\right), \tag{2.3}$$

where $J\left(t\right)$ is the overall level of investment (gross fixed capital formation), allows to calculate the investments in the maintenance of fixed assets $Ju\left(t\right)$. The balance can also be written in nominal terms

$$pJ\left(t\right) = p_m\left(t\right)Jm\left(t\right) + p_u\left(t\right)Ju\left(t\right), \tag{2.4}$$

where $pJ\left(t\right)$ is the overall level of investments (gross fixed capital formation) in the current prices, $p_m\left(t\right)$ and $p_u\left(t\right)$ are the deflators of the corresponding elements of investments. In this version of the model, however, we assume them being equal.

We introduce the human capital per capita denoted by $H(t)$. As was noted in the introduction, in this version of the model we consider it as an exogenous variable.

Next, we introduce the total amount of payments made by the Producer to its employees, which is called wage and mixed income in the system of national accounts. We denote this variables by $W\left(t\right)$.

As was noted in the introduction, we assume that the production function of the agent we consider has a constant elasticity of substitution. Namely, we assume the following structure of the production function:

$$Y(t) = A\left(\alpha\left(u_M(t)\, M(t)\right)^{\theta} + (1-\alpha)\left(u_H(t)\, H(t)\, R_{\max}(t)\right)^{\theta}\right)^{\frac{1}{\theta}}, \tag{2.5}$$

where $u_M(t)$ and $u_H(t)$ are the utilization rates, respectively, of fixed assets and human capital. We define these indicators in the following non-linear way:

$$u_M(t) = \left(\frac{Ju(t)}{M(t)}\right)^{b}, u_H(t) = \left(\frac{R(t)}{R_{\max}(t)}\right)^{a}. \tag{2.6}$$

We substitute (2.6) into (2.5) and also substitute A with several normalization coefficients in order to avoid the dimensionality problem. As a result, the production function takes the form

$$\frac{Y(t)}{Y0} = \left[\alpha\left(\frac{Ju(t)}{J0}\right)^{b\rho}\left(\frac{M(t)}{M0}\right)^{(1-b)\rho} + (1-\alpha)\left(\frac{H(t)}{H0}\right)^{\rho}\left(\frac{R(t)}{R0}\right)^{b\rho}\left(\frac{R_{\max}(t)}{Rmax0}\right)^{(1-b)\rho}\right]^{\frac{1}{\rho}}. \tag{2.7}$$

The volume of production $0 \leq Y(t)$ and the volume of products sold $0 \leq Yp(t)$ may vary, which leads to the introduction of stock $0 \leq Z(t)$ in the model. Its change can be calculated as

$$\frac{\mathrm{d}}{\mathrm{d}t} Z(t) = Y(t) - Yp(t). \tag{2.8}$$

Denote the loans attracted by the Producer in national currency (rubles) by $L(t)$. The average terms for which loans are attracted will be denoted by $(\beta_l(t))^{-1}$. The variable $\beta_l(t)$ is the inverse duration and interpreted as the average frequency of the loans return. Then the dynamics of ruble loans is defined by the equation

$$\frac{\mathrm{d}}{\mathrm{d}t} L(t) = K(t) - \beta_l(t)\, L(t), \tag{2.9}$$

where $0 \leq K(t)$ is the flow of newly attracted loans and $r_L(t)$ is the effective interest rate on loans.

Producer also attracts loans in foreign currency, which are nominated in dollars in the Russian statistical data. We denote the corresponding value by $vL(t)$. The dynamics of foreign currency loans is described by the equation similar to (2.9):

$$\frac{\mathrm{d}}{\mathrm{d}t} vL(t) = vK(t) - \beta_{vl}(t)\, vL(t). \tag{2.10}$$

Here $\beta_{vl}(t)$ is the reverse durations of currency loans, newly attracted foreign currency loans are denoted by $0 \leq vK(t)$ and the effective interest rate on foreign currency loans is denoted by $r_{vl}(t)$.

The taxes Producer pays can be aggregared into four groups: value added tax, labor taxes, property taxes and income taxes. The rates of these aggregated taxes are denoted by $\tau_y(t)$, $\tau_r(t)$, $\tau_{tm}(t)$, $\tau_{pr}(t)$ correspondingly. For servicing operations related to loans and investments, the producer uses current account $N(t)$. In this version of the model it is assumed that its volumes are proportional to ruble and foreign currency loans and fixed assets. These proportions are defined by the coefficients ν_l, ν_{vl}, ν_m:

$$N(t) \geq \nu_l L(t) + \nu_{vl} w_{vl}(t)\, vL(t) + \nu_m p_m(t)\, M(t), \tag{2.11}$$

where $w_{vl}(t)$ denotes the exchange rate.

Other expenses of the Producer are denoted by $OC_o(t)$ and are considered as the exogenous variable. We also denote the profit after taxation by $\Pr(t)$. The financial balance of the Producer can be written as

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}N(t)=K(t)-\beta_l(t)L(t)-r_l(t)L(t)-(1+\tau_{pr}(t))\Pr(t)+\\&w_{vl}(t)(vK(t)-\beta_{vl}(t)vL(t)-r_{vl}(t)vL(t))-OC_o(t)\\&+(1-\tau_y(t))p_y(t)\,Yp(t)-p_j(t)\,Ju(t)-p_m(t)\,Jm(t)-\\&\tau_{tm}(t)\,p_{tm}(t)\,M(t)-(1+\tau_r(t))\,w_r(t)\,R(t)\,.\end{aligned} \tag{2.12}$$

The above relationships represent the limitations imposed on the Producer's ability to choose the values of its planned variables (controls):

$$\begin{aligned}&Ju(t)\,,Jm(t)\,,K(t)\,,L(t)\,,M(t)\,,N(t)\,,\Pr(t)\,,\\&R(t)\,,\,Yp(t)\,,Z(t)\,,vK(t)\,,vL(t)\,,Y(t)\,.\end{aligned} \tag{2.13}$$

According to the principle of rational expectations, when planning its control variables, the producer can rely on an accurate forecast of information variables:

$$\begin{aligned}&H(t)\,,OC_o(t)\,,R_{\max}(t)\,,\beta_l(t)\,,\beta_{vl}(t)\,,\delta_{ah}(t)\,,\delta_{am}(t)\,,p_j(t)\,,p_m(t)\,,p_{tm}(t)\,,\\&p_y(t)\,,r_l(t)\,,r_{vl}(t)\,,\tau_h(t)\,,\tau_{pr}(t)\,,\tau_r(t)\,,\tau_{tm}(t)\,,\tau_y(t)\,,w_h(t)\,,w_r(t)\,,w_{vl}(t)\,.\end{aligned}$$

As in [25], the goal of the producer in the model is to maximize the total discounted utility from the profit after taxation. We assume that the discount rate in the utility function is equal to the GDP deflator:

$$\int_{t0}^{T}\frac{\mathrm{e}^{-\Delta\,t}}{1-\eta}\left(\frac{\Pr(t)}{p_y(t)}\right)^{1-\eta}\mathrm{d}t. \tag{2.14}$$

The problem is supplemented with terminal condition, which can be interpreted as a growth condition for some linear combination of the phase variables:

$$\Omega(t0)\,\gamma\le\Omega(T)\,, \tag{2.15}$$

where $\Omega(t)=aL(t)\,L(t)+aM(t)\,M(t)+aN(t)\,N(t)+aZ(t)\,Z(t)+avL(t)\,vL(t)$. This is the analogue of no Ponzi condition written in terms of the Producer's own capital which is a difference between its assets and liabilities.

As in [25], we derive the solution of the maximization problem of the function (2.14) under constraints (2.1) – (2.12) and the terminal condition (2.15) with respect to the variables (2.13) using the Lagrange method. We obtain the system of sufficient optimality conditions, which can be divided into several groups: equations for primal variables, differential equations for dual variables, complementary slackness conditions and the terminal condition. This system is then tranformed to the more tractable form using the method described in detail in [26].

First, we transform it to the discrete time by replacing derivatives with increments (backward increments are used for direct variables and forward increments are used for dual variables). Second, some direct variables are replaced by their values in the previous period. Third, differential equations for the dual variables are replaced by expressions of corresponding separatrices. Forth, the complementary slackness conditions are replaced by their more regular approximations (the procedure is called the relaxation of complementary slackness conditions). After all these transformations, we proceed to the dynamic system which can also be presented by several groups of expressions.

The first group consists of several expressions defining growth rates of different exogenous variables (in different forms) and two more auxiliary variables:

$$
\begin{aligned}
&g_{pm}\left(t\right)=\frac{p_m\left(t\right)-p_m\left(t-1\right)}{p_m\left(t-1\right)}, g_{wh}\left(t\right)=\frac{w_h\left(t\right)-w_h\left(t-1\right)}{w_h\left(t-1\right)},\\
&g_{py}\left(t\right)=\frac{p_y\left(t\right)-p_y\left(t-1\right)}{p_y\left(t-1\right)}, g_{taupr}\left(t\right)=\frac{\tau_{pr}\left(t\right)-\tau_{pr}\left(t-1\right)}{\tau_{pr}\left(t-1\right)+1},\\
&g_{tauh}\left(t\right)=\frac{\tau_h\left(t\right)-\tau_h\left(t-1\right)}{1+\tau_h\left(t-1\right)}, g_{rmax}\left(t\right)=\frac{R_{\max}\left(t\right)-R_{\max}\left(t-1\right)}{R_{\max}\left(t-1\right)},\\
&g_{ttauy}\left(t\right)=-\frac{\tau_y\left(t\right)-\tau_y\left(t-1\right)}{-\tau_y\left(t-1\right)+1},\\
&RR(t)=\left(\frac{H(t-1)}{H0}\right)^{\frac{\theta}{-b\theta+1}}\left(\frac{Y(t)}{Y0}\right)^{-\frac{\theta}{-b\theta+1}}\left(\frac{Rmax(t)}{Rmax0}\right)^{\frac{(1-b)\theta}{-b\theta+1}}\times\\
&\times\left(\frac{bY(t)\,py(t)\,(1-\alpha)\,(1-\tau_y(t))}{wr(t)\,R0\,(1+\tau_r(t))}\right)^{\frac{b\theta}{-b\theta+1}},\\
&RR2(t)=-pj(t)\,Ju(t)-(1+\tau_r(t))\,wr(t)\,R(t)-OCo(t)+\\
&+Y(t)\,(1-\tau_y(t))\,py(t)-\Pr(t)\,(\tau_{pr}(t)+1)\,.
\end{aligned}
$$

The second group consists of expressions defining “real” variables such as GDP and its components included in the model, fixed capital stock and the number of employed.

Real GDP in the model is defined as

$$
\begin{aligned}
&\left(\frac{Y(t)}{Y0}\right)^{\frac{(1-b)\theta}{-b\theta+1}}=\alpha\left(\frac{M(t-1)}{M0}\right)^{\frac{(1-b)\theta}{-b\theta+1}}\left(\frac{\alpha bpy(t)\,(1-\tau_y(t))}{pj(t)\,J0}\right)^{\frac{b\theta}{-b\theta+1}}+\\
&+(1-\alpha)\left(\frac{Rmax(t)}{Rmax0}\right)^{\frac{(1-b)\theta}{-b\theta+1}}\left(\frac{H(t-1)}{H0}\right)^{\frac{\theta}{-b\theta+1}}\left(\frac{bpy(t)\,(1-\alpha)\,(1-\tau_y(t))}{wr(t)\,R0\,(1+\tau_r(t))}\right)^{\frac{b\theta}{-b\theta+1}}.
\end{aligned}
$$

The number of employed $R\left(t\right)$ and the level of investments in the maintenance of fixed assets $Ju\left(t\right)$ are adjusted at this stage to better take into account the particularities of Russian macroeconomic dynamics during the recent years. The expression which defines the number of employed persons derived as first order optimality condition is denoted by $R^*(t)$. It is adjusted to better take into account the situation of partial employment. This leads to the following expressions:

$$
\begin{aligned}
&R^*(t)=\left(\frac{H(t-1)}{H0}\right)^{\frac{\theta}{-b\theta+1}}\left(\frac{Rmax(t)}{Rmax0}\right)^{\frac{(1-b)\theta}{-b\theta+1}}R0\left(\frac{Y(t)}{Y0}\right)^{-\frac{\theta}{-b\theta+1}}\times\\
&\times\left(\frac{bY(t)\,py(t)\,(1-\alpha)\,(1-\tau_y(t))}{wr(t)\,R0\,(1+\tau_r(t))}\right)^{\frac{1}{-b\theta+1}},\\
&R\left(t\right)=a_rRmax\left(t\right)+b_rR^*\left(t\right),
\end{aligned}
$$

Similarly, the expression which defines the investments in the maintenance of fixed assets obtained from first order optimization conditions is denoted by $Ju^*(t)$. It is adjusted to take into account the exogenous constraint related to the measures introduced to limit the

COVID-19 spread. As was mentioned in the introduction, we use the COVID-19 stringency index denoted by $covid(t)$ to measure the strictness of the introduced measures. The resulting expressions are the following:

$$Ju^*(t) = \left(\frac{Y(t)}{Y0}\right)^{-\frac{\theta}{-b\theta+1}} \left(\frac{M(t-1)}{M0}\right)^{\frac{(1-b)\theta}{-b\theta+1}} J0 \left(\frac{\alpha b Y(t)\, py(t)\,(1-\tau_y(t))}{pj(t)\, J0}\right)^{\frac{1}{-b\theta+1}},$$
$$Ju\,(t) = \min\{M(t-1)\,(mm\,(1-covid(t)))^{\frac{1}{b}}\,, Ju^*(t)\}.$$

The profit of producer $\Pr(t)$, investments in building up fixed assets $Jm(t)$ and the change of stocks $Z(t)$ are defined by the following relations:

$$\Pr(t) = \left(\frac{-\Delta - g_{taupr}(t) + \rho(t)}{\eta} + \left(-\frac{1}{\eta}+1\right) g_{py}(t)\right) \Pr(t-1) + \Pr(t-1)\,,$$
$$Jm(t) = b_2\left(\delta_{am}(t) + \frac{-\delta_{am}(t) - \tau_{tm}(t) + g_{pm}(t) - \frac{RR(t)\alpha(-1+b)(\tau_y(t)-1)Y(t)py(t)}{M(t-1)pm(t)}}{\delta_{am}(t)\,num+1}\right) M(t-1) -$$
$$b_2 g_{pm}(t)\, M(t-1) - a_2\left((\delta_{am}(t)\,num+1)\,\rho(t) + \delta_{am}(t) + \tau_{tm}(t) - g_{pm}(t)\right) M(t-1) +$$
$$+\frac{RR(t)\,\alpha\,(-1+b)\,(\tau_y(t)-1)\,Y(t)\,py(t)}{pm(t)} + \frac{cc_2 RR2(t)}{pm(t)},$$
$$Z(t) = Z(t-1) + (b_5 - a_5\,(-g_{py}(t) - g_{ttauy}(t) + \rho(t)))\,Z(t-1) + \frac{cc_5 RR2(t)}{py(t)\,(1-\tau_y(t))}.$$

The expression defining the stock of the fixed capital $M(t)$ is also adjusted – we introduce its smoothed version denoted by $Msm(t)$. This smoothed variable is compared with the statistical values of $M(t)$.

$$M\,(t) = Jm\,(t) - \delta_{am}\,(t)\,M\,(t-1) + M\,(t-1)\,,$$
$$Msm(t) = \kappa Msm(t-1) + (1-\kappa)M(t).$$

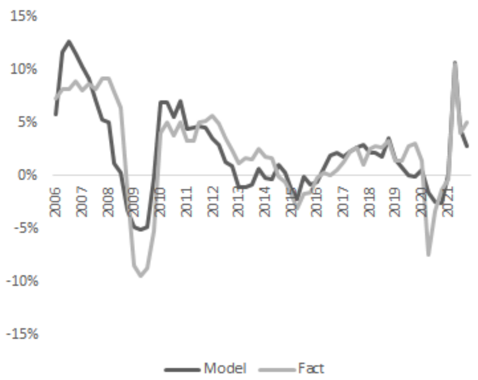
The last group are the equations defining financial variables, both stocks and flows.

$$vK(t) = b_3\left(\beta_{vl}(t) - \frac{rvl(t)+gwvl(t)}{\beta_{vl}(t)\,nuvl-1} - gwvl(t)\right) vL(t-1) -$$
$$a_3\left((\beta_{vl}(t)\,nuvl-1)\,\rho(t) + rvl(t) + gwvl(t)\right) vL(t-1) - \frac{cc_3 RR2(t)}{wvl(t)},$$
$$K(t) = \left(b_4\left(-\frac{rl(t)}{\eta(t)\,nul-1} + \eta(t)\right) - a_4\left((\eta(t)\,nul-1)\,\rho(t) + rl(t)\right)\right) L(t-1) +$$
$$+ (-1 + cc_1 + cc_2 + cc_3 + cc_5)\,RR2(t),$$
$$N\,(t) = \nu_l L\,(t-1) + \nu_{vl} w_{vl}\,(t)\,vL\,(t-1) + \nu_m p_m\,(t)\,M\,(t-1)\,,$$
$$L\,(t) = K\,(t) - \beta_l\,(t)\,L\,(t-1) + L\,(t-1)\,,$$
$$vL\,(t) = vK\,(t) - \beta_{vl}\,(t)\,vL\,(t-1) + vL\,(t-1)\,.$$

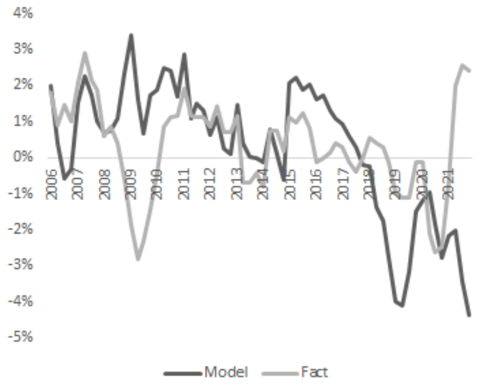
3. THE MODEL: IDENTIFICATION AND RESULTS

The presented version of the model is identified on the Russian macroeconomic data from 2005q1 to 2021q4. The sources of the data are essentially the same as in [25] – Rosstat data is used for GDP and its components, capital stock etc., Bank of Russia data is used for the variables related to the interactions with the banking sector, and The Federal Treasury data is used for the taxation data. The seasonality identification procedure described in [23] is applied where necessary. The new data sources include the COVID-19 stringency database [17] and the human capital development database [27]. The first dataset is available on a daily basis, and we use the quarterly average value. The second dataset is available on a yearly basis, and we extrapolate the given values for the quarters.

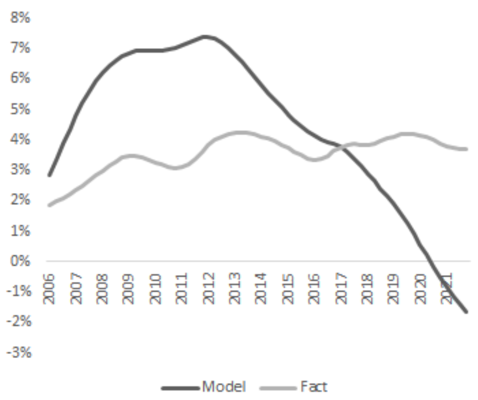
We identify the parameters of the model in two ways. The first one is the standard dynamic system approach used in [25]. We calculate the growth rates of eight variables (Y, R, M, L, vL, N, Pr, W) to the corresponding quarter of the previous year. The function of errors is the sum of squares of the discrepancies between the statistics and the model data with some weights. We put the weight 25 for the GDP as the most important variable and 1 for all other variables. Using the command lsqnonlin of the MATLAB Optimization package, we find the set of model parameters which minimizes this function of errors. The accuracy of data fit is shown on the plots below. As we can see, the model performs rather well even during the COVID-19 pandemic.



(a) GDP

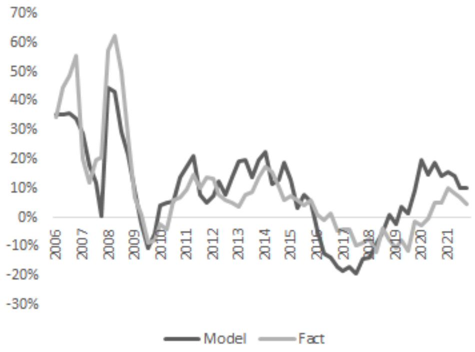


(b) Number of employed

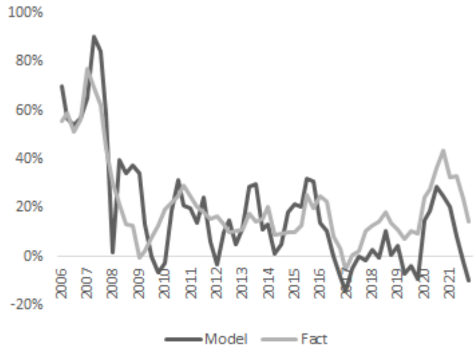


(c) Fixed assets

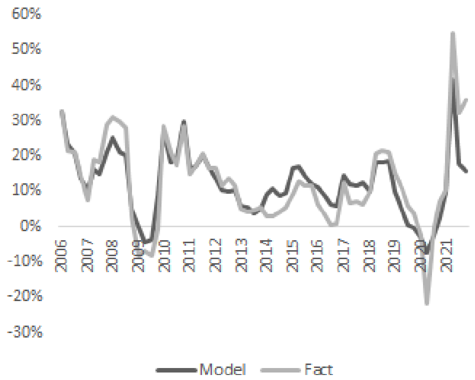
(d) Ruble loans



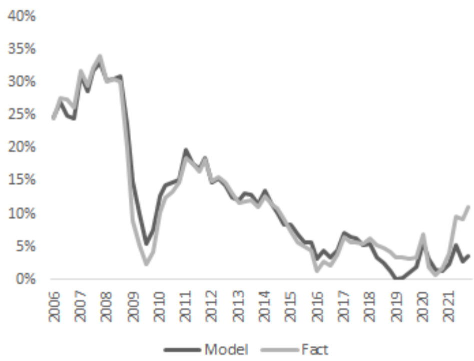
(e) Foreign currency loans



(f) Current accounts



(g) Profit



(h) Wages

Mean average percentage errors of forecasts are shown in the table 3.1.

Table 3.1. Mean absolute percent errors of the model variables, dynamic system

Y	R	M	L	vL	N	Pr	W
1.77	1.01	8.38	8.91	8.82	12.45	5.13	1.13

The second method of the parameters identification we apply is based on the idea of multistep forecasting proposed in [24]. In this framework the parameters of the model are identified in such a way that the error of forecasts for a given length of forecasting is minimized. We consider the forecasts up to 6 quarters ahead and again using the command lsqnonlin of the MATLAB Optimization package find a set of parameters which minimizes the sum of squares of forecast errors. As for the previous case, we minimize the total error of growth rates of the modelled variables compared to their values 4 quarters ago. Mean absolute percentage errors of the forecasts for a given variable and a given number of quarters ahead is presented in the table 3.2. For the comparison, we also present mean absolute percentage errors of the simple autoregressive AR(1) models. As we can see, for the variables vL, Pr, W the model performs better than AR(1) for the whole period, for R, L, N the model performs better or about the same for long-term forecasts, for M the model and AR(1) perform about the same, and for Y the model performs worse than AR(1).

Table 3.2. Mean absolute percent errors of the model variables, multistep forecasting

		1	2	3	4	5	6
Y	model	3.58	3.41	3.30	3.31	3.21	3.09
Y	AR(1)	1.12	1.58	1.80	2.17	2.29	2.35
R	model	0.89	0.87	0.89	0.87	0.83	0.78
R	AR(1)	0.48	0.65	0.72	0.85	0.83	0.91
M	model	0.08	0.16	0.23	0.31	0.37	0.42
M	AR(1)	0.08	0.16	0.23	0.30	0.35	0.40
L	model	2.66	3.57	4.49	5.21	5.50	5.63
L	AR(1)	2.12	3.35	4.25	5.11	5.71	6.29
vL	model	3.06	3.72	3.90	4.05	3.81	4.72
vL	AR(1)	3.43	4.74	6.21	7.27	8.20	8.78
N	model	10.48	10.18	10.06	10.10	9.80	9.97
N	AR(1)	4.30	6.57	8.48	10.24	10.17	9.82
Pr	model	3.21	3.99	4.71	5.86	5.63	5.45
Pr	AR(1)	4.43	5.81	6.42	7.88	8.39	9.08
W	model	1.03	0.99	1.00	0.98	0.93	0.87
W	AR(1)	1.36	1.77	2.32	3.00	3.25	3.74

4. CONCLUSION

We propose the new version of the model of the production side of the Russian economy. Introduction of several new features such as more complex production function and the exogenous COVID-19-related restrictions allowed to reproduce the wide set of macroeconomic variables with rather high accuracy even during the COVID-19 pandemic. One of the possible future directions of research is the introduction of sanctions-related restrictions the Russian economy currently faces into the model.

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